SCHOOL OF COMPUTER SCIENCE

Multidimensional Adaptive Social Networks In Understanding Extremism

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Abstract

Extremism is found prevalent in many areas of society, ranging from political beliefs, religious ideology to even food preferences and opinions in sport. Such extremism in opinions is undesired, as it often leads to clusters of individuals holding similar views with low tolerance to differing clusters. Here we study how extreme ideas and fragmentation arise within a society of individuals considering multiple topics when connecting to other individuals. Our study utilises a multi-dimensional adaptive network as an extension of an existing adaptive network. Our model holds the assumption that each individual conforms to their local social norm for each topic being considered (e.g., if two topics such as politics and sport are considered, then each individual gradually adopts the political beliefs and sports beliefs they are exposed to). Our results show that when individuals focus on connecting with respect to similarity in opinion, then they should focus on topics wherein other individuals are most similar to them to reduce fragmentation; when individuals focus on connecting with respect to diversity in opinion, they should focus on topics wherein other individuals local social norm is most different to their opinion in order to form a strongly connected society of individuals.

Dedication and Acknowledgements

To my supervisor, Professor Seth Bullock, for his invaluable supervision and support.

Declaration

I declare that the work in this dissertation was carried out in accordance with the requirements of the University's Regulations and Code of Practice for Taught Programmes and that it has not been submitted for any other academic award. Except where indicated by specific reference in the text, this work is my own work. Work done in collaboration with, or with the assistance of others including AI methods, is indicated as such. I have identified all material in this dissertation which is not my own work through appropriate referencing and acknowledgement. Where I have quoted or otherwise incorporated material which is the work of others, I have included the source in the references. Any views expressed in the dissertation, other than referenced material, are those of the author.

Harleen Gulati, Wednesday 1st May, 2024

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Ethics Statement

- "This project did not require ethical review, as determined by my supervisor, Professor Seth Bullock"

Chapter 1

Introduction

In the interconnected world of today, it has become instant and easy to access a range of information and opinions of others through communication media. As a result of this, extremism of society has become prevalent from views within political ideology - where "political polarization is on the rise across the world" (Gidron et al., 2019) - to religious extremism which has "grown over the last fifty years" (Halimi and Sudiman, 2021). Alongside this, society has been observed to fragment into different communities which strongly oppose one-another such as the split of vegans and anti-vegans where "anti-vegan individuals perceive vegans as 'radical', 'arrogant', 'angry', 'condescending' and 'intransigent'" (Demir, 2024).

These levels of fragmentation and extremism are undesired as they have "concerned many theorists of democracy, who have argued that exposure to a diverse range of viewpoints is crucial for developing well-informed citizens" who are "tolerant to the ideas of others". Fragmentation within society only finds itself worsening, as fragmented communities have "like-minded voices in a kind of echo-chamber" which "may contribute towards polarization towards the extremes". (Bright, 2018) Thus, extremism only gets more extreme and can subsequently hinder different fragmented groups from even tolerating differing ideas.

Thus, understanding what behaviors or patterns lead to extremism can help us as individuals and as a society refrain from following these patterns and consequently be more tolerant of and have greater exposure to differing views.

Plenty of existing research endeavours to understand the arise of extremism within societies from sharing and spreading of opinions (Sayama, 2020) (Gross and Blasius, 2008). The name often given to research done in this area is opinion dynamics: investigating the spreading of opinions in a collection of human beings. (Xia et al., 2011). Opinion dynamics heavily utilizes networks which in simple terms is a set of vertices connected via edges (Dorogovtsev and Mendes, 2002). It can be seen why opinion dynamics would benefit greatly from the use of networks. An intuitive example of this is a group of friends who each have an opinion in a field such as sport. We can then view each vertex of a network as an individual and assign each vertex a state capturing their opinion in sport, whilst the edges between two individuals reflect a friendship. The edges may also be weighted, reflecting how close two friends are. The larger the weight of an edge, the closer the two friends are.

With time however it is often found that our opinions change and who we feel close to also changes. Thus in our intuitive broad example of friendships, we would benefit greatly if the network modelled this change in opinion (i.e., change in the state of each vertex) and change in friendships (i.e., change in the edge weights and edge links) with respect to time. This would allow us to see how opinions spread and shape a society with time. This modelling can be achieved with the use of agent-based adaptive networks which are networks whose links change adaptively with respect to their states, resulting in a dynamic interplay between the state and topology of the network (Gross and Blasius, 2008). In essence, agent-based adaptive networks are networks whose states (i.e., values the vertices take) and topology (structure of the network - edge weights and edge links) both change with respect to time.

Indeed, opinion dynamics utilises agent-based adaptive network models to demonstrate how extreme ideas and opinions arise in society over time. Though adaptive networks have been modelled as random graphs (the change in state and topology can be modelled as a random process), it is becoming increasingly recognised that the topology and evolution of these networks are governed by robust organising principles (Albert and Barabási, 2002). Referring back to our continuous friendship example, though the changes in opinions and friendships of each individual could be perceived as random there is usually influence from underlying principles such as perhaps how often these friends meet (e.g., two individuals meeting more often are more likely to develop a friendship), how agreeable their opinions are (e.g., we are more keen to develop a friendship to those similar to us) and many more underlying factors.

Opinion dynamics research focuses on possible underlying principles which could govern change in opinion and connections: an example of this is the principle of homophily - the tendency to bond with individuals similar to yourself (Maia et al., 2021). Many agent-based adaptive network models utilise the principle of homophily (Sayama, 2020). Intuitively speaking, in our example of friendships, the principle of homophily suggests individuals become closer to individuals who share a similar opinion. A society where each individual has a large homophily (i.e., where each individual only becomes close to other individuals if their opinion is very similar) can lead to a subset of individuals with strong links with each other, but weak or no links with other individuals. For example, a subset of individuals may have a very close friendship, but each individual within this subset has no or a distant friendship with individuals outside of the subset. This can lead to the subset of individuals forming a separate fragment of the network - the network then with time becomes fragmented and small groups of individuals who oppose one another may be observed.

Thus, it can be seen how homophily could contribute to fragmentation of society. Alongside homophily, many other governing principles can guide the evolution of opinions and connections with time. Another example is conformity (users update their opinion with respect to the local neighbourhood) which guides how the opinions of individuals change with respect to time, and is found itself used in many agent-based adaptive network models (Das et al., 2014). Indeed, in a group of friends, an individual may change their opinion to look similar to their friends.

Perhaps however the society of friends we are considering are open to and take an interest in views which are different from the rest; perhaps individuals within our example become closer to individuals who hold different opinions. Such a principle has been captured in existing models and is referred to as attention to novelty; the willingness and interest to connect to those with differing views has been shown to form networks which are well-connected (Sayama, 2020). In our recurring example, if the individuals form friendships based on opinions different to their own as well as opinions similar to theirs, it is less likely for the fragmentation phenomena we discussed earlier to be observed since individuals find themselves more flexible in forming links with other individuals as they are less fixated on similarity in opinion.

Plenty more governing principles could be considered and applied to the evolution of a social network. For example, in our example of friendship we could apply the principles of conformity, homophily and attention to novelty to assess how the society of friends evolves with time with respect to their opinion in sport.

It is noteworthy to observe the principle of homophily focuses on strengthening ties between individuals sharing similar opinions. In our example, we focus on individuals opinion in sport however each individual in the real-world holds many different opinions (sports, politics, religion, diet etc.). If two individuals then say have a similar opinion in sport but differ greatly in their political opinion, would the principle of homophily strengthen their connection based on their similarity or would they drift apart due to their difference in political beliefs? It would be interesting to dive into this area deeper, since in the real-world it is indeed the case that individuals have multiple opinions.

Thus, it would be interesting to investigate the behavior of agent-based adaptive network models to multi-dimensional opinions: this would mirror the real-world closer and further investigate how individuals react and respond to settings where their connection to others is no longer influenced by just one opinion, but multiple opinions.

