Exploring the Log-Linear Rule of Credit Rating Deterioration

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Abstract

This study models the expected frequencies of credit rating deterioration for publicly listed corporates. A deterioration is defined as a downgrade of one or more rating notches. Firms with a fixed initial credit rating are treated as a uniformly ordered sample, ranked by credit quality estimates derived from a distance-to-default (DD) framework. Two hypotheses are examined. The asset-level hypothesis posits that, within each rating category, the logarithm of the expected deterioration frequency (EDF) is linearly related to the uniform rank. The portfolio-level hypothesis states that the logarithm of the expected deterioration frequency over the tail of the sample is linearly related to the uniform rank. Strong empirical support for both hypotheses is provided, based on goodness-of-fit measures, visual evidence, and superior performance relative to the commonly used logit and probit specification.

Key words: credit risk, rating deterioration frequency, rating transition frequency, credit quality ranking, distance-to-default

JEL code: G12, G32, G33

1 Introduction

Credit rating agencies publish average one-year default frequencies for companies across different credit ratings. The logarithms of these frequencies increase linearly as the credit rating declines. In other terms, as credit ratings decline, the average default experience worsens exponentially. The rating agencies also maintain annual time series data for average default frequencies for each rating. Within a specific speculative-grade rating, and assuming the years are organized as a uniform sample sorted by the default frequencies, the logarithms of default frequencies exhibit a linear relation across the sample. In other words, for a fixed rating, the average annual default experience can be explained using an exponential rule. These observations can be easily reconstructed from the agency summaries, for instance using the S&P report by Kraemer et al. (2022), and they are also discussed in the empirical studies by Cappon et al. (2018) and Harju (2023). One may think of these observations being related to the negative Matthew effect – for corporates already experiencing financial pressure, problems accumulate through a snowball effect, and the challenges expand at an exponential pace. Motivated by these observations, we arrive at a fundamental question: if the average default experience varies exponentially with credit ratings, could a similar exponential pattern also explain the differences in credit rating deterioration frequencies among corporates within the same rating category? Furthermore, since logistic and probit models are widely used by practitioners in credit analysis, how do they fit into this framework, and are they optimal?

In this research letter, I propose a tentative answer for the questions concluding the previous paragraph. The context is the credit rating deteriorations of public corporates. The public corporates are a suitable domain for this study because the ranking of these entities by their credit quality is well studied. The firm specific DD statistic is proven in empirical tests to have an impressive ability to rank the corporates in their credit quality (see e.g., Bharath and Shumway (2008) and Kealhofer (2003)). The first objective is to use DD to rank the corporate population from the highest to the lowest credit quality, and to produce a uniformly distributed ranked sample within each credit rating. The following two hypotheses are examined per rating. The asset level hypothesis – the logarithm of the EDF is linearly related to the uniform rank. The portfolio level hypothesis – the logarithm of the expected tail deterioration frequency (ETDF) is linearly related to the uniform rank. The ETDF is defined by

$$\text{ETDF}(r) = \mathcal{E}(\mathbf{1}^d : R < r) \tag{1}$$

for all values $r \in [0, 1]$ for the uniform rank R. The indicator variable $\mathbf{1}^d$ gets values in $\{0, 1\}$ and signals the occurrence of a rating deterioration over the one-year risk horizon. The empirical analysis indicates that the log-linear EDF model estimated by maximum likelihood (MLE) and the ETDF model estimated via regression exhibit stronger predictive power for EDFs than the DD-based logistic regression that is used as a benchmark. Moreover, the log-linear EDF model demonstrates a slight performance advantage over the log-linear ETDF specification.

From an econometric perspective, the key idea is to estimate EDFs at a fixed risk horizon in two steps: first, to construct a sufficient statistic that ranks firms uniformly by credit quality, and second, to specify a functional form for the EDF estimator that uses this uniform rank as its covariate. The study by Harju (2023) discusses this approach in the context of default probability estimation and advocates the use of log-linear EDF or log-linear ETDF specifications. By contrast, the standard practice for EDF estimation at a fixed horizon is to employ logistic or probit models, the latter arising naturally from barrier-type credit models. Representative studies include Kaplan and Urwitz (1979), Blume et al. (1998), Frey and McNeil (2003),Albanese and Chen (2006), Altman and Rijken (2006), and Edirisinghe et al. (2022).

