Parallel HW 1

2.14)

~~1) ~~proof of minimal routing~~ → min distance is # bit diff~~
~~- construction of hypercube s.t. each connected node (vertex) differs from its neighbor (edge) by exactly one bit. → Hamming distance is ~~valid~~ the distance between two nodes.~~

1) - H(A, B) = distance between A and B because of construction of H.C.
- minimal routing (E-cube) can be reduced from H(A, B).

d

d, d-1

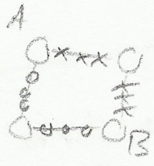
d, d-1, d-2

2)

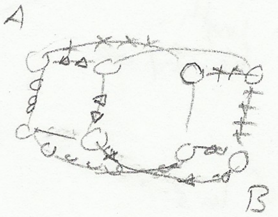
d=1

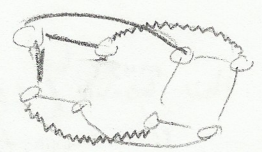
P(A, B) = 1

d=2

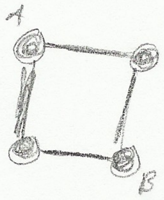


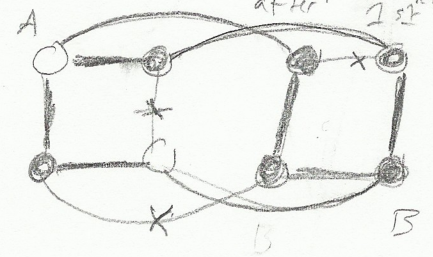
P(A, B) = 2





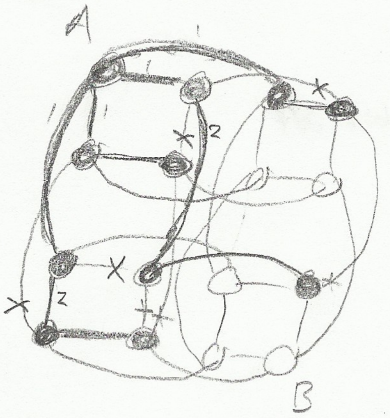
Algo:
- for each edge connected to A, start a path - each path will connect to a distinct vertex and there will be d paths. Color the end vertex black.
- for each path in turn, select an edge to follow to the next vertex. if the next vertex is not B, then remove all edges connected to a vertex that is black.
-



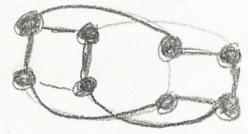


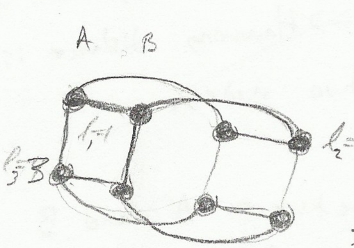
- d edges removed
- each path removed 1 after 1st step.

DFS version



All nodes shortest path





1st path – d edges

$d 2^{d-1}$ total

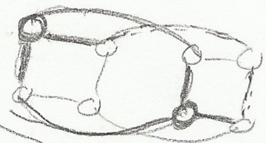
2nd path – d edges

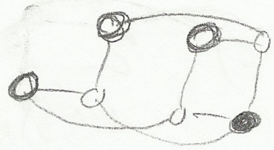
3rd path – d edges

d^d edges after d paths.

~~$t = \ldots\ l_1 = 8 = 2^d ?$~~

• At most d paths because only d edges leave the start node and lead into the end node.





7.12)

* cycle goes through some intermediate B.

4D H(A,B) is min length... but B can be any node.

h=3

3 paths three of len 3

h=4

X cases: shorter path back |

~~equal length path back~~ trivially 2x even

longer path back

$l_1 + l_2$ cases:

One path even, one odd — paths of diff length

~~both even~~ → trivially even } 2 paths of same length

~~both~~ odd → trivially even

this must be because $l_1 \neq l_2$ — remaining + only 2

diff lengths between A and B?