We build upon Sayama's agent-based adaptive network model (Sayama, 2020) and begin by replicating this model. We then extend Sayama's model giving each node a multi-dimensional opinion and investigate how different behaviors of the individuals lead to different topologies being evolved. As Sayama did, we also conduct systematic parametric-sweep numerical experiments for one dimensional (the original model) and two dimensional cases to elucidate the effects of individual behavior (conformity, homophily, attention to novelty and in multi-dimensional settings a ‘homophily updater’ and an ‘attention to novelty updater’ which decide which dimension the individual focuses on when strengthening their ties to similar and differing views respectively) on social networks (Sayama, 2020).

Chapter 2

Background

2.1 Introduction to networks

Many real-world phenomena, from the Internet to networks of friendships, disease transmissions and even terrorism (Newman et al., 2006) can be modeled, analyzed and understood with the framework of networks. From a formal point of view, a network is a set of vertices (also referred to as nodes) connected via edges and can hold many properties such as having weighted edges (edges with a strength attached), unweighted edges, directed edges (edges which can only go in a specified direction), loops (an edge with both ends attached to the same vertex) and even multiple edges between a given pair of vertices (Dorogovtsev and Mendes, 2002). It can then be seen how networks can easily model the real-world: for example, in the case of disease transmissions each vertex (also referred to as a node) can be viewed as an "infectious" or "noninfectious" individual and edges between nodes represent which individuals are in close contact with other individuals. Another example is the example discussed earlier: each node being viewed as an individual with an opinion captured by the node state and an edge between a pair of nodes implies a friendship between these individuals.

2.2 Complex Networks

In the real-world, it is often observed however that the structure of a network (the nodes it finds itself connected to) changes with time: chemical systems, neural networks, social interacting species and the Internet are but a few examples of this; thus, to better mirror the nuances of the real-world there has been a great interest in the research of complex networks: networks whose structure is irregular, complex and dynamically evolving with time (Boccaletti et al., 2006). Such networks represent a wide range of systems in nature and society and though traditionally such networks have been modeled as random graphs, it is becoming increasingly recognising that the topology and evolution of these networks are governed by robust organizing principles (Albert and Barabási, 2002). Referring back to the friendship example we had introduced earlier, indeed in the real-world friendships are never static and change with time - though the change in friendships can be seen as random, it is usually influenced by underlying principles such as perhaps how often individuals meet (perhaps individuals meeting up more often are more likely to develop a friendship), how agreeable individuals are (perhaps we are more keen to develop a friendship to those similar to us) and many more underlying principles.

2.3 Adaptive Networks

It is noteworthy to observe that when mentioning complex networks above, we discussed networks whose structure (topology) changes dynamically with time. However, complex networks can indeed also refer to dynamical networks where the nodes are individual dynamical systems coupled with static links (Gross and Sayama, 2009). An example of this can be the friendship example continued, where the friendships (i.e., links between individuals) remains the same however each friend/individual changes their opinion with respect to time (again governed by perhaps underlying principles such as who the individual is friends with and what their friends opinions are). However, if we consider the real-world and our recurring example of friendships, it is often not the case that only the opinions of individuals change or only the people they are friends with change. It is usually the case that both change with time - the people we are friends with influence our opinion and also perhaps who we decide to become friends with in the future. Our opinions also influence who we choose as our friends and perhaps who we decide we no longer wish to conduct a friendship with. What we have described in our friendship example is an adaptive network - a network whose links change adaptively with respect to its states, resulting in a dynamic interplay between the state and the topology of the network (Gross and Blasius, 2008).

2.4 Opinion Dynamics

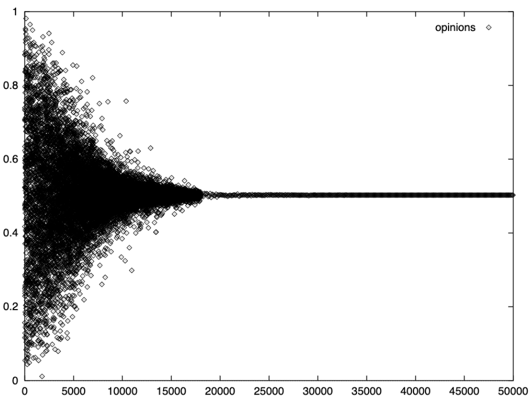

Figure 2.1: Time chart of opinions ($d = 5$, $\mu = 0.5$, $N = 2000$. One time unit corresponds to sampling 100 pairs of agents. Figure extracted from (Deffuant et al., 2000)

This phenomena observed is hardly new and seen in almost all real networks and consequently occurs in many disciplines (Gross and Sayama, 2009). A key discipline which has greatly benefited from adaptive networks is opinion dynamics: a research field utilizing mathematical and physical models and agent-based computational modeling tools to investigate the spreading of opinions in a collection of human beings (Xia et al., 2011). Opinion dynamics is an important area of research because insights about the dynamics of opinion formation are relevant for our understanding of a number of social and economical processes (Brede, 2019), ranging from political campaigns (Javarone, 2014; Hegselmann et al., 2014), the spread of technology (Laciana and Rovere, 2011) and applications in financial markets (Easley and Kleinberg, 2010). The importance of opinion dynamics also extends to insights in information dissemination, viral marketing and targeting messages to users in a network (Das et al., 2014) and the understanding of the birth of extreme ideas and opinions in a society where individuals simply try to conform to social norms within their neighborhood (Sayama, 2020).

Early research within opinion dynamics began with a focus on binary opinions which social actors update under either social influence or according to their own experience; these binary opinion models were well studied and most often displayed uniformity of opinions when interactions occurred across the whole population (Li et al., 2019), (Yildiz et al., 2013) - i.e. a "herd" behaviour (Deffuant et al., 2000). Extending upon this research, Deffuant proposed a model of N agents which rather than holding binary opinions, held continuous opinions; these agents would randomly meet and readjust their opinions when their difference in opinions was smaller than some magnitude D (the readjustment would result in both agents moving towards the opinion of the other agent) (Deffuant et al., 2000). Figure 2.1 shows an example of this model being implemented for N=2000, D=0.5 and $\mu = 0.5$ (where μ is the rate of convergence of one agent to the opinion of another agent provided their opinions are within the given threshold of D=0.5); the figure shows the opinions of all agents eventually converging (Deffuant et al., 2000).

As in the friendship example, it is often the case wherein we are connected to some individuals by a pre-exisiting friendship; our opinion is influenced by those we have a connection or link to and thus rather than having two agents randomly meet as was the case in (Deffuant et al., 2000), the real-world is better mirrored if connections between individuals and how they influence the opinion of individuals with time is taken into consideration.

2.5 Adaptive networks in social dynamics

It can be thus seen easily the benefit adaptive networks provide to opinion dynamics - a high-level possible intuition is the nodes in the network each have a 'state': each node captures an individual and the 'state' represents the opinion of this individual. The edges between the nodes capture the degree to which two nodes interact with each other (and if the edge is weighted, the weight of the edge could capture how frequent these interactions are). Then, with time and governing principles, opinions and connections find themselves changing and we can observe how the structure of a society changes (e.g., observe if 'social bubbles' have been formed) via adaptive networks.

The high-level intuition provided above has found itself implemented extensively in research - the intuition remains the same, however different adaptive networks may act on different governing principles to evaluate and reassess the change of the states and topology of the network. For example, users may iteratively update their respected opinions in accordance to a model termed the BIASED-VOTERMODEL (BVM) for opinion updates (Das et al., 2014) - here the BVM model acts as the governing principles on whose basis the users update their respective opinions. Another example is letting agents chat in a model system with simple local rules (Rosvall and Sneppen, 2006) and a third example is a model where network evolution is jointly determined by the distance between individual opinions and network structure (Dong et al., 2022). Stochastic-agent based models where social evolution is driven by the rewiring or breakage of social contracts also exist (Maia et al., 2021) and many more examples of agent-based models for opinion dynamics exist (Brede, 2019), (Li et al., 2023).

