# The Determinants of Short-Term Interest Rate Changes

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**Abstract**

This study investigates the predictive power of the Fed funds futures market for short-term U.S. Treasury rate changes. In light of the significant monetary policy adjustments by the Federal Reserve during and after the Covid-19 pandemic, this research shows how target rate factors derived from Fed funds futures market quotes can be used to model short-term rate changes. These factors track movements of the short-term rates that are accountable for the monetary policy adjustments and speculations related to such adjustments. By analyzing data from 2013 to 2023, the study finds that target rate factors can explain a large portion of short-term rate changes. Specifically, they account for 52% of the variance in 1-month rates, 79% in 3-month rates, and 90% in 6-month rates at a 10-day frequency, and 41%, 66%, and 82% respectively at a 5-day frequency. Additionally, the study discusses risk estimation during the volatile period of 2020-2022, emphasizing the importance of considering monetary policy related factors in risk analysis.

Keywords: short-term rates, monetary policy actions, target rates, target rate factors, futures markets.

JEL code: E43, E42, G12, G13, G32

## 1 Introduction

In the analysis of market price movements, a common practice involves identifying factors that explain the price fluctuations of assets. These factors are typically modeled as stochastic variables, and they serve to explain the sources of market risk. In the case of U.S. Treasury bonds, and related derivatives, the factors typically explain the movements of the U.S. Treasury rates. The rate dynamics is heavily influenced by the monetary policy targeting that is implemented by the Federal Reserve system (Fed) in the United States. This relationship becames particularly transparent following the events precipitated by the Covid-19 pandemic. In early 2020, in response to the global crises Fed lowered the target rates close to zero, and subsequently in 2022 adjusted upwards. The effective overnight Fed Funds rate (EFFR) is depicted in Image 1. Essentially all the variation of the EFFR rate can be accounted for the jumps taking place on the dates following the Federal Open Market Committee (FOMC) meetings. The monetary policy adjustments are decided in these meetings. Image 1 also depicts the time series of the key short term rates subject to study in this manuscript.

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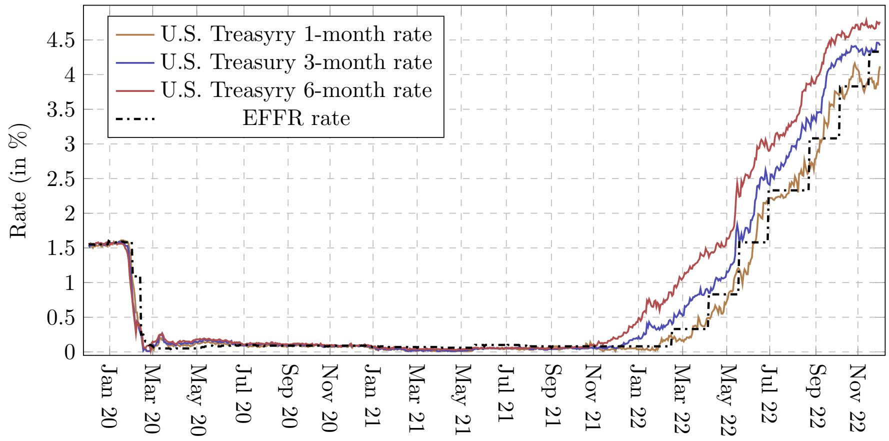


Image 1: U.S. Treasury short-term rates and EFFR rate time series over the period 2020-2022. The time stamps are the month ending business days.

The time series dynamics of all the short-term rates exhibit a strong and positive dependence, subject to some lags. The rate movements are also positively related to the EFFR rate movements. However, the EFFR rate changes only in response to updated monetary policy targets, and it is not influenced by market liquidity related fluctuations. This characteristic makes the EFFR rate particularly useful for connecting the short-term rate changes to monetary policy adjustments. The Fed funds futures market allows investors to speculate about upcoming changes in monetary policy. A daily futures rate curve can be inferred from Fed funds futures quotes. Using a standard compounding argument, the futures curve allows for making predictions about short-term rate changes. Fundamentally, this framework links the Fed monetary targeting to short-term rate changes. These remarks naturally lead to the following questions: First, is there information available in the Fed funds futures market that allows for making more accurate predictions about the short-term rate changes than one can make by using the information from the U.S. treasury bond market alone? Second, how much of the time-series variation in the U.S. Treasury short-term rates is attributable to monetary policy changes?

In this research document, I aim to provide answers to the questions concluding the opening paragraph. The analysis is based on a factor model for the short-term rates changes. The primary market factors of the model are target rate factors, which explain the components in the short-rate changes that are directly connected to the monetary policy adjustments, or are connected to the changes in the views about the future monetary policy adjustments. The parameters of these factors are estimated on the Fed funds futures quotes. In this framework, it is demonstrated that in average, the short-term rate change predictions inferred from the calibrated target rate factor model are more accurate than the predictions inferred directly from the U.S. Treasury term structure. The main result of this manuscript concerns the ability of the target rate factor model to reduce the variance of the short-term rate change time series of various frequencies. The period for the empirical study is from 2013 to 2023. At a 10-day frequency, the target rate factors account for a reduction of 52% in the variance of the 1-month rate, 79% in the 3-month rate, and 90% in the 6-month rate. At a 5-day frequency, the variance reduction stands at 41% for the 1-month rate, 66% for the 3-month rate, and 82% for the 6-month rate. In contrast, at a 1-day frequency, the variance reduction is less significant across these rates. The residual rate-change, after the target rate factor contributes are controlled, is captured with a systematic market factor and idiosyncratic component, which measure the common and rate specific fluctuations that are related to market liquidity instead monetary policy. The last part of the manuscript discusses risk estimation in this framework during the 2020-2022 period.

The economic framework that connects the monetary policy targeting to the futures and rates market movements has been discussed widely in the literature. Bernanke and Blinder (1992) demonstrate that the Fed funds rate is an important indicator of the executed monetary policy, and the changes in the rate reflect the changes in the policy. The use of Fed funds futures as predictors of the monetary policy adjustments has been explored by Krueger and Kuttner (1996), Sack (2004) and Gurkaynak et al. (2007). The study by Evans and Marshall (1998) shows that the monetary policy actions have significant influence on various interest rates. Also, Kuttner (2001) demonstrate that the unexpected changes in the Fed funds futures market has strong impacts on the short-term interest rates, and a diminishing impact on longer-term rates. Heitfield and Park (2019) show that the SOFR futures are effective predictors for the realized short-term SOFR rates. In the context of financial modeling in the valuation and risk applications, the monetary policy adjustments taking place on pre-determined dates still remains relatively unexplored. Piazzesi (2001, 2005) constructs a pure jump process for Federal Reserve target rates, wherein the jump intensity is contingent upon forthcoming FOMC meeting dates and macroeconomic factors. Kim and Wright (2014) investigate a jump diffusion short rate process, with jumps occurring at predetermined times. Backwell and Hayes (2022) introduce a pure jump process for short rates, allowing for both predictable and unpredictable jump occurrences. Harju (2024) utilizes the Fed funds futures market for the calibration of a short rate model with predictable jump times. Expanding upon the Heath-Jarrow-Morton (HJM) framework, Gellert and Schlogl (2021) present a term structure model wherein the short rate is influenced by jumps at both predictable and unpredictable times, alongside a residual diffusion process. This study is inspired by Fontana et al. (2020), which introduces a comprehensive class of term structure models within the HJM framework, accommodating complex short rate dynamics.

**Data Used in Empirical Study.**

The EFFR quotes used in this study are publicly available from the internet pages of Federal Reserve Bank of New York. The historical EFFR quotes have spikes at the end of each month. These spikes are attributed to regulatory reporting requirements, banks adjusting balance sheets to appear more favorable, and increased liquidity needs, as discussed by Baig and Winters (2021). These aspects should not be considered as monetary policy adjustments, and for this reason, the month end business day EFFR rate quotes are replaced with the quotes from the previous business day, with the exception that the month end business day is the first business day following a FOMC meeting. This pattern largely disappeared after 2018. The Fed funds futures price quotes are available from CME group, and the U.S. Treasury rates from the U.S. Treasury. In each case, the end-of-day quotes are used.

**Code for Used in Empirical Study.**

Python code, together with data inputs, are stored in the public repository GITHUB REPO (TODO).

**Conventions.**

The study focuses on the short-term rates of maturities in 1-, 3-, and 6-months. The 360-days in a year convention is used, and the 1-, 3-, and 6-month tenors mean the 30-, 90-, and 180-day tenors.

