Theory

1) **Random Variables**

T: A person has travelled

C: A person has caught corona.

D: A person has caught disease other than corona.

M: A person has mild corona.

S: A person has severe case of corona.

X: A person has Died

a) $P(T \wedge (C \vee D)) = 0.825$

or $P(T \wedge C) \vee P(T \wedge D) = 0.825$

b) $P(M/T) = 0.15$

$P(S/T) = 0.22$

c) $P(D/T) = 0.485$

d) $P(D \wedge X/T) = 0.24$

e) $P(S \wedge -T) = 0.025$

f) $P(S/T) = 0.457$

g) $P(X \wedge C) = 0.059$

h) $P(M \vee S) = 0.70$

i) $P(T/S) = 0.80$

j) $P(C) = 0.5$

b) to verify these proposition create a valid probability distribution, we need to check them against the axiom of probability. The three axiom of prob are:

i) Non-negativity:- All the probabilities are non-negative.

We can see in the part a) that all the values are greater than zero.

ii) Normality:- Sum of probability of a certain event (one that always happens) is 1.

$P(M/T) + P(S/T) + P(D/T) \leq 1$

iii) Additivity:- If two or more events are mutually exclusive then $P(\cup A_i) = \sum P(A_i)$

Data isn't sufficient to infer if there are events are exclusive or not.

c)

T	C	D	M	S	X	Probability
0	0	0	0	0	0	—
0	0	0	0	1	0	0.025
0	1	0	0	0	1	0.059
1	0	0	1	0	0	—
1	0	0	0	1	0	—
1	0	0	0	0	1	—

0 — means false

1 — means True

Not enough data to fill all the entries

d) Since, the joint distribution table isn't complete we can't really say ~~the~~ with confirmation about the Independence.

$$P(A, B/C) = P(A/C) * P(B/C)$$

If A & B are independent when C is known to occur.

M an S (Mild and severe cases of corona) are likely Conditionally Independent given C (having corona) as the severity of the disease does not necessarily depend on each other once it's established that the person has corona.

D (other diseases) might be conditionally independent from C (caught corona) and S (severe corona) given T assuming the diseases caught are unrelated.

These are assumptions based on the nature of the events not rigorous mathematical conclusions.

2)

a) Yes, switching to the other unopened door to maximize your chance of winning the key.

- By staying to the original choice, to win our choice has to be correct i.e $P = 1/3$ (staying)

- By switching and winning, our first choice has to be incorrect.

$$P(\text{first choice wrong}) = 2/3$$

Therefore switching increases the probability.

b) Case 1: ~~I chose wrong door, the other person can choose correct door with 2/3~~

~~$P(w) = \frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$~~

Yes, switching door is still the optimal strategy as it takes advantage of the higher prob that the car is behind the other unopened door.

The key is that when I initially chose a door, there was 2/3 chance that the ~~car~~ key was behind one of the other doors. If he revels on lost life, the prob of that the key is behind intially chosen door remains 1/3. But the prob that the car is behind the other unopened door increases to 2/3

$P(\text{win}) = 2/3$

Switching is still optimal.

c)

If ~~win~~ we win by switching then my first choice has to be wrong which is by probability 2/3

$P(\text{win by switching} / \text{revealed wrong door}) = 2/3$

d) Switch : $E[\text{Key}] = \frac{2}{3} \times 1 + \frac{1}{3} \times 0 = 2/3$

Stick : $E[\text{Key}] = \frac{1}{3} \times 1 + \frac{2}{3} \times 0 = 1/3$

Switching would ~~b~~ maxime winning