What is noteworthy to observe is the array of social dynamics which come into play for different models when deciding how the network evolves. For example, the BVM model briefly mentioned is derived from understanding how users update their opinion with respect to their neighbor's opinion (Das et al., 2014). The stochastic-agent based model discussed focuses on bounded confidence - opinions which are continuously valued and influenced only by neighboring nodes whose opinion are within their confidence bound (Li et al., 2023) - and homophily (tendency to bond with individuals similar to yourself) mechanisms (Maia et al., 2021) and some models may even focus on agents who rewire their connections with a given goal in mind (Brede, 2019).

Most models thus focus on conformity (users updating their opinion with respect to their local neighborhood) and homophily (users strengthening their connections to other users with similar opinions). It can be observed nonetheless in the real-world that if an individual is different to us, we do not necessarily shun them out but perhaps their difference in opinion is intriguing to us and we wish to get to know them further. Some models do account for this by including an attention to novelty influence in their social dynamics/governing principles where individuals with a high attention to novelty are more likely to connect with individuals with differing views (Sayama, 2020).

2.6 Multiple Opinions

What can be observed here is how most models mentioned and most models researched focus on the 'opinion' of each individual being a single value. However, in the real-world we have a collection of opinions: from our food preferences, political ideologies, religious beliefs and even our favorite flavor of ice-cream. Hence, it can be seen that our opinions are multi-dimensional. It would thus be interesting to observe how with time our connection and opinions change if we look into multi-dimensional opinions. It will be interesting to see what governing principles are chosen to drive social evolution, the justification of these principles and the societies they lead to.

Some work in literature has been done in multi-dimensional opinion dynamics: for example, a novel multidimensional extension of an existing model where it is assumed opinions in each dimension are interdependent (Parsegov et al., 2017). Another example is multidimensional opinions under bounded confidence intervals (Lorenz, 2007). Some examples do not work on the foundation of an existing model with one dimensional opinions, but rather propose a model directly for multi-dimensional opinions (Gubanov et al., 2021). Most examples encountered however focus on multi-dimensional opinions in bounded confidence models (Lorenz, 2008). These models tend to evolve into a consensus (well-connected network) under certain conditions as the dimensionality increases (e.g., the model described in (Lorenz, 2007) 'fosters a consensus when inter-related issues are bought into discussion and prevents a consensus when independent issues are brought into discussion' and the model described in (Lorenz, 2008) 'fosters consensus if issues are under a budget constraint but diminishes consensus otherwise').

Thus, there is limited research still in multi-dimensional opinion dynamics which deviate away from bounded confidence models - such models have individuals only affected by their neighbors if their opinion is within a confidence bound (Li et al., 2023) however as discussed previously, further social dynamics can be considered such as attention to novelty. Also, as individuals ourselves we do not necessarily shun out those whose opinion is not within a given 'bound' to ours (sometimes, like attention to novelty describes, we may be intrigued by individuals differing from ourselves and other times we may be highly conformable and be easier willing to accept our neighborhood's local opinion). Additionally, dimensions can also be independent of one another (e.g., political ideology and food preferences do not seem as if they'd overlap). Thus, it is worth factoring these points into account.

We thus build upon (Sayama, 2020) 'computational agent-based model of adaptive social network dynamics investigating the non-trivial social dynamics by which extreme ideas can arise among well-meaning individuals'. We choose Sayama's model because it focuses on non-trivial social dynamics to guide the evolution of the network: conformity, homophily and neophily (attention to novelty) which are dynamics that can be seen in the real-world. We begin by replicating the results produced by Sayama - as Sayama did, we 'conduct systematic parameter-sweep numerical experiments and regression analyses to elucidate effects of several individual behaviors (conformity, homophily and novelty) on ideological dynamics on social networks' (Sayama, 2020). We then extend to the multi-dimensional case, tweaking the original governing principles which drive social evolution to fit the multi-dimensional cases and like before, 'conducting systematic parameter-sweep numerical experiments to observe the effects of conformity, homophily and novelty on the evolution of the network' (Sayama, 2020).

Chapter 3

Base Model

The base model is an adaptive network and captures individuals in a society, wherein each individual holds an opinion and is derived from (Sayama, 2020). Below the structure of the adaptive network and the principles which govern the evolution of opinions and connections within the network is detailed. The structure of the model and the mathematical formulae discussed below all are from (Sayama, 2020).

3.1 Structure of the Model

This paragraph summarises the structure of the model and the information within this paragraph is derived from (Sayama, 2020). The network consists of N nodes. Each node in the network has its own state ,$x_i \in \mathbb{R}$, and the node and its state represent an individual and their opinion. Between nodes there exist weighted directed edges: in particular $w_{ij} \in \mathbb{R}_0$ is the weight of an edge from node j to node i which captures individual j conveying their opinion to individual i; the weight of this edge captures the strength of the connection from individual j to individual i. The base model works on a basic assumption as follows: for each node i in the network, this node conforms gradually to its perceived weighted average state (i.e., for all the nodes j conveying their opinion to node i, node i will gradually adopt the weighted average state of all nodes j) whilst also changing edge weight based on its state.

For example, if an individual A is receiving opinions from other individuals (say B and C), then this individual finds themselves gradually adopting an average of the opinions they are exposed to. If the strength of the connection from individual B to individual A is greater than the strength of the connection from individual C to individual A, then the exposure to individual B's opinion has a greater effect on individual A, and thus the assumption works on individual A slowly adopting the weighted average perceived opinion.

3.2 Principles Governing Co-Evolution of Opinions and Connections

The principles and their mathematical formulae discussed in this section are all derived from (Sayama, 2020).

There are 3 key principles which guide the co-evolution of the states of the nodes (i.e. opinions of individuals) and the edge weights (i.e. connections between individuals). These 3 principles are as follows: 1. Social Conformity 2. Homophily 3. Attention to Novelty. Before delving into the mathematical operation of these principles, we denote $\langle x \rangle_i$ to refer to the weighted average opinion perceived by node i.

Thus $\langle x \rangle_i = \frac{\sum_{j \in N_i} w_{ij} x_i}{\sum_{j \in N_i} w_{ij}}$

Note N_i refers to the set of nodes j which convey their opinion to node i.

Mathematically the principles of [1] social conformity and [2] homophily and attention to novelty operate as follows:

$\frac{dx_i}{dt} = c(\langle x \rangle_i - x_i) + \epsilon$ [1]

$\frac{dw_{ij}}{dt} = hF_h(x_i, x_j) + aF_a(\langle x \rangle_i, x_j)$ [2]

where:

$F_h(x_i, x_j) = \theta_h - |x_i - x_j|$ [3]

$F_a(\langle x \rangle_i, x_j) = |\langle x \rangle_i - x_j| - \theta_a$ [4]

ϵ is a small random fluctuation term to account for randomness observed in the real-world.

3.3 Principle 1 - Conformity

We begin by discussing principle [1] derived from (Sayama, 2020): $\frac{dx_i}{dt} = c(\langle x \rangle_i - x_i) + \epsilon$.

For a given node i, [1] tells us the rate at which the state of node i moves towards its perceived weighted average state $\langle x \rangle_i$. The parameter c measures the strength of conformity of a node i: the larger the strength of conformity, the larger [1] is and thus the faster the state of node i will move in the direction of its perceived weighted average state. Likewise, the smaller the strength of conformity is, the smaller [1] is and the state of node i moves slowly in the direction of the perceived weighted average state. In terms of viewing this from the lens of individuals, the larger the conformity of an individual, the quicker this individual adopts the views of the opinions they are surrounded by and the less the conformity of an individual, the longer they take to adopt the views of the opinions they are surrounded by. Thus, after a small time step, the opinion of node i becomes $x_i + c(\langle x \rangle_i - x_i) + \epsilon$ where x_i was the original opinion of node i. We observe if c = 1 and $\epsilon = 0$ then node i would completely adopt its perceived average state.