# 2 Estimation of Future Rate Curves

Suppose that there are $k$ FOMC meetings scheduled in a forward-looking time window. The valuation date is denoted by $t$ and the numbers of days from $t$ to the FOMC meetings are denoted by $N_i$ for $i \in \{1, \dots, k\}$. The current overnight rate is $r_{0t}$. The model assumes that the daily futures curve is piece-wise constant, and takes jumps on the days following the FOMC meetings. The estimates of the jump sizes on the valuation date are denoted by $j_{it}$ for $i \in \{1, \dots, k\}$. Now, the daily futures curve value estimated on date $t$ for the $N$ days tenor point is defined by

$$f_{Nt} = r_{0t} + \sum_{i=1}^{k} j_{it}\mathbf{1}(N > N_i), \tag{1}$$

where $\mathbf{1}(N > N_i)$ is the indicator function that has value 1 if $N > N_i$ and zero otherwise.

The Fed funds futures contracts are issued for each reference calendar month. The payouts are determined by the arithmetic averages of the EFFR rates during the reference months. In the case where the reference month is the current month, the symbol $\mathcal{T}_0^-$ is used for the set of historic dates in the current month, and $\mathcal{T}_0^+$ is used for the remaining dates in the current month. In particular, the closing EFFR rates are available for the dates in $\mathcal{T}_0^-$. Now, for the contract with reference month the current month, indexed by $m = 0$, the price is estimated by

$$\Pi_t(m=0) = 1 - \frac{1}{\#\mathcal{T}_0^- + \#\mathcal{T}_0^+}\Big(\sum_{u\in\mathcal{T}_0^-} r_u + \sum_{u\in\mathcal{T}_0^+} f_{ut}\Big),$$

where # counts the number of dates in the set. In the case the reference month is later than the current month, $m > 0$, the price is estimated by taking the average of the daily futures rates during the reference month. Therefore, the price estimate reads

$$\Pi_t(m) = 1 - \frac{1}{\#\mathcal{T}_m}\sum_{u\in\mathcal{T}_m} f_{ut},$$

where $\mathcal{T}_m$ denotes the set of all dates in the reference month.

In what follows, the 360-dates in a calendar year day-counting convention is being applied. Given the daily futures curve $f_{ut}$, the no-arbitrage constraints forces a relation between the interest rate $r_{Nt}$ for the $N$-day tenor, and the daily futures values. This relation is given by

$$1 + \frac{N \cdot r_{Nt}}{360} = \prod_{u\in\mathcal{D}(t,N)}\Big(1 + \frac{f_{ut}}{360}\Big), \tag{2}$$

where $\mathcal{D}(t, N)$ is the set of days between the dates $t + 1$ and $t + N$, including both end points.

Two estimators for the parameters of the daily forward curve are considered. In the first case, $r_{0t}$ is set to match the time-$t$ value of the EFFR rate, and the jumps $j_{it}$ are estimated by minimizing the sum of square errors function between the Fed funds futures price quotes, $\check{\Pi}_t(m)$, and their model estimates over the current, and the 7 consecutive reference months:

$$(j_{1t}, \dots, j_{kt}) = \mathrm{argmin}_{j_{1t},\dots,j_{kt}} \sum_{m=0}^{k}\Big(\check{\Pi}_t(m) - \Pi_t(m)\Big)^2,$$

where $k$ is the number of FOMC meetings that are scheduled for the following 7-months period. In a typical case, there are 5 FOMC meetings during this period, and 5 jump values are fitted. This strategy for the parameter estimation was introduced by Heitfield and Park (2019) in the context of SOFR futures.

The second estimator is based on matching the market quotes of short-term rates with the corresponding model predicted rates in (2). The rates selected for the calibration are the 1-, 3-, and 6-month rates, or in the notation introduced above, the rates with $N$ equal to 30-, 90-, and 180-days. The trading activity causes the market quotes of these rates fluctuate strongly, and it is not realistic to assume that this calibration could produce a meaningful estimate for each jump value. The parsimonious strategy where only $r_{0t}$ and the common jump size $j_t = j_{1t} = \cdots = j_{kt}$ are subject to estimation is more stable. These parameters are estimated by

$$(r_{0t}, j_t) = \mathrm{argmin}_{r_{0t}, j_t} \sum_{N\in\{30,90,180\}} (\check{r}_{Nt} - r_{Nt})^2.$$

This estimation does not require quotes from the Fed funds markets. The primary motivation to introduce the second estimator is to test whether the information available from the Fed funds futures market can be used to make more accurate predictions about the future market movements. The left panel of Image 2 depicts calibrated futures curves on March 3rd 2022, and the corresponding interest rate curves that are estimated by using these two methodologies.

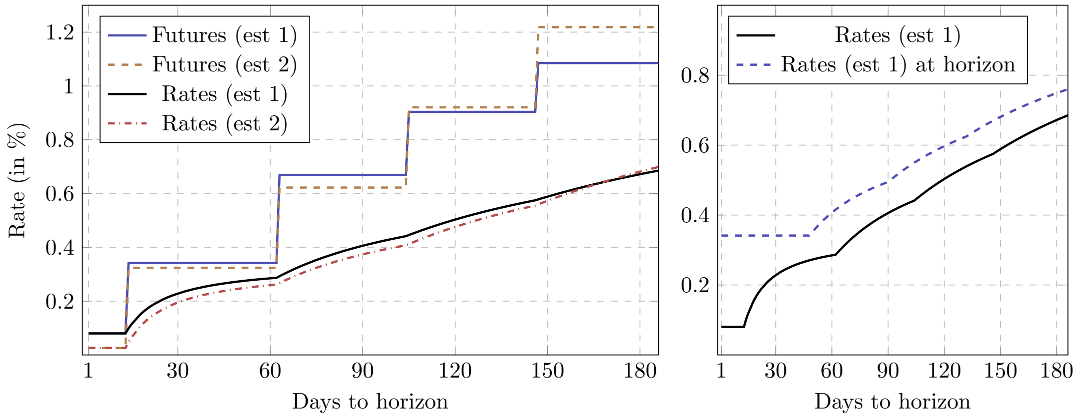


Image 2: Calibrated futures curves and the rates inferred from the futures curve on 03-Mar-2022 on the left panel. Predicted rates for the 10-business dates risk horizon on the right panel.

The calibrated futures curves have significant upwards jumps following the FOMC meetings. These jumps signal the forward-looking market expectations of the policy changes in early 2022 when the EFFR rate was almost zero. In the right panel, the curve is presented together with a prediction of the curve in the risk horizon that is 10 days ahead of the evaluation date, which will be discussed in the next section.

# 3 Rate Change Prediction

The futures curve allows for making predictions of the rates at future risk-horizons. In the risk horizon that is $x$ days ahead from the evaluation date $t$, the prediction of the $N$-days tenor rate, denoted by $r_{Nt}(x)$, can be inferred from the equation

$$1 + \frac{N \cdot r_{Nt}(x)}{360} = \prod_{u \in \mathcal{D}(t+x, N+x)} \left(1 + \frac{f_{ut}}{360}\right). \tag{3}$$

The right panel of Image 2 presents the interest rate term structure inferred by using the first parameter estimation strategy on March 3rd 2022 together with the estimate of the term structure at the risk horizon 10-business days ahead. The gap between the two curves represents the predicted movement of the curve at a given tenor. The first FOMC meeting date following the evaluation date is March 16th, and so the meeting takes place before the risk horizon March 17th. Therefore, the shortest tenors are influenced very heavily by the meeting. The gap remains wide over longer tenors as a consequence of additional FOMC meetings for which rate hikes have been calibrated. The estimate of the expected movement of the $N$-days rate between the evaluation date $t$ and the risk-horizon $t + x$ is denoted by

$$\phi_{Nt}(x) = r_{Nt}(x) - r_{Nt}.$$

This quantity is model dependent as it depends on how the futures curve is estimated.

Next, we turn into empirical analysis and proceed to test the model's ability to predict the changes of the market rate quotes. In particular, we wish to test if the model with the primary estimator can make more accurate predictions than the common approaches, where the expected rate movements are inferred from the rate curves. However, fitting a rate curve is a very subjective problem, and we have to be very simplistic here. Two basic curve estimators are considered. The first one fits a linear curve for the short duration rates, by means of ordinary linear regression on the 30, 90 and 180-days tenors, and the corresponding rate quotes. The second fits a linear spline on the same quotes and uses extrapolation to extend the curve outside the 30-to-180-days domain. The predicted $N$-days tenor rate in the $x$-days risk horizon, is solved from

$$\left(1 + \frac{x \cdot r_{xt}}{360}\right)\left(1 + \frac{N \cdot r_{Nt}(x)}{360}\right) = 1 + \frac{(N + x) \cdot r_{N+x,t}}{360} \tag{4}$$

In what follows, Benchmark 1 and Benchmark 2 refer to the models to predict the rate quote changes by solving $r_{Nt}(x)$ from the equation (4), and in which the interest rate curve is fitted by using the line estimator in the first case and the spline estimator in the second. Benchmark 3 model solves $r_{Nt}(x)$ from (3) and the futures curve is calibrated by using the secondary futures curve estimate. The standard deviations of the observed rate quote changes as well as the predicted rate changes in the 1-, 5-, and 10-business day frequencies are presented in Table 1.

| Tenor | Horizon | Observed | Bench.1 | Bench.2 | Bench.3 | Model |
|---|---|---|---|---|---|---|
| 1Mo | 1d | 3.99 | 0.59 | 0.94 | 0.74 | 0.88 |
| | 5d | 8.09 | 2.48 | 4.1 | 3.11 | 3.7 |
| | 10d | 12.4 | 4.91 | 7.86 | 5.51 | 6.49 |
| 3Mo | 1d | 2.72 | 0.59 | 0.64 | 0.6 | 0.6 |
| | 5d | 7.15 | 2.48 | 2.67 | 2.48 | 2.49 |
| | 10d | 10.76 | 4.91 | 5.11 | 4.88 | 4.77 |
| 6Mo | 1d | 2.77 | 0.59 | 0.55 | 0.59 | 0.44 |
| | 5d | 7.05 | 2.48 | 2.24 | 2.46 | 1.82 |
| | 10d | 10.97 | 4.91 | 4.42 | 4.87 | 3.49 |

Table 1. Standard deviations of the rate change and predicted rate change time series. The dataset used for the analysis comprises data from 2013 to 2023.