The EDF models provide corporate bond market makers, portfolio managers, and risk managers with essential information on the relative frequencies of credit deteriorations across issuers and market conditions. The central finding of this study is that the frequency decays exponentially in the uniform rank variable, yielding a parsimonious and transparent characterization of rating deterioration risk.

Data Used in Empirical Study.

The dataset comprises the inputs required to estimate DDs, together with one-year rating transition data, covering more than 57000 firm–quarter observations over the period 2010–2024. Rating transitions are identified using a hierarchical fallback procedure that sources transition data sequentially from S&P, Moody's, Fitch, and Egan-Jones. The sample includes corporate issuers across all industries and geographic regions.

Code Used in Empirical Study.

All data and the Python scripts used to produce the tables and figures in this study are publicly available in the repository referenced in Harju (2026).

2 Distance-to-Default

The distance to default (DD) for publicly traded corporates is defined as the volatility-adjusted log inverse leverage. For a one-year horizon, DD is given by

$$\text{DD} = \frac{1}{\sigma}\Big(\log\Big(\frac{A}{L}\Big) + \mu - \frac{1}{2}\sigma^2\Big) \tag{2}$$

where σ is the annual asset volatility, A is the total market value of assets L is the value of debt and μ is the annualized asset drift. In this study, debt L is defined as the sum of the book value of short-term debt and one half of the book value of long-term debt. The parameters A, σ and μ cannot be observed directly. A standard estimation strategy for A and σ is taken where the market capitalization of the corporate is set equal to the value of a call option on A with strike L and volatility σ. This relation depends on both A and σ, and therefore is not sufficient alone. This study uses the estimator for A and σ discussed by Kealhofer (2003) and Vassalou and Xing (2004). The estimation is based on an iterative strategy that links the asset volatility σ to the standard deviation of the stock returns. Further implementation details are provided in Vassalou and Xing (2004) and Harju (2023). The asset drift is defined as the sum of the risk-free rate and the asset risk premium, $\mu = r + \pi_a$. The U.S. one-month treasury rate is used for r, and π_a is set to 0.0343 for the A-rated-, 0.0355 for the BBB-rated-, and 0.0270 for the speculative-grade entities. These estimates were found by Zhan et al. (2009). In rare occurrences, this calibration strategy produces asset volatility estimates that are unrealistically high or low. To mitigate the influence of these outliers, for each rating category (A, BBB, BB, and B), the lowest and highest 1% of observations based on the asset volatility are excluded from the sample.

The empirical distribution of the logarithms of the DDs can be approximated with a good accuracy with the logistic distribution. The distribution function is defined by

$$\mathcal{L}(\log(\mathrm{DD})) = \Big(1 + \exp\Big(-\frac{\log(\mathrm{DD}) - \mu}{s}\Big)\Big)^{-1},$$

where μ is the location parameter and $s > 0$ is the scale parameter. Table 1 summarizes the results of the distributional fit. Parameter estimation is performed by minimizing the Kolmogorov-Smirnov distance.

Rating	μ	s	KS-distance	Sample Size	Deterioration Freq
A	2.17	0.28	0.01	9948	6.21
BBB	1.98	0.28	0.02	20631	3.11
BB	1.59	0.29	0.02	16963	4.80
B	1.13	0.36	0.02	9752	4.90

Table 1. Fitted parameters, Kolmogorov-Smirnov distances, sample sizes and empirical deterioration frequencies in percentage unit.

The sample sizes for the highest and the lowest credit rating are considerable smaller than the four largest categories A, BBB, BB, and B, and will not be included in this analysis.