3.4 Principle 2 - Homophily and Attention to Novelty

We now discuss principle [2] derived from (Sayama, 2020) : $\frac{dw_{ij}}{dt} = hF_h(x_i, x_j) + aF_a(\langle x \rangle_i, x_j)$ where

$F_h(x_i, x_j) = \theta_h - |x_i - x_j|$ [3]

and

$F_a(\langle x \rangle_i, x_j) = |\langle x \rangle_i - x_j| - \theta_a$ [4]

3.4.1 Discussing h and a

For nodes i and j, [2] captures the rate of change of the strength of a connection from node j to node i. The parameters h and a involved in [2] measure the strength of homophily and attention to novelty of node j respectively. As mentioned in the background, homophily focuses on similarity in opinions and attention to novelty focuses on the difference in the perceived weighted average opinion of individual i and the opinion of individual j (i.e. if the perceived weighted average opinion of individual i is different to the opinion of individual j then this implies individual i is exposed to a greater array of opinions from other individuals, thus they stand out from individual j as 'intriguing' leading individual j to strengthen the bond between them). The larger the parameter h is, the larger the strength of homophily is for node j, thus implying node j to require the state of node i to be very similar to its own state in order for the connection to be strengthened. For h smaller, node j is more flexible in how similar the state of node i is. If parameter a is large, this implies the attention of novelty of node j is large, hence node j is likely to strengthen their connection to node i for differences in the state of node j to the local perceived weighted average of node i. If however a is small, then node j has a small attention to novelty and may not be as willing to strengthen the connection to node i for differences in the state of node j to the local perceived weighted average of node i.

3.4.2 Discussing [3] and [4]

We observe the parameters both being multiplied to functions [3] and [4] respectively, both derived from (Sayama, 2020). Firstly, we discuss [3]: $F_h(x_i, x_j) = \theta_h - |x_i - x_j|$. To capture the effect [3] produces on [2], we assume the following: θ_h, $aF_a(\langle x \rangle_i, x_j)$ and h to be fixed values. Then as $|x_i - x_j|$ increases, $F_h(x_i, x_j)$ decreases which implies $hF_h(x_i, x_j)$ decreases which thus implies [2] decreases (working under our assumption). Thus, as the difference in states between node i and node j increases, we deduce the rate at which the connection is strengthened from node j to node i decreases (or perhaps could even be negative and thus the connection weakens). Now, keeping our previous assumptions intact but replacing the assumption of θ_h fixed with the assumption of $|x_i - x_j|$ fixed, we deduce that as θ_h increases F_h increases and hence hF_h increases and hence [2] increases, thus the rate at which our connection is strengthened increases. Hence, [3] determines the rate of change of the edge connection from node j to node i with respect to the difference in state between node i and node j, wherein this difference has a negative effect on the strength of the connection.

Next, we discuss [4]: $F_a(\langle x \rangle_i, x_j) = |\langle x \rangle_i - x_j| - \theta_a$. To capture the effect [4] produces on [2], we likewise hold similar assumptions: $hF_h(x_i, x_j)$, θ_a and a are fixed values. Then as $|\langle x \rangle_i - x_j|$ increases, $F_a(\langle x \rangle_i, x_j)$ increases which implies $aF_a(\langle x \rangle_i, x_j)$ increases which thus implies [2] increases (working under our assumption). Thus, as the difference in state between node j and the local weighted average opinion perceived by node i increases, we deduce the rate at which the connection is strengthened from node j to node i increases. Thus, the more different the perceived weighted average opinion of individual i is from individual j, the more it stands out and comes under the attention to novelty eye of node j. Now, likewise to before, keeping our previous assumptions intact but replacing the assumption of θ_a fixed with the assumption of $|\langle x \rangle_i - x_j|$ fixed, we deduce that as θ_a increases, $F_a(\langle x \rangle_i, x_j)$ decreases and hence $aF_a(\langle x \rangle_i, x_j)$ decreases and hence [2] decreases, thus the rate at which our connection is strengthened decreases (or becomes negative and thus the connection weakens). Hence, [4] determines the rate of change of the edge connection from node j to node i with respect to the difference in the local perceived weighted state of node i and the state of node j, wherein this difference has a positive effect on the strength of the connection.

3.4.3 Reiterating effect of h and a

To reiterate the effect of the parameters h and a observe if we have $hF_h(x_i, x_j) = h(\theta_h - |x_i - x_j|)$ and we have h_2 larger than h, then for the same θ_h, to have $hF_h(x_i, x_j) = h_2 F_{h_2}(x_i, x_j)$ we would require $|x_i - x_j|$ to be smaller. Thus, as discussed previously, the larger h is, the more similarity is required in the opinion of two individuals for a connection to be strengthened. Likewise, if we have $aF_a(\langle x \rangle_i, x_j) = a(|\langle x \rangle_i - x_j| - \theta_a)$ and we have a_2 larger than a, then for the same θ_a, to have $aF_a(\langle x \rangle_i, x_j) = a_2 F_{a_2}(\langle x \rangle_i, x_j)$ we would require $\langle x \rangle_i - x_j|$ to be larger. Thus, as discussed previously, the larger a is, the greater the attention to novelty of individual j is and the less difference is required between the perceived weighted average opinion of individual i and the opinion of individual j for a connection to be strengthened.

3.5 Base Model Summarised

To summarise we have an adaptive network from (Sayama, 2020) as follows:

A network of N nodes, weighted directed edges between the nodes representing opinions being conveyed, principles for the co-evolution of the states and connections of the nodes. The principles are: social conformity, homophily and attention to novelty respectively whose mathematical underpinnings we have just discussed. In essence, conformity captures individuals gradually adopting the weighted average of the opinions they are exposed to, homophily captures strengthening (or weakening) connections based on similarity in opinions and attention to novelty captures strengthening (or weakening) connections based on differences in the average opinion one individual is exposed to and the opinion of another individual.

3.6 Implementing the Model

Following how Sayama implemented the base model (Sayama, 2020) we do as follows:

We begin by implementing the adaptive network in Python 3.7 with NetworkX. The following parameter combinations are to be tested $\{c, h, a, \theta_h, \theta_a\} \in 0.01, 0.03, 0.1, 0.3$ for $n = 100$ (i.e. for a network with 100 nodes). We conduct numerical simulations to systematically test all combinations of these parameter values. For each combination, we do five independent simulation runs thus resulting in 4*4*4*4*4*5 = 5020 total number of simulations.

Initially, the network is configured so each pair of nodes are connected by a directed edge with weights randomly sampled from the uniform distribution [0,1]; each node has a random node state sampled from the standard normal distribution. Each simulation run is conducted for 1000 time steps where each time step lasts for 0.1 seconds. In each time step, we change the state of the nodes and the edge weights between nodes as governed by the principles discussed earlier. So, for the principle of conformity, if x_i is the current state of node i, then in a given time step, $x_i = x_i + (c(\langle x \rangle_i - x_i) * 0.1)$. Similarly, for the principle of homophily, if w_{ij} is the opinion being conveyed from node j to node i then in a given time step $w_{ij} = w_{ij} + (hF_h(x_i, x_j) * 0.1)$ and likewise for the principle of attention to novelty, $w_{ij} = w_{ij} + (aF_a(\langle x \rangle_i, x_j) * 0.1)$. We are multiplying by 0.1 because each time step lasts for 0.1 seconds. Once a time step has been complete, we add a small effect of randomness to the state of each node - this randomness is denoted as ϵ which we draw from a normal distribution with mean 0 and standard deviation 0.01.

3.7 Outcome Measures

As Sayama did, for each simulation run, once 1000 time steps have been complete, we deduce outcome measures for the final evolved adaptive network (Sayama, 2020). These outcome measures are as follows and are derived from (Sayama, 2020):

1. Average edge weight
2. Number of communities
3. Modularity of communities
4. Range of average community states
5. Standard deviation of average community states

3.7.1 Outcome Measure 1

Outcome measure 1 calculates the arithmetic average of all the edge weights in the network; a larger average edge weight is indicative of on average stronger connections between individuals.

3.7.2 Outcome Measure 2 and Outcome Measure 3

Networks find themselves naturally dividing into communities or modules (Newman, 2006). The modularity of a network 'measures the extent to which edge density is higher within parts than between parts' (Skerman, 2015) and captures how much communication there is between individuals of differing communities. The larger the modularity is, the more sparse the connections are between communities. Likewise, the smaller the modularity, the more dense the connections are between communities. Understanding the structure of communities and the modularity of networks can give us information on how opinions and connections have evolved with respect to time. For example, the more communities seen suggest lots of 'social bubbles' have been formed, suggesting there are clusters of individuals with differing views. A larger modularity is indicative of greater fragmentation within a network, since it suggests different communities have little communication between each other whereas a smaller modularity suggests well-connected communities tolerant to differing views.

