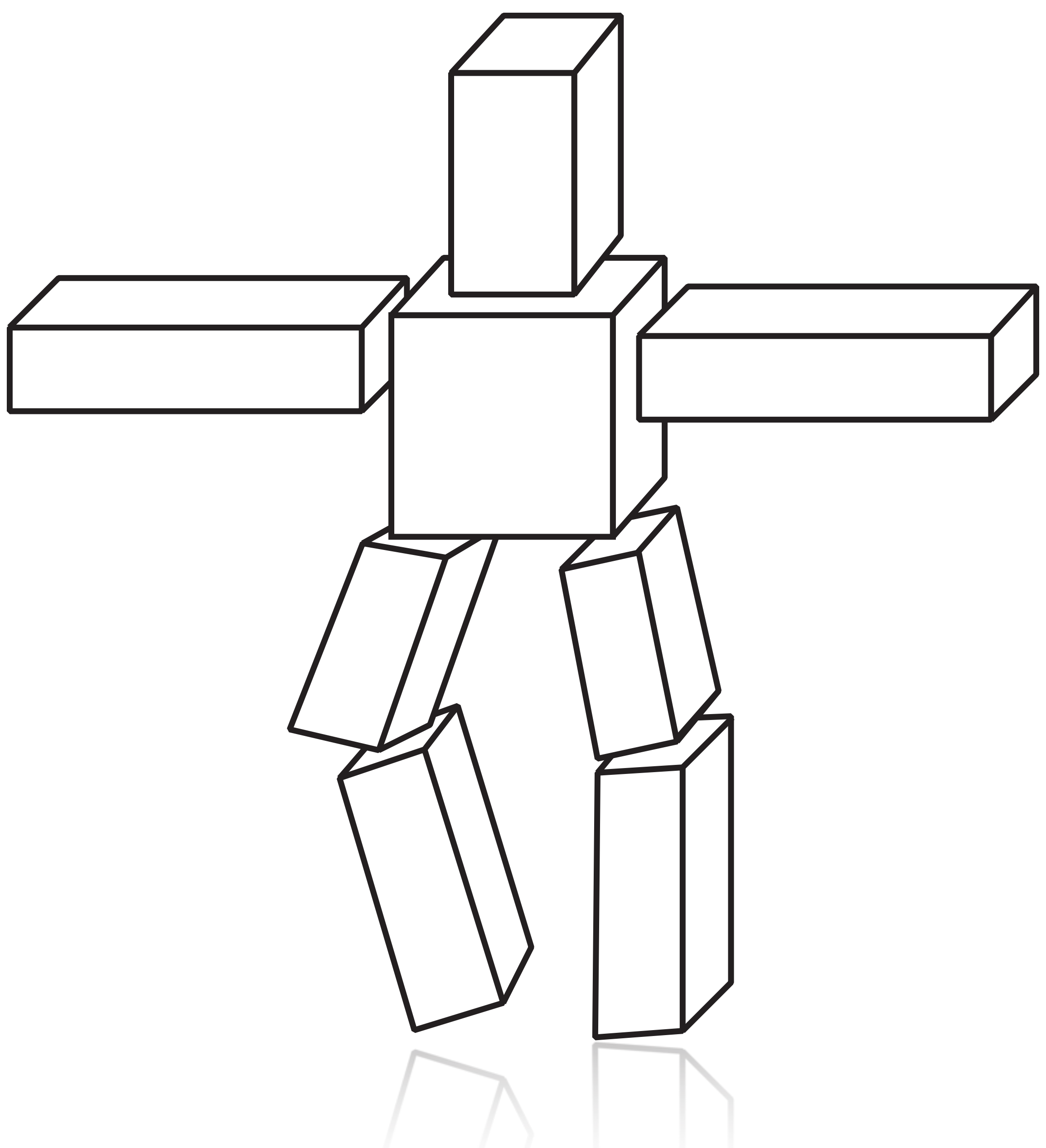
Introduction to Animation

Computer Graphics
CMU 15-462/15-662

Increasing the complexity of our models

Transformations



Geometry

Materials, lighting, ...

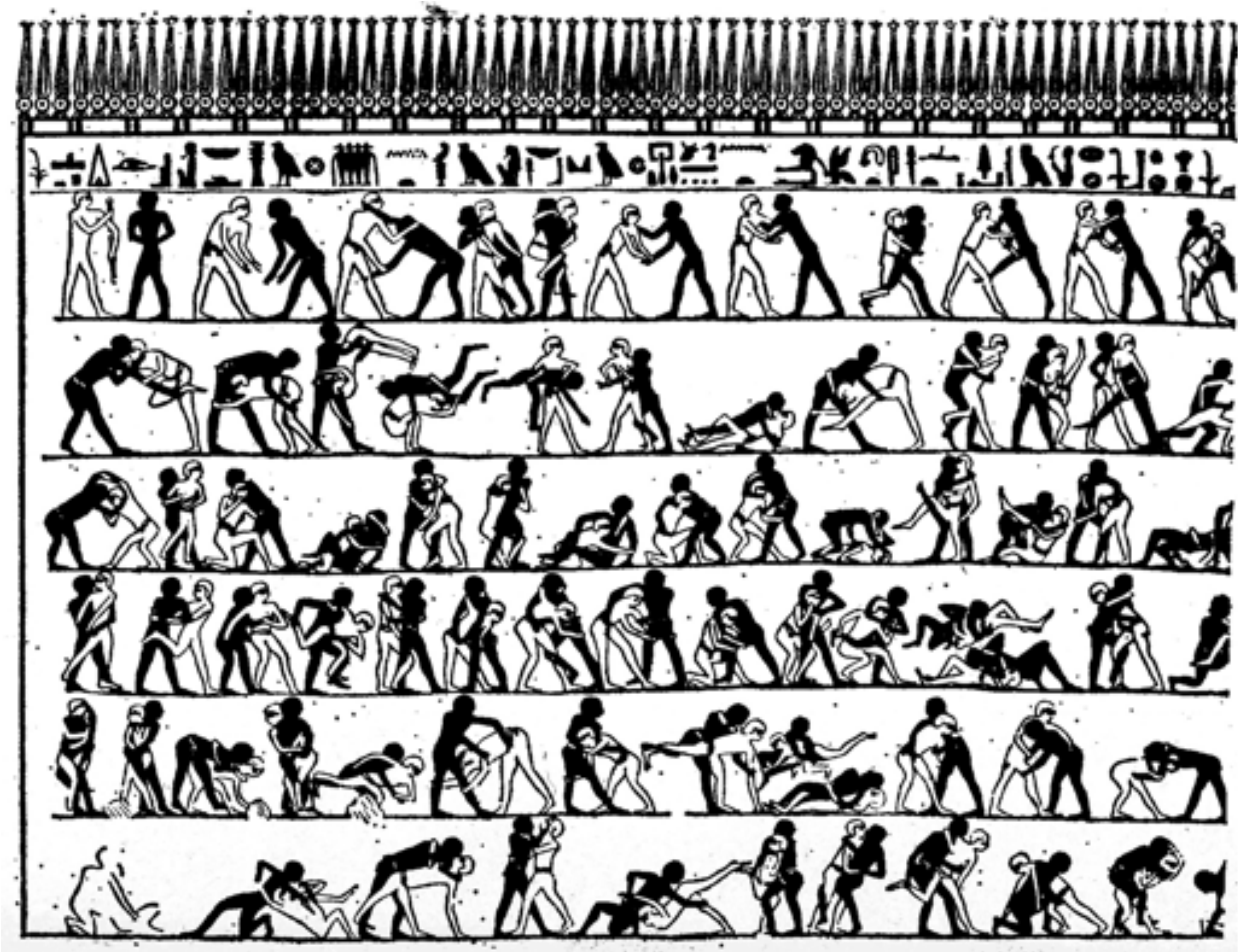
Increasing the complexity of our models

...but what about motion?

First Animation

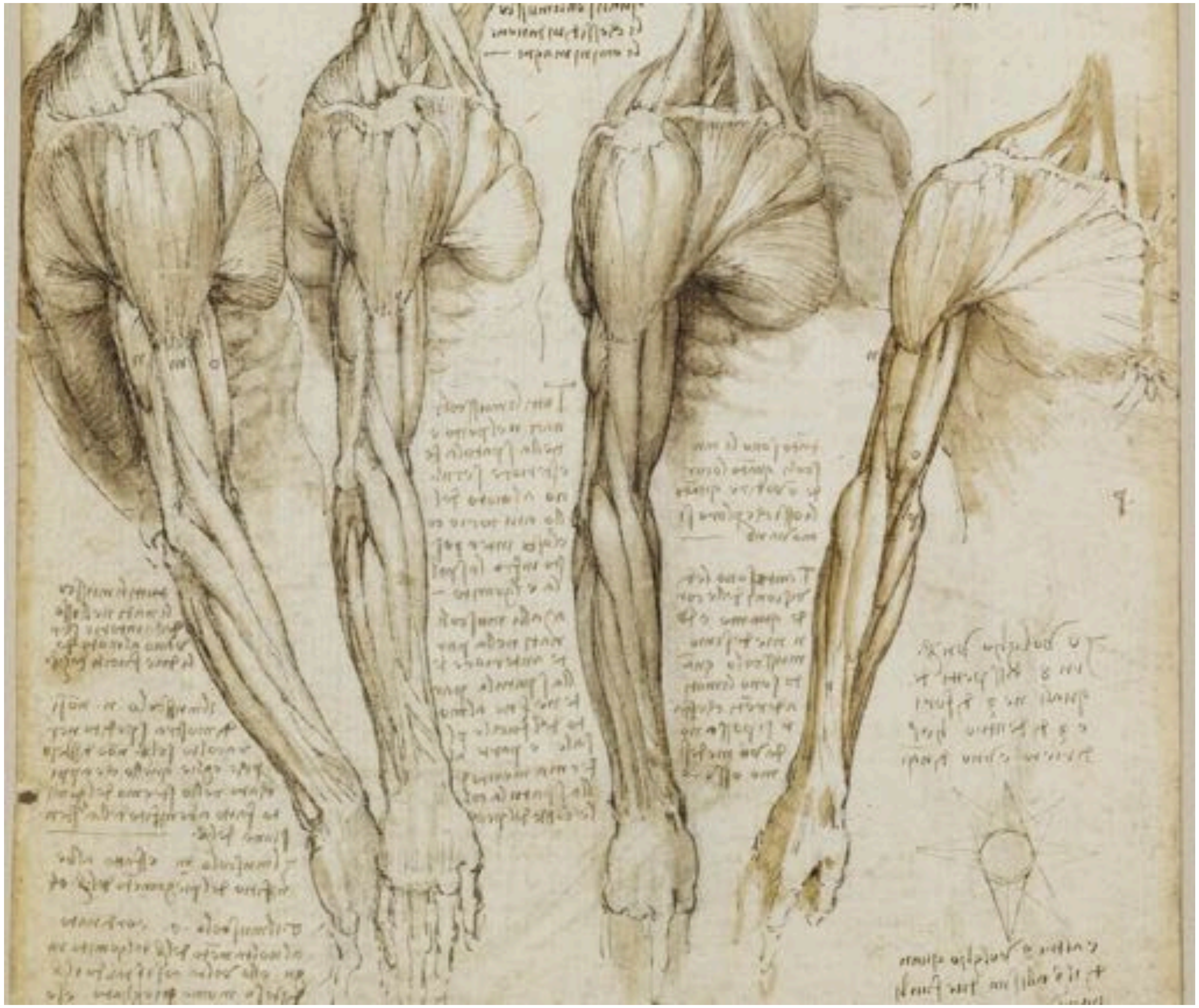
(Shahr-e Sukhteh, Iran 3200 BCE)

History of Animation



(tomb of Khnumhotep, Egypt 2400 BCE)

History of Animation



Leonardo da Vinci (1510)

History of Animation

Claude Monet, “Woman with a Parasol” (1875)