3 Models and Parameter Estimation

The benchmark model in this study uses DD as its explanatory variable, whereas the proposed models are based on the uniform rank $R = 1 - \mathcal{F}(\mathrm{DD})$, where $\mathcal{F}$ denotes the empirical distribution function of DD. Under this transformation, firms are ranked from highest to lowest credit quality. The benchmark logistic model is chosen for the benchmark

$$\mathrm{EDF}(\mathrm{DD}) = \Big(1 + \exp(\alpha - \beta \cdot \mathrm{DD})\Big)^{-1}. \tag{3}$$

Another common choice is the probit specification, however, based on the AIC scores, the logistic model is much stronger predictor for the EDF within the studied rating categories, and therefore the logistic specification is selected as the benchmark.

A log-linear EDF model is defined as

$$\mathrm{EDF}(r) = \exp(\alpha + \beta r). \tag{4}$$

The ETDF corresponding to this EDF, as defined in(1), is obtained by

$$\mathrm{ETDF}(r) = \frac{\mathcal{E}(\mathbf{1}^d \cdot \mathbf{1}(R < r))}{\mathcal{P}(R < r)} = \frac{1}{r}\int_0^r \mathrm{EDF}(x)dx = \frac{1}{\beta r}\Big(\exp(\alpha + \beta r) - \exp(\alpha)\Big). \tag{5}$$

The log-linear ETDF model has the specification

$$\mathrm{ETDF}(r) = \exp(\alpha + \beta r). \tag{6}$$

Starting from the ETDF (6), the implied EDF is obtained via

$$\mathrm{EDF}(r) = \frac{d}{dr}(r \cdot \mathrm{ETDF}(r)) = (1 + \beta r)\exp(\alpha + \beta r).$$

The benchmark and log-linear EDF models are estimated by maximizing the Bernoulli likelihood with deterioration probabilities determined by (3) and (4). The log-linear EDF model, and the log-linear ETDF model can be given an alternative curve-fitting estimation approach. Given a sample of N observations, the empirical ETDF at k/N for any $k \in \{1, \dots, N\}$, is defined by

$$\mathrm{ETDF}\left(\frac{k}{N}\right) = \frac{1}{k}\sum_{i=1}^{k} \mathbf{1}_i^d \tag{7}$$

where the deterioration indicators $\mathbf{1}_i^d$ are valued in $\{0, 1\}$ and indexed so that $R_i \leqslant R_{i+1}$ for all i. Note that (7) is an expanding average over the indicators, and therefore the ETDF estimates of the lowest k/N have high uncertainty. To address this, we compute the empirical ETDFs over the domain $[0.2, 1]$ which results in the ETDF estimates for the 80% of the empirically observed DDs. The value of the empirical ETDF at 1 is equal to the EDF mean over the full sample. The model mean EDFs are $(\exp(\alpha + \beta) - \exp(\alpha))/\beta$ and $\exp(\alpha + \beta)$ in the cases of the log-linear EDF, and the log-linear ETDF hypotheses. The parameters are estimated with the constraint that the empirical mean EDF matches the model mean EDF. This results in a one-parameter estimation problem for (5) and (6). The remaining parameter is calibrated by minimizing the least square errors sum for matching the logarithms of the model ETDFs on the empirical log-ETDFs over the domain $[0.2, 1]$.

Four models have been introduced: the benchmark logistic regression model (Logistic), a log-linear EDF model with an MLE estimator (Model 1), a log-linear EDF model with a constrained least squares estimator (Model 2), and a log-linear ETDF model with a constrained least squares estimator (Model 3). For each rating under study, the parameter estimates, the AIC scores computed over the full sample, and the R-squared scores for the log-ETDF fit over the 80% of the lowest rated sample, are presented in table 2.

rating	model	alpha	beta	AIC	R-squared
A	Logistic	-2.12	0.07	4587.27	0.83
	Model 1	-3.26	0.90	4591.56	0.94
	Model 2	-3.24	0.86	4591.62	0.95
	Model 3	-3.25	0.47	4591.76	0.95
BBB	Logistic	-2.10	0.20	5498.30	0.87
	Model 1	-4.98	2.52	5421.09	0.90
	Model 2	-4.88	2.38	5421.98	0.95
	Model 3	-4.92	1.45	5428.59	0.94
BB	Logistic	-1.40	0.35	6122.39	0.89
	Model 1	-4.98	3.14	5974.43	0.85
	Model 2	-4.82	2.91	5976.92	0.95
	Model 3	-4.86	1.82	5989.60	0.95
B	Logistic	-1.40	0.56	3501.96	0.76
	Model 1	-5.12	3.35	3451.51	0.79
	Model 2	-4.96	3.13	3452.80	0.91
	Model 3	-4.96	1.95	3463.99	0.92

Table 2. Fitting statistics.