The standard deviations of the observed rate changes are considerably higher than the standard deviations of the predicted rate changes. Largely, this difference should be explainable by the reality that the US government bills are traded in high volumes, and the trading activity naturally adds to the time series volatility of the rates. Another important observation is that the 1-Month rate tends to be more volatile than the 3-month and 6-month rates.

For each tenor $N$ and risk horizon $x$, we define the prediction error at valuation time $t$ as the absolute difference between the observed rate change $\Delta(\check{r}_{Nt})(x)$ and the predicted mean rate change

$$e_{Nt}(x) = |\Delta(\check{r}_{Nt})(x) - \phi_{Nt}(x)|.$$

To simplify the exposure, the following normalized error score is introduced

$$e^*_{Nt}(x) = \frac{\sum_t e_{Nt}(x)}{\sum_t |\Delta(\check{r}_{Nt})(x)|},$$

where the sum runs over all the times in the panel dataset. The score measures the average prediction error, relative to the average rate movement over the full data panel. If the value is less than one, then one may think that the model has the ability to reduce the average magnitude of the rate fluctuation for the frequency determined by the risk horizon. The error scores for each tenor and risk horizon and presented in Table 2.

| Tenor | Horizon | Bench.1 | Bench.2 | Bench.3 | Model |
|---|---|---|---|---|---|
| 1Mo | 1d | 1.03 | 1.03 | 1.04 | 1.03 |
| | 5d | 0.94 | 0.92 | 0.93 | 0.89 |
| | 10d | 0.89 | 0.89 | 0.88 | 0.86 |
| 3Mo | 1d | 1.0 | 1.0 | 1.0 | 0.99 |
| | 5d | 0.9 | 0.9 | 0.9 | 0.86 |
| | 10d | 0.81 | 0.81 | 0.8 | 0.78 |
| 6Mo | 1d | 1.01 | 1.01 | 1.01 | 1.0 |
| | 5d | 0.92 | 0.92 | 0.92 | 0.88 |
| | 10d | 0.86 | 0.85 | 0.86 | 0.83 |

Table 2. Normalized error scores for each model computed over the full panel dataset.

In the case of 1-day risk horizon, the error scores are essentially indistinguishable from 1. This is the case for all the benchmarks, and the model. For the longer horizons, the model and the benchmarks are able to reduce the average rate fluctuation score, and the model beats the benchmark in each case. The observed rate changes have considerable amount of noise caused by market activity, and over longer risk horizons, the predicted movements become stronger relative to the amplitude of this noise.[1]

Next, the dataset is divided into two components. The first one has the periods which start at no later than 10 days before an upcoming scheduled FOMC meeting, and the second one has all the remaining data. So, the first dataset has the periods that are close to a FOMC meeting, or the FOMC meeting takes place in the period. The scores are computed in both data subsets, and the normalizations are performed by using the data available in the subset. Starting from the latter case, the test for the normalized prediction scores is performed, and the results are presented in Table 3.

| Tenor | Horizon | Bench.1 | Bench.2 | Bench.3 | Model |
|---|---|---|---|---|---|
| 1Mo | 1d | 1.04 | 1.04 | 1.03 | 1.02 |
| | 5d | 0.94 | 0.91 | 0.94 | 0.92 |
| | 10d | 0.87 | 0.84 | 0.89 | 0.88 |
| 3Mo | 1d | 1.0 | 1.0 | 1.0 | 0.99 |
| | 5d | 0.87 | 0.87 | 0.87 | 0.85 |
| | 10d | 0.78 | 0.78 | 0.77 | 0.78 |
| 6Mo | 1d | 1.01 | 1.01 | 1.01 | 1.0 |
| | 5d | 0.88 | 0.88 | 0.88 | 0.86 |
| | 10d | 0.8 | 0.79 | 0.8 | 0.82 |

Table 3. Normalized error scores for each model computed over the periods that are not close to a FOMC meeting.

The results are inconclusive, and do not provide evidence for improved prediction performance for the model against the benchmarks. However, the error scores for the 5-day and 10-day suggest that the model and the benchmarks reduce the average magnitude of the rate fluctuation. Next, the test is executed in the case of the periods that are close to a FOMC meeting, and the results are collected in Table 4.

| Tenor | Horizon | Bench.1 | Bench.2 | Bench.3 | Model |
|---|---|---|---|---|---|
| 1Mo | 1d | 1.02 | 1.01 | 1.05 | 1.04 |
| | 5d | 0.96 | 0.96 | 0.89 | 0.81 |
| | 10d | 0.95 | 1.06 | 0.81 | 0.79 |
| 3Mo | 1d | 1.0 | 1.01 | 1.01 | 0.99 |
| | 5d | 0.96 | 0.97 | 0.98 | 0.88 |
| | 10d | 0.89 | 0.91 | 0.9 | 0.79 |
| 6Mo | 1d | 1.01 | 1.01 | 1.01 | 0.99 |
| | 5d | 1.02 | 1.01 | 1.03 | 0.93 |
| | 10d | 1.01 | 0.99 | 1.01 | 0.87 |

Table 4. Normalized error scores for each model computed over the periods that are close to a FOMC meeting.

The model clearly beats the benchmarks in the 5-days and 10-days risk horizons. These remarks are not surprising. It is reasonable to expect that the corrections to traded rates that are related to the expected monetary policy changes are consentrated to the periods close to the FOMC meeting. Also, the Fed funds futures market volumes become higher as the time to the FOMC meeting gets shorter, and for this reason the information inferred from the Fed funds markets gets better. The sets of errors $e_{Nt}(x)$ for the benchmarks and the model are very volatile, and the differences between the means are not claimed to have statistical significance with high confidence.

Next, we set up a game-theoretic setting and study the frequencies of how often the model is able to beat the benchmarks in prediction accuracy. In this framework, it is easier to make statistically

[1] For instance, in a simplified case where the noise is normally distributed, and there is a linear drifting rate in time, the amplitude of the noise would have a square-root time dependence, and the fraction between the drift rate and the amplitude gets higher over longer time horizons. For this technical reason, it is easier to pick signals over longer horizons in a heavily noised environment.

significant claims. The dataset is partitioned onto 10 calendar years from 2013 to 2022. Every test year, the model competes against each of the benchmark in the number of periods where the model outperforms the benchmark. For instance, in the case of 5-day frequency, there are 52 prediction errors for the model and each benchmark every year. If the model and a benchmark both win 13 rounds, then a tie is declared. Otherwise, the winner is declared. The winner makes the more accurate prediction more often. We also apply the binomial test. If with 95% confidence, the null hypothesis that the model and a benchmark have the equal chances of winning can be rejected, then a winner is declared with statistically significant advantage. These estimations are performed in each of the 10 years in the dataset. It is often the case that one or more ties are observed, and in this case, less than 10 periods are declared a winner. In the statistically significant counts, there is always less than 10 periods with a winner. The results are presented in Table 5.

| Tenor | Horizon | Bench.1 × Model | Bench.2 × Model | Bench.3 × Model |
|---|---|---|---|---|
| 1Mo | 1d | 1/9 (0/8) | 4/6 (0/4) | 1/9 (0/0) |
| | 5d | 0/10 (0/1) | 3/7 (0/0) | 3/6 (0/1) |
| | 10d | 2/8 (0/2) | 5/3 (0/0) | 4/5 (0/2) |
| 3Mo | 1d | 0/10 (0/8) | 0/10 (0/8) | 0/10 (0/8) |
| | 5d | 2/8 (0/3) | 2/7 (0/4) | 0/8 (0/3) |
| | 10d | 1/8 (0/1) | 1/8 (0/1) | 1/8 (0/1) |
| 6Mo | 1d | 0/10 (0/8) | 0/10 (0/8) | 0/10 (0/8) |
| | 5d | 2/8 (0/4) | 2/8 (1/4) | 2/8 (0/4) |
| | 10d | 2/8 (0/4) | 2/7 (1/4) | 2/8 (0/3) |

Table 5. Results of the game theoretic frequency test. The results of the statistically significant competition are presented in parentheses.

