Algorithmic Robotics
COMP/ELEC/MECH 450/550
Homework 3

DUE: November 14th at the beginning of class (1pm). Submit your answers as a PDF to Canvas.

Please read the honor code and the additions described in the course syllabus. Present your work and your work only. You must *explain* all of your answers. Answers without explanation will be given no credit.

1. **(30 points)** An assumption when using a PRM based approach for motion planning is that the configuration space is not *pathologically* difficult.
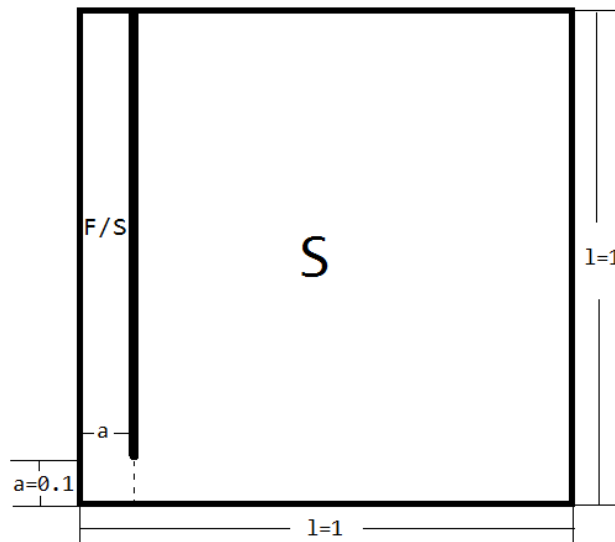

Figure 1: The Configuration Space for Problem 1

For the configuration space shown in Figure 1:

(a) **(5 points)** Estimate the ε-goodness of the space, i.e., what is the largest ε such that this space is ε-good?

(b) **(5 points)** Alice the world-renowned roboticist claims that the 0.4-lookout of S is a 0.6 fraction of S e.g., $\beta = 0.4, \alpha = 0.6$. Do you think this is true? Why or why not?

(c) **(10 points)** For a space with low $(\varepsilon, \alpha, \beta)$-expansiveness, what kind of sampling techniques would you expect to work well? Explain.

(d) **(10 points)** Imagine that you extrude this configuration space to 3 dimensions. Essentially, this is a 3D configuration space composed of stacked 2D slices, each slice being the configuration space shown in Figure 1. How will the $(\varepsilon, \alpha, \beta)$ values be affected by this change? Will they increase, decrease or remain unchanged? Explain.

2. **(15 points)** Figure 2 shows an indoor vacuum robot has a differential drive chassis with two wheels, each with a radius of r, that are separated by a distance L.
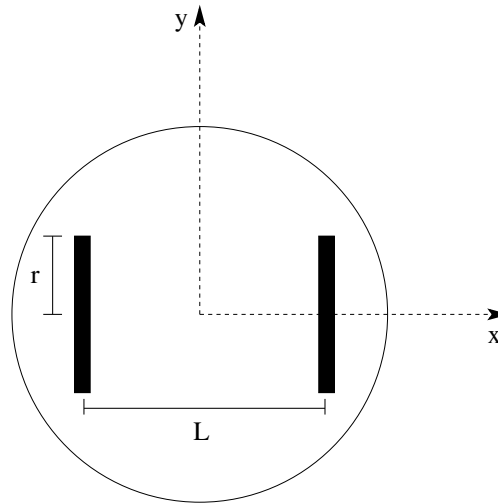

Figure 2: The Indoor Vacuum Robot

Each wheel is controlled independently with its own motor. Presume that you can specify the torque for each motor, and the motors can directly change the velocities (u_l and u_r) of your Roomba. Then the dynamics of your Roomba can be specified with the following differential equations:

$$\dot{x} = \frac{r}{2}(u_l + u_r)\cos\theta$$
$$\dot{y} = \frac{r}{2}(u_l + u_r)\sin\theta$$
$$\dot{\theta} = \frac{r}{L}(u_r - u_l)$$

   (a) **(5 points)** What is the configuration space of the robot?
   (b) **(5 points)** What is the control space of the robot?
   (c) **(5 points)** Is this a non-holonomic or a holonomic robot?

3. **(15 points)** Recall the visibility graph method. Similar to the PRM, the visibility graph also captures the continuous space using a graph structure. Compare these two methods. For each method, provide at least one scenario in which it would work well while the other would not. Justify your answer.

4. **(15 points)** Explain why the Brushfire Algorithm does not scale to higher dimensions.

5. **(25 points)** Bob the world-famous roboticist is tasked with designing the motion planning system for a mobile manipulator with a 3D workspace that consists of a holonomic base that can move freely in the plane and a 6 degree-of-freedom arm. Wanting to impress his boss, Bob decides to use a different planner for the base and the arm. He has a trapezoidal decomposition planner and an RRT planner available. Which planner would you recommend to Bob for each of the components (the base and the arm)? Explain your answer.