$l = \underbrace{H(A,B)} + \underbrace{H(A,B) + 2}$

$= 2H(A,B) + 2$ → is even

* cases
- paths are eq len $l_1 = l_2 = H(A,B)$, $l = 2H(A,B)$
- paths are not eq len → len of l_2 is $H(A,B)+2$

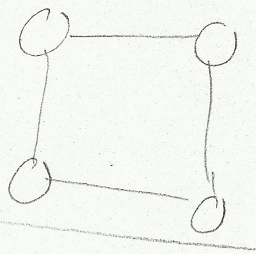
$l = l_1 + l_2 = H(A,B) + H(A,B) + 2 = 2H(A,B) + 2$

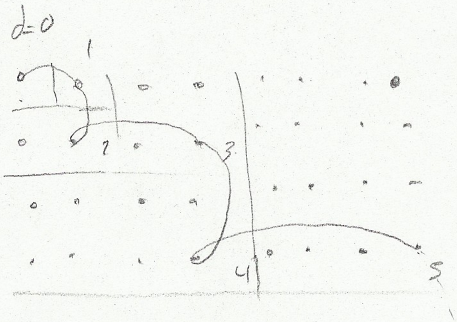
show True for each H(A,B') and H(B',B).

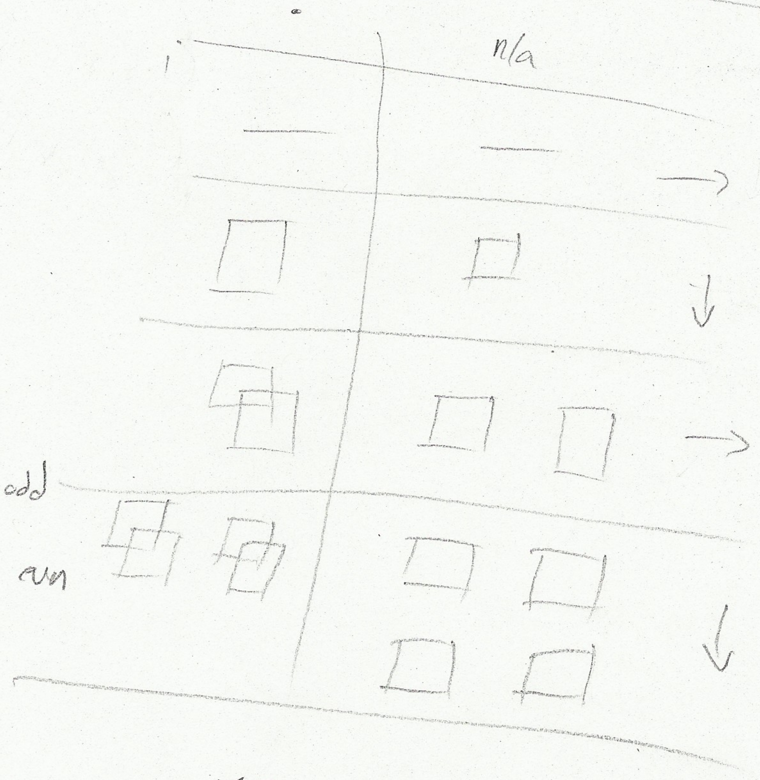
2 classes of len.

\# nodes = $2[2^{d-1}] = 2^d$

\# edges ≠







$$R(n,d) = \begin{cases} 0 & d = 0 \\ R(n-d+1,\ d-1) & d \text{ is odd} \\ R(n/2 + n\%d,\ d-1) & d \text{ is even} \end{cases}$$

x=

$x = + \log_2 d$ per odd dim $\% \log_2 d$

$y = + \log_2 d$ per even dim

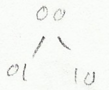
$(x, y) \rightarrow n$

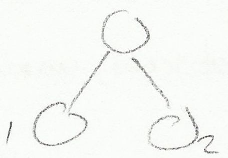
$n = \log_2 d \cdot y + x$

$2^d - 1$ node complete binary tree onto d-dim H.C. ($= 2^d$ node).

dialation - max # links in dest(E') that source(E) is mapped onto.