Algorithms exist to calculate the number of communities and the modularity of communities within a network Aksoylar et al. (2017), (Ding et al., 2020), (Chaudhary and Singh, 2020). To keep consistency with Sayama (Sayama, 2020) we detect the community structure using the Louvain modularity maximisation method Blondel et al. (2008). Since this method works only on undirected graphs, we convert our directed graph to an undirected graph by averaging the weight of two directed edges between a pair of nodes into one undirected weight. So for example the undirected weight of an edge w_{ij} would become $\frac{w_{ij}+w_{ji}}{2}$ for the directed weights w_{ij} and w_{ji}. If there exists only one directed edge between a pair of nodes, we set this to be the undirected weight.

3.7.3 Outcome Measure 4 and Outcome Measure 5

Once all the communities have been found, we then find the average state of each community. Outcome measure 4 then calculates the difference between the smallest and largest average community states and outcome measure 5 calculates the standard deviation of the average community states (e.g. if we have 3 communities each with an average community state of x_1 , x_2 and x_3 then let $\bar{x} = \frac{x_1+x_2+x_3}{3}$ giving us the standard deviation to be $\frac{1}{3}\sum_{j=1}^{3}(x_j - \bar{x})^2$ and the range to be $max(x_1, x_2, x_3) - min(x_1, x_2, x_3)$.

Outcome 4 and outcome 5 give an indication of how extreme the network has become; the larger the range and standard deviation, the more extreme ideas are produced in the social network.

3.7.4 Summary of Outcome Measures

Thus, in essence outcome measure 1 gives an idea as to how well connected individuals are within the network. Outcome measures 2 and 3 help characterise the levels of fragmentation within the network and outcome measures 4 and 5 characterise the extreme ideas produced within the network.

Chapter 4

Extended Model

4.1 Structure of the Model

In the base model we derived from Sayama, (Sayama, 2020) we focused on an adaptive network with N nodes, wherein each node has a state and directed weighted edges between pairs of nodes. The state of each node x_i was a real-valued number (i.e. $x_i \in \mathbb{R}$). The nodes represented individuals and their states the opinion of an individual; the directed weighted edges represented the conveying of an opinion from one individual to another, as well as the strength of connection between two individuals.

We now extend the base model to form our extended model as follows: our extended model is an adaptive network of N nodes, wherein each node represents an individual. Directed weighted edges exist between pairs of nodes conveying the sharing of an opinion of one individual to another individual as well as indicating the strength of a connection between two individuals. However, unlike in the base model, the state of each node is as follows: $x_i \in [x_{i1}, ..., x_{iD}]$.

Thus, each node can hold multiple opinions (e.g., a political opinion, a religious opinion, a food preference and so forth). The opinion in dimension 1 (i.e. x_{i1}) may represent political opinions, the opinion in dimension 2 (i.e. x_{i2}) may represent religious opinions and so forth. Thus, each dimension represents a different 'topic' (such as politics is represented in dimension 1, food in dimension 2 etc) and the state of opinions within a given dimension represent the opinion of individuals with respect to that given topic. An edge existing from node j to node i thereby captures individual j sharing all their opinions to individual i (i.e. individual i is exposed to the political, religious, food preference and so forth opinions of individual j).

The base model worked on the basic assumption of the state of each node i conforming gradually to its perceived weighted average opinion with time. For the extended model, we firstly make the assumption each dimension is independent (for example, say dimension 1 captures political opinions and dimension 2 captures food preferences then it is perhaps assumed the political opinion of an individual and their food preference hold little dependence or overlap between them). From this assumption, we then assume for each $d \in 1, ..., D$ the state of each node i in dimension d conforms gradually to its perceived weighted average opinion in dimension d (i.e., the political opinion of an individual changes slowly to adopt the political opinions this individual is surrounded by and likewise the religious opinion of an individual changes slowly to adopt the religious opinions they are surrounded by). We can view this as extending the assumption in the base model and fusing this with the assumption of each dimension being independent to have the assumption in the base model hold for each dimension in the extended model.

4.2 Principles Governing Co-Evolution of Opinions and Connections

The three key principles of conformity, homophily and attention to novelty guide the co-evolution of opinions and connections as they did in the base model - however due to individuals holding multiple opinions, the mathematical underpinnings of these principles had to be tweaked. Before we discuss the mathematical underpinnings of each principle, we introduce some useful notation below:

x_{id} denotes the state of node i in dimension d (i.e. the opinion of individual i with respect to the topic relevant to the dth dimension e.g., politics).

$\langle x \rangle_{id}$ denotes the perceived weighted average opinion of node i in dimension d and is as follows

$\langle x \rangle_{id} = \frac{\sum_{j \in N_i} w_{ij} x_{jd}}{\sum_{j \in N_i} w_{ij}}$ where N_i is the set of nodes sending their opinion to node i

$[\epsilon_1, ..., \epsilon_D]$ denotes small random fluctuations in each dimension, to account for the randomness of the real-world

4.3 Principle 1 - Conformity

4.3.1 Recap of Conformity

In the base model, where the states are one-dimensional (i.e. each individual holds a single opinion) our basic assumption is the state of each node gradually conforms to their perceived weighted average state; the principle of conformity guided the rate at which this gradual adoption happened, and was mathematically expressed as follows:

$\frac{dx_i}{dt} = c(\langle x \rangle_i - \langle x \rangle_i) + \epsilon$

wherein c measures the strength of conformity: the larger c is, the faster the opinion of node i moves in the direction of their perceived weighted average state and likewise the smaller c is, the slower the opinion of node i moves in the direction of their perceived weighted average state. We accounted for random fluctuations in the real-world with the small term ϵ.

4.3.2 Conformity in the Extended Model

In the extended model, our basic assumption now is for each dimension d in $1, ..., D$, the state of each node in that given dimension conforms to their perceived weighted average state within that dimension. Likewise as before, the conformity of an individual will guide at which rate this gradual adoption happens. We assume the conformity of an individual to be the same value independent of the dimension the individual finds themselves in and thus for $din1, .., D$ we mathematically express the principle of conformity for multiple opinions as follows:

$\frac{dx_{id}}{dt} = c(\langle x \rangle_{id} - x_{id}) + \epsilon_d$

wherein c holds the same effect and implication as it did in the base model. The larger c is, the greater the strength of conformity of an individual is. For say 2 dimensions (e.g., say the first dimension represents political opinion and the second dimension represents food preference) the larger the conformity of an individual is, the quicker they adopt the political and food preference opinions they are exposed to and likewise the smaller the conformity of an individual, the slower this individual adopts the political and food preferences they are exposed to. Once again ϵ_d accounts for random fluctuations in opinions in the dth dimension.

4.4 Principle 2 - Homophily and Attention to Novelty

4.4.1 Recap of Homophily and Attention to Novelty

In the base model, the principles of homophily and attention to novelty guided the evolution of the edges between nodes and the mathematical underpinnings of these principles were as follows:

$\frac{dw_{ij}}{dt} = hF_h(x_i, x_j) + aF_a(\langle x \rangle_i, x_j)$ where

$F_h(x_i, x_j) = \theta_h - |x_i - x_j|$ and $F_a(\langle x\rangle_i, x_j) = |\langle x\rangle_i - x_j| - \theta_a$

These principles measured the rate of change of the edge from node j to node i with respect to the homophily, attention to novelty, θ_h and θ_a parameters of node j, the difference in state of node j and node i and the difference in the perceived average state of node i and the state of node j.

