2.15)

Required For 2 HC partitions

<u>each part</u>	<u>total (x2)</u>
2^{d-1} nodes	2^d ✓
+	
degree = $(d-1)$	$\boxed{(d-1)}$
+	
$(d-1) \times 2^{d-2}$ edges	$(d-1) \times 2^{d-1}$ ✓

$\Rightarrow$

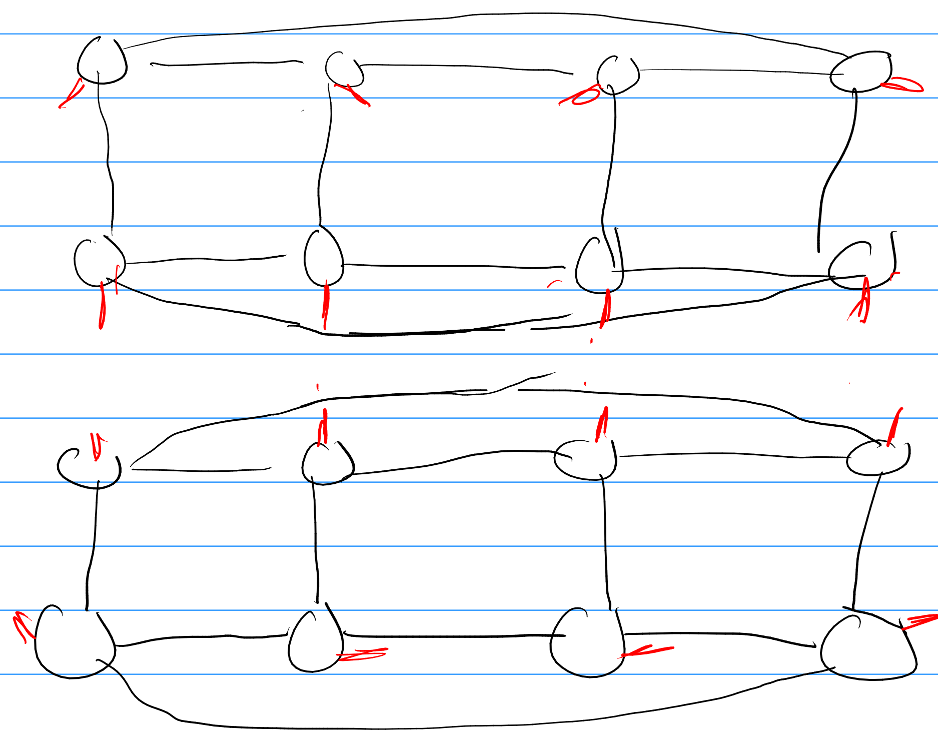
2x H.C. of dim $d-1$

goal is to establish the # of edges and degree as sufficient conditions, implying partitions are HCs and use proof by contradiction.

12

8

12



$d=4$

cut $> 8 = 2^{d-1}$

#

<u>Handshaking lemma</u>

$\sum \deg(v) = 2 \cdot |E|$

$|V|(d-1) \neq 2|E|$

$|V| = 2^d$

$\rightarrow |E| \neq \frac{2^d(d-1)}{2} = 2^{d-1}(d-1)$

Assume we can partition the graph by cutting $x \leq 2^{d-1}$ edges, leaving the two evenly sized partitions as non-hypercubes.

edges $(d-1)2^{d-1}$

After cutting the graph, it will have $z = d \times 2^{d-1} - x$ edges left, evenly divided between the two partitions.

The <u>handshaking lemma</u> states:

$$\sum \deg(v) = 2|E|.$$

$$z = d \times 2^{d-1} - x$$
$$z \geq d \times 2^{d-1} - 2^{d-1}$$
$$z \geq (d-1) \cdot 2^{d-1}$$
$$2z \geq (d-1) 2^d$$

Therefore, $\sum \deg(v) = 2z$.

Being that neither partition is a hypercube, $\sum \deg(v) \neq |V| \cdot (d-1)$

And being that we are only cutting edges, we will remain with the original number of nodes. $|V| = 2^d$.

$$\sum \deg(v) \neq 2^d \cdot (d-1)$$

$\Rightarrow 2^d (d-1) \neq 2z$ along with our formula for $2z$ above

$\Rightarrow 2z > 2^{d-1}(d-1)$

$z > 2^{d-2}(d-1)$

cut edges $\leq 2^{d-1}$

And because we started with a Hypercube, no node has degree of more than d.

$$\sum \deg(v) \le 2|E|$$

$$|V| d \le 2z$$

$$d \cdot 2^d \le 2z$$

$$d \cdot 2^{d-1} \le z$$

$$d \cdot 2^{d-1} - z \le 2^{d-1}$$

$$d \cdot 2^{d-1} - 2^{d-1} \le z$$

$$(d-1)\,2^{d-1} \le z$$

$$z \ge d \cdot 2^{d-1}$$

$$z > 2^{d-2}(d-1)$$

$$d \times 2^{d-1} - z \le 2^{d-1}$$

$$d \times 2^{d-1} - d \times 2^{d-1} \le d^{d-1}$$