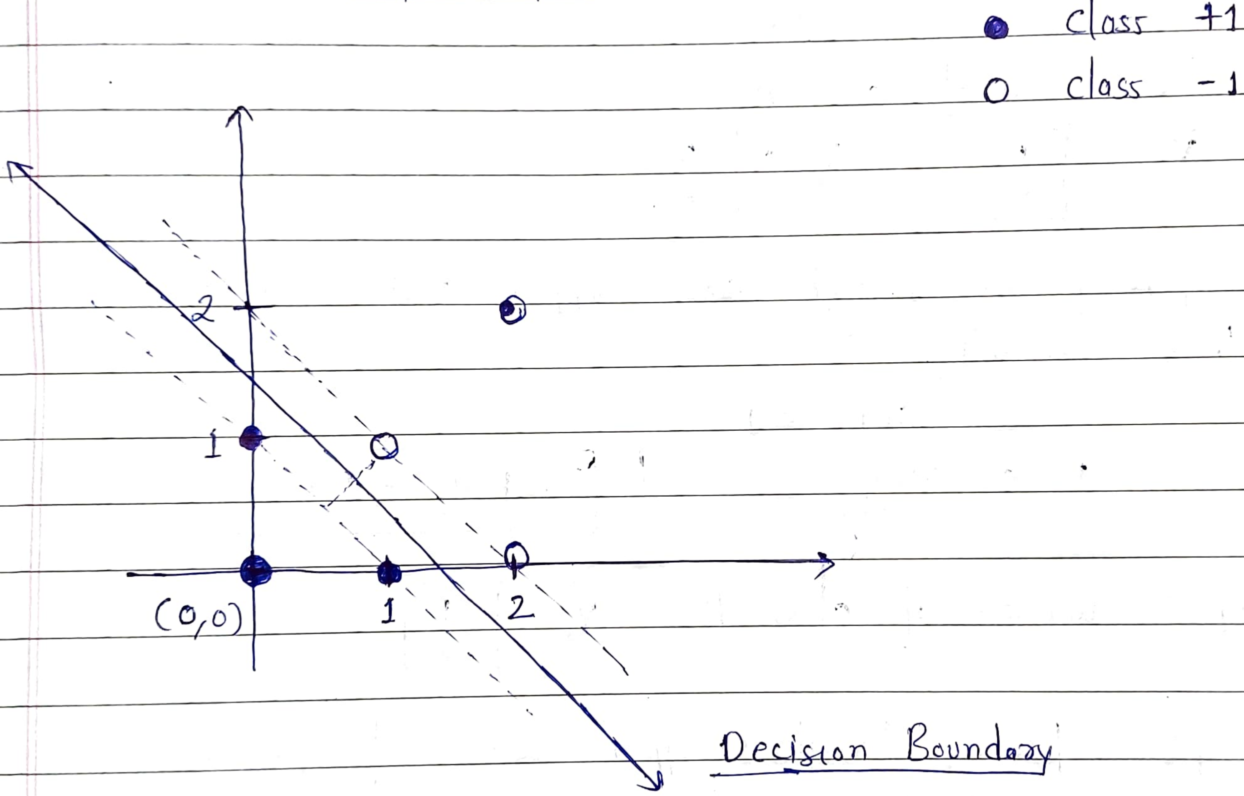
Sec-A

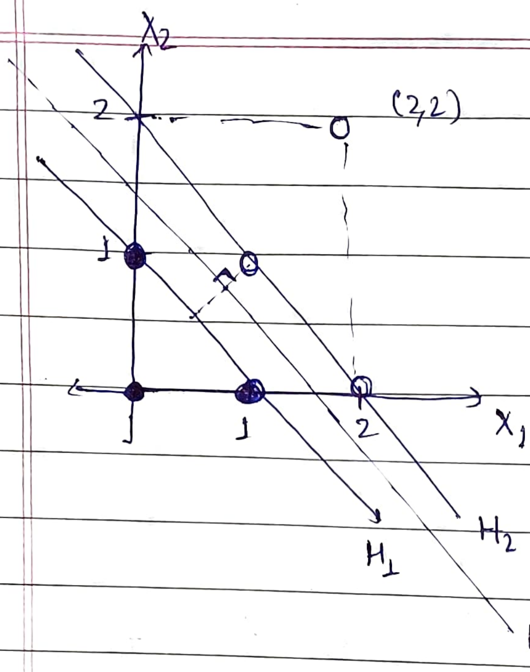
1)

x_1	x_2	class
0	0	+1
1	0	+1
0	1	+1
1	1	-1
2	2	-1
2	0	-1



- Yes, the points are linearly separable.
We can clearly see from the plot that Decision boundary exists and its separates the points of class A from class B.