The in-sample AIC score suggest that Model 1 is the strongest. The logistic model suffers from the highest loss of information. Image 1 depicts the empirical log-ETDFs and the log-ETDFs based on the estimated models as a function of the rank R.

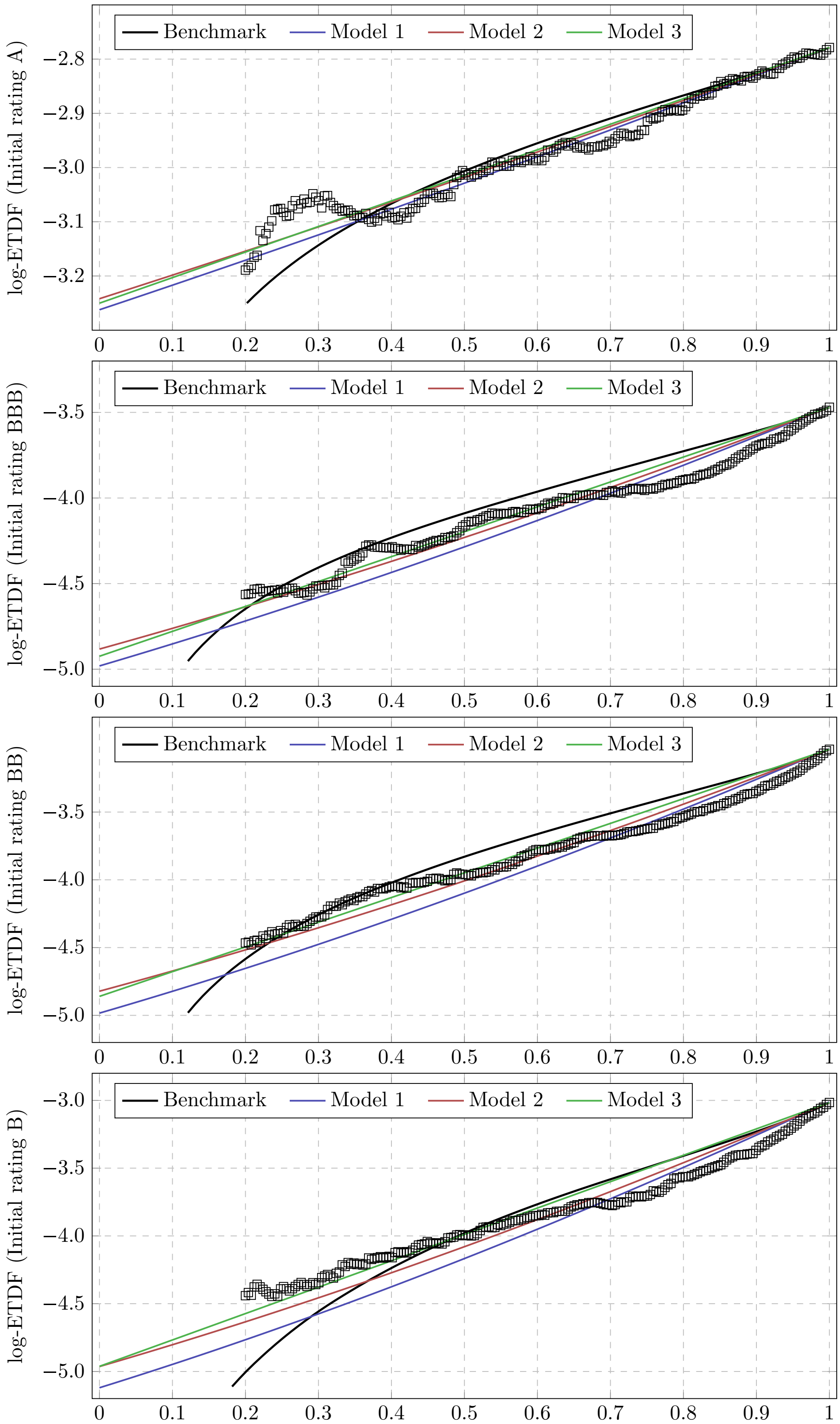


Image 1. Empirical log-ETDF, and the log-ETDF estimates for all the models and ratings studied.

The empirical relationship between R and the log-ETDF is largely linear. With the exception of the Logistic benchmark, all models show strong agreement on the linear log-ETDF pattern. Model estimates converge at $R = 1$, indicating that all models, including the logistic, correctly reproduce the empirical mean EDF. The log-ETDF values have more uncertainty as the rank R decreases. There is a good confidence about the ETDF for the lowest credit quality estimates, and lower confidence for the highest credit quality estimates. If R were ranked in ascending credit quality, extrapolation over the exponentially growing ETDF tail would be required.

4 Accuracy Testing

All models, including the benchmark, employ the same explanatory variable (DD), which implies an identical ranking of estimated EDFs across models. Consequently, discrimination-based tests, such as the conventional ROC test, are unable to detect performance differences among the models. This section therefore employs bucketing tests to assess each model's ability to match empirical EDFs across the full panel dataset and credit ratings.

First, an in-sample bucketing test is orgnized separately for each rating category. Model parameters are estimated using the full panel dataset, as described in Section 2. The dataset is then partitioned into ten equally sized subsets according to the increasing rank of EDF values; these subsets are referred to as buckets. For each model and each bucket, predicted EDFs are computed for all constituents, and bucket-level averages of both model-predicted EDFs and empirical default frequencies are