The model seems to perform much stronger than the benchmarks even in the 1-day risk horizon. The statistically significant scores are overwhelmingly in favour of the model. Note that the sample sizes get smaller in the 5-days and 10-days horizons, and for this reason, we see less results that are statistically significant.

# 4 Empirical Notes About Uncertainties of Rate Changes

The remaining of this manuscript focuses on finding risk factors that explain the risky rate changes. This section discusses the associated risk sources in broad terms. First, we investigate how accurately the predicted EFFR changes match with the actual EFFR changes. To this end, we consider all the pairs of times, $(t, T_k)$, where $t$ is any date in the full panel data set, and $T_k$ are the scheduled FOMC meeting dates that take places within the 7 month period following the time $t$. With each pair $(t, T_k)$, we associate the error function

$$e(t, T_k) = \Delta(\text{effr})_{T_k+1,t} - \sum_{i=1}^{k} \mathbf{1}(T_i \geqslant t) j_{it}$$

where $\Delta(\text{effr})_{T_k+1,t}$ is the actual change of the EFFR between $t$ and the day following the FOMC meeting on $T_k$. The parameters $j_{1t}, \ldots, j_{kt}$ are the jumps estimated at $t$, and the sum counts the aggregate jump over all jumps between $t$ and $T_k + 1$. The latter is the model prediction of the EFFR change between $t$ and $T_k + 1$. Next, a categorical variable $\mathcal{C}_i$ is defined for each pair $(t, T_l)$ such that

$$\mathcal{C}(t, T_k) = \sum_{i=1}^{10} i \cdot \mathbf{1}\Big(i - 1 \leqslant \frac{T_k - t}{20} < i\Big).$$

So, more practically, we divide the sample of all pairs $(t, T_k)$ into 10 buckets, and the bucket $i$ contains all the pair for which the time to meeting, $T_k - t$, is at least $20 \cdot (i-1)$ days but strictly less than $20 \cdot i$ days. The standard deviations of the errors $e(t, T_k)$ are computed in each category. Image 3 illustrates these standard deviations when they are plotted as functions of the median time to meeting in each bucket. These medians are equal to $10 + 20 \cdot (i - 1)$ for $i \in \{1, \ldots, 10\}$.

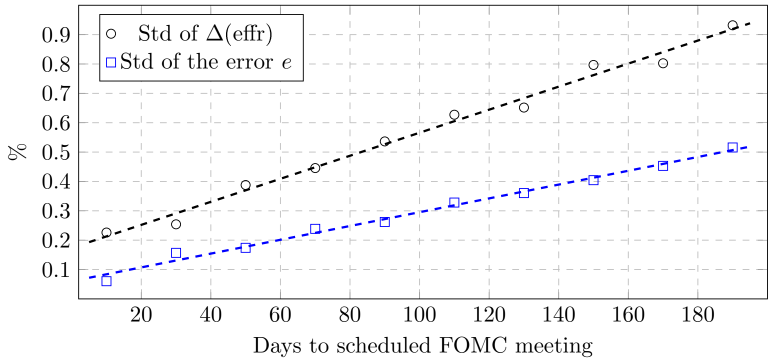


Image 3. The dependence between the standard deviation of the EFFR change prediction errors (in percentage) and time (in days).

The circle marks depict the standard deviations of the historical EFFR changes in different categories, and the square marks depict the standard deviations of the errors. The lines are the regression lines that are fitted on the marks. The slopes are 0.004 and 0.002. The error can be primarily attributed to the following to channels of uncertainty. First, the inferences about the meeting decisions become sharper as the meeting gets closer. Second, the average number of meetings scheduled within a time window increases as the time window gets longer. One should note that the uncertainty does not converge to zero as the time horizon tends to zero. In fact, there is always quite considerable level of uncertainty left about the EFFR changes even right before the FOMC meetings. The prediction error for the EFFR changes right before the meeting have standard deviation about 0.02 percent. One should also note that the errors have a linear dependence on time to meeting. Therefore, the underlying population of both EFFR movements and the prediction errors have a fat-tailed probability law.

There is another more subtle risk source associated with the prediction of the short-term rates. The FOMC meeting frequency is typically 6 weeks, and the meetings dates are announced months before they take place. In some rare occurrences, a meeting has been cancelled and replaced with one or more emergency meetings. This happened following the dot-com bubble burst in 2001, financial crises in 2008, and the COVID-19 pandemic in 2020. In the latter case, the meeting of March 18, 2020 was replaced with two emergency meetings which took place in the first half of March 2020. Both resulted in decisions for significant rate cuts. For the following example, we control the uncertainty of the meeting dates and work with the actual dates of the meeting that were organized. Now, we study the short-term rate changes over the fixed 1-week (5-business days) frequency. For each short-term rate under study, we compute the rate change time series. The aim is to analyze the historical data and test whether the realized rate-change standard deviations have dependency on the distance to the upcoming FOMC meeting. In this case, all the historical rate-changes are attributed to one of the four buckets. The first one contains the rate-changes over the periods that had a meeting, the second bucket has the rate-changes for the periods that started between 1 and 2 weeks before a meeting, the third between 3 and 4 weeks before a meeting, and the last one over 4 weeks before a meeting. There is unusually strong regime change between the historical rate change standard deviations before and after the Covid-pandemic. Therefore, we divide the dataset onto two parts: the first one has the observations before 2020 and the other one after 2020. The results are presented in Table 6.

| Dataset | Tenor | Bucket 1 | Bucket 2 | Bucket 3 | Bucket 4 |
|---|---|---|---|---|---|
| Before 2020 | 1Mo | 3.40 | 4.69 | 4.02 | 4.38 |
| | 3Mo | 3.52 | 3.05 | 2.62 | 3.06 |
| | 6Mo | 3.85 | 3.27 | 2.97 | 3.27 |
| After 2020 | 1Mo | 14.9 | 14.4 | 12.2 | 8.71 |
| | 3Mo | 17.8 | 10.1 | 11.9 | 5.31 |
| | 6Mo | 15.9 | 14.0 | 7.75 | 6.03 |

Table 6. Standard deviations of historical short-term rate changes in various categories determined by the time to a FOMC meeting.

The 3-month and the 6-month rates exhibit a downward sloping dependence on the distance to a FOMC meeting. The study is inconclusive in the case of the 1-month rate.

In conclusion to this empirical section, we made three observations that will become important for the risk model framework. First, the inferences about the EFFR changes based on the Fed funds futures market capture roughly half of the standard deviation of the EFFR rate changes, and additional risk factors are required to control the remaining uncertainties in the EFFR movements. Second, in very rare circumstances, FOMC meetings have been rescheduled. And finally, the short term rate change standard deviation gets stronger as the FOMC meetings approach.

# 5 Target Rate Factors

A risk model is proposed for the short-term rates that captures the risk associated with the monetary policy changes. The model introduces target rate factors, which are designed to capture the risk related to the uncertainty of upcoming FOMC meeting decisions. Additionally, after accounting for the risk driven by the target rate factors, the residual risk is examined. This residual risk is expected to be related to the natural fluctuations of the rates caused by the market activity. Specifically, we aim to understand the relative magnitudes of the risks associated with monetary policy targeting compared to those related to market trading activity.

The next goal is to represent the predicted rates as linear functions of the overnight EFFR rate, and the anticipated jumps. To this end, the relationship in (2) is approximated by

$$r_{Nt} = \frac{1}{N}\Big(N r_{0t} + \sum_{i\geqslant 1}(N-(N_{it}+1))^{+} j_{it}\Big), \tag{5}$$

where $N_{it}$ for $i \geqslant 1$ represents the number of days until the scheduled FOMC meetings, counted from the estimation date $t$. The notation $(X)^{+} = \max(X, 0)$ is used. This approximation is established by setting to zero the terms in equation (3) that are of order two or higher in $r_{0t}$ and the jumps $j_{it}$. It is important to remember that for the FOMC meeting scheduled to happen in $N_{it}$ days, given the information available at $t$, the resulting jump is realized at the quoted rate in $N_{it}+1$ days. To simplify the notation, we use $N^{j}_{it} = N_{it}+1$ and consider $N^{j}_{it}$ as the days until the $i$'th jump. The relation (5) has a simple interpretation: the coefficient $(N - N^{j}_{it})^{+}$ is the number of days the $N$-days futures curve is influenced by the jump $j_{it}$, assuming that the meeting takes place. In particular, $(N - N^{j}_{it})^{+} = 0$ if $N^{j}_{it} \geqslant N$, meaning that only the jumps occurring before the horizon $N$ contribute to the rate.