Intuitively, as discussed in the base model section, the parameter h measured the strength of homophily of node j - the larger h is, the greater similarity in opinion is required from node i for the connection to be strengthened. Parameter a measured strength of attention to novelty, and the larger a is the greater attention to novelty node j possesses, thus less difference is required between the perceived weighted average state of node i and the state of node j for a connection to be strengthened. Parameters θ_h and θ_a guided the rate at which the connection was strengthened with a larger θ_h suggesting node j to strengthen the connection to node i quicker and a larger θ_a suggesting node j to strengthen the connection to node i slower. Understanding why these parameters have these effects from the mathematical equations was discussed in the base model section.

4.4.2 Homophily and Attention to Novelty in Extended Model

We would like to retain the principles of homophily and attention to novelty in the extended model; thus we require each node to have a homophily and an attention to novelty which we assume is independent of the dimension they find themselves in. The homophily in the base model captured how similar an individual requires the opinions of another individual to be in order for connections to become strengthened. Consider now two individuals: individual A and individual B. Then in the base model, we could easily find the difference in their opinion and with respect to the homophily of individual A, strengthen the connection from individual A to individual B accordingly. However, suppose individual A and individual B both have opinions in politics and sport (so we are in the case of our extended model with two dimensions). There will be a difference in the political opinions of individual A and B and likewise difference in the opinion of sport of individual A and individual B. The question arising is how does individual A choose to strengthen their connection to individual B with respect to their homophily. Do they choose to focus on the dimension where the opinions differ the most, and as a consequence the connection finds itself not strengthening as much as was possible than if they had focused on the dimension where the opinions were most similar. Or does individual A take account of all the differences in opinions in each dimension and use all the differences in opinions (e.g., by taking an average of the differences) to decide upon how much to strengthen their connection?

A similar question arises when focusing on the attention to novelty. Continuing with our two individuals A and B, suppose individual A has some attention to novelty. Now suppose the political opinions individual B is exposed to is very different from the political opinion of individual A and likewise the sport opinions individual B is exposed to is similar to the sport opinion of individual A. The same question finds itself arising: which opinion does individual A focus on with respect to their attention to novelty. Do they focus on the dimension wherein individual B's exposed opinion is similar to theirs, thus resulting is less strong of a connection than if they had focused on a dimension wherein individual B's exposed opinion was much different to theirs. Or, do they look at the difference between theirs and individual B's exposed opinion in all dimensions to guide their strengthening or weakening of their bond to individual B?

To answer these questions, we consider all possible options which individuals could take and refer to these options as policies (and assume each individual abides by the same policy) and introduce two new parameters to capture these options. The first parameter we refer to as a 'homophily updater' and the second a 'attention to novelty updater'. These parameters guide in assisting individuals on which dimensions to focus on when changing their connection to other individuals. we focus on the following cases:

1. Homophily Updater = MAX , Attention to Novelty Updater = MIN
2. Homophily Updater = MIN, Attention to Novelty Updater = MAX
3. Homophily Updater = MIN, Attention to Novelty Updater = MIN
4. Homophily Updater = MAX, Attention to Novelty Updater = MAX

As a simplification, we write H:X, A:Y for X, Y $\in$ [MAX MAX, MAX MIN, MIN MAX, MIN MIN] where H denotes the homophily updater, A denotes the neophily updater and X and Y are the respective values of these updaters. Suppose we have two individuals A and B and two topics: politics and sport. For the homophily updater = MIN we have as follows: individual A observes the difference in their political opinion to the political opinion of individual B and the difference in their sports opinion to the sports opinion of individual B. Individual A then chooses the dimension wherein the difference in opinion is the smallest (call this dimension d_1). For homophily updater = MAX, individual A would choose the dimension where the difference in the opinion is maximum (call this dimension d_2). Likewise, individual A observes the difference in their political opinion to the perceived weighted average political opinion of individual B and the difference in their sport opinion to the perceived weighted average sport opinion of individual B. For the neophily updater = MAX, individual A chooses the dimension where this difference is maximal (call this dimension d_3) and for neophily updater = MIN, individual A chooses the dimension where this difference is minimal (call this dimension d_4). Suppose we have H:MAX, A:MIN. So individual A chooses dimension d_2 and dimension d_4 when changing their connection with respect to homophily and attention to novelty. Then, individual A changes their connection with rate $\frac{dw_{ij}}{dt} = hF_h(x_{id_1}, x_{id_1}) + aF_a(< x >_{id_4}, x_{jd_4})$ where $F_h(x_{id_1}, x_{id_1})$ and $F_a(< x >_{id_4}, x_{jd_4})$ are unchanged from the base model.

Thus, the policies determine which dimension individuals focus on when changing their connections with respect to homophily and neophily.

4.5 Summary of Extended Model

To summarise we have:

A network of N nodes, wherein each node holds multiple opinions. There exist weighted directed edges between the nodes representing opinions being conveyed; the principles of social conformity, homophily and attention to novelty still guide the co-evolution of states and edge weights. The principle of conformity now captures each node gradually adopting the weighted average of the opinions they are exposed to in each dimension (e.g., the political opinion of an individual slowly conforms to the political opinions they are exposed to, likewise the sports opinion of an individual slowly conforms to the sports opinions they are exposed to). The principles of homophily and attention to novelty are now introduced to two new parameters: a homophily updater and an attention to novelty updater which guide which dimension individuals focus on when strengthening their bonds with respect to similarity in opinion and which dimension individuals focus on when strengthening their bonds with respect to attention to novelty. The extended model focuses on the following cases (which we term as policies): (H:MAX, A:MIN), (H:MIN, A:MAX), (H:MIN, A:MIN) and (H:MAX, A:MAX) wherein the first principle has individuals focusing on dimensions where other individuals differ the most when they connect with respect to homophily, and focusing on dimensions where the perceived weighted average opinion of other individuals is similar to theirs when connecting with respect to attention to novelty. The second principle has individuals focusing on dimensions where other individuals are most similar when they connect with respect to homophily, and focusing on dimensions where the perceived weighted average opinion of other individuals is most different to theirs when connecting with respect to attention to novelty. Likewise, the third and fourth principles have homophily updater = attention to novelty updater = MIN and homophily updater = attention to novelty updater = MAX.

4.6 Implementing the Extended Model

As in the base model, we implement the extended model in Python 3.7 using NetworkX. We implement four instances of the extended model: the first instance focuses on the H:MAX, A:MIN, the second on the H:MIN, A:MAX, the third on the H:MIN, A:MIN and the fourth on the H:MAX, A:MAX policy. For each policy, we implement the extended model for D= 2, 5 and 10 dimensions. For each policy and dimension, the following parameter combinations are tested $c, h, a, \theta_h, \theta_a \in \{0.01, 0.03, 0.1, 0.3\}$ for $n = 100$ (i.e. for a network with 100 nodes). We conduct numerical simulations to systematically test all combinations of these parameter values. As we had in the base model, the network is configured so each pair of nodes are connected by a directed edge with weights randomly sampled from the uniform distribution [0,1]. Each node x_i has a D dimensional random node state as follows $[x_{i1}, .., x_{iD}]$ where each x_{ij} for $j \in 1, .., D$ is sampled from the standard normal distribution.

For each parameter combination, we do 5 independent simulation runs. Each simulation run is conducted for 1000 time steps where each time step lasts for 0.1 seconds. In each time step, we change the state of the nodes and the edge weights between nodes as governed by the policies discussed earlier. So, for the principle of conformity, if $[x_{i1}, .., x_{iD}]$ is the current state of node i, then in a given time step for each $d \in 1, .., D$, $x_{id} = x_{id} + (c(\langle x \rangle_{id} - x_{id}) * 0.1)$. Then based on what policy we are abiding by, we choose our dimensions d and p as the dimensions to focus on for homophily and attention to novelty (e.g., if in the H:MAX, A:MIN case then d would be the dimension where two individuals differ the most in opinion and p would be the dimension where one individuals perceived weighted average opinion is most similar to another individual). Thus, for the principle of homophily, if w_{ij} is the opinion being conveyed from node j to node i then in a given time step $w_{ij} = w_{ij} + (hF_h(x_{id}, x_{jd}) * 0.1)$ and likewise for the principle of attention to novelty, $w_{ij} = w_{ij} + (aF_a(< x >_{ip}, x_{jp}) * 0.1)$. We are multiplying by 0.1 because each time step lasts for 0.1 seconds. Once a time step has been complete, we add a small effect of randomness to the state of each node - this randomness is denoted as $\epsilon = [\epsilon_1, .., \epsilon_d]$ where each ϵ_i is drawn from a normal distribution with mean 0 and standard deviation 0.01.