History of Animation

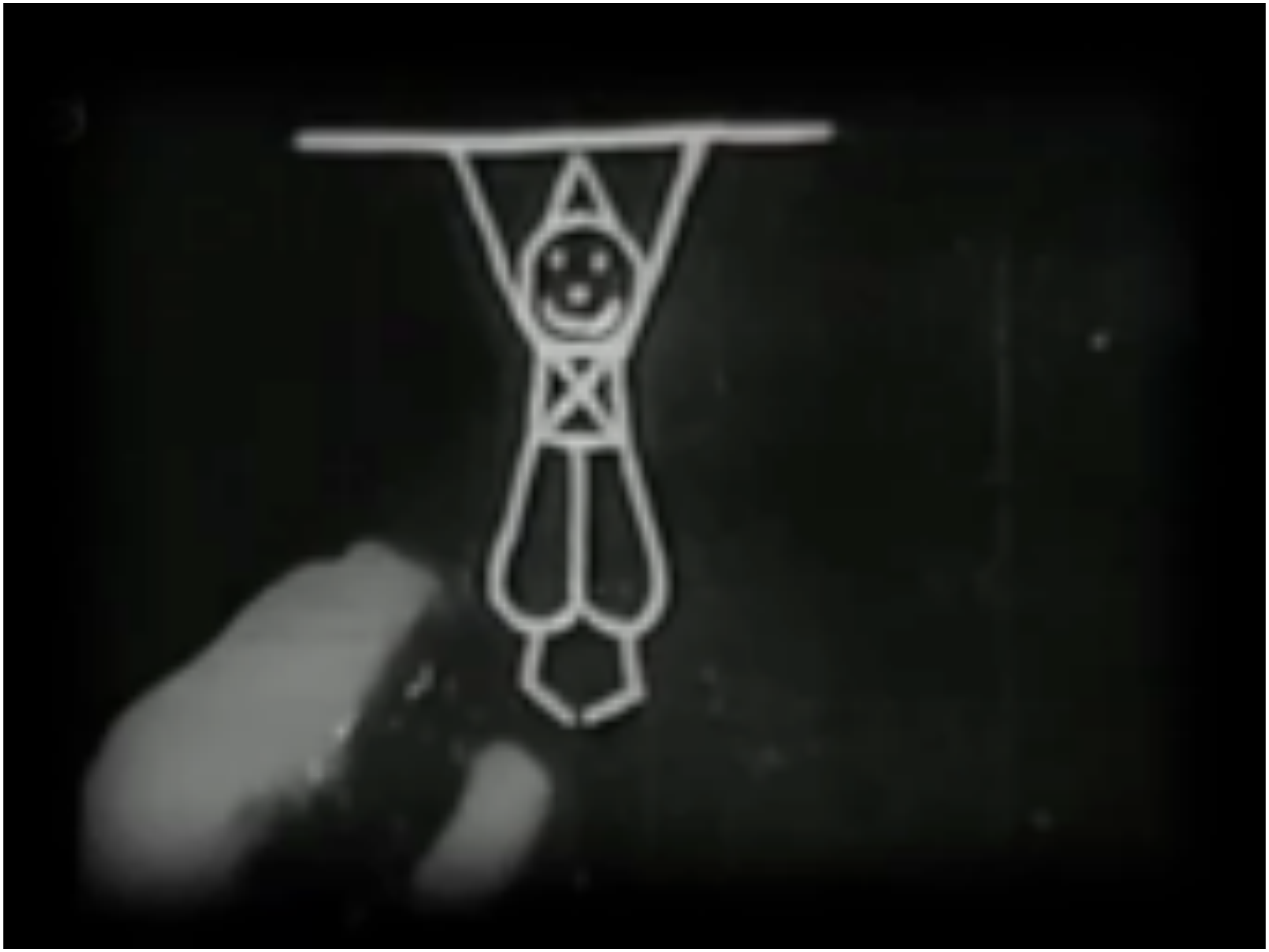
(Phenakistoscope, 1831)

First Film

- Originally used as scientific tool rather than for entertainment
- Critical technology that accelerated development of animation

Eadweard Muybridge, "Sallie Gardner" (1878)

First Animation on Film



Emile Cohl, “Fantasmagorie” (1908)

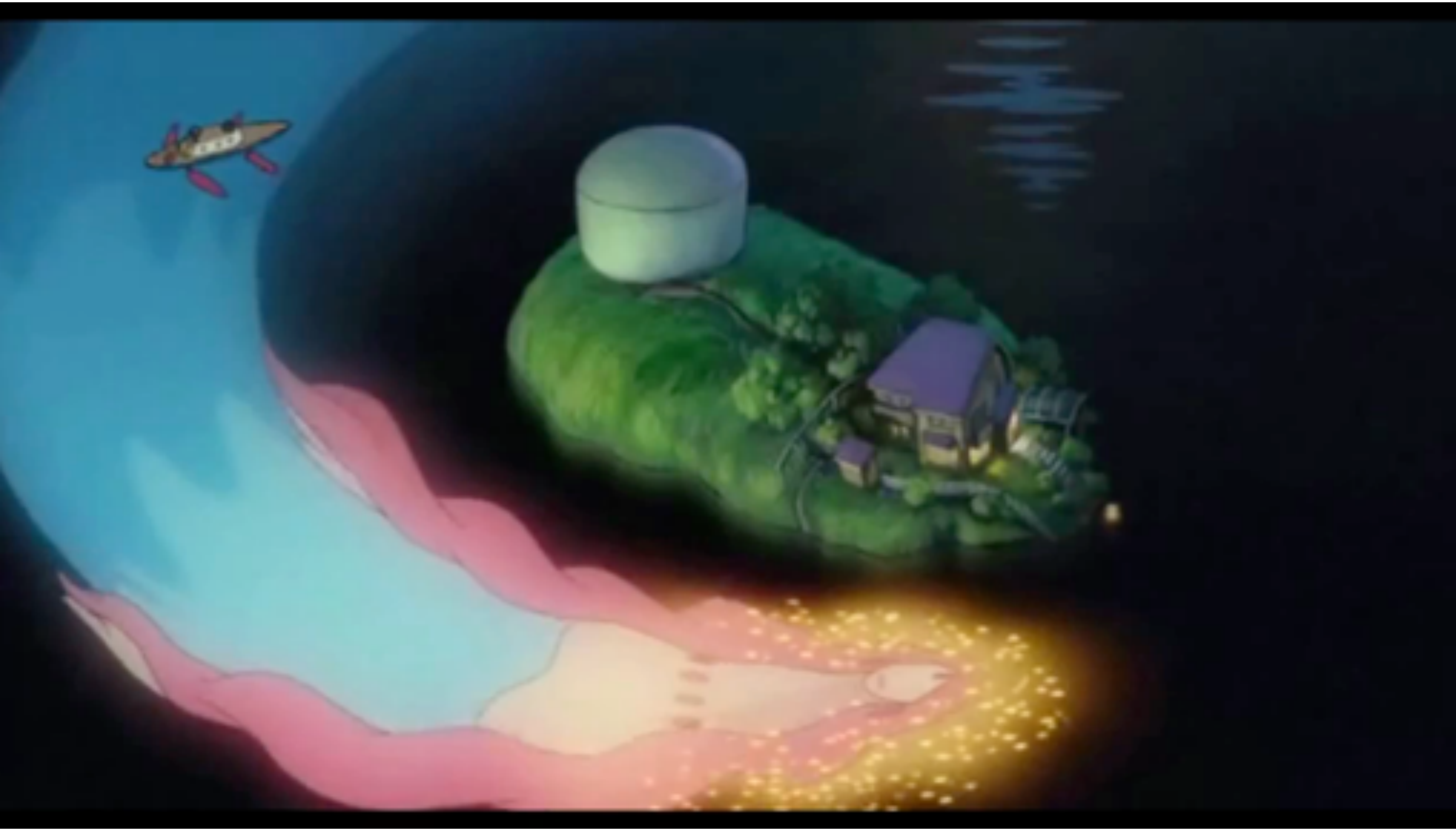
First Feature-Length Animation

Lotte Reiniger, “Die Abenteuer des Prinzen Achmed” (1926)

First Hand-Drawn Feature-Length Animation

Disney, “Snow White and the Seven Dwarves” (1937)

Hand-Drawn Animation - Present Day



Studio Ghibli, “Ponyo” (2008)

First Computer-Generated Animation

- New technology, also developed as a scientific tool
- Again turbo-charged the development of animation

John Whitney, "Catalog" (1961)

First Digital-Computer-Generated Animation

Ivan Sutherland, “Sketchpad” (1963)

First 3D Computer Animation

William Fetter, "Boeing Man" (1964)

Early Computer Animation

Nikolay Konstantinov, “Kitty” (1968)

Early Computer Animation

Ed Catmull & Fred Park, “Computer Animated Faces” (1972)

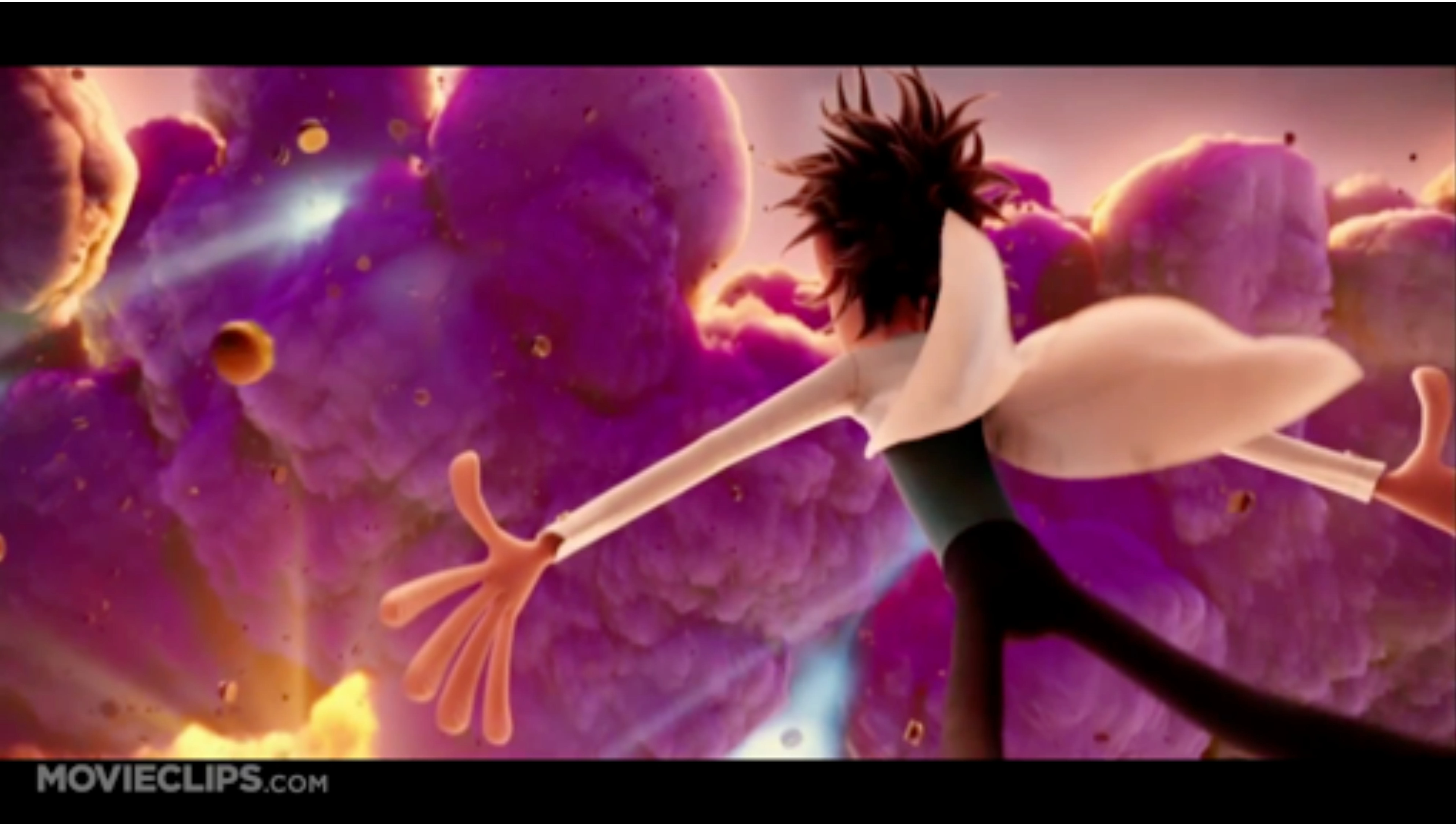
First Attempted CG Feature Film

NYIT [Williams, Heckbert, Catmull, ...], "The Works" (1984)

First CG Feature Film

Pixar, “Toy Story” (1995)

Computer Animation - Present Day



Sony Pictures Animation, “Cloudy With a Chance of Meatballs” (2009)