2)



- Support vectors of class A

$$\vec{V_1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \vec{V_2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

hyperplane passing through ~~class~~ support vectors of. class A.

∴ Using geometry

$$x+y = 1$$

$$H_1 : x+y-1 = 0 \ / \ \boxed{x_1 + x_2 - 1 = 0}$$

- Support vectors of class B. $\vec{V_1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Hyperplane passing through support vectors of class B.

$$x+y = 2$$

$$\boxed{H_2 : x_1 + x_2 = 2}$$

- Draw a ⊥ line from (1,1) to H_1 and find the mid point

$$\left(\frac{1}{2}, \frac{1}{2}\right) \bullet \text{———} \bullet \text{———} \bullet (1,1)$$

$$(3/4, 3/4)$$

Decision boundary will pass through (3/4, 3/4) & parallel to H_1 & H_2

eq.

$$2x + 2y - 3 = 0$$

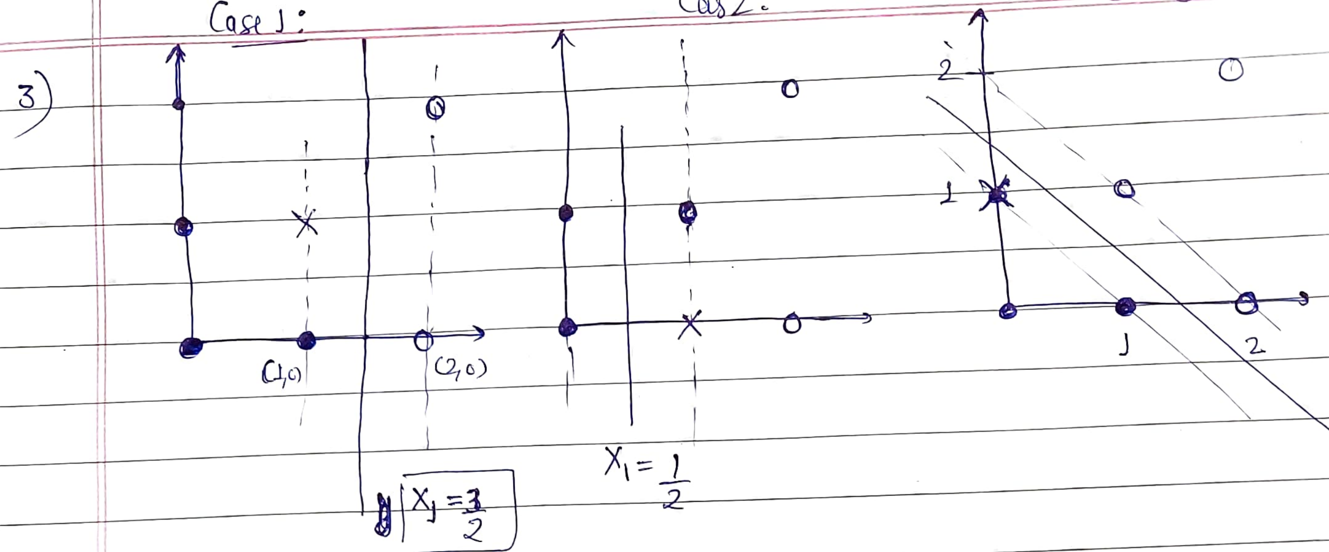
$$\boxed{H_0 : 2x_1 + 2x_2 - 3 = 0}$$

$$\boxed{w = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \quad b = -3}$$

$$H_0 = \begin{bmatrix} 2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} - 3 = 0 \qquad w^T b = 0$$

3)

Case 1: Cas 2: Case 3:



Case 1:-

- If we remove support vector of class B. the decision boundary will change and optimal margin will increase to 1 from $\frac{1}{\sqrt{2}}$ in previous ~~...~~ setting. New decision boundary $X_1 = \frac{3}{2}$.

Case 2:-

- If we remove one support vector of class A. the decision boundary will change and optimal margin will increase to 1 from $1/\sqrt{2}$. New decision boundary $x_1 = 1/2$

Case 3:-

- If we remove (0,1) from the class A, the decision boundary will remain same & optimal margin will also remain $\frac{1}{\sqrt{2}}$.

4) In any general dataset, if we remove non-supporting vectors then the decision boundary hyperplane will not get affected and optimal margin will remain same. Since, there lies no points b/w the optimal margin, if we remove support vectors, that means we ~~are~~ choosing points further away from the current support vector. Hence, the optimal margin will either remain same or increase if we remove one of the support vectors from the dataset.