4.7 Outcome Measures

For each simulation run, once 1000 time steps have been complete, we deduce outcome measures for the final evolved adaptive network. These outcome measures are as follows:

1. Average edge weight
2. Number of communities
3. Modularity of communities
4. Range of average community states
5. Standard deviation of average community states

The first three outcome measures were derived exactly as they been in the base model. Details of how they have been derived are written in the base model, and in essence outcome 1 summaries on average how well connected individuals are, outcomes 2 and 3 summarise the levels of fragmentation present within a network.

Outcomes 4 and 5 in the base model were derived as follows: for each community, the average opinion within the community was found and then the range and standard deviations of these average opinions were calculated. The aim was to capture extreme ideas being produced in different communities, with a larger range and standard deviation indicative of greater variation in different community ideas and thus greater extremism within the network. To capture this essence for the extended model we do as follows: for each community, we calculate the average community state per dimension (e.g., say we have three communities, let $[\bar{a_{i1}}, .., \bar{a_{iD}}]$, $[\bar{b_{i1}}, .., \bar{b_{iD}}]$, $[\bar{c_{i1}}, .., \bar{c_{iD}}]$ denote the average community state of each dimension for each community). Then, we calculate the range and standard deviation of each dimensions average community state as we had done for the base model. Thus we have D ranges and D standard deviations, where range and standard deviation i denotes the range and standard deviation of the average community states in dimension i. We then take the average of all the ranges as outcome measure 4 and the average of all the standard deviations as outcome measure 5.

Chapter 5

Base Model - Results

Figure 5.1 focuses on results derived by (Sayama, 2020) for a network of n = 100 nodes. Figure 5.1 shows the relationships between the five model parameters and the five outcome measures; each dot in Figure 5.1 corresponds to the average of five identical simulation runs (note dots may lie on top of each other for same values). Each column focuses on a specific parameter value and each row on a specified outcome measure. Each graph then shows the relationship between the parameter value and the outcome measure (e.g., the top left graph shows increasing conformity decreases average edge weight). Figure 5.1 also consists of trend lines obtained by linear regression for statistically significant correlations with $p < 10^{-4}$.

Figure 5.2 follows the same format as figure 5.1, however figure 5.2 is derived from the base model implementation for a network of $n = 100$ nodes. The slight difference here is each dot in figure 5.2 now represents a single simulation run, rather than the average of five simulation runs.

Figure 5.3 shows the result for multiple linear regression of each outcome measure on model parameters and their interactions derived from (Sayama, 2020) (so for example, the first column in Figure 5.3 finds the coefficients $\alpha, \beta, ...$ best fitting the following model: average edge weight = $\alpha c + \beta a + ...$ for all parameters and their interactions from the observed data of all simulation runs). Statistically significant coefficients - i.e. coefficients which have a greater impact on a given outcome measure - are indicated with asterisks (***: $p < 10^{-4}$, **: $p < 10^{-3}$, *: $p < 10^{-2}$). The smaller p is, the greater the impact of the parameter on the final outcome measure. Coefficients of model parameters with the largest and second largest effect sizes are indicated in bold (red: positive, blue: negative).

Figure 5.4 follows the same format as figure 5.3, however figure 5.4 is derived from the base model implementation for a network of $n = 100$ nodes.

We observe both Figure 5.1 and figure 5.2 show the same general patterns for all graphs (e.g., as conformity increases, average edge weight decreases, θ_h and θ_a both show little effect in comparison to c, h, and a for all outcome measures, as homophily increases, number of communities increases etc).

Figure 5.3 and figure 5.4 both show similar values for the model parameters with the largest effect sizes; both tables reinforce h and a to be the two parameters with the largest effects on social network evolution. Small discrepancies can be noticed (for example, the coefficient of $a\theta_a$ for outcome measure modularity has a value of 0.033 in (Sayama, 2020) however has a value of 0.1 from our implementation). Another example is the coefficient of $c\theta_a$ for outcome measure range of average community states has a value of 7.8 in (Sayama, 2020) however has a value of 4.8 from our implementation. These discrepancies may be a result of randomness in the model (initially edge weights and states of nodes are assigned random values from a uniform [0,1] and standard normal distributions respectively). Most values however are similar in our implementation and (Sayama, 2020), especially the statistically significant values.

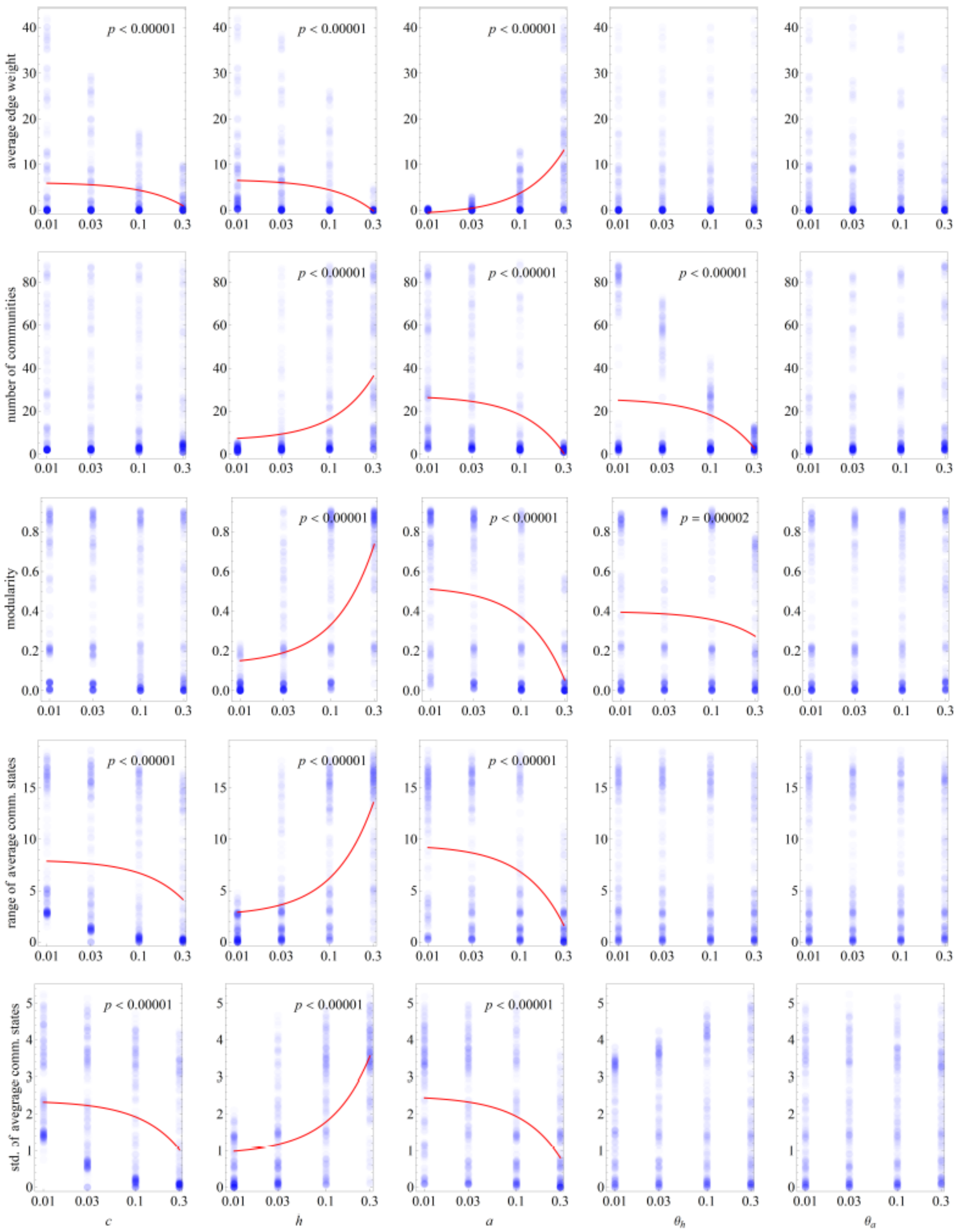

Figure 5.1: Relationships between model parameters and outcome measures for networks of n = 100 derived from (Sayama, 2020)

