Homework 2

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COMP550

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**1. Prove thatand regular matrix multiplication satisfy the properties listed above.**

**(a) Closure:**

For set *T*, we have and

We know thatis rotation matrices so that it from a group, alsois a rotated translation vector and plus another vector, by the fact that translation vectors and vector addition from a group we know thatis from a group. Therefore, we have from a group. Since all set *T* satisfy the closure property, the multiplication of set *T* and regular matrix also satisfy the closure property.

**(b) Associativity:**

Suppose we have: , and

On the other hand:

As we can see that . So that the set *T* satisfies the associativity property.

**(c) Identity:**

*I* is an identity matrix in *T*.

Therefore

and

Thus we have

So the set *T* has the identity property.

**(d) Inverse:**

For, and

In order to have, we need to show that

and

For :

Thus we have

So that if , we have

Also if we could let , then we have

Then

Thus, we can find a inverse matrixforin Set *T*

**2.**

**(a) What is a rotation of radians about the axis [0 0 1]*T* as a unit quaternion, *q1* ?**

Suppose we have a unit quaternion, *q1* = (cos(θ), sin(θ)N).

We have quaternion rotation equation:

Since the rotation is about axis [0 0 1]*T* , we could write *q1* as:

**(b) Given the quaternion , what is ?**

We have and

So that:

Therefore, is:

**3. Determine the topology and dimension of the manipulator’s configuration space.**

**(a) A manipulator with two prismatic points.**

Two prismatic joints connect two rigid bodies. The movement of two bodies is defined by the linear sliding movement of two prismatic points. Thus, we could use the two prismatic joints to describe the movement of the manipulator. So that the DoF of this manipulator is 2, and the topology of the manipulator is .

**(b) A manipulator with three revolute joints.**

The manipulator has three revolute joints. The configuration space should be the semi-circle of that three links can cover. Three angles, and can specify the configuration of this manipulator. Thus, the dimension of configuration space is 3. The topology should be .

**(c) A manipulator with two revolute joints and a prismatic joint.**

The manipulator has 2 revolute joints and 1 prismatic joint. The configuration space is the semi-circle that all links could cover. The revolute joint and prismatic joint is considered as 1 dimension each. Thus, we have 3 dimension of configuration space for this manipulator. The topology should be .

**4. Figure shows a three-link kinematic chain in 2D. The lengths of link and are and , respectively. The joint angles of the chain are and .**

**(a) Determine the topology and dimension of the configuration space for this manipulator.**

Since the joint angles of the chain are and , there only 2 parameters to specify the configuration of the object. The dimension of the configuration space is 2, and the topology is .

**(b) Determine the Homogeneous coordinates of the point 2 in the local frame of link .**

In the local frame of link , the homogeneous coordinate should be:

**(c) Determine the forward kinematic of this three-link chain. That is, calculate the homogenous coordinates of the point 2 in the local frame of link .**

We know that the homogenous transformation for 2D chains is:

So that we could have the homogenous coordinates

**(d) Determine the homogeneous transformation from the local frame of to the local frame of .**

We could have the transformation matrix

We use the same way to get the transformation matrix :

Then movesfrom its local frame to the local frame of.

**(e) Show that.**

So far we know that:

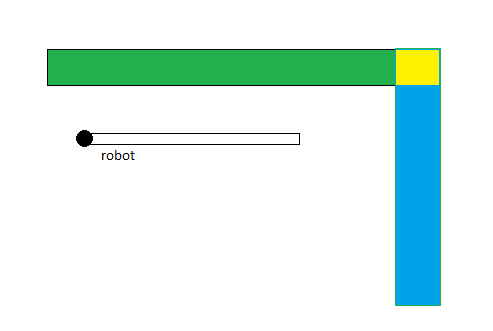
and

Therefore, equals:

**5.**

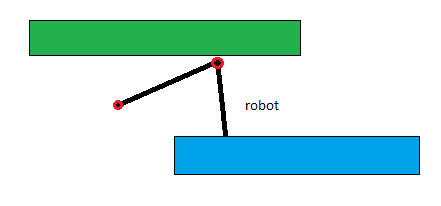
**(a) Consider workspace obstacles A and B. If, do the configuration space obstacles QA and QB always overlap?**

No, we know that if two configuration space obstacles are overlap, then the robots can reach to both obstacles at the same time.



Imagine a workspace like above image. We have obstacle A in green and obstacle B in blue. The yellow part is the overlap of two obstacles. For the robot with one revolute joint, it will reach obstacle A in some angle. But it will never reach obstacle B and obstacle A at the same time. So that the configuration space obstacles QA and QB are not overlap under this situation.

**(b) If, is it possible for the configuration space obstacles QA and QB overlap?**



Now two obstacles are not overlap. Consider the above image, obstacle A in green and obstacle B in blue. We have a 2 dimension robot with two red revolute joints. Under some certain angle, we could have this robot to reach both obstacles even if two obstacles are not overlap. Thus, the configuration space obstacles QA and QB are overlap.

**6. Suppose five polyhedral bodies float freely in 3D world. They are each capable of rotating and translating. If these are treated as “one” composite robot, what is the topology of the resulting configuration space (assume that the bodies are not attached to each other)? What is the dimension of the composite configuration space?**

Since each polyhedral body floats freely in 3D world and able to rotating and translating. So we need three dimension to describe the rotation of the polyhedral body, (), and three dimension to describe the translating, (*x, y, z*). Therefore, the dimension of configuration space for each polyhedral body should be 6. Since bodies are not attached to each other, then there is no constrains between polyhedral bodies. Thus, the dimension of composite configuration space for total 5 polyhedral bodies should be . The topology of the resulting configuration space should be