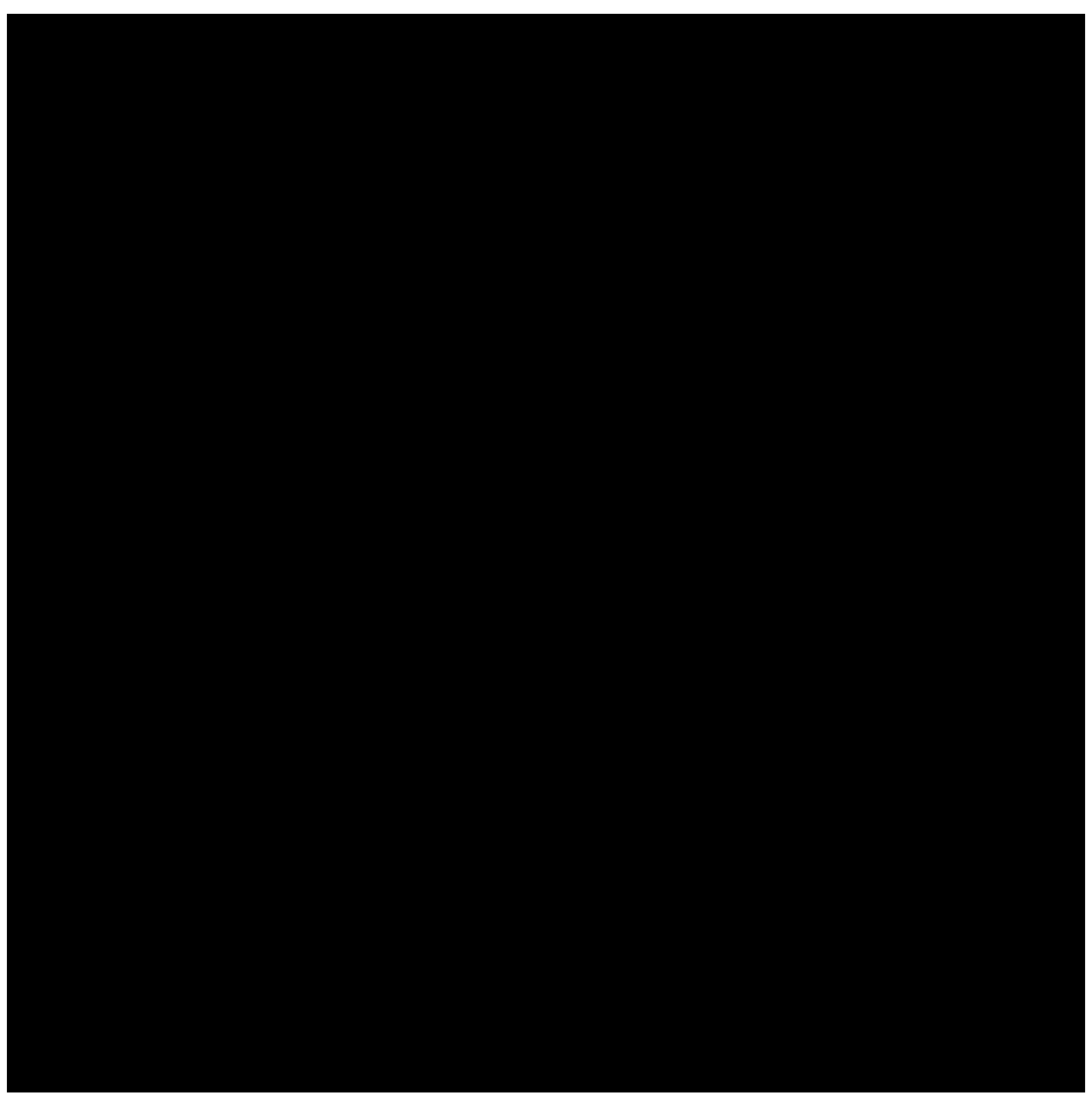
Zoetrope - Solid Animation

Zoetrope - 3D Printed Animation

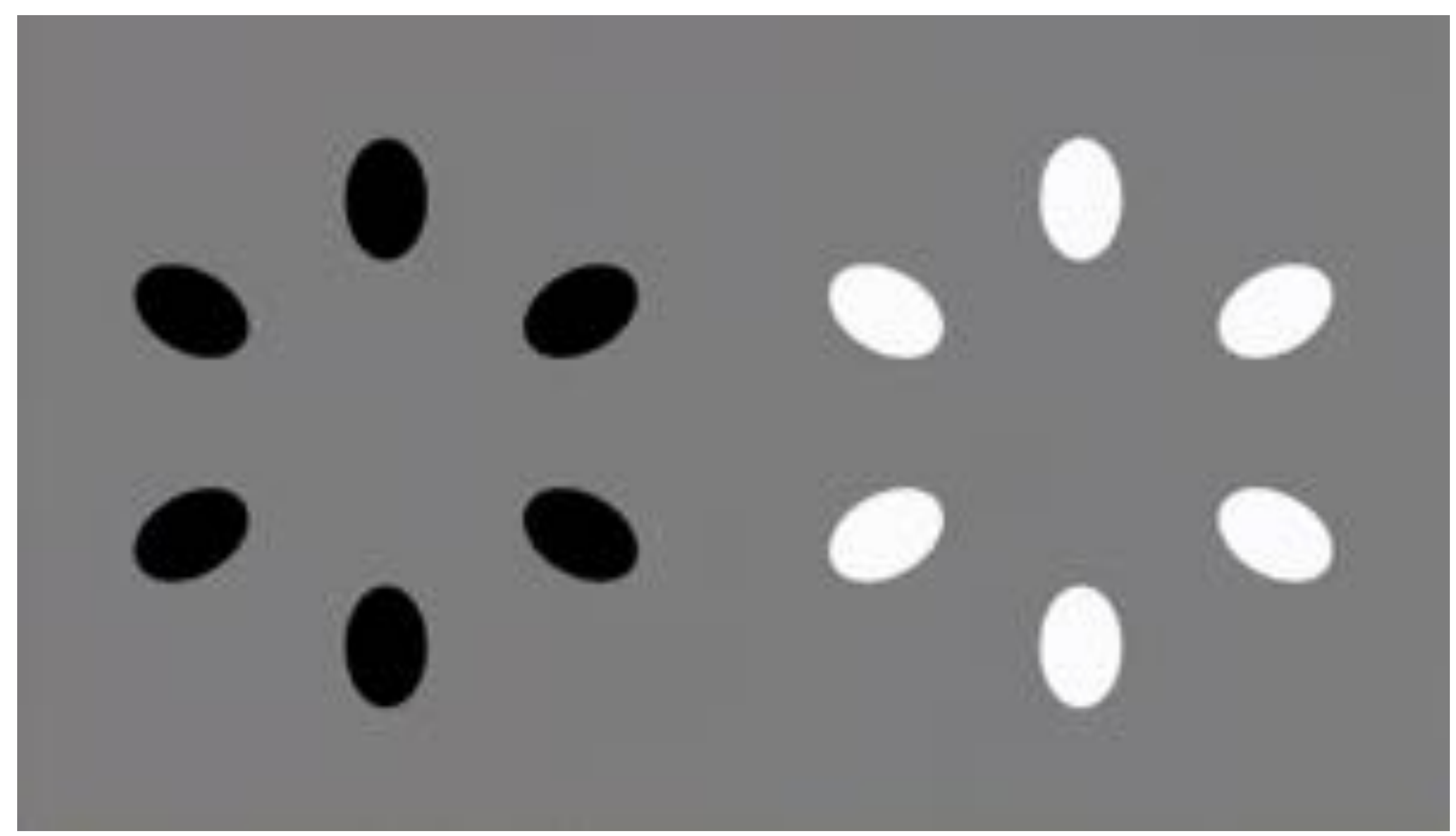
John Edmark — BLOOMS

Perception of Motion

- Original (but debunked) theory: persistence of vision ("streaking")
- The eye is not a camera! More modern explanation:
  - beta phenomenon: visual memory in brain—not eyeball
  - phi phenomenon: brain anticipates, giving sense of motion



beta

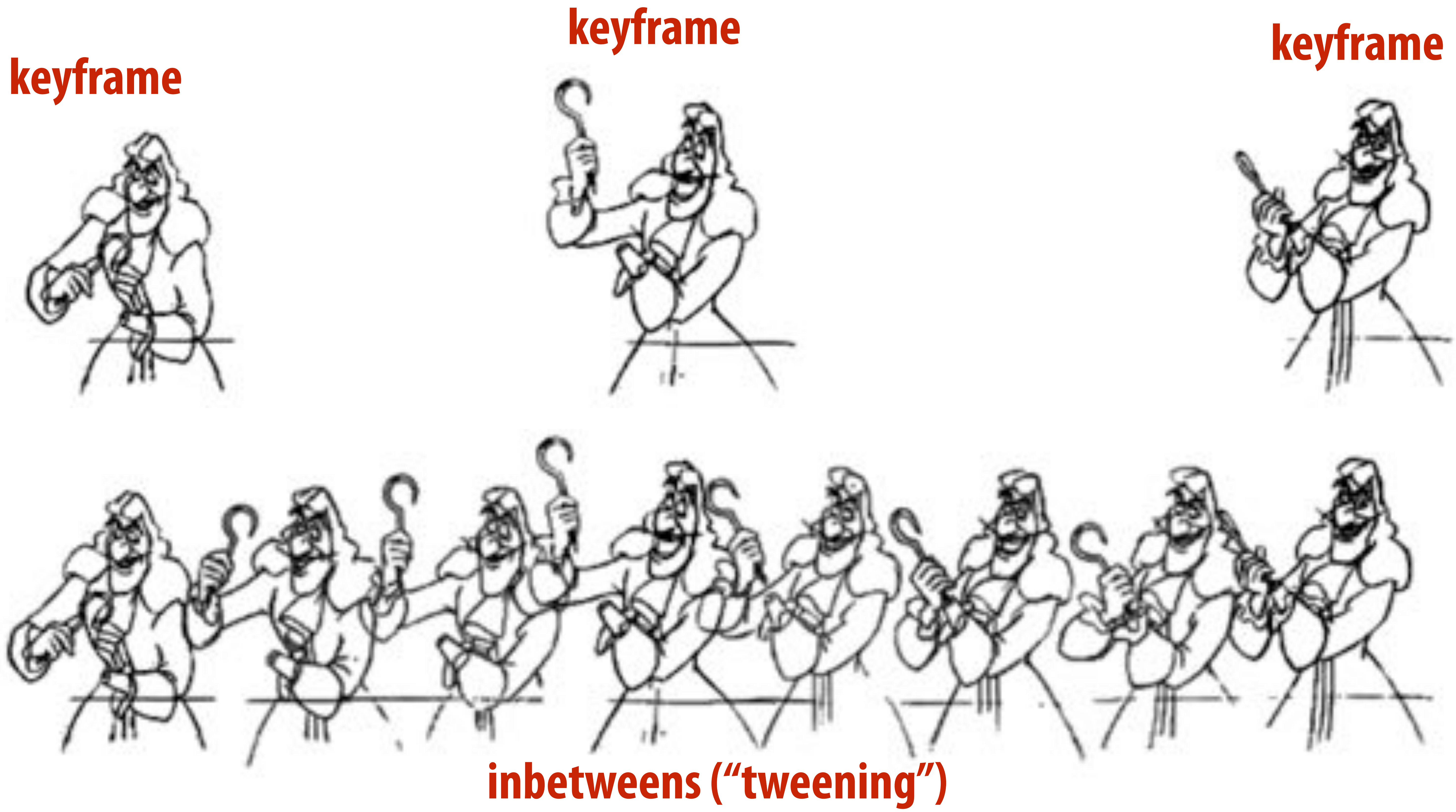


credit: Akiyoshi Kitaoka

phi

Generating Motion (Hand-Drawn)

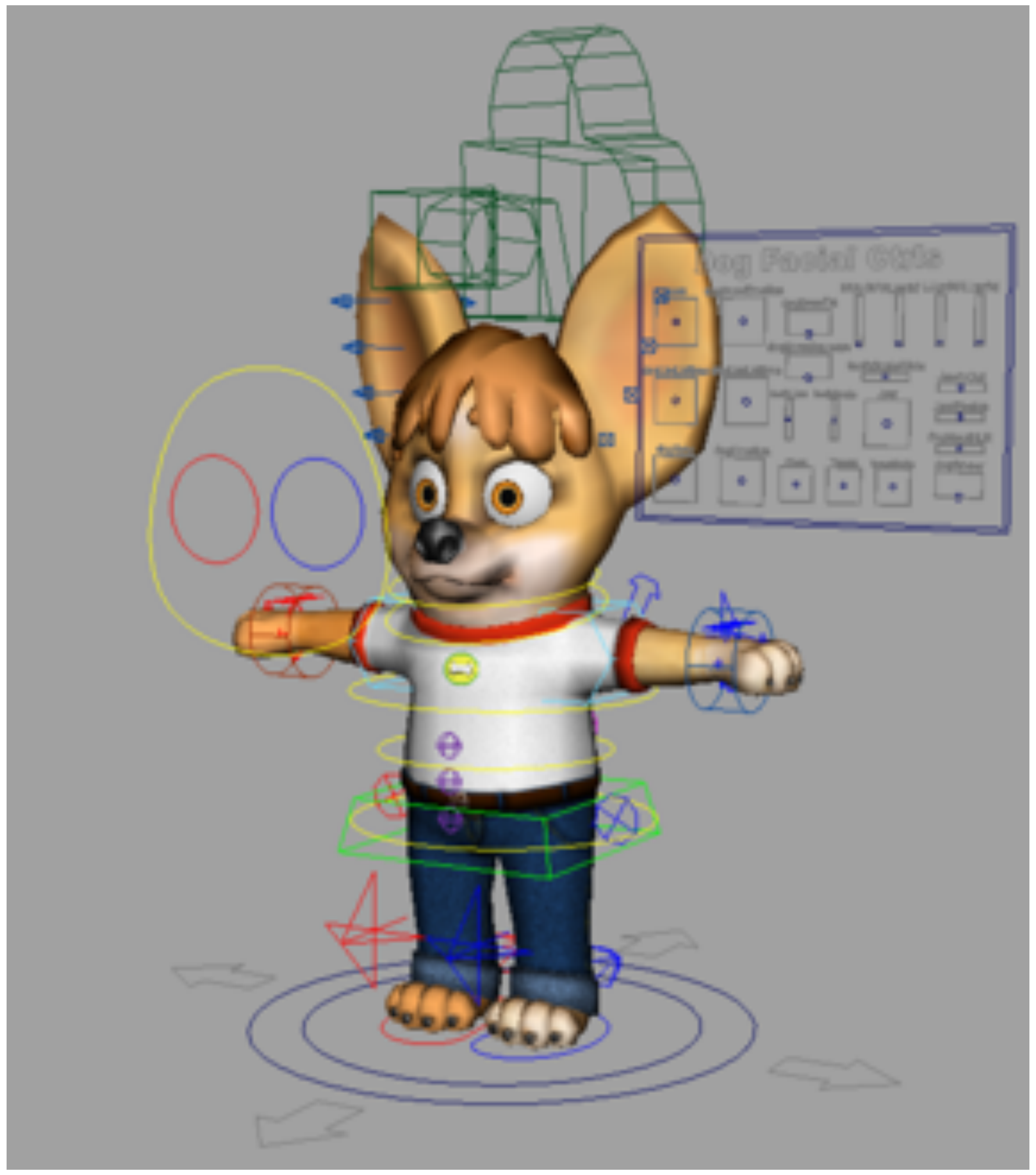
- Senior artist draws keyframes
- Apprentice draws inbetweens
- Tedious / labor intensive (opportunity for technology!)