Suppose that $x > 0$ is the number of days to the risk horizon. Let $r_0(x)$ and $j_{it}(x)$ for $i \geqslant 1$ denote the values of the overnight EFFR rate, and the jumps at the risk horizon $t + x$. The index $i$ refers to the $i$'th FOMC meeting counted from time $t$. Assume that $x < N_2$, meaning the risk horizon is earlier than the second scheduled FOMC meeting. This is not a real limitation of the study, as it does not make sense to consider risk horizons longer than six weeks in the analysis of short-term rates. Now, the $N$-day rate at the risk horizon $t + x$ is modeled by:

$$r_{Nt}(x) = \begin{cases} \dfrac{1}{N}\Big(N r_{0t}(x) + \displaystyle\sum_{i\geqslant 1}(N + x - N^{j}_{it})^{+} j_{it}(x)\Big) & \text{if } x < N^{j}_{1t}, \\ \dfrac{1}{N}\Big(N r_{0t}(x) + \displaystyle\sum_{i\geqslant 2}(N + x - N^{j}_{it})^{+} j_{it}(x)\Big) & \text{if } x \geqslant N^{j}_{1t}. \end{cases}$$

In the first case, the risk horizon is earlier than the jump resulting in from the first FOMC meeting counted from $t$. In the second case, the term $j_{1t}(x)$ does not appear because the jump associated with the first meeting already happened before the risk horizon. In this case, the EFFR rate at the risk horizon, denoted by $r_{0t}(x)$, is instructive to be considered as the sum

$$r_{0t}(x) = \tilde{j}_{1t}(x) + \tilde{r}_{0t}(x)$$

where $\tilde{j}_{1t}(x)$ is the jump, i.e., the change of EFFR between the dates $t + N^{j}_{1t}$ and $t + N_{1t}$. Note that the change of EFFR over the full interval can be broken down as:

$$r_{0t}(x) - r_{0t} = \tilde{j}_{1t}(x) + (\tilde{r}_{0t}(x) - r_{0t}).$$

The last part on the right-hand side of the equation is the residual change of EFFR after the jump is accounted for. Let us define the piecewise constant function:

$$\phi(N, x, N_{it}^j) = \begin{cases} x & \text{if } N_{it}^j \leqslant N, \\ N + x - N_{it}^j & \text{if } N < N_{it}^j < N + x, \\ 0 & \text{if } N_{it}^j \geqslant N + x. \end{cases}$$

This function counts the difference between the sensitivities of the $N$-days rate on the $i$'th jump at the risk horizon $x + t$ and at time $t$:

$$\phi(N, x, N_{it}^j) = (N + x - N_{it}^j)^+ - (N - N_{it}^j)^+.$$

Now, with the notation $\Delta(r_{Nt})(x) = r_{Nt}(x) - r_{Nt}$, it is straightforward to establish that:

$$\Delta(r_{Nt})(x) = \frac{1}{N}\Big(N(r_{0t}(x) - r_{0t}) + \sum_{i \geqslant 1}(N + x - N_{it}^j)^+(j_{it}(x) - j_{it}) + \sum_{i \geqslant 1}\phi(N, x, N_{it}^j)j_{it}\Big)$$

in the case $x < N_{1t}^j$, and:

$$\begin{aligned} \Delta(r_{Nt})(x) \;=\; & \frac{1}{N}\Big(N(\tilde{r}_{0t}(x) - r_0) + N(\tilde{j}_{1t}(x) - j_{1t}) + \sum_{i \geqslant 2}(N + x - N_{it}^j)^+(j_{it}(x) - j_{it}) \\ & +\; \min(N, N_{1t}^j)j_{1t} + \sum_{i \geqslant 2}\phi(N, x, N_{it}^j)j_{it}\Big) \end{aligned}$$

in the case $x \geqslant N_{1t}^j$. These relations also break down to risky and deterministic components as

$$\Delta(r_{Nt})(x) = \Delta(r_{Nt}^{\times})(x) + \phi_{Nt}(x),$$

where

$$\Delta(r_{Nt}^{\times})(x) = \begin{cases} (r_{0t}(x) - r_{0t}) + \sum_{i \geqslant 1}\dfrac{(N + x - N_{it}^j)^+}{N}(j_{it}(x) - j_{it}) & \text{if } x < N_{1t}^j, \\ (\tilde{r}_{0t}(x) - r_{0t}) + (\tilde{j}_{1t}(x) - j_{1t}) + \sum_{i \geqslant 2}\dfrac{(N + x - N_{it}^j)^+}{N}(j_{it}(x) - j_{it}) & \text{if } x \geqslant N_{1t}^j. \end{cases}$$

The deterministic parts are linear approximations of the predictions that were discussed in Section 3.

So far, we established a risk model that attributes all the short-term rate changes to the factors that are inferred from the Fed funds futures market quotes, which are directly related to changes in monetary policy targets. Now, we turn to the historical time series of the actual observed rate changes and examine how much of the time series variation this model captures. Let $\Delta(\check{r}_{Nt})(x)$ denote the $N$-day rate changes over the interval from $t$ to $t + x$, and

$$\varepsilon_{Nt}^{\times}(x) = \Delta(\check{r}_{Nt})(x) - \Delta(r_{Nt}^{\times})(x) - \phi_{Nt}(x)$$

denote the residuals. The components $\phi_{Nt}(x)$ are inferred from the calibrated jumps at time $t$, and $\Delta(r_{Nt}^{\times})(x)$ are inferred from the differences between the jump parameters calibrated on the futures data at the horizon $t+x$ and at $t$. The special jump factor $\tilde{j}_{1t}(x) - j_{1t}$ is the actual jump that was observed after the meeting. The residuals should be considered as fluctuations that are related to the trading activity in the market segment, and under the hypothesis that the Fed funds futures prices carry the best available information about the upcoming changes of the monetary economics, the residuals $\varepsilon_{Nt}^{\times}(x)$ should be independent of the views related to the changes of monetary policies. Let $\mu_t(x)$ denote the average change of the short-term rates over the time interval from $t$ to $t + x$. This level effect is considered as the common market factor explaining the systematic movement of all the short-term rates. The rate specific error is estimated by:

$$\varepsilon_{Nt}(x) = \varepsilon_{Nt}^{\times}(x) - \mu_t(x).$$

We have now established a linear factor model to explain the rate changes $\Delta(r_{Nt})(x)$. The factors are the overnight rate change $\Delta(r_{0t})(x) = r_{0t}(x) - r_{0t}$ for $x < N^j_{1t}$ or $\Delta(r_{0t})(x) = \widetilde{r}_{0t}(x) - r_{0t}$ otherwise, the first jump factor $\Delta(j_{1t})(x) = j_{1t}(x) - j_{1t}$ for $x < N^j_{1t}$ or $\Delta(j_{1t})(x) = \widetilde{j}_{1t}(x) - j_{1t}$ otherwise, the remaining jump factors $\Delta(j_{it})(x) = j_{it}(x) - j_{it}$ for $i > 1$, and the level factor $\mu_t(x)$. These factors explain the systematic changes in the rates, while the idiosyncratic changes specific to each of the rates are explained by the residual $\varepsilon_{Nt}(x)$.

Image 4 depicts the time series of standard deviations for the 3-month rate at a 5-day frequency, as well as the standard deviations of the risky rate change

$$\overline{\Delta(\check{r}_{Nt})}(x) = \Delta(\check{r}_{Nt})(x) - \phi_{Nt}(x),$$

for $N = 90$ and $x = 5\text{d}$, and the residuals $\varepsilon^{\times}_{90,t}(5\text{d})$ and $\varepsilon_{90,t}(5\text{d})$. These standard deviations are computed over windows of 100 weekly observations, equivalent to 500 business days.

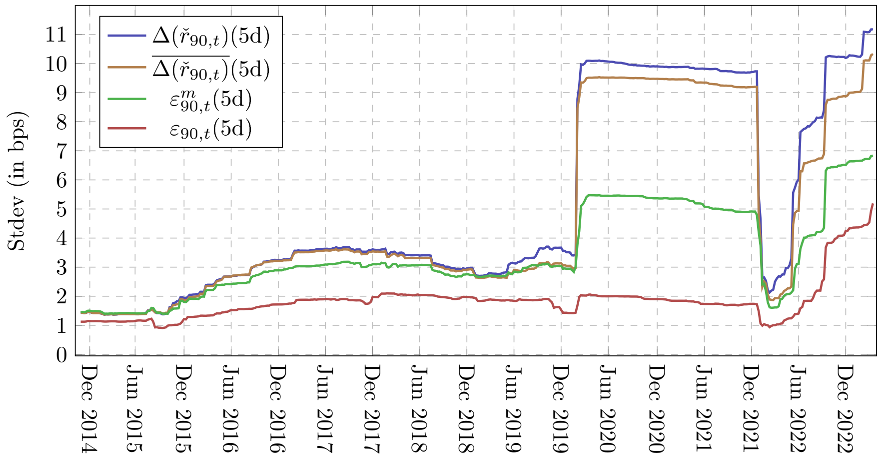


Image 4: Rolling standard deviations of the 3-months rate change, risky rate change, and the residuals over in the weekly frequency.

