2.12)

For any cycle in a graph, assume the starting node is A and there is some intermediate node B.

Using the properties from 2.14, we can show that no cycle is odd length in a hypercube. We also use the assumption of minimal routing on p. 64.

Taking the two sub-paths between A -> B and B -> A there are two cases.

Case 1) equal length paths:

$$l = H(A,B) + H(B,A) = 2H(A,B)$$

This is true because path lengths are equal, and is always even.

Case 2) not-equal length paths:

$$\begin{aligned} l &= l_1 + l_2 \\ &= H(A,B) + H(B,A) + 2 \\ &= H(A,B) + H(A,B) + 2 \\ &= 2(H(A,B) + 1) \end{aligned}$$

Which is also even.

This step can be conducted recursively for each sub-path A -> B' -> B component A -> B' and B' -> B until their Hamming distance is 1, in which case we are done with that sub path.

Therefore, there are no odd-length paths in a hypercube graph.