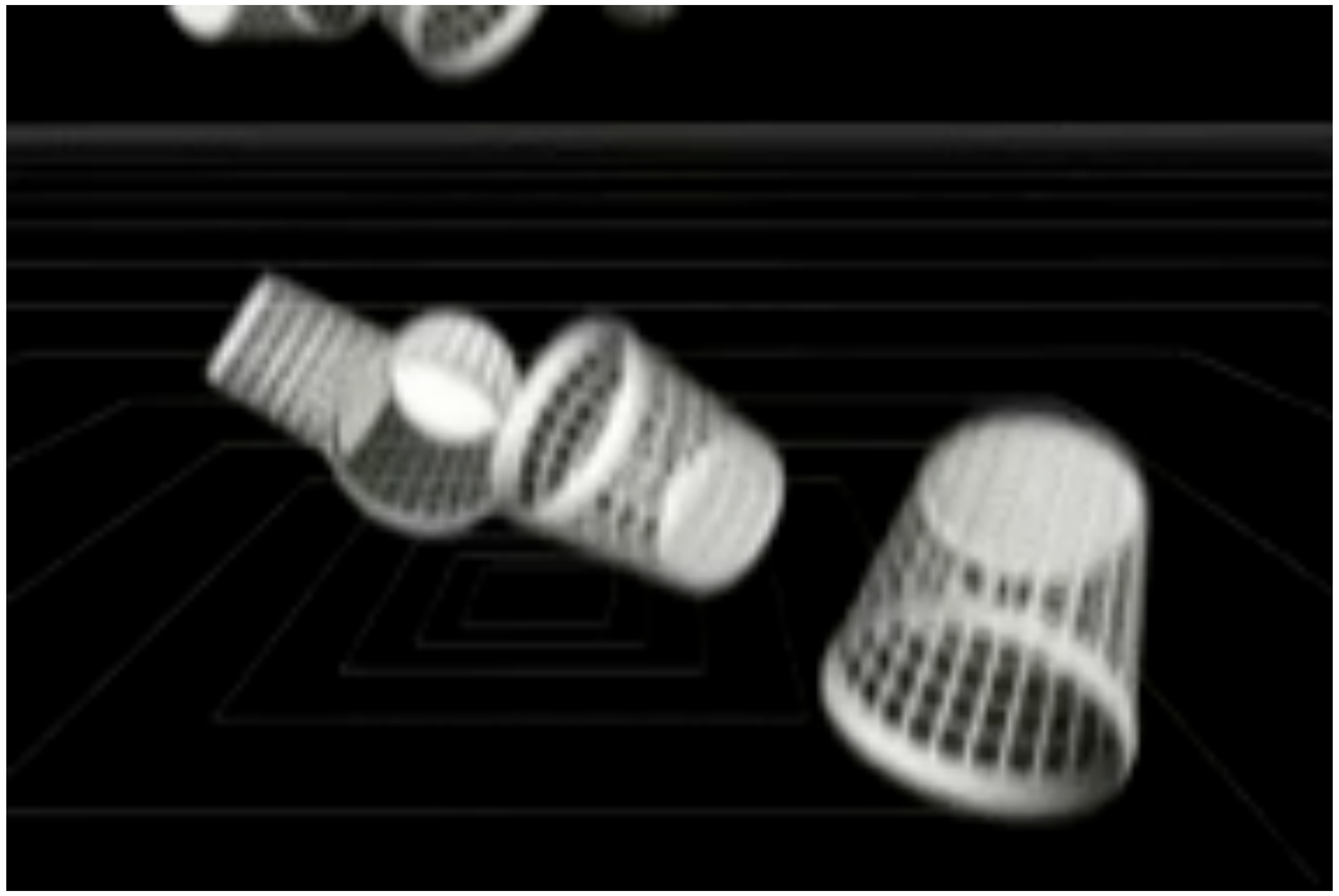


How do we describe motion on a computer?

Basic Techniques in Computer Animation

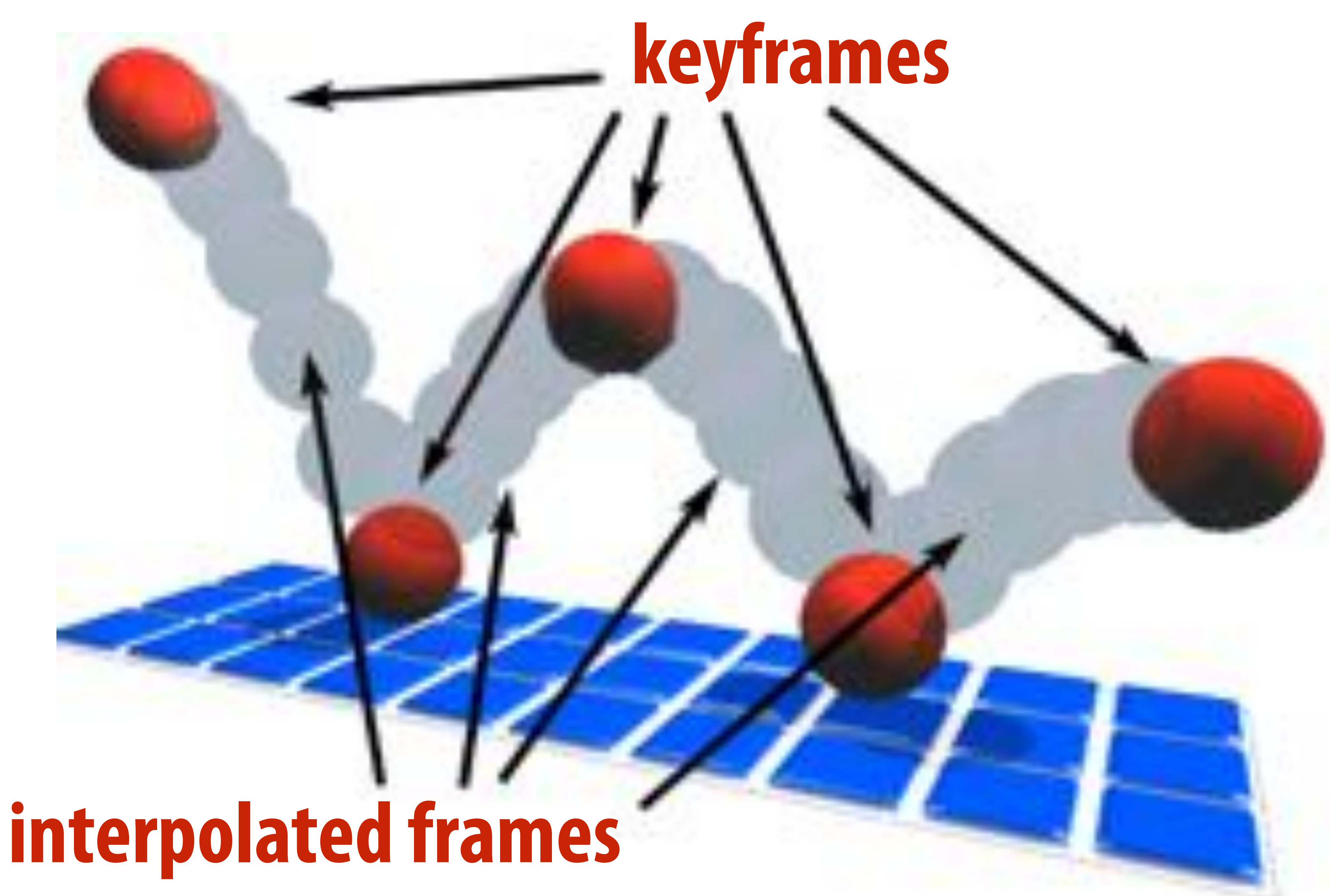
- Artist-directed (e.g., keyframing)
- Data-driven (e.g., motion capture)
- Procedural (e.g., simulation)





Keyframing

- Basic idea:
  - specify important events only
  - computer fills in the rest via interpolation/approximation
- "Events" don't have to be position
- Could be color, light intensity, camera zoom, ...



How do you interpolate data?

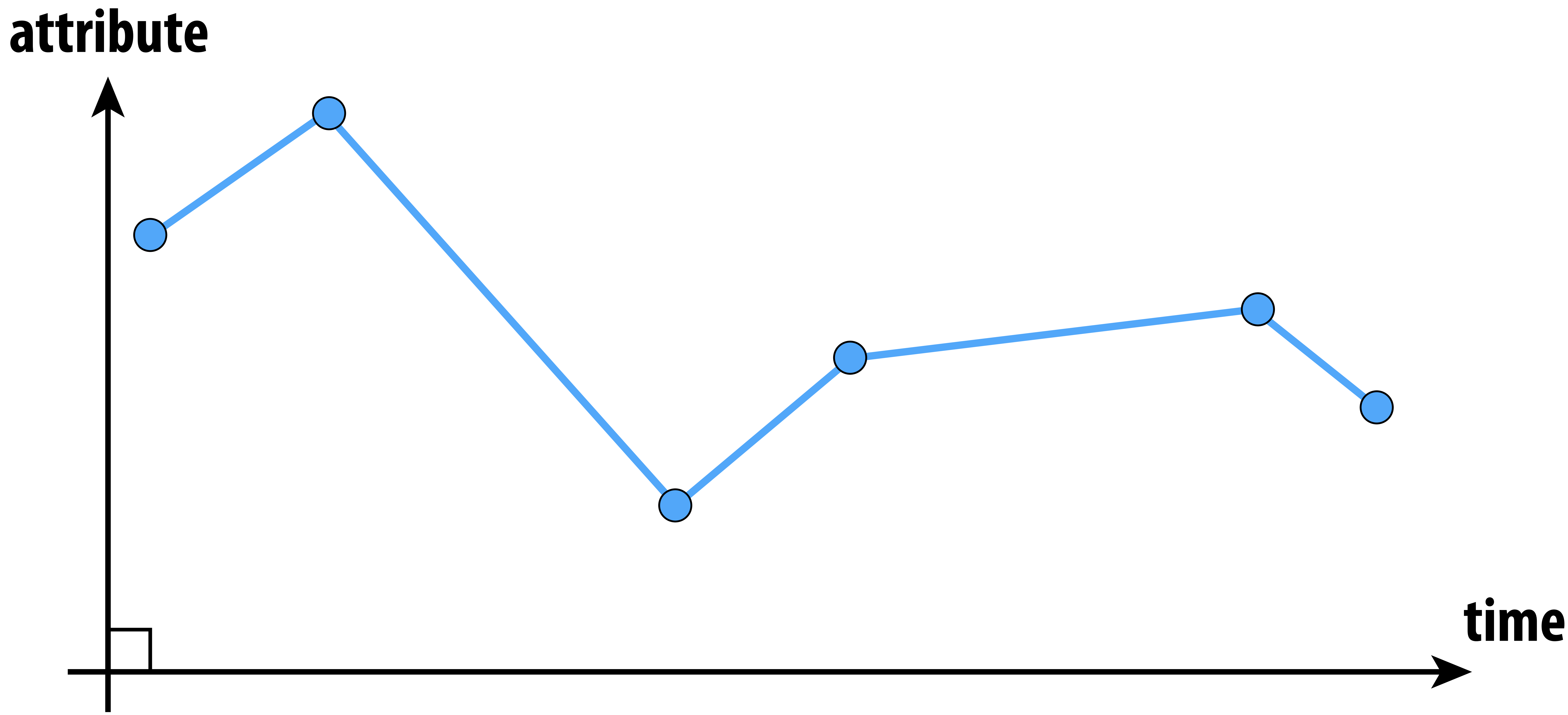
Spline Interpolation

- **Mathematical theory of interpolation arose from study of thin strips of wood or metal ("splines") under various forces**

(Good summary in Levin, "The Elastica: A Mathematical History")

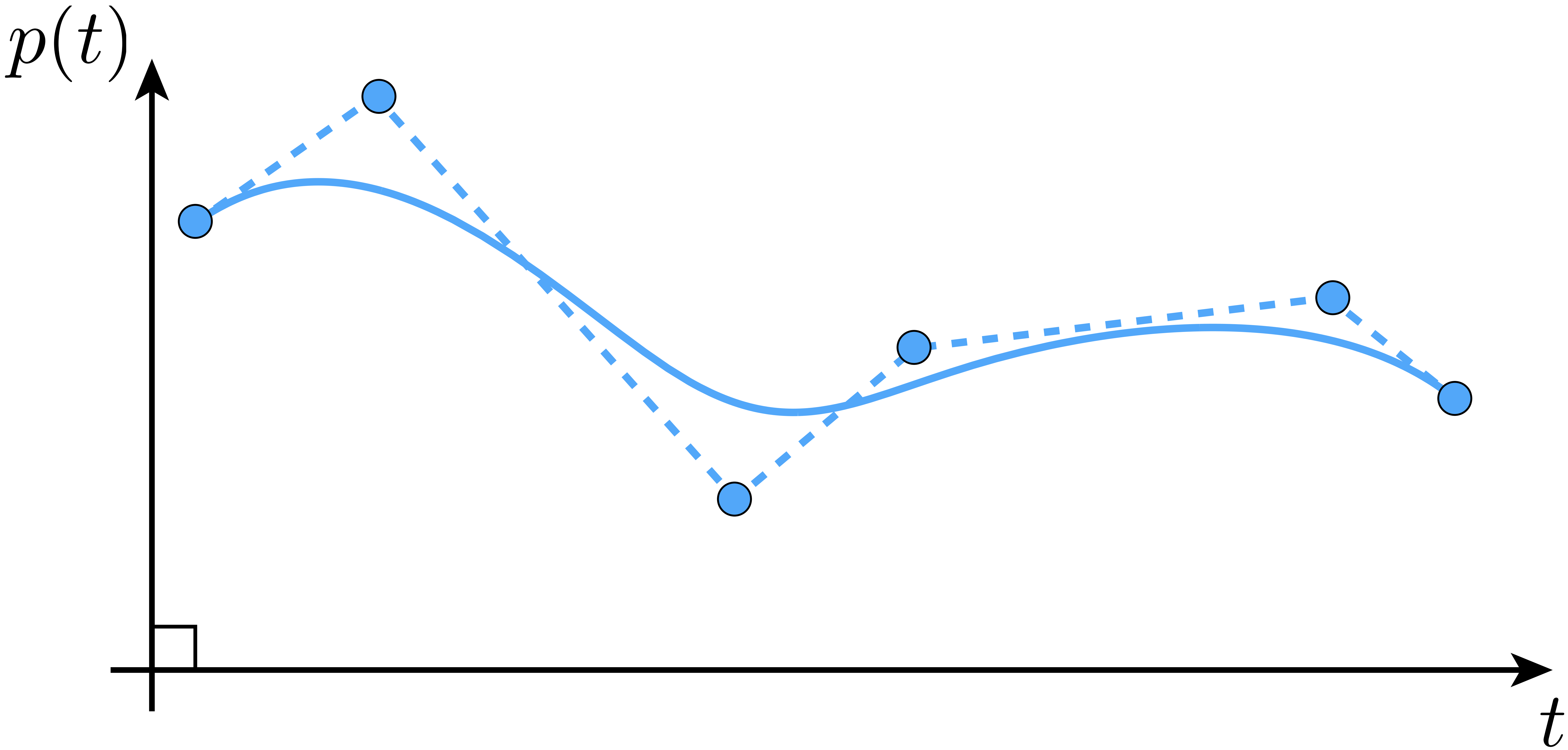
Interpolation

- Basic idea: “connect the dots”
- E.g., piecewise linear interpolation
- Simple, but yields rather rough motion (infinite acceleration)



Piecewise Polynomial Interpolation

- **Common interpolant: piecewise polynomial "spline"**



Basic motivation: get better continuity than piecewise linear!