There is a clear difference between the standard deviation of the rate change before 2020 and after. The rate cuts in 2020 following the onset of COVID-19 led to a rapid increase in the standard deviations[2]. Additionally, the rate hikes starting in 2022 and continuing until the end of the estimation period further contributed significantly to the standard deviation.

To evaluate the model's ability to reduce variance numerically, we extend the discussion from the previous paragraph to all tenors and frequencies. Table 7 presents the standard deviations of $\Delta(\check{r}_{Nt})$, $\overline{\Delta(\check{r}_{Nt})}$, $\varepsilon^{\times}_{Nt}$, $\varepsilon_{Nt}$, the target rate factors $\Delta(r^{\times}_{Nt})$, and the level factor $\mu_t$ for each tenor and risk horizon. The upper panel provides estimates of these statistics from the full dataset. The dataset is further divided into periods containing a jump following an FOMC meeting and periods without a jump.

[2] The immediate drop in standard deviations in early 2022 is attributed to a purely mechanistic reason: the rate cuts from 2020 leaving the estimation window.

| Tenor | Horizon | $\Delta(\check{r}_{Nt})$ | $\overline{\Delta(\check{r}_{Nt})}$ | $\varepsilon^{\times}_{Nt}$ | $\varepsilon_{Nt}$ | $\Delta(r^{\times}_{Nt})$ | $\mu_t$ |
|---|---|---|---|---|---|---|---|
| All periods | | | | | | | |
| 1Mo | 1d | 3.99 | 3.96 | 3.99 | 2.74 | 1.26 | 1.96 |
| | 5d | 8.09 | 7.34 | 6.2 | 4.22 | 3.5 | 3.26 |
| | 10d | 12.21 | 10.43 | 8.45 | 5.08 | 5.68 | 4.52 |
| 3Mo | 1d | 2.72 | 2.58 | 2.3 | 1.7 | 1.53 | 1.96 |
| | 5d | 7.16 | 6.4 | 4.18 | 2.69 | 4.41 | 3.26 |
| | 10d | 10.77 | 8.8 | 4.9 | 2.91 | 6.63 | 4.52 |
| 6Mo | 1d | 2.77 | 2.67 | 1.93 | 1.73 | 2.32 | 1.96 |
| | 5d | 7.04 | 6.5 | 2.98 | 2.83 | 6.25 | 3.26 |
| | 10d | 10.99 | 9.63 | 3.49 | 3.48 | 8.81 | 4.52 |
| Periods without a jump | | | | | | | |
| 1Mo | 1d | 4.0 | 3.98 | 3.96 | 2.74 | 1.18 | 1.88 |
| | 5d | 7.88 | 7.2 | 6.38 | 4.36 | 2.9 | 3.08 |
| | 10d | 11.06 | 9.54 | 9.2 | 5.51 | 2.32 | 4.79 |
| 3Mo | 1d | 2.68 | 2.53 | 2.18 | 1.7 | 1.53 | 1.88 |
| | 5d | 6.28 | 5.46 | 3.9 | 2.79 | 3.92 | 3.08 |
| | 10d | 8.93 | 6.39 | 5.14 | 3.12 | 3.4 | 4.79 |
| 6Mo | 1d | 2.75 | 2.65 | 1.88 | 1.73 | 2.33 | 1.88 |
| | 5d | 6.43 | 5.88 | 2.71 | 2.84 | 6.05 | 3.08 |
| | 10d | 8.55 | 6.94 | 3.35 | 3.67 | 6.49 | 4.79 |
| Periods with a jump | | | | | | | |
| 1Mo | 1d | 3.6 | 3.3 | 4.72 | 2.49 | 2.73 | 3.56 |
| | 5d | 8.98 | 8.09 | 5.15 | 3.37 | 5.78 | 4.08 |
| | 10d | 14.33 | 12.17 | 6.49 | 4.02 | 9.6 | 3.82 |
| 3Mo | 1d | 3.52 | 3.55 | 4.44 | 1.76 | 1.62 | 3.56 |
| | 5d | 10.61 | 9.99 | 5.44 | 2.05 | 6.44 | 4.08 |
| | 10d | 13.91 | 12.47 | 4.27 | 2.37 | 10.74 | 3.82 |
| 6Mo | 1d | 3.09 | 3.05 | 2.92 | 1.8 | 1.89 | 3.56 |
| | 5d | 9.63 | 9.11 | 4.14 | 2.81 | 7.24 | 4.08 |
| | 10d | 15.0 | 13.79 | 3.77 | 3.02 | 12.49 | 3.82 |

Table 7. Standard deviations of rate changes, risky rate changes, residuals and factors.

Across all periods, a substantial reduction in the variance is evident after accounting for the predicted rates and the contributions of the target rate factors. At a 10-day frequency, the target rate factors account for a reduction of 52% in the variance of the 1-month rate, 79% in the 3-month rate, and 90% in the 6-month rate. At a 5-day frequency, the variance reduction stands at 41% for the 1-month rate, 66% for the 3-month rate, and 82% for the 6-month rate. In contrast, at a 1-day frequency, the variance reduction is less significant across these rates. Rate changes associated with FOMC meeting-related jumps have higher standard deviations compared to periods without such jumps. Moreover, the model's efficacy in reducing standard deviation is markedly greater during periods with jumps. The standard deviation of the target rate factor increases with frequency and is much higher during periods with jumps. In contrast, the standard deviation of the level factor also increases with frequency, but there is no clear difference between periods with and without jumps.

The study of rate change standard deviations is further broken down by year. Table 8 presents the standard deviations of the rate changes $\Delta(\check{r}_{Nt})$, the target rate factor residuals $\varepsilon^{\times}_{Nt}$, and the total residuals $\varepsilon_{Nt}$ for all tenors and frequencies.

| Ten. | Hor. | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 | 2019 | 2020 | 2021 | 2022 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Standard deviations of the rate changes $\Delta(\check{r}_{Nt})$ | | | | | | | | | | | |
| 1Mo | 1d | 2.14 | 1.2 | 1.74 | 2.63 | 3.98 | 3.01 | 3.09 | 3.31 | 1.21 | 6.3 |
| | 5d | 3.91 | 2.57 | 2.84 | 4.35 | 4.5 | 4.98 | 4.97 | 12.56 | 2.17 | 13.5 |
| | 10d | 7.16 | 1.45 | 4.61 | 6.05 | 6.34 | 5.74 | 6.66 | 22.28 | 3.08 | 19.96 |
| 3Mo | 1d | 0.99 | 0.87 | 1.49 | 2.1 | 2.18 | 1.9 | 2.26 | 3.4 | 0.8 | 5.53 |
| | 5d | 1.67 | 1.22 | 2.45 | 3.82 | 3.29 | 2.61 | 3.6 | 13.2 | 1.37 | 12.76 |
| | 10d | 1.56 | 1.22 | 3.34 | 5.46 | 5.91 | 3.49 | 5.5 | 21.57 | 1.44 | 15.6 |
| 6Mo | 1d | 0.96 | 0.77 | 1.81 | 1.97 | 1.78 | 1.8 | 2.25 | 3.26 | 0.88 | 5.03 |
| | 5d | 1.39 | 1.23 | 3.92 | 3.73 | 2.97 | 2.84 | 4.19 | 11.88 | 1.3 | 10.69 |
| | 10d | 1.16 | 1.68 | 5.89 | 4.85 | 4.11 | 4.08 | 5.73 | 20.66 | 1.84 | 15.17 |
| Standard deviations of the target rate factor residuals $\varepsilon^{\times}_{Nt}$ | | | | | | | | | | | |
| 1Mo | 1d | 2.19 | 1.29 | 1.85 | 2.67 | 3.97 | 3.02 | 3.33 | 2.9 | 1.24 | 6.51 |
| | 5d | 3.73 | 2.8 | 2.88 | 4.34 | 4.22 | 4.8 | 4.86 | 4.19 | 2.13 | 11.52 |
| | 10d | 6.91 | 1.61 | 4.89 | 5.87 | 5.76 | 5.02 | 7.13 | 7.31 | 3.02 | 18.76 |
| 3Mo | 1d | 1.0 | 0.86 | 1.44 | 2.04 | 2.12 | 1.85 | 2.18 | 3.0 | 0.81 | 4.66 |
| | 5d | 1.62 | 1.35 | 2.19 | 3.39 | 2.83 | 2.69 | 3.31 | 6.67 | 1.37 | 8.98 |
| | 10d | 1.48 | 1.36 | 2.72 | 4.36 | 4.82 | 3.3 | 4.48 | 5.3 | 1.35 | 11.38 |
| 6Mo | 1d | 0.93 | 0.76 | 1.74 | 1.92 | 1.59 | 1.7 | 2.05 | 2.31 | 0.88 | 3.28 |
| | 5d | 1.22 | 1.2 | 2.9 | 2.92 | 2.13 | 2.42 | 3.16 | 5.31 | 0.92 | 3.91 |
| | 10d | 1.05 | 1.64 | 4.38 | 3.46 | 2.49 | 3.11 | 4.1 | 4.23 | 1.04 | 5.93 |
| Standard deviations of the total residuals $\varepsilon_{Nt}$ | | | | | | | | | | | |
| 1Mo | 1d | 1.27 | 0.93 | 1.46 | 1.78 | 2.58 | 1.89 | 2.32 | 1.58 | 0.87 | 4.88 |
| | 5d | 2.45 | 1.72 | 2.16 | 2.47 | 2.75 | 3.12 | 2.92 | 3.1 | 1.33 | 7.69 |
| | 10d | 4.44 | 1.18 | 2.99 | 2.79 | 3.93 | 2.95 | 4.0 | 3.06 | 1.96 | 11.47 |
| 3Mo | 1d | 0.8 | 0.64 | 1.08 | 1.32 | 1.71 | 1.19 | 1.52 | 1.12 | 0.64 | 3.19 |
| | 5d | 1.42 | 0.75 | 1.53 | 1.84 | 2.14 | 1.75 | 1.45 | 2.21 | 0.95 | 5.84 |
| | 10d | 2.32 | 0.75 | 1.95 | 2.11 | 3.13 | 1.29 | 1.92 | 1.97 | 1.16 | 6.39 |
| 6Mo | 1d | 0.77 | 0.68 | 1.17 | 1.48 | 1.7 | 1.41 | 1.64 | 1.3 | 0.67 | 2.81 |
| | 5d | 1.31 | 1.55 | 2.05 | 2.46 | 1.74 | 2.07 | 2.68 | 1.5 | 0.88 | 5.19 |
| | 10d | 2.26 | 1.13 | 3.13 | 2.98 | 2.15 | 2.04 | 3.52 | 1.65 | 1.09 | 7.58 |