<u>d=2</u>



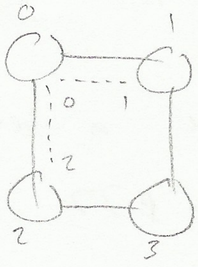


1@0
2@1

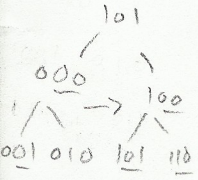
3

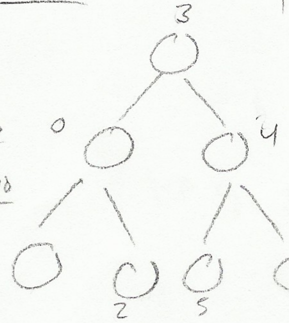
$\Longrightarrow$

1@0
2@1



<u>d=3</u>

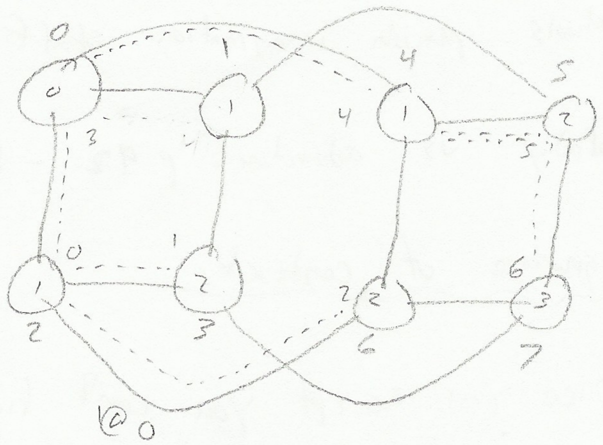




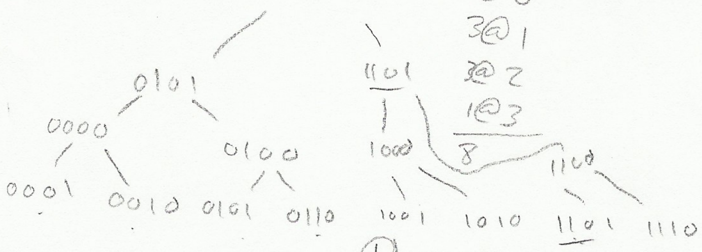
1@0
2@1
4@2

7

$\Longrightarrow$



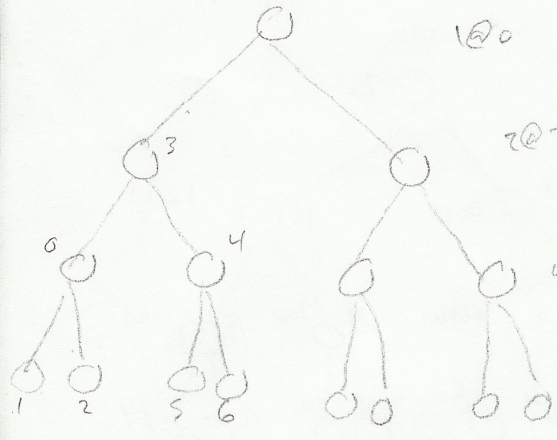
Addressing?
Huffman?