Splines

- In general, a spline is any piecewise polynomial function
- In 1D, spline interpolates data over the real line:

$$(t_i, f_i), \quad i = 0, \ldots, n$$

"knots"

values

$$t_i < t_{i+1}$$

- "Interpolates" just means that the function exactly passes through those values:

$$f(t_i) = f_i \quad \forall i$$

- The only other condition is that the function is a polynomial when restricted to any interval between knots:

polynomial

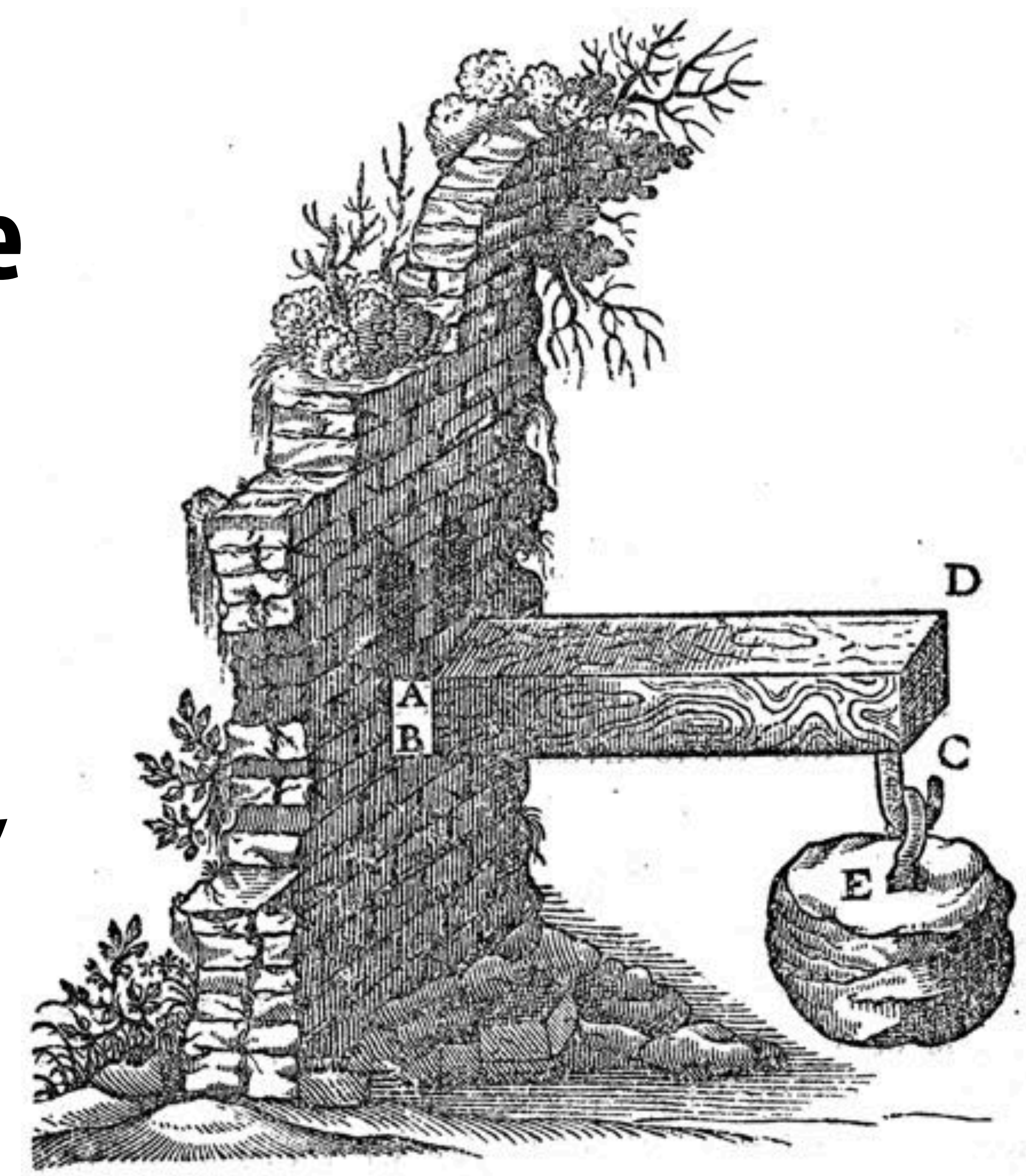
degree

$$\text{for } t_i \leq t \leq t_{i+1},\, f(t) = \sum_{j=1}^{d} c_i t^j =: p_i(t)$$

coefficients

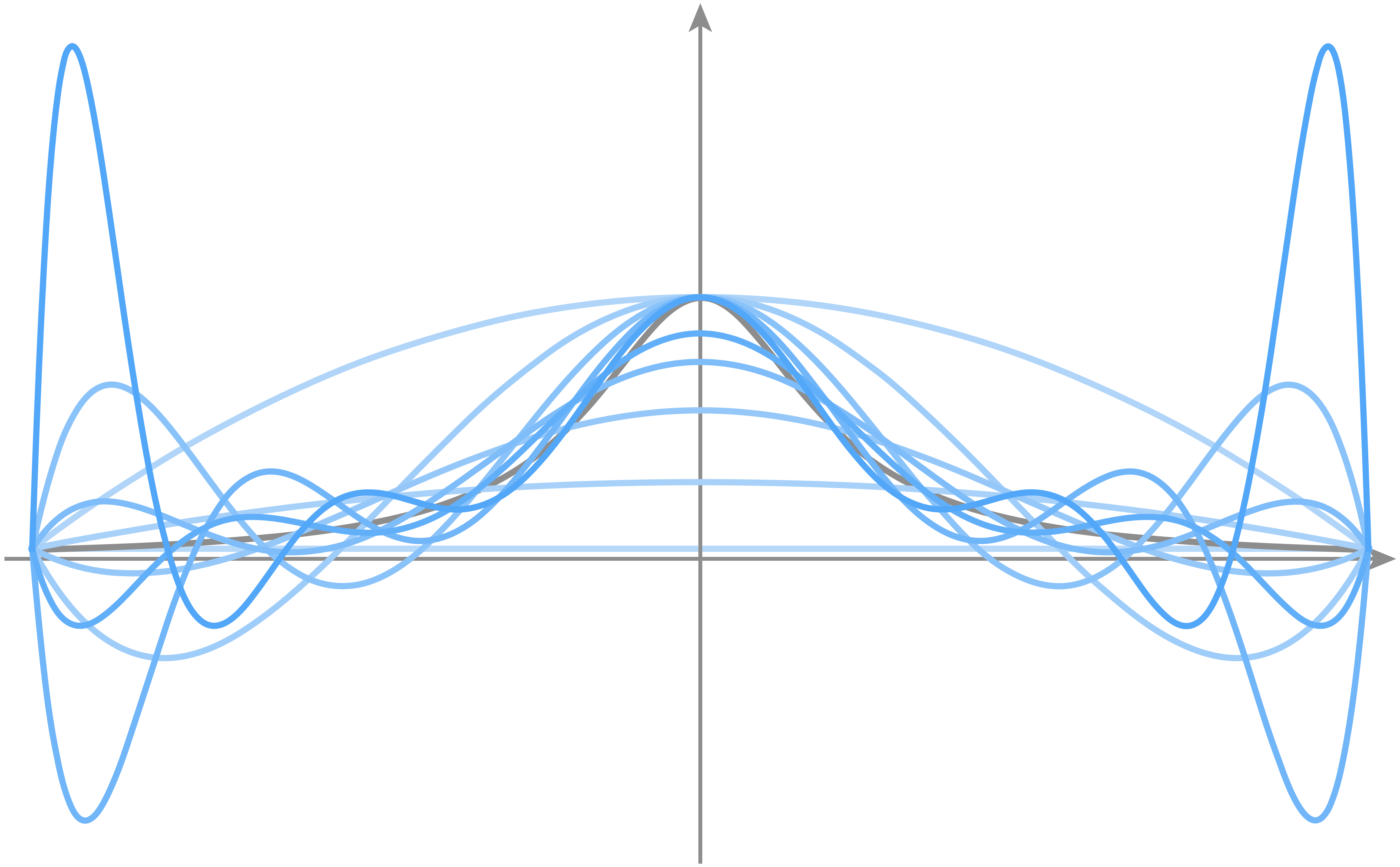
What's so special about cubic polynomials?

- Splines most commonly used for interpolation are cubic (d=3)
- Piecewise cubics give exact solution to elastic spline problem under assumption of small displacements
- More precisely: among all curves interpolating set of data points, minimizes norm of second derivative (not curvature)
- Food for thought: who cares about physical splines? We're on a computer! And are interpolating phenomena in time
- Motivation is perhaps pragmatic: e.g., simple closed form, decent continuity
- Plenty of good reasons to choose alternatives (e.g., NURBS for exact conics, clothoid to prevent jerky motion, ...)
- Also...



Runge Phenomenon

- Tempting to use higher-degree polynomials, in order to get higher-order continuity
- Can lead to oscillation, ultimately worse approximation:

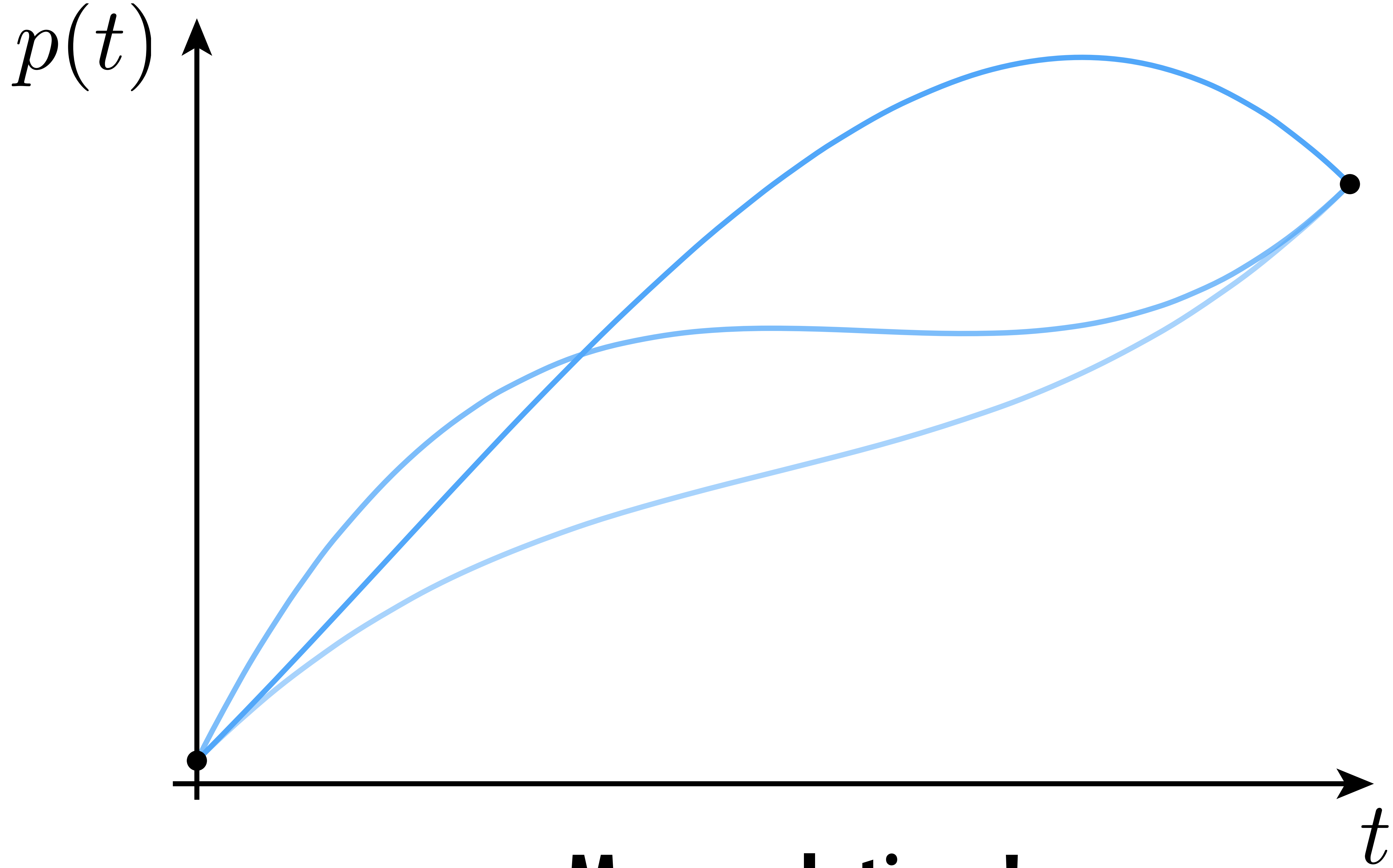


Fitting a Cubic Polynomial to Endpoints

- **Consider a single cubic polynomial**

$$p(t) = at^3 + bt^2 + ct + d$$

- **Suppose we want it to match given endpoints:**



Many solutions!