Table 8. Standard deviations of the rate changes and residuals broken down by year.

The highest rate change standard deviations were observed in 2020, when the Fed cut target rates, and in 2022, when they increased target rates. The high standard deviation in 2020 is largely attributed to the short term rate market's response to aggressive changes in the target rate policy, and the target rate factor model effectively demonstrates strong variance reduction. Similarly, the variance reduction in 2022 is also highly significant. During these periods of high volatility, also significant market level fluctuations are evident, as indicated by the variance reduction associated with the level factor.

Table 9 presents the correlations for the target rate factor $\Delta(r^{\times}_{Nt})(x)$ and the residual $\varepsilon_{Nt}(x)$ pair, target rate factors and level factor $\mu_t(x)$ pair, and the level factor and the residual pair. The correlations are computed over the full dataset.

| Tenor | Horizon | $\Delta(r^{\times}_{Nt})(x), \mu_t(x)$ | $\Delta(r^{\times}_{Nt})(x), \varepsilon_{Nt}(x)$ | $\mu_t(x), \varepsilon_{Nt}(x)$ |
|---|---|---|---|---|
| 1Mo | 1d | -0.26 | -0.07 | 0.43 |
| | 5d | 0.06 | 0.07 | 0.37 |
| | 10d | 0.12 | -0.01 | 0.55 |
| 3Mo | 1d | -0.21 | 0.06 | -0.22 |
| | 5d | 0.07 | 0.08 | -0.02 |
| | 10d | 0.13 | 0.04 | -0.18 |
| 6Mo | 1d | -0.14 | -0.08 | -0.46 |
| | 5d | 0.09 | -0.27 | -0.53 |
| | 10d | 0.13 | -0.12 | -0.65 |

Table 9. Correlations between target rate factor contributions to rates, level factors and residuals.

The target rate factor contributions and the level factor exhibit essentially zero correlation. This is also true for the target rate factor and the residual pair. The residual of the 1-month rate and the corresponding level factor are positively correlated, whereas for the 6-month rate, the dependence is negative. This suggests the presence of a systematic dependence that can be attributed to a slope market factor. This study, however, focuses on the target rate factors and does not explore the market factor residual further.

Another topic specific to short-term rates is the heavy-tailed nature of rate change distributions. The excess kurtosis statistic is often used to approximate this heavy-tailedness. Table 10 presents the corresponding values.

| Tenor | Horizon | $\Delta(\check{r}_{Nt})$ | $\overline{\Delta(\check{r}_{Nt})}$ | $\varepsilon^{\times}_{Nt}$ | $\varepsilon_{Nt}$ | $\Delta(r^{\times}_{Nt})$ | $\mu_t$ |
|---|---|---|---|---|---|---|---|
| 1Mo | 1d | 46.88 | 46.4 | 47.64 | 39.9 | 172.94 | 27.63 |
| | 5d | 18.73 | 26.05 | 16.15 | 19.02 | 132.46 | 10.88 |
| | 10d | 21.07 | 24.12 | 12.92 | 15.84 | 81.06 | 6.11 |
| 3Mo | 1d | 26.58 | 24.42 | 35.99 | 22.21 | 102.85 | 27.63 |
| | 5d | 51.12 | 60.96 | 33.92 | 26.25 | 83.07 | 10.88 |
| | 10d | 28.83 | 43.53 | 15.39 | 8.29 | 68.88 | 6.11 |
| 6Mo | 1d | 28.26 | 30.85 | 13.7 | 21.35 | 126.09 | 27.63 |
| | 5d | 43.65 | 54.36 | 12.96 | 16.9 | 67.72 | 10.88 |
| | 10d | 25.49 | 34.31 | 2.97 | 11.91 | 38.79 | 6.11 |

Table 10. Excess kurtosis of rate changes, risky rate changes, residuals and factors.

The kurtosis of the rate changes and the risky rate changes is high for all tenors and horizons. The target rate factors demonstrate some ability to reduce this kurtosis, unlike the level factor, which shows no such effect. The target rate factor contributions are themselves highly leptokurtic, which is expected since these factors undergo large jumps following monetary policy changes and exhibit much less movement during stable periods. Significant kurtosis is also observed in the level factors and residuals, which is natural given their substantial movements during turbulent periods.

The analysis presented in this section relies on historical time series of observed factor changes. A natural extension of this study is to develop a predictive model for the distributions of the rate changes. In such a model, the factors are treated as random variables, and the parameters of their underlying probability laws are estimated using historical time series of factor movements and residuals. An additional complexity in this context is the uncertainty related to the FOMC meeting dates. During periods of high market distress, emergency FOMC meetings have sometimes replaced scheduled ones. Consequently, the parameters $N_{it}$, which measure the time to the FOMC meeting from date $t$, should also be treated as random variables. The historical record of rescheduled FOMC meetings is sparse, complicating the estimation of the uncertainty in $N_{it}$.

# 6 Case Studies

Some risk applications of the target rate factors models are presented. The goal is to demonstrate how the target rate factors can be used to break down the market risk into components. A volatility metric is used for the risk estimation. The volatility represents an estimate of the standard deviation for the rate change between the risk horizon and the valuation time. The volatility $\sigma$ is subject to the relation

$$\sigma^2 = X^t \cdot \mathcal{COV} \cdot X + \mathcal{V}ar(\varepsilon)$$

where $\mathcal{COV}$ is the covariance matrix of the factors $\Delta(r_{0t})(x)$, $\Delta(j_{1t})(x), \ldots, \Delta(j_{4t})(x)$ and $\mu_t(x)$ defined in Section 5. The component $\mathcal{V}ar(\varepsilon)$ is the variance of the residual component, which is treated as an independent random variable. In the case $x < N^j_{1t}$, i.e., there is no jump before the risk horizon, the factor loading vector $X$ is the column vector with the components

$$X_0 = 1, X_i = \frac{(N + x - N^j_{it})^+}{N} \text{ for } i \in \{1,2,3,4\}, \text{ and } X_5 = 1,$$

and in the case $x \geqslant N^j_{1t}$, the components are

$$X_0 = 1, X_1 = 1, X_i = \frac{(N + x - N^j_{it})^+}{N} \text{ for } i \in \{2,3,4\}, \text{ and } X_5 = 1.$$

The last component $X_5 = 1$ is the loading on the level factor. The volatility breaks down to systematic and idiosyncratic components according to

$$\sigma^s = \frac{X^t \cdot \mathcal{Cov} \cdot X}{\sigma} \quad \text{and} \quad \sigma^\varepsilon = \frac{\mathcal{Var}(\varepsilon)}{\sigma}$$

The systematic volatility estimate $\sigma^s$ breaks down to factor components according to

$$\sigma^s_i = \frac{X^t_i(\mathcal{Cov} \cdot X)}{\sigma}$$

for all $i \in \{0, \dots, 5\}$. These components represent the factor contributions to the risk. All the components $\sigma^s_0, \dots, \sigma^s_5$ and $\sigma^\varepsilon$ sum to $\sigma$.