1@0
3@1
3@2
1@3

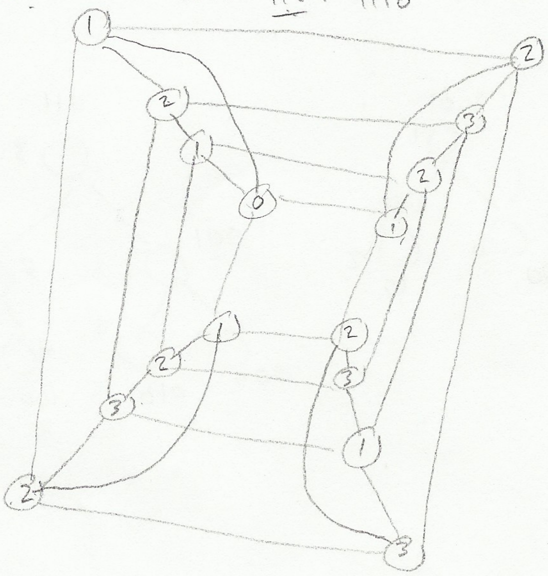
8

<u>d=4</u>



1@0
2@2
4@3
5@4

15



1@0
4@1
6@2
4@3
1@4

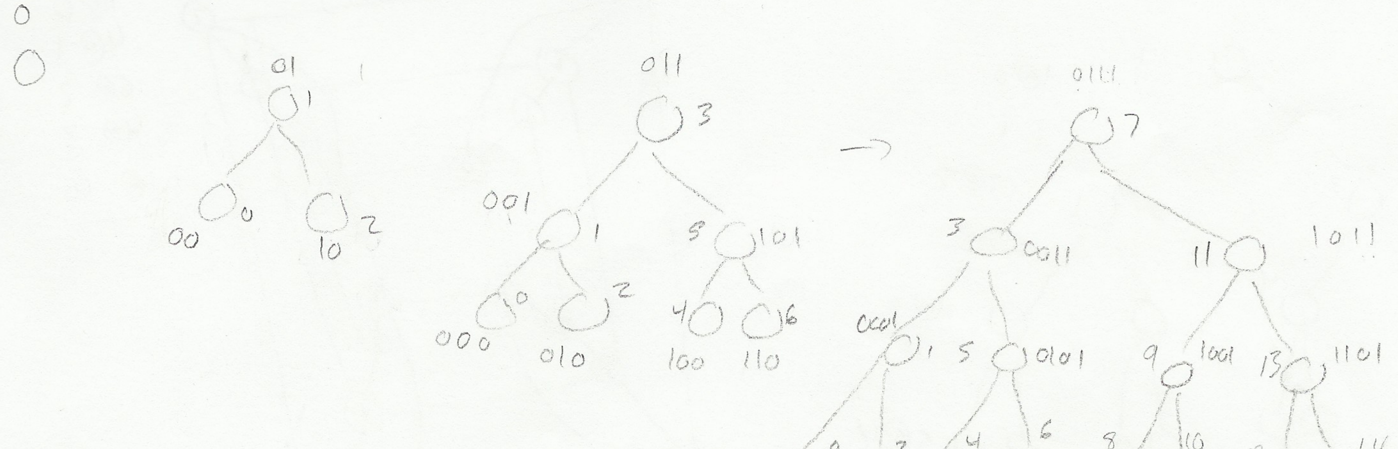
16

* Lin Array into HC
p.87 RCG

8:45
Coffman 8:52 16
9:12 535B

9:00

Coffman 9:14 465A - 9:48
SB TC 10:22 535



01
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0111
7
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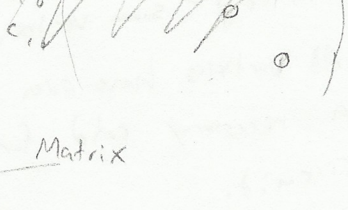
0011
11
1011
0001
1
5
0101
9
1001
13
1101
0
2
4
6
8
10
12
14
0000
0010
0100
0110
1000
1010
1100
1110

- H.C. has d dimensions with one edge per dim from each vertex, so the source and dest can be at most d edges away. In order for

- can be 0 or 1 unit away from source per dim.
- min distance between two nodes is d.
- if a path to dest crosses $> \frac{2^d}{2} = 2^{d-1}$ nodes, there is a repeated node in the path. (bipartate)
- if there are repeated nodes, then there must be an even # of edges by this proof* and it can be ignored.
- Sum of distance vector is distance

"Simple path" has no repeated verticies

Paths are "independent" if they have no internal verticies in common.



~~$(0, \dots, 0)$ d-tuple.~~

Sign(π) is

$(0, \dots, 0)$ d-tuple

Incidence Matrix

$d = 1$

$$\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$d = 2$

$\frac{d \cdot 2^d}{2}$ rows

$$\begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 0 & -1 & 0 \end{pmatrix}$$

1 2

3 4

$(1,0,0)$ $(1,1,0)$ $(0,1,0)$

$(0,1,1)$ $(1,1,1)$

A

Eulerian Cycle

Sign(π) permutations?

A ⊕ to $\vec{0}$.

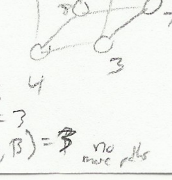
~~or A to B~~?