Cubic Polynomial - Degrees of Freedom

- **Why are there so many different solutions?**
- **Cubic polynomial has four degrees of freedom (DOFs), namely four coefficients (a,b,c,d) that we can manipulate/control**
- **Only need two degrees of freedom to specify endpoints:**

$$p(t) = at^3 + bt^2 + ct + d$$

$$p(0) = p_0 \qquad \Rightarrow d = p_0$$

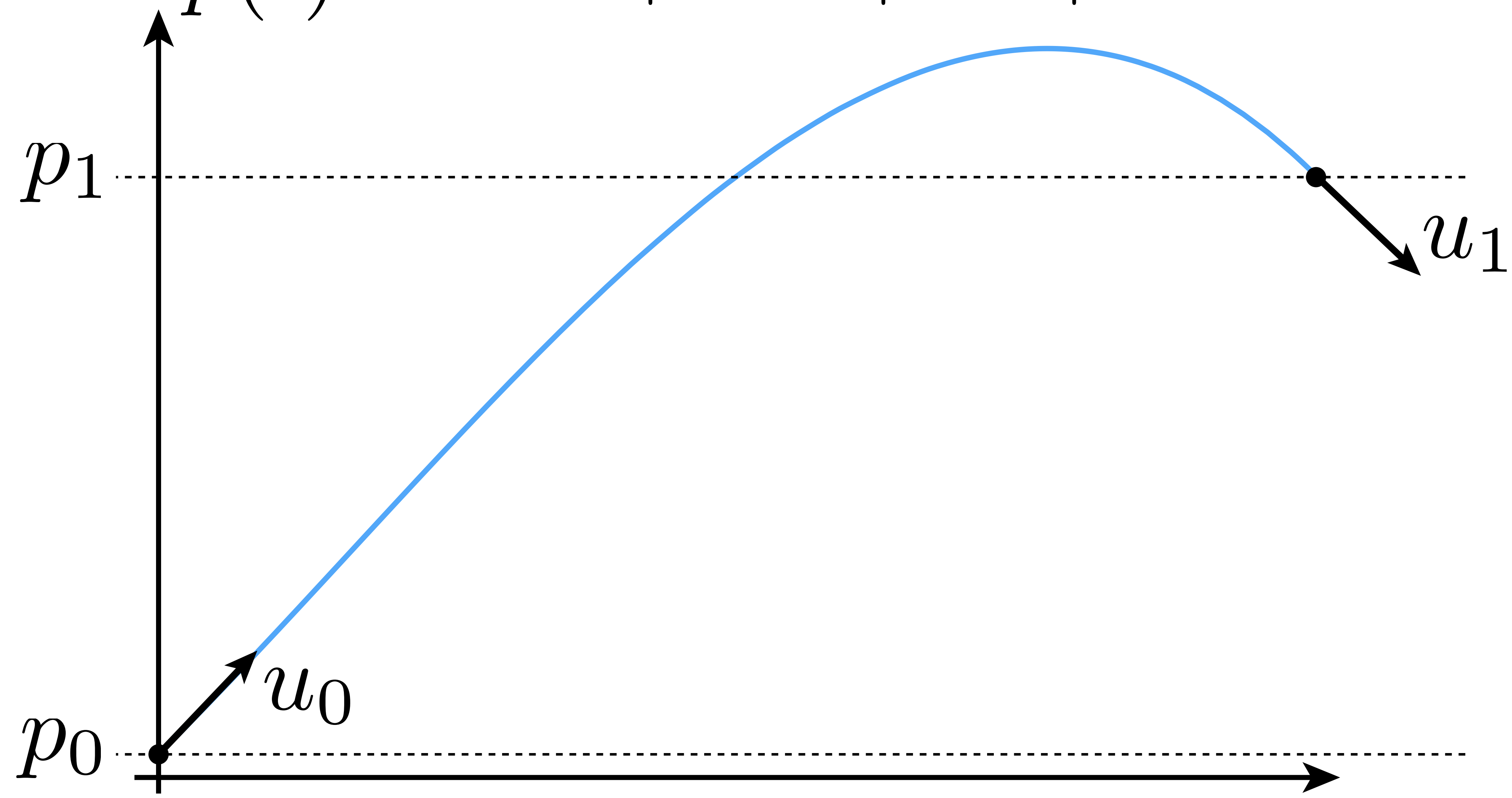
$$p(1) = p_1 \qquad \Rightarrow a + b + c + d = p_1$$

- **Overall, four unknowns but only two equations**
- **Not enough to uniquely determine the curve!**

Fitting Cubic to Endpoints and Derivatives

- **What if we also match derivatives at endpoints?**

$$p(t) = at^3 + bt^2 + ct + d$$



$$p(0) = p_0 \quad \Rightarrow d = p_0$$

$$p(1) = p_1 \quad \Rightarrow a + b + c + d = p_1$$

$$p'(0) = u_0 \quad \Rightarrow c = u_0$$

$$p'(1) = u_1 \quad \Rightarrow 3a + 2b + c = u_1$$

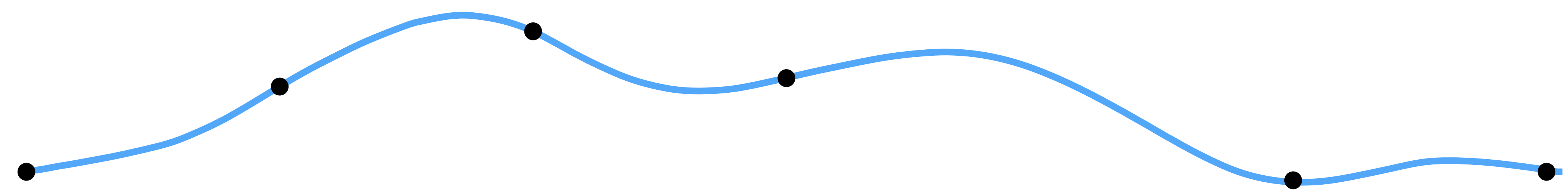
Splines as Linear Systems

- This time, we have four equations in four unknowns
- Could also express as a matrix equation:

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} p_0 \\ p_1 \\ u_0 \\ u_1 \end{bmatrix}$$

- Often, this is the game we will play:
  - each condition on spline leads to a linear equality
  - hence, if we have m degrees of freedom, we need m (linearly independent!) conditions to determine spline

Natural Splines



- Now consider piecewise spline made of cubic polynomials p_i
- For each interval, want polynomial "piece" p_i to interpolate data (e.g., keyframes) at both endpoints:

$$p_i(t_i) = f_i, \;\; p_i(t_{i+1}) = f_{i+1}, \;\; i = 0, \ldots, n-1$$

- Want tangents to agree at endpoints ("C^1 continuity"):

$$p'(t_{i+1}) = p'_{i+1}(t_{i+1}), \;\; i = 0, \ldots, n-2$$

- Also want curvature to agree at endpoints ("C^2 continuity"):

$$p''(t_{i+1}) = p''_{i+1}(t_{i+1}), \;\; i = 0, \ldots, n-2$$

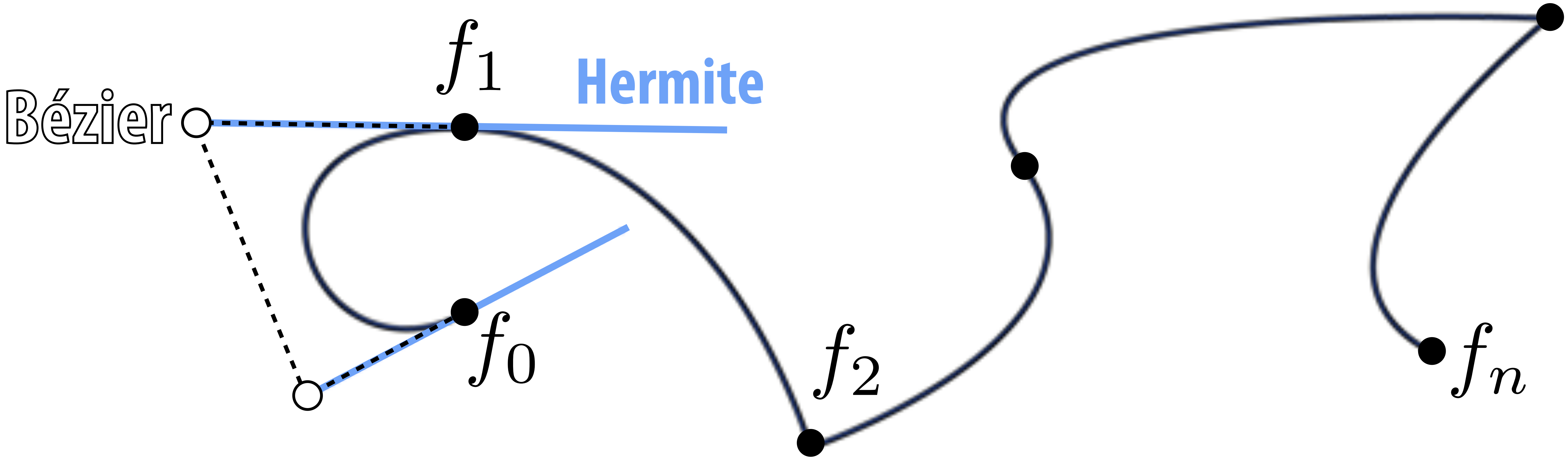
- How many equations do we have at this point?
  - 2n+(n-1)+(n-1) = 4n-2
- Pin down remaining DOFs by setting curvature to zero at endpoints (this is what makes the curve "natural")

Spline Desiderata

- In general, what are some properties of a "good" spline?
  - INTERPOLATION: spline passes exactly through data points
  - CONTINUITY: at least twice differentiable everywhere
  - LOCALITY: moving one control point doesn't affect whole curve
- How does our natural spline do?
  - INTERPOLATION: yes, by construction
  - CONTINUITY: C^2 everywhere
  - LOCALITY: no, coefficients depend on global linear system
- Many other types of splines we can consider
- Spoiler: there is "no free lunch" with cubic splines: can't simultaneously get <u>all three</u> properties

Review: Hermite/Bézier Splines

- Discussed briefly in introduction to geometry
- Each cubic "piece" specified by endpoints and tangents:



- Equivalently: by four points (Bézier form); just take difference!
- Commonly used for 2D vector art (Illustrator, Inkscape, SVG, ...)
- Can we get tangent continuity?
- Sure: set both tangents to same value on both sides of knot!
  - E.g., f_1 above, but not f_2

Properties of Hermite/Bézier Spline

- More precisely, want endpoints to interpolate data:

$$p_i(t_i) = f_i,\ p_i(t_{i+1}) = f_{i+1},\ i = 0, \ldots, n-1$$

- Also want tangents to interpolate some given data:

$$p_i'(t_i) = u_i, p_i'(t_{i+1}) = u_{i+1}, i = 0, \ldots, n-1$$

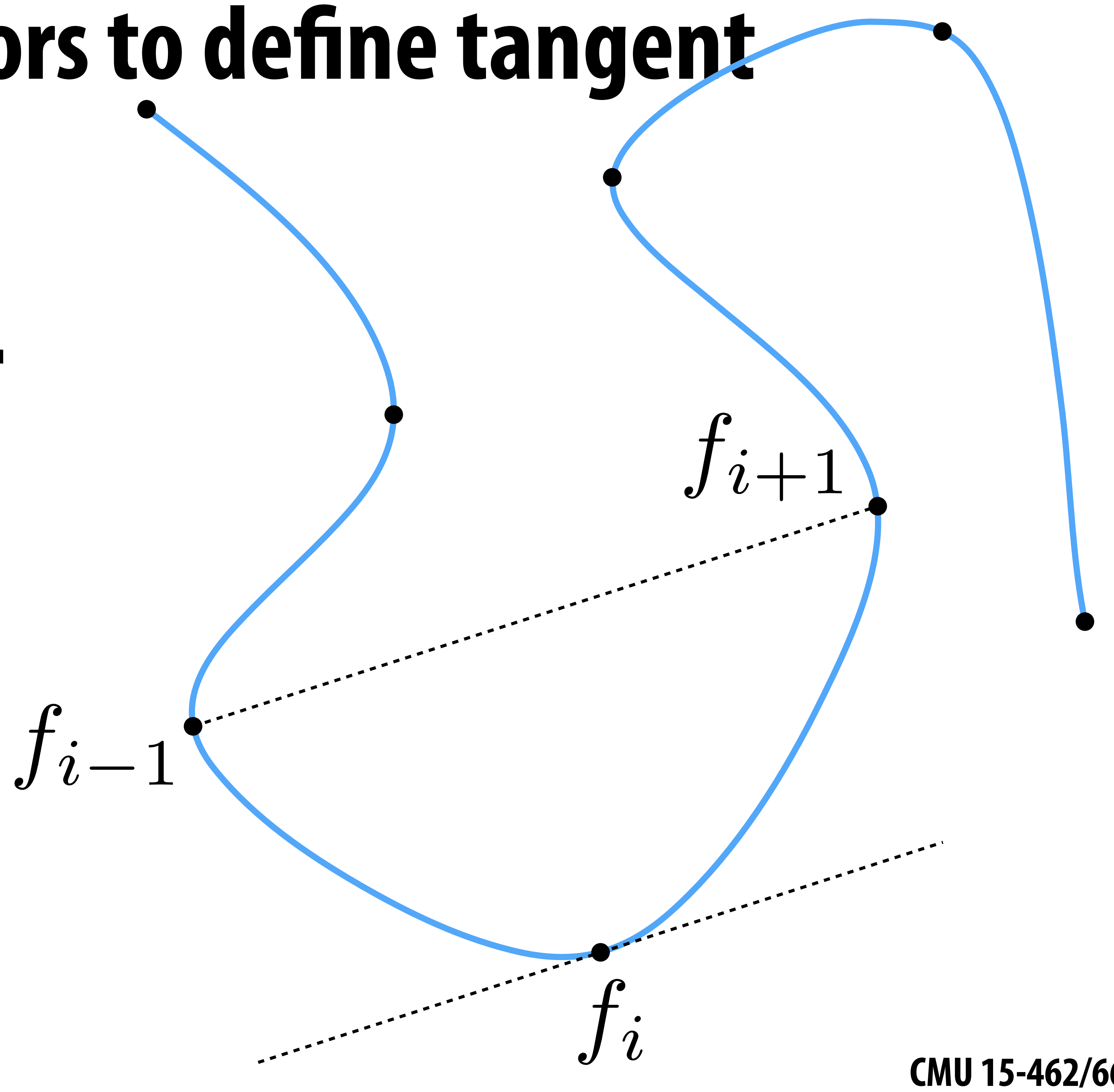
- How is this different from our natural spline's tangent condition?
- There, tangents didn't have to match any prescribed value—they merely had to be the same. Here, they are given.
- How many conditions overall?
  - 2n + 2n = 4n
- What properties does this curve have?
  - INTERPOLATION and LOCALITY, but not C^2 CONTINUITY

Catmull-Rom Splines

- Sometimes makes sense to specify tangents (e.g., illustration)
- Often more convenient to just specify values
- Catmull-Rom: specialization of Hermite spline, determined by values alone
- Basic idea: use difference of neighbors to define tangent

$$u_i := \frac{f_{i+1} - f_{i-1}}{t_{i+1} - t_{i-1}}$$

- All the same properties as any other Hermite spline (locality, etc.)
- Commonly used to interpolate motion in computer animation.
- Many, many variants, but Catmull-Rom is usuallygood starting point



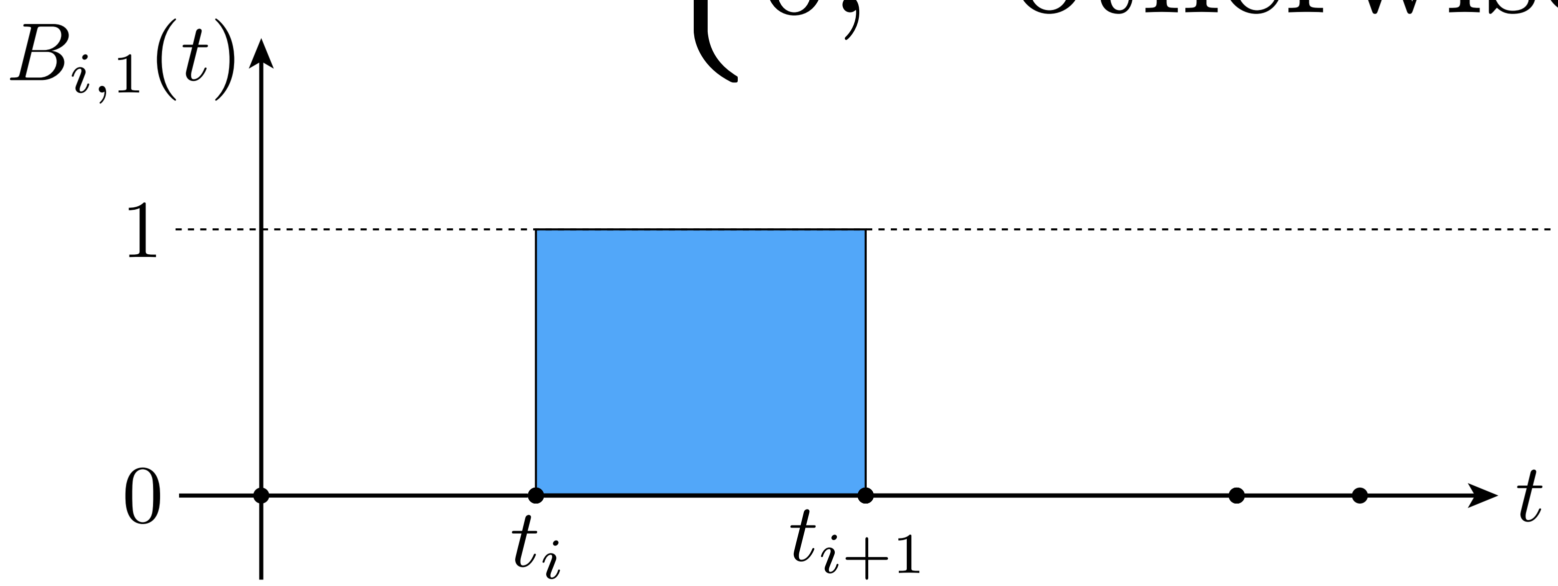
Spline Desiderata, Revisited

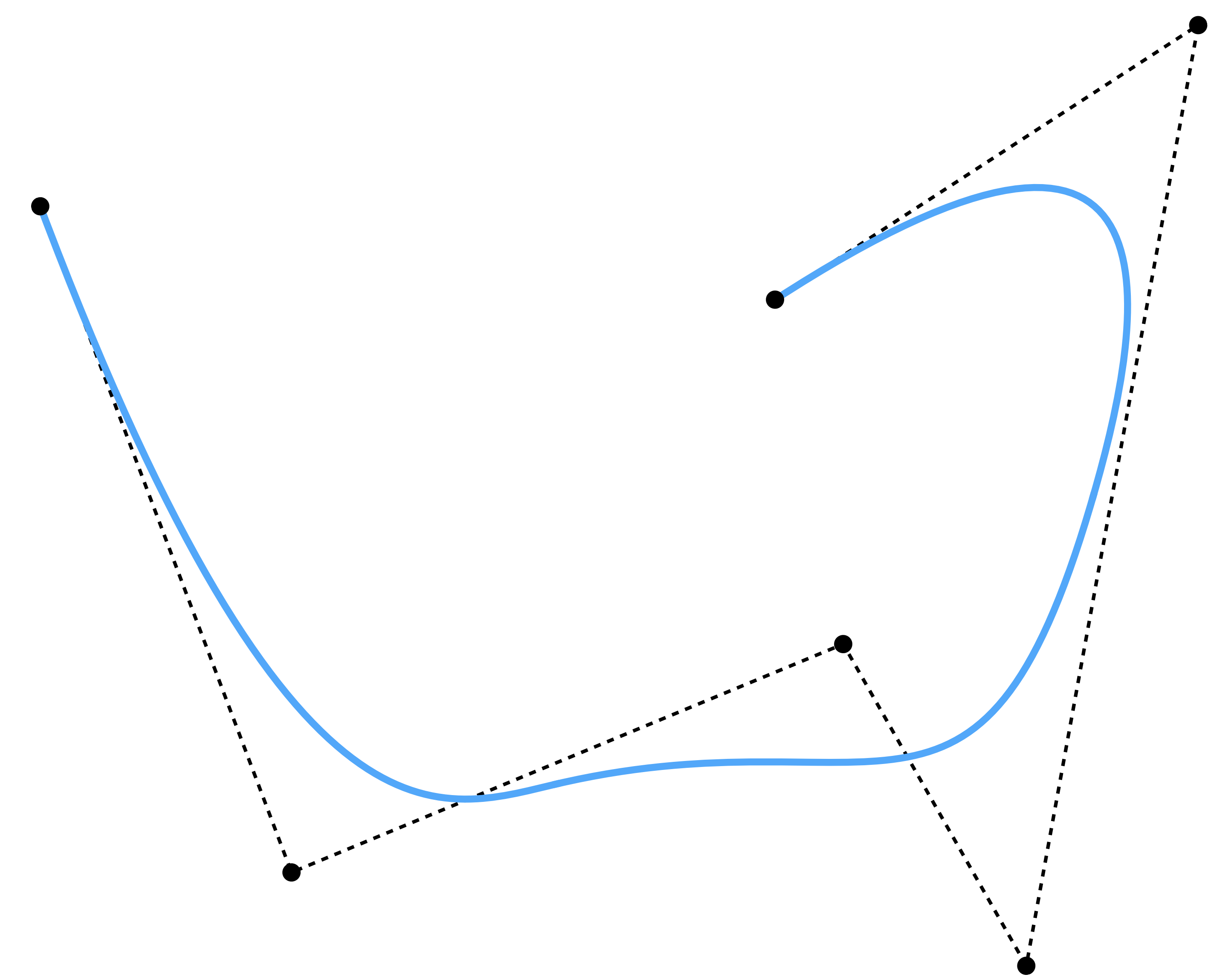
	INTERPOLATION	CONTINUITY	LOCALITY
natural	YES	YES	NO
Hermite	YES	NO	YES
???	NO	YES	YES

B-Splines

- **Get better continuity and local control by sacrificing interpolation**
- **B-spline basis defined recursively:**

$$B_{i,1}(t) := \begin{cases} 1, & \text{if } t_i \leq t < t_{i+1} \\ 0, & \text{otherwise} \end{cases}$$





linear interpolation

$$B_{i,k}(t) := \frac{t - t_i}{t_{i+k-1} - t_i} B_{i,k-1}(t) + \frac{t_{i+k} - t}{t_{i+k} - t_{i+1}} B_{i+1,k-1}(t)$$

- **B-spline itself is then a linear combination of bases:**

$$f(t) := \sum_i a_i B_{i,d}$$

degree

Spline Desiderata, Revisited

	INTERPOLATION	CONTINUITY	LOCALITY
natural	YES	YES	NO
Hermite	YES	NO	YES
B-splines	NO	YES	YES

Ok, I get it: splines are great.
But what exactly are we interpolating?

Simple example: camera path

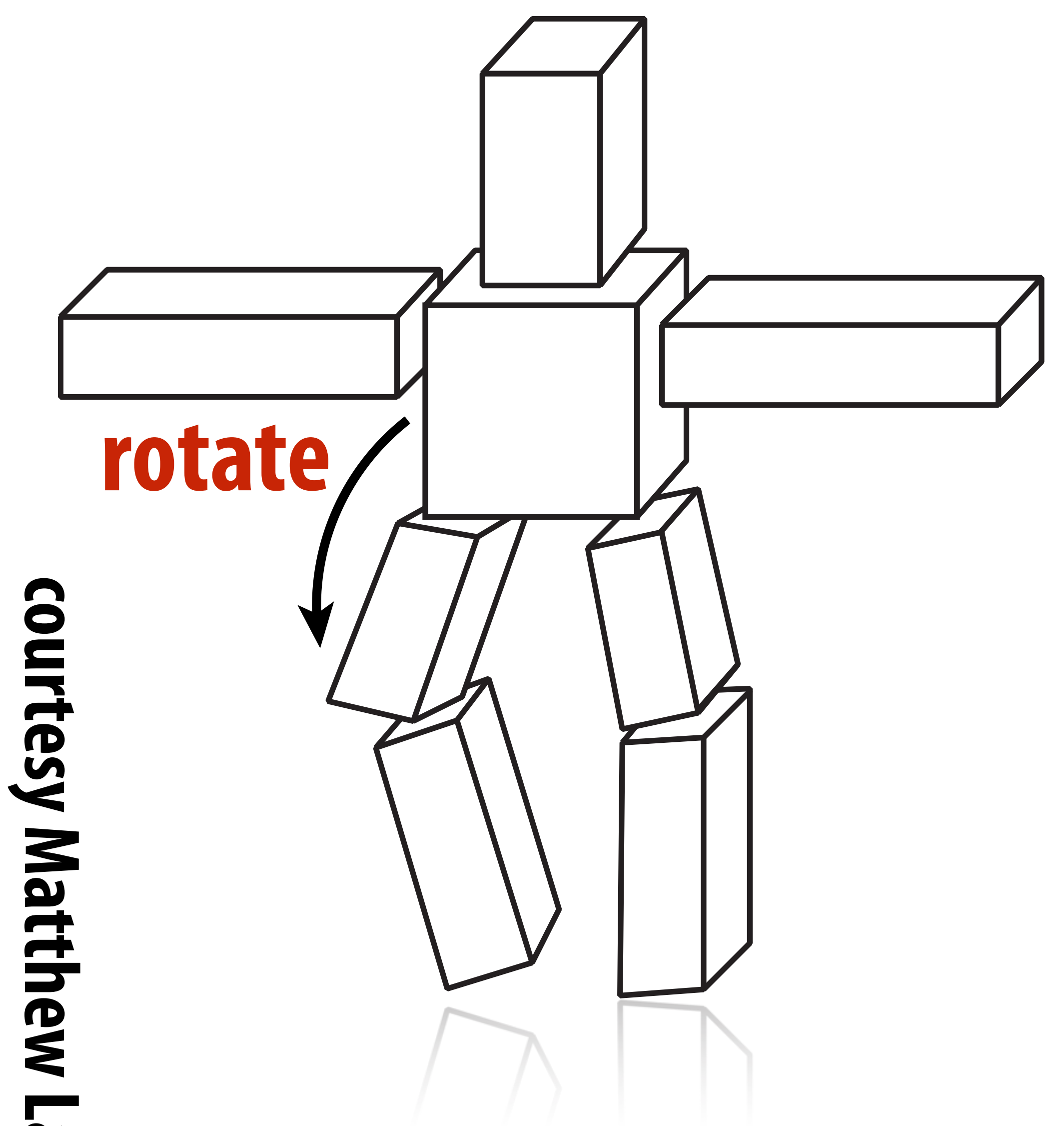
- Animate position, direction, "up" direction of camera
  - each path is a function f(t) = (x(t), y(t), z(t))
  - each component (x,y,z) is a spline