calculated. For each bucket, relative prediction errors are defined as the absolute value of the difference between the average model-predicted EDF and the average empirical frequency, normalized by the latter. An overall accuracy metric is obtained by summing the relative errors across all buckets, allowing direct comparison across models. In addition, a pairwise model comparison is performed. For each of the six model pairs, a p-value is computed for the null hypothesis that the model performing worse according to the accuracy metric has a lower mean prediction error. This procedure yields model-specific accuracy scores and associated p-values for the comparison tests assessing statistical significance. However, due to the high variance of prediction errors, these p-values are generally large. To improve statistical power, we compute average accuracy scores across ratings for each model and combine the corresponding p-values using Stouffer's method to obtain a single model-specific significance measure. All in-sample results are reported in Table 3.

	L	1	2	3	L vs 1	L vs 2	L vs 3	1 vs 2	1 vs 3	2 vs 3
A	145.78	111.49	112.51	115.66	9.04	10.00	11.33	37.90	16.07	5.26
BBB	267.87	182.75	191.57	204.77	7.97	6.54	8.01	28.52	17.74	11.09
BB	199.86	191.10	166.26	163.43	44.32	25.86	21.41	9.36	21.08	43.20
B	308.97	244.36	239.49	280.07	7.23	8.79	28.63	42.66	21.00	5.73
Average	230.62	182.42	177.45	190.98	1.49	0.83	2.36	37.67	16.86	1.68

Table 3. Results of the in-sample performance test. L denotes Logistic model, while 1, 2, and 3 denote Model 1, Model 2, and Model 3. All columns are in percentage units.

The results indicate that all proposed models outperform the Logistic benchmark model. Stouffer's combined test statistic suggests that this improvement is statistically significant at the 95% confidence level. The results do not provide sufficient evidence to distinguish statistically between the performances of Models 1 and 2. However, Model 2 has advantage over Model 3.