A 00 01

10 B 11

Cycle via some intermediate B

H(A, B)



A B

H(A,B)=3

P(A,B) = 3

P(len=3 (A,B)) = 3 no more paths

H(A,B) = 2

P(A,B) = 3

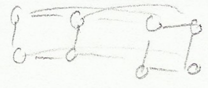
$P_{len=2}(A,B) = 2$

len of last path = 2+2=4

Parallel ~~Pa~~ HW 1

2.12)

$\begin{array}{r} 64 \\ 16 \\ \hline 80 \end{array}$



$d = \log_2 p$

$p = 2^d$

$\boxed{\begin{array}{l} \text{Longest Path: } d \\ \#\ \text{verticies: } 2^d \\ \#\ \text{edges/vertex: } d \\ \#\ \text{edges: } \frac{p \log p}{2} = \frac{dp}{2} = \frac{d \cdot 2^d}{2} = d \cdot 2^{d-1} \end{array}}$

					$\times 2 +$
$\#e =$	1	4	12	32	80
$d =$	1	2	3	4	5

$e(d) = \begin{cases} 1 & d = 1 \\ 2e(d-1) + 2^{d-1} & \text{otherwise} \end{cases}$

Assume there is an odd length path

$\lfloor \ell/2 \rfloor \neq \lceil \ell/2 \rceil$; $\ell \bmod 2 \neq 0$.

Assume no dupe vertex in path?
- infinite length cycle for deterministic routing.

Pigeonhole principle?

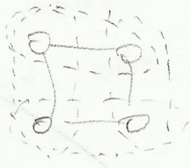
Eulerian Trail (circuit):
Visits each edge once + Start + end = same vertex.
- All verticies have even degree is a necessary cond for an eulerian circuit.
- No more than 2 verticies may have an odd degree in a trail.

travel some # of steps along any dimension => have to take same # of steps in opposite direction to return.

d=1

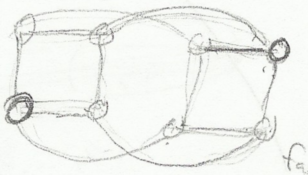
m = d

d=2



4 cases of d=1?

d=3

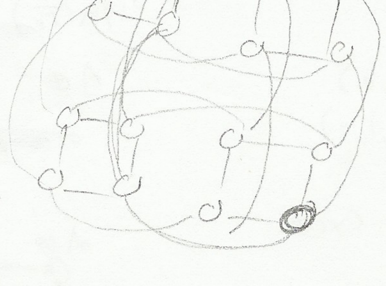


3 options from start.

max 3 hops to farthest node (no dupes)

(passes $> \frac{2^d}{2} = 2^{d-1}$ nodes → dupe vertex.)

an entire plane/cube/object of one lower dim $(d-1)$



Shortest path

- each node has exactly one different bit from its neighbor
- only one step per dimension in H.C.

$4 \times 2^4 > (4-1) \times 2^{4-1}$

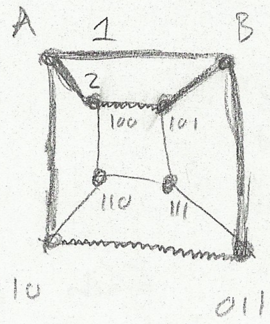
$4 \times 16 > 3 \times 8$

$64 > 24$ ✓

d=3

change LSB first, MSB last.

A=000



B=001

$A = 000$ $A'_1 = 001$ $A'_2 = 010$ $A'_3 = 100$

$\underline{A \oplus A' = 1}$

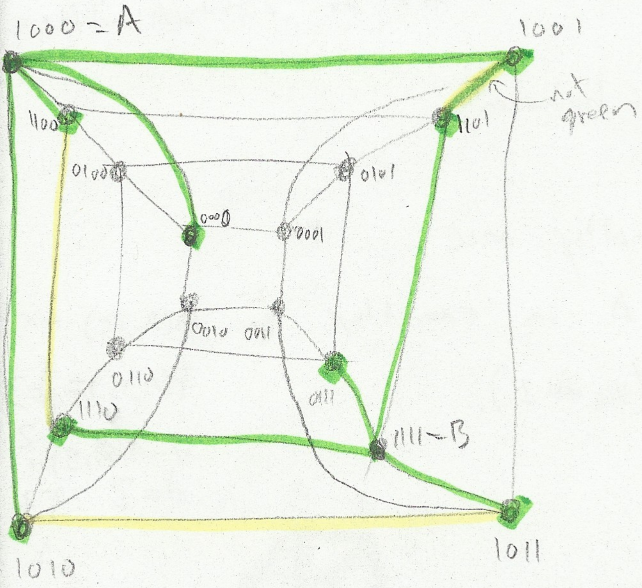
$B = 001$ $B'_1 = 000$ $B'_2 = 011$ $B'_3 = 101$

$\underline{B \oplus B' = 1}$

$B'_1 = A$; $A'_1 = B$ ✓

Because we do not violate minimal routing, we only travel along edges toward B (except in leaving A to A').