Zaha Hadid Architects—City of Dreams Hotel Tower

Character Animation

- Scene graph/kinematic chain: scene as tree of transformations
- E.g. in our "cube man," configuration of a leg might be expressed as rotation relative to body
- Animate by interpolating transformations
- Often have sophisticated "rig":

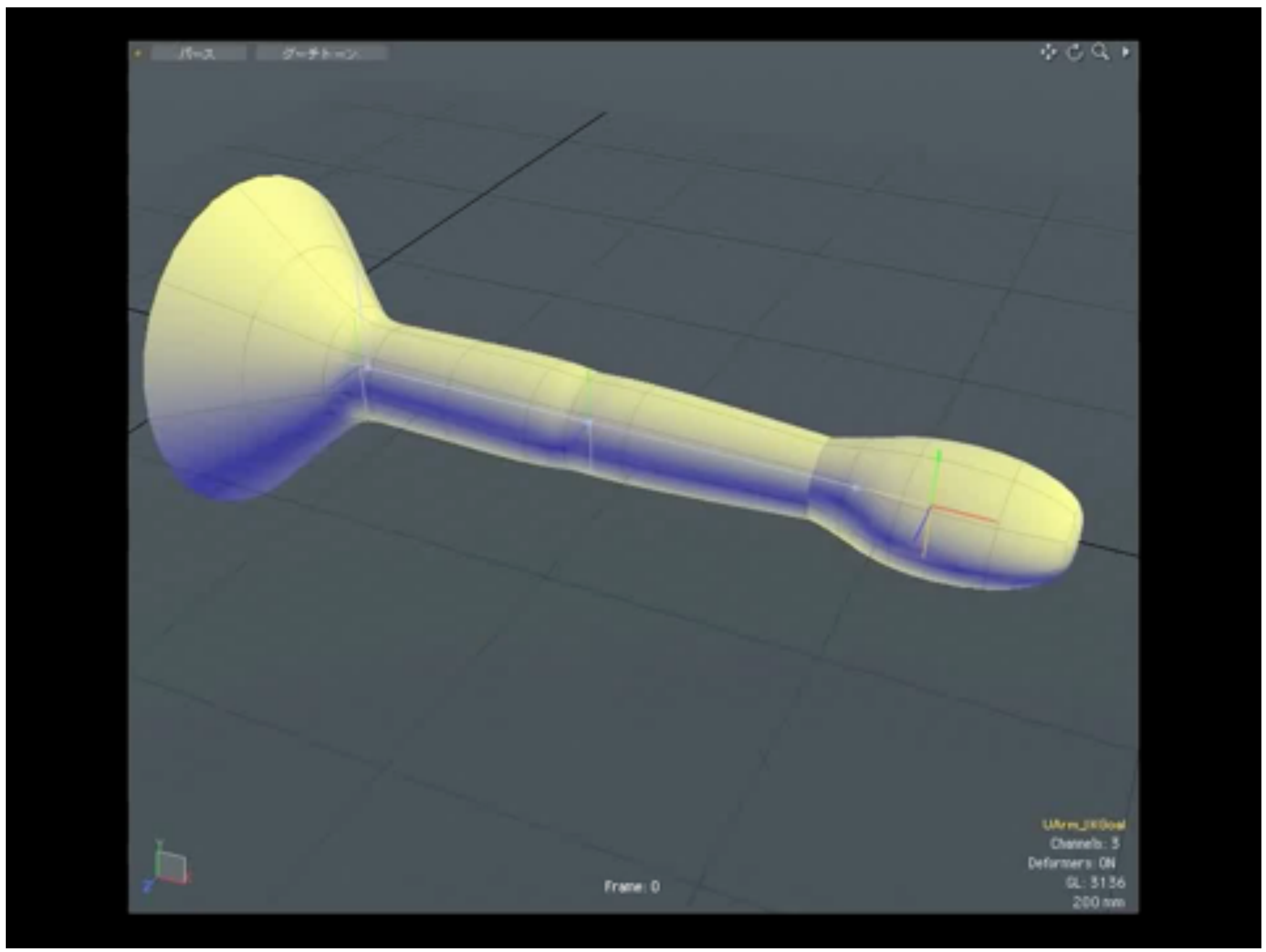
courtesy Matthew Lailler



Even w/ computer "tweening," a lot of work to animate!

Inverse Kinematics

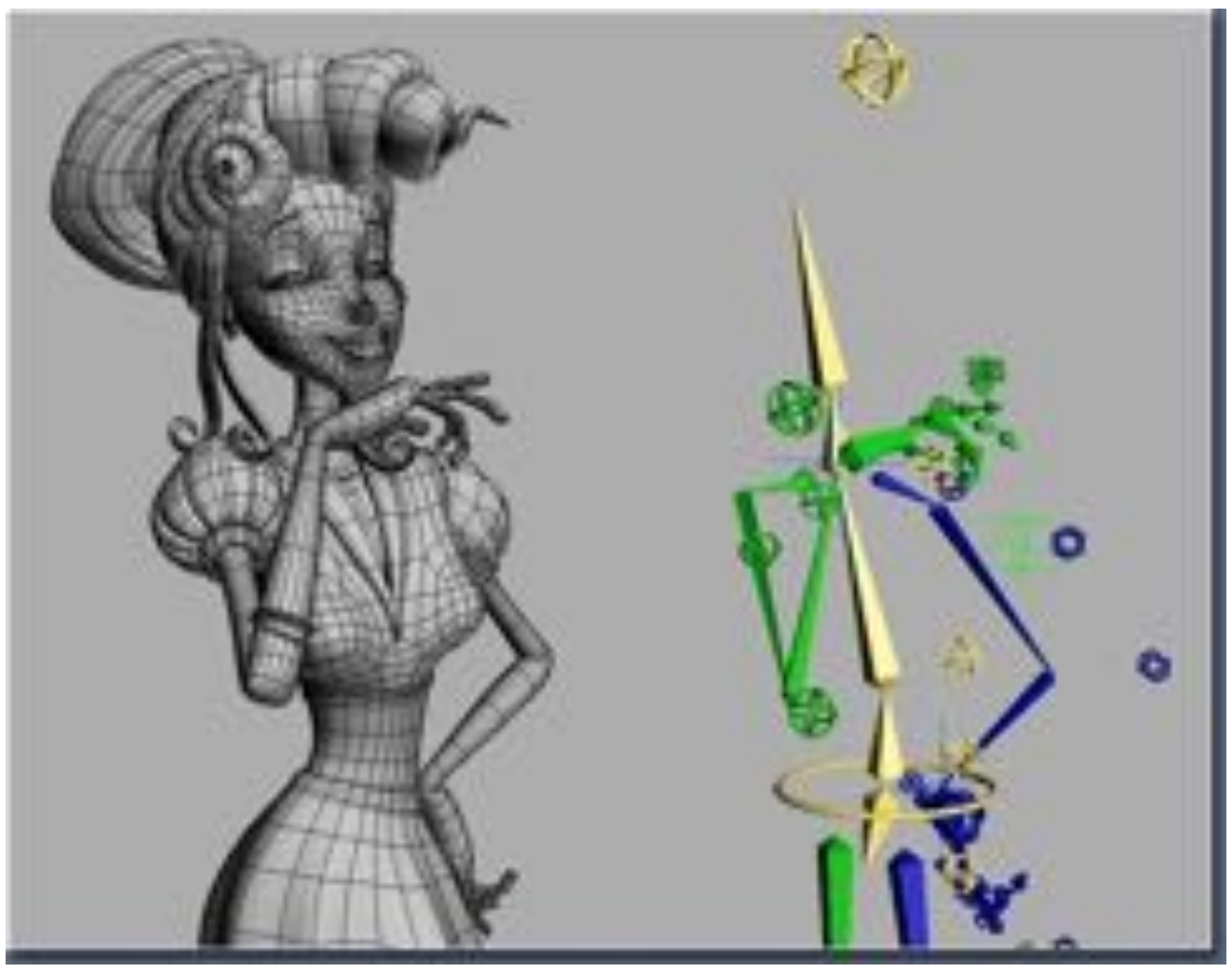
- Important technique in animation & robotics
- Rather than adjust individual transformations, set “goal” and use algorithm to come up with plausible motion:

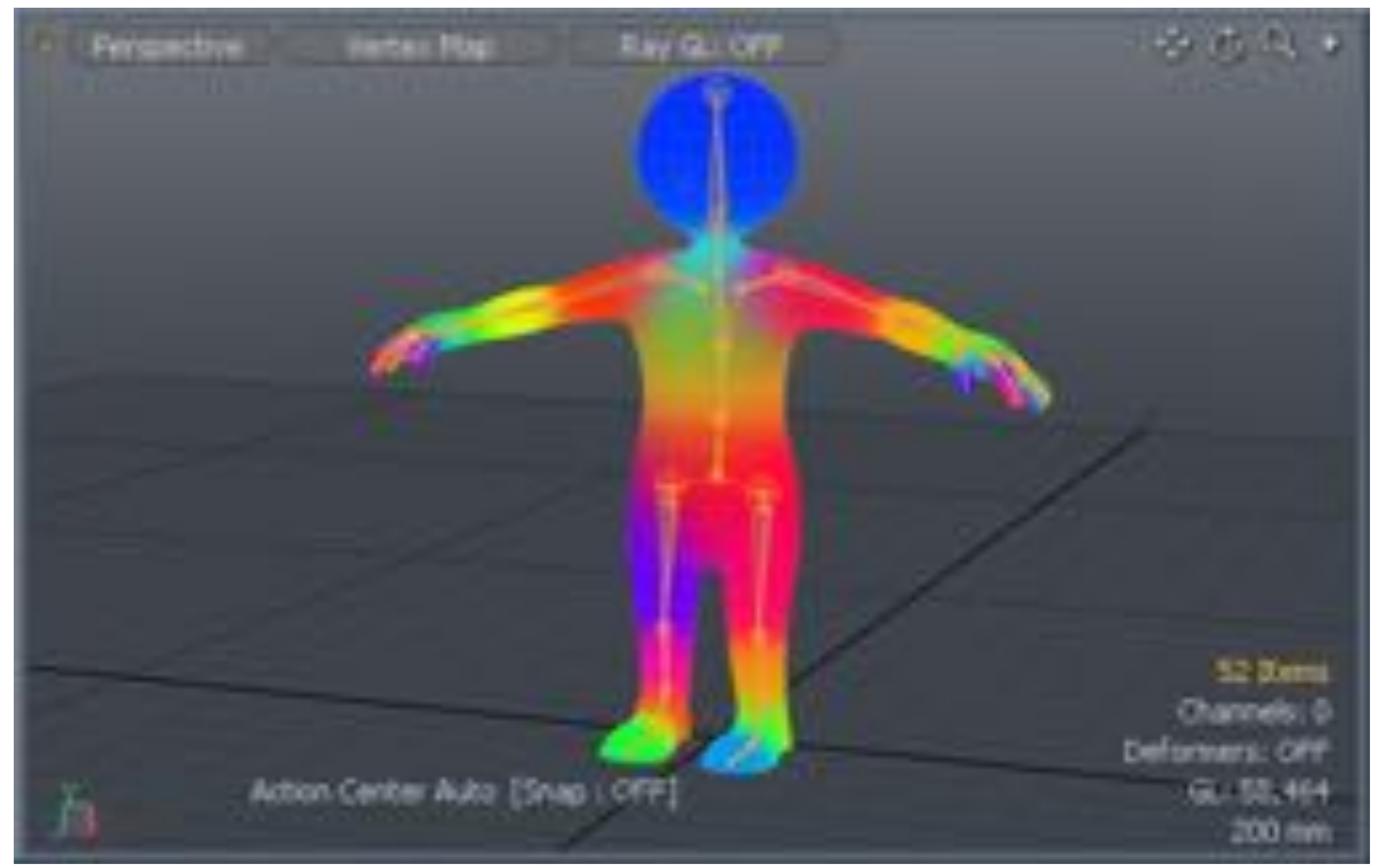


Many algorithms—basic idea: numerical optimization/descent

Skeletal Animation

- Previous characters looked a lot different from “cube man”!
- Often use “skeleton” to drive deformation of continuous surface
- Influence of each bone determined by, e.g., weighting function:





(Many other possibilities—still active area of R&D)

Blend Shapes

- Instead of skeleton, interpolate directly between surfaces
- E.g., model a collection of facial expressions:

courtesy Félix Ferrand

- Simplest scheme: take linear combination of vertex positions
- Spline used to control choice of weights over time

Coming up next...

- Even with "computer-aided tweening," animating everything by hand takes a lot of work!
- Will see how data, physical simulation can help