Next we conduct an out-of-sample test using a bootstrap-based procedure applied separately to each credit rating. A total of 100 bootstrap iterations are performed. In each iteration, the dataset is randomly partitioned into a training set containing 70% of the observations, and a testing set containing the remaining 30%. Model parameters are estimated on the training set. The testing set is then partitioned into ten buckets following the same procedure as in the in-sample analysis. Now, for each bucket, relative prediction errors are defined as the absolute difference between the average model-predicted EDF and the average empirical frequency, normalized by the bucket specific average empirical frequency computed using the full dataset. The overall accuracy metric is the sum of the relative errors across all buckets. Table 4 reports, for each rating and each model, the average scores over the iterations. Similar to the in-sample test, the p-values for the model comparison tests are computed in each iteration. The Stouffer p-values are the calculated for each model and rating. The resulting statistics are presented in Table 4.

	L	1	2	3	L vs 1	L vs 2	L vs 3	1 vs 2	1 vs 3	2 vs 3
A	207.71	194.46	195.99	196.89	0.00	0.00	0.00	0.05	0.00	0.00
BBB	333.16	276.34	280.10	290.83	0.00	0.00	0.00	0.88	0.00	0.00
BB	294.30	265.74	254.56	263.14	0.00	0.00	0.00	0.00	20.72	0.00
B	391.46	339.05	343.13	367.59	0.00	0.00	0.00	4.37	0.00	0.00

Table 4. Results of the out-of-sample performance test. All columns are in percentage units.

Consistent with the in-sample findings, all models outperform the benchmark in the out-of-sample test. The improvements are statistically significant across all ratings and models. Models 1 and 2 outperform the log-linear ETDF specification.

5 EDFs in Closed Form

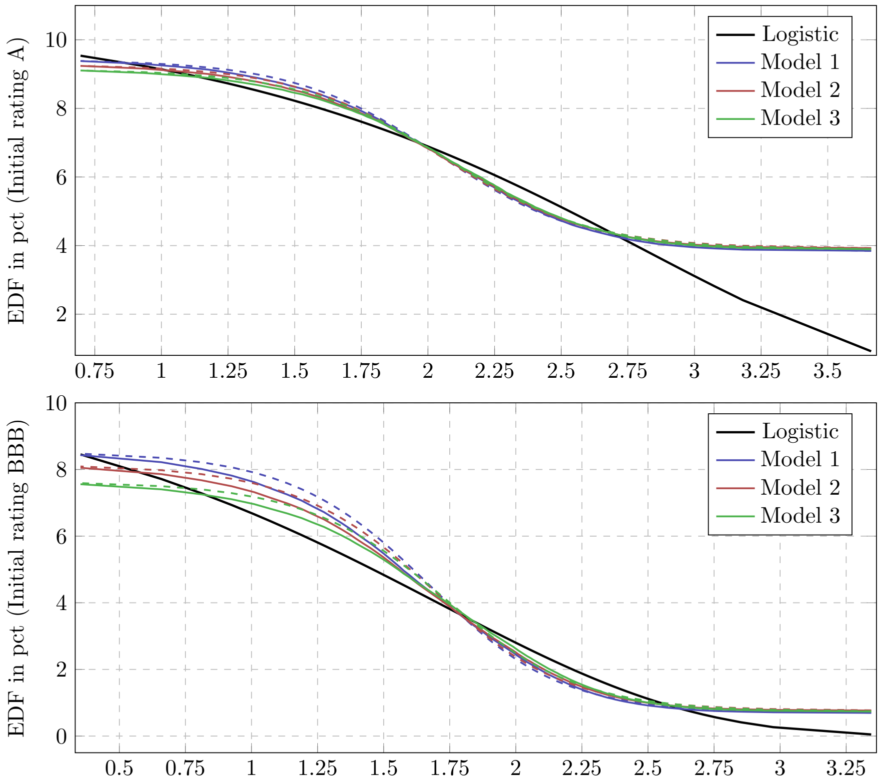
In Models 1–3, EDFs are expressed as functions of the rank variable $R = 1 - \mathcal{F}(\mathrm{DD})$ where $\mathcal{F}$ denotes the empirical distribution function of the DDs. Equivalently, $R = 1 - \mathcal{G}(\log(\mathrm{DD}))$, with $\mathcal{G}$ is the empirical distribution function of log-DDs. As noted in Section 2, the distribution of log-DDs is well approximated by a logistic law. Therefore, we adopt the approximation $R = 1 - \mathcal{L}(\log(\mathrm{DD}))$ where $\mathcal{L}$ is the logistic distribution function with parameters estimated in Section 2. Under this approximation, EDFs can be expressed directly as functions of log(DD). For the log-linear EDF model,