Recall that the meaning of the factors in the case where the risk horizon $x$ is subject to $x < N^j_{1t}$, or $x \geqslant N^j_{1t}$ varies. In the former case, the covariance matrix and the residual variance are estimated using a 3-years historical sample of all the factor movements which contained a FOMC meeting. Since there are 8 FOMC meetings each year, this sample contains 24 observations of factor movements. In the latter case, the factor movements over the periods which do not contain FOMC related jumps are selected in the historical 3-years window. The components of the covariance matrix and the variance are estimated by using the standard unweighted biased mean estimator.

Next, we explore the rate cuts in 2020 and the rate hikes in 2022, depicted in Image 1, with more details. The full risk breakdown for the 5-day risk horizons, estimated on 27-Nov-2019 and on 26-Feb-2020 are presented in Table 11.

| Tenor | $\phi$ | $\Delta(r)$ | $\Delta(j_1)$ | $\Delta(j_2)$ | $\Delta(j_3)$ | $\Delta(j_4)$ | TRF | Level | $\varepsilon$ | $\sigma$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 27-Nov-2019 | | | | | | | | | | |
| 1Mo | 0.35 | 0.3 | 0.07 | 0.0 | 0.0 | 0.0 | 0.37 | 1.55 | 2.79 | 4.72 |
| 3Mo | -0.06 | 0.21 | 0.54 | 0.1 | 0.0 | 0.0 | 0.85 | 1.71 | 1.74 | 4.31 |
| 6Mo | -0.31 | 0.24 | 0.59 | 0.6 | 0.25 | 0.06 | 1.74 | 1.32 | 2.12 | 5.18 |
| 26-Feb-2020 | | | | | | | | | | |
| 1Mo | -1.69 | 0.42 | -0.1 | 0.0 | 0.0 | 0.0 | 0.32 | 1.74 | 2.66 | 4.72 |
| 3Mo | -1.63 | 0.26 | 0.21 | 0.15 | 0.0 | 0.0 | 0.62 | 1.88 | 1.65 | 4.15 |
| 6Mo | -1.79 | 0.28 | 0.4 | 0.75 | 0.32 | 0.07 | 1.82 | 1.29 | 2.08 | 5.19 |

Table 11. Risk estimates in the beginning of Covid pandemic.

The column $\phi$ has the predicted rate changes over the 5-day risk horizon. The columns $\Delta(r)$ and $\Delta(j_1), \dots, \Delta(j_4)$ are the factor contributions to volatility from the overnight rate change and from the target rate factors. The TFR column has the total contributions from these risk sources. Level and $\varepsilon$-columns have the contributions from the level factor and from the idiosyncratic risk, and $\sigma$ has the total standard deviation estimates. The first scheduled FOMC meeting after the risk estimation date 26-Feb-2020 is 16-Mar-2020. For this reason, the parameters for the risk metrics on 26-Feb-2020 are estimated using a sample of periods that does not contain the FOMC meeting related jumps. However, there was an unannounced FOMC meeting on 03-Mar-2020 where significant rate cuts were announced. The model calibration fails to capture the elevated risk associated with the target rate policy change: the total risk estimates, and the total contributions from the target rate factors are very close to the corresponding values estimated one quarter earlier, on 27-Nov-2019. The predicted jump sizes on 26-Feb-2020 are $-7$, $-14$, $-15$, and $-10$ basis points for the 4 scheduled FOMC meetings after the estimation date. These values do not reflect the actual decision, which moved the EFFR rate from 158 bps on 26-Feb-2020 to 109 on 4-Mar-2020. The actual observed rate changes were $-59$ bps for the 1-month rate change, $-81$ bps for the 3-months rate, and $-74$ for the 6-months rate. In comparison to the predicted metrics, these magnitudes of these events are more extreme than 10-sigma. As a conclusion, based on the risk metrics derived from the rates quotes, and the Fed funds futures quotes, the market quotes did not reflect the concern of immediate rate cuts during the early stages of the Covid pandemic.

The first post-pandemic rate hike were decided in the FOMC meeting on 16-Mar-2022. The EFFR rate moved from 8 bps to 33. More rate hikes were decided in the subsequent meetings in 2022. A particularly large increase in EFFR was seen after the meeting on 21-Sep-2022, after which the EFFR rate moved from 233 bps to 308 bps. Table 11 presents risk metrics in the cases of two 5-days risk horizon with estimation dates 16-Mar-2022, and 21-Sep-2022.

| Tenor | $\phi$ | $\Delta(r)$ | $\Delta(j_1)$ | $\Delta(j_2)$ | $\Delta(j_3)$ | $\Delta(j_4)$ | TRF | Level | $\varepsilon$ | $\sigma$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 16-Mar-2022 | | | | | | | | | | |
| 1Mo | 0.87 | -0.29 | 7.81 | 0.0 | 0.0 | 0.0 | 7.52 | 4.56 | 3.31 | 15.4 |
| 3Mo | 5.35 | -0.27 | 8.13 | 1.41 | 0.0 | 0.0 | 9.28 | 4.35 | 2.15 | 15.78 |
| 6Mo | 4.08 | -0.23 | 8.15 | 2.72 | 0.21 | 0.18 | 11.04 | 3.9 | 2.06 | 17.0 |
| 21-Sep-2022 | | | | | | | | | | |
| 1Mo | 2.51 | -0.24 | 7.73 | 0.0 | 0.0 | 0.0 | 7.49 | 4.82 | 4.03 | 16.33 |
| 3Mo | 9.94 | -0.26 | 8.14 | 1.67 | 0.01 | 0.0 | 9.57 | 4.75 | 2.6 | 16.91 |
| 6Mo | 5.92 | -0.25 | 8.14 | 2.53 | 0.16 | 0.23 | 10.81 | 4.51 | 3.43 | 18.76 |

Table 11. Risk estimates during the post-pandemic rate hike period.

In the first test case, the predicted jumps associated with the 4 of the closest FOMC meetings are 26, 41, 34 and 27 bps. The target rate factor contribution to risk is no significantly higher than in the case studies in the beginning of the Covid pandemic. This is a consequence of wide speculation about the upcoming monetary policy targets. In addition, the level and the residual contributions are high which is caused by high volatilities in the short rate markets during this period. In the case 21-Sep-2022 we see very similar risk profile. The predicted jump sizes are 75, 69, 48, and 20 bps. In both of these tests cases, the risk contribution from the overnight rate change is negative. The negative sign arises because the historical samples of $\Delta(r_{0t})(x)$ and $\Delta(j_{1t})(x)$ have negative correlation.

In all test cases, the target rate factor contribution to the risk increases as the tenor gets longer. This phenomenon has a natural interpretation in the model: the longer tenors are sensitive to target rate factors explaining the risk of the monetary policy adjustments further in the future. This model feature produces a risk profile discussed in Section 4.

# 7 Discussion

This manuscript focuses on constructing a theoretical framework to examine the impact of monetary policy adjustments on short-rate movements. To achieve this objective, a factor model is developed that incorporates factors related to market movements driven by speculation on forthcoming FOMC decisions as well as rate changes directly associated with these decisions. Additionally, the factor model can be employed to estimate risk metrics for short-term rates. Section 6 briefly discusses this application, albeit with a heavily simplified parameter estimation problem, which presents a rather complex econometric challenge. Firstly, the volatility of the target rate factors raises questions: Is this volatility dependent on the anticipated magnitude of upcoming target rate adjustments, and does it vary with the time remaining until the meeting? Secondly, the time series data for the target rate factors exhibit fat-tailed characteristics, necessitating the use of specialized distributions for accurate probability law estimation. Thirdly, the manuscript addresses uncertainties related to FOMC meeting dates, noting the complexity of the estimation problem due to the infrequency of rescheduled meetings.

This work contributes to the stochastic rate modelling by proposing several key features. Firstly, a short rate process should incorporate jumps at predetermined points in time. Also, the model should include a component with stochastic dynamics that adjust the short rate value in response to evolving expectations about the central bank target rates. It is also observed that short-term rates are subject to market liquidity related fluctuations, captured by the level factor, as well as idiosyncratic variations. Notably, the 1-month rate exhibits particularly strong idiosyncratic fluctuations. These findings favor stochastic modelling that captures the full dynamics of the forward curve, such as the HJM framework, as rate-specific idiosyncrasies are challenging to model within the traditional short rate framework.

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