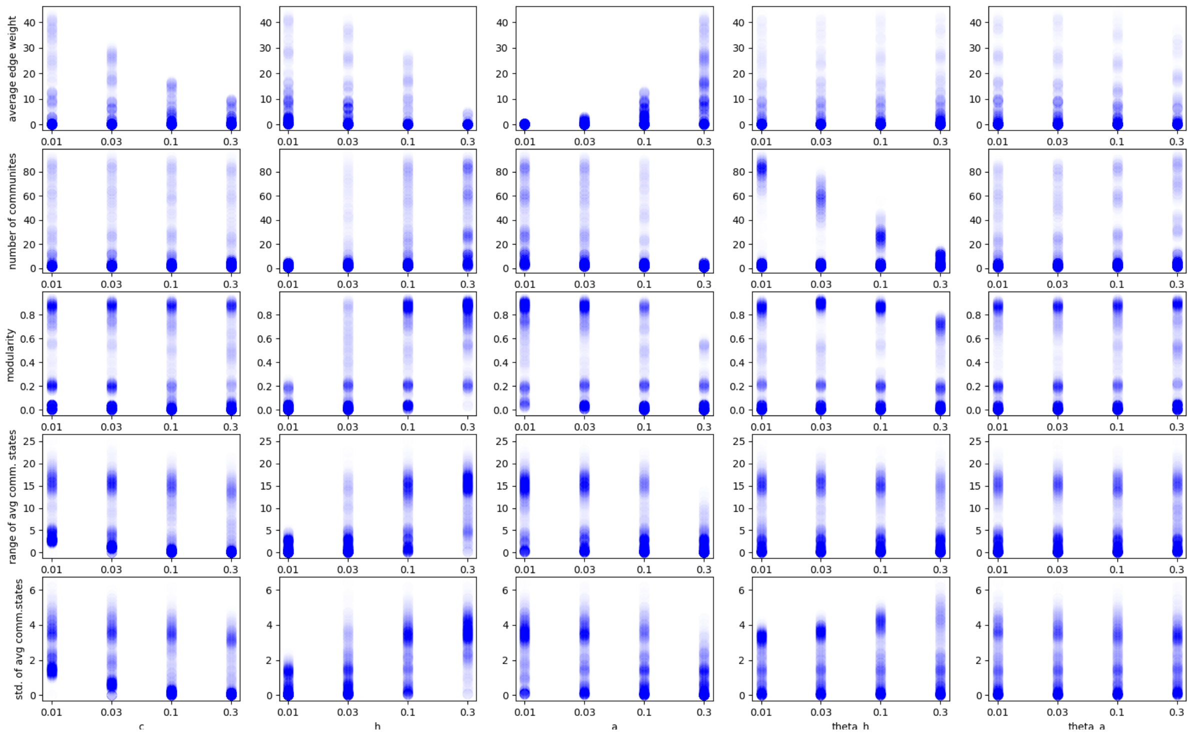

Figure 5.2: Relationships between model parameters and outcome measures for networks of n = 100 : base model implementation

Outcome measure	average edge weight	number of communities	modularity	range of average comm. states	std. of average comm. states
const.	-0.336133	17.9348	0.306332	6.36077	1.92395
c	-6.32563^{***}	-19.9322	-0.311658^{***}	-14.0533^{***}	-4.52717^{***}
h	$\mathbf{-11.8516}^{***}$	$\mathbf{196.224}^{***}$	$\mathbf{2.52259}^{***}$	$\mathbf{48.7035}^{***}$	$\mathbf{9.21682}^{***}$
a	$\mathbf{100.97}^{***}$	$\mathbf{-90.2584}^{***}$	$\mathbf{-1.44367}^{***}$	$\mathbf{-23.1149}^{***}$	$\mathbf{-5.0213}^{***}$
θ_h	0.139315	-66.0141^{***}	-0.161914^{***}	-2.06788^{***}	0.98742
θ_a	-1.68908^{***}	26.8527^{***}	0.206154^{***}	0.440309^{*}	-0.482658^{*}
ch	96.777^{***}	-2.40873	0.496937	1.44904	3.96937
ca	-196.254^{***}	62.63	1.21425^{*}	32.9133^{***}	4.85103
$c\theta_h$	0.938596	23.7479	-1.42933^{*}	-31.1381^{**}	-11.4987^{***}
$c\theta_a$	1.65429	22.5155	0.415723	7.85521	3.23167
ha	-234.986^{***}	-421.495^{***}	-3.68595^{***}	-88.4928^{***}	-11.1493^{***}
$h\theta_h$	4.31408	-498.796^{***}	-1.75052^{***}	-29.9663^{**}	2.28346
$h\theta_a$	32.3537^{***}	47.8377	0.356691	8.55397	1.59301
$a\theta_h$	18.5702	447.439^{***}	1.27031^{*}	20.2363	-3.5686
$a\theta_a$	-77.7543^{***}	-79.4616	0.0335362	8.32262	4.16891
$\theta_h\theta_a$	-6.09341	-72.9776	-0.39431	-1.1345	0.91514

Figure 5.3: Multiple linear regression of each outcome measure on model parameters and their interactions derived from (Sayama, 2020)

Outcome Measure	Average edge weight	Number of communities	Modularity	Range of average comm states	Std of average comm states
$const$	-0.37	18.03	0.31	6.13	1.87
c	-6.20 (***)	-19.75	-0.33 (***)	-13.22 (***)	-4.39 (***)
h	**-11.73 (***)**	**196.07 (***)**	**2.50 (***)**	**48.56 (***)**	**9.40 (***)**
a	**100.99 (***)**	**-90.78 (***)**	**-1.45 (***)**	**-22.19 (***)**	**-4.82 (***)**
. θ_h	0.15	-66.39 (***)	-0.18 (***)	-1.63 (***)	1.02
θ_a	-1.60 (***)	27.34 (***)	0.17 (***)	1.26 (*)	-0.27 (*)
ch	96.97 (***)	-2.77	0.50	0.55	3.73
ca	-196.57 (***)	63.75	1.25 (*)	31.40 (***)	4.67
$c\theta_h$	0.90	23.11	-1.35 (*)	-32.73 (**)	-11.54 (***)
$c\theta_a$	0.79	16.71	0.39	4.84	2.54
ha	-235.61 (***)	-421.59 (***)	-3.70 (***)	-90.33 (***)	-11.75 (***)
$h\theta_h$	4.27	-495.48 (***)	-1.68 (***)	-29.81 (**)	2.03
$h\theta_a$	31.65 (***)	40.76	0.38	10.30	0.95
$a\theta_h$	18.17	447.42 (***)	1.26 (*)	18.80	-4.04
$a\theta_a$	-76.29 (***)	-78.05	0.10	5.48	3.61
$\theta_h\theta_a$	-5.69	-69.87	-0.29	-1.05	1.18

Figure 5.4: Multiple linear regression of each outcome measure on model parameters and their interactions : base model implementation

Chapter 6

Extended Model - Results

Four instances of the extended model are implemented. Each instance adheres to a different policy H:X, A:Y for X, Y $\in$ [MAX MAX, MIN MIN, MAX MIN, MIN MAX] where H denotes the homophily updater and A denotes the attention to novelty updater. For H = MAX, nodes focus on the dimension with the greatest difference in state when connecting with respect to homophily and for H = MIN, they focus on the dimension with the smallest difference in state. For A = MAX, nodes focus on the dimension with the greatest difference in their state and the perceived weighted average state of the node they are changing their connection with when connecting with respect to neophily, and for A = MIN, they instead focus on the dimension with the least difference. For example, H:MAX, A:MIN operates as follows: for nodes i and j, node j focuses on the dimension with the greatest difference in