$$\mathrm{EDF}(\log(\mathrm{DD})) = \exp(a + b \cdot (1 - \mathcal{L}(\log(\mathrm{DD}))))$$

while for the log-linear ETDF model,

$$\mathrm{EDF}(\log(\mathrm{DD})) = (1 + \beta \cdot (1 - \mathcal{L}(\log(\mathrm{DD})))) \exp(\alpha + \beta \cdot (1 - \mathcal{L}(\log(\mathrm{DD}))))$$

For each rating studied, the EDFs are plotted as functions of log-DD in Image 2.



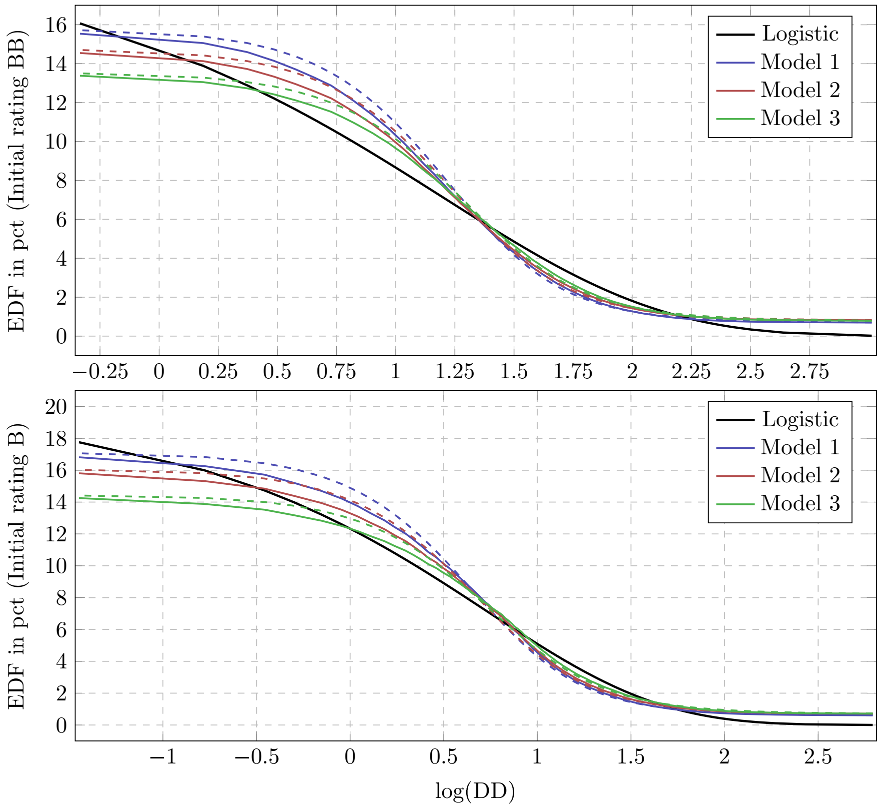


IMAGE 2. The solid lines depict the model EDF predictions as functions of log-DD. The dashed lines are the corresponding EDF estimates based on the original model estimated using the rank variable R.