Then take each pair of A' and B' that are the closest (smallest H(A', B')) and repeat the process to find A'' and B''. If a pair of A' and B' are equal, a path is found. ~~If a repeat value is produced, discard it.~~ ~~for~~, only consider A' and B' that do not violate minimal routing). If there are multiple, pick the one with the smallest LSB difference (eg. min $\|B' - A'\|$).



A = 1000 $A'_1 = 1001$, 1010, 1100, 0000

B = 1111 $B'_1 = 1110$, 1101, 1011, 0111

Parallel HW1

2/10/2011

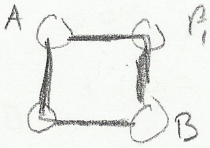
2.14)

H.C. # edges: $d \cdot 2^{d-1}$
verticies: 2^d

2) $\boxed{P(A,B) = d}$

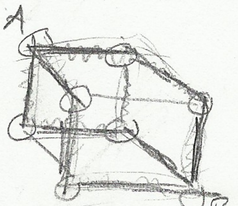
Assume minimal + deterministic routing (p.64 bottom).
2.14

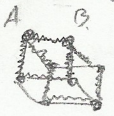
$d=2$



$d \cdot 2^{d-1} - d^d \overset{?}{=} d$
$2 \cdot 2 - 2^2 = 0$ ✓

$d=3$



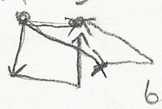


Each of the d edges connected to each vertex follows exactly one dimension. By following the first available dim (assuming each node is addressed uniquely by a d-tuple, and first available is the one with the lowest dimension left to right. Available=

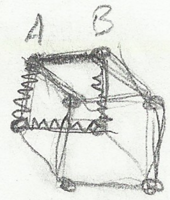
- Each vertex has d edges connected to it.
- Each of those d edges occupies a unique dimension (spatial).
- There are d! potential paths between A and B [of len H(A,B)].
- If we follow each dimension exactly once we will arrive at B from A in exactly H(A,B) edges, at a minimum (via #1).

H(A,B) min
$P(A,B) = \cancel{d}\,3$
$-1 = 2$
$d - H(A,B) = 1$
$d = 3 \quad = 2$

-



because we assume minimal routing, x will not send it along an axis "away" from B.



3

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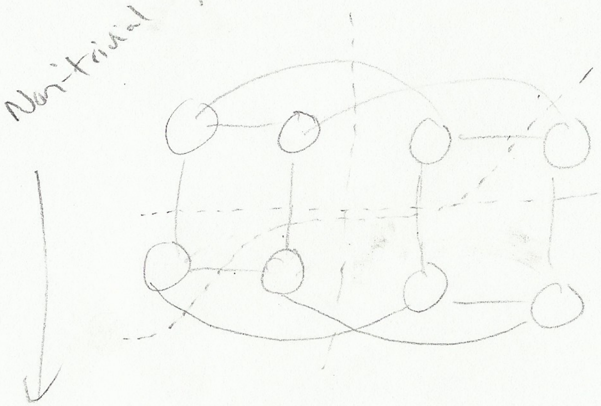
will get at least 1 edge closer with each hop.

Non-trivial graph



2^{d-1} links in p-dim hypercube

$2^{d}-1$ links in non-hypercube

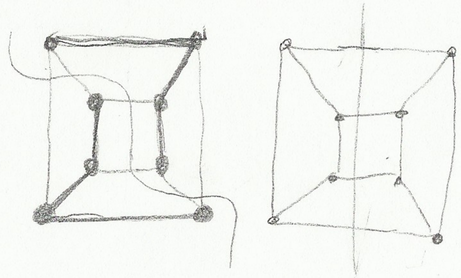
cut-vertex if:

$K(G-v) > K(G)$

and G becomes disconn

bridge: same but for edge.

Assume that non (d-1) HC partitions can be made from a d dim HC with 2^{d-1} links or fewer cut.



bipartate ⟺ no odd len cycles

2-colorable ⇒ bipartate

equal cardinality in partitions means balanced bipartate.

not 2-regular

path

2-regular

(d-1)-regular

cycle

non-isomorphism?