Several observations emerge from Figure 2. The gaps between the solid and dashed lines are the EDF estimation errors arising from the logistic approximation of the empirical log-DD distribution. In each case, the log-linear EDF model predicts higher deterioration probabilities for the firms with the shortest DD. The shape the log-DD to EDF curve is very similar between the three tested models. The benchmark logistic model has a markedly different profile. The variations of the EFDs over different ratings increase as the rating declines. Within the closed-form framework developed above, these observations can be examined quantitatively, allowing the derivation of fundamental summary statistics for each model. The standard summaries are reported in Table 5.

rating	model	lower bound	upper bound	mean	stdev
A	Logistic	0.00	100.00	6.21	1.60
	Model 1	3.83	9.42	6.21	1.60
	Model 2	3.91	9.28	6.21	1.54
	Model 3	3.88	9.14	6.21	1.52
BBB	Logistic	0.00	100.00	3.11	1.80
	Model 1	0.69	8.54	3.12	2.16
	Model 2	0.76	8.15	3.11	2.04
	Model 3	0.73	7.64	3.11	1.94
BB	Logistic	0.00	100.00	4.80	3.36
	Model 1	0.69	15.79	4.81	4.06
	Model 2	0.81	14.76	4.80	3.79
	Model 3	0.78	13.55	4.80	3.54
B	Logistic	0.00	100.00	4.90	3.93
	Model 1	0.60	17.11	4.92	4.40
	Model 2	0.70	16.06	4.90	4.13
	Model 3	0.70	14.44	4.90	3.79

Table 5. Summary statistics of the EDF estimates in percentage units.

The lower and upper bounds of the log-linear EDF model are equal to $\exp(\alpha)$ and $\exp(\alpha+\beta)$. This can be obtained by selecting the lower and upper bounds for the logistic distribution function $\mathcal{L}$ (i.e., 0 and 1) in the functional specification of EDF. The mean can be computed directly by

$$\mathcal{E}(\mathrm{EDF}) = \int_{-\infty}^{\infty} \exp(\alpha + \beta \cdot \mathcal{L}(x))\ell(x)dx = \int_{0}^{1} \exp(\alpha + \beta y)dy = \frac{\exp(\alpha+\beta) - \exp(\alpha)}{\beta},$$

where $\ell(x)$ is the density function of the log-DDs, and the coordinate transformation $y = \mathcal{L}(x)$ has been applied. The variance can be computed by starting with $\mathcal{Var}(\mathrm{EDF}) = \mathcal{E}(\mathrm{EDF}^2) - (\mathcal{E}(\mathrm{EDF}))^2$, and applying the same coordinate transformation, that results in

$$\mathcal{Var}(\mathrm{EDF}) = \frac{\exp(2\alpha+2\beta) - \exp(2\alpha)}{2\beta} - \Big(\frac{\exp(\alpha+\beta) - \exp(\alpha)}{\beta}\Big)^2.$$

In the case of the log-linear ETDF model, the upper and lower bounds for the EDF are $\exp(\alpha)$ and $(1+\beta)\exp(\alpha+\beta)$. The mean is equal to $\exp(\alpha+\beta)$, i.e., the upper bound of ETDF, and the variance is equal to

$$\mathcal{Var}(\mathrm{ETF}) = \frac{1}{2}(\beta - 1 + \frac{1}{2\beta})\exp(2(\alpha+\beta)) - \frac{1}{4\beta}\exp(2\alpha).$$

The latter can be checked by using the same coordinate transformation as in the case of the log-linear EDF model, and an integration by parts.

6 Conclusions

A review of the historical summaries published by major credit rating agencies indicates that, within each rating category, average default experience follows a log-linear pattern. This empirical regularity—also highlighted in Harju (2023)—motivates a closer examination of log-linear structures in credit risk analysis. The present study empirically investigates rating deterioration frequencies across the largest rating categories, leveraging the substantial amount of publicly accessible transition data.

The theoretical framework ranks firms within each rating category using the DD metric, and establishes a uniform ordering from highest to lowest credit quality. Within this setting, two hypotheses are examined: the log-EDF, or the log-ETDF are linearly related to the uniform rank. Based on goodness-of-fit metrics and visual inspection, the two models approximate the log-ETDF curves with impressive accuracy. Moreover, both specifications exhibit superior explanatory power relative to standard logit and probit models that use DD as the covariate, in both in-sample and out-of-sample tests. Finally, the log-linear EDF model demonstrates a modest performance advantage over the log-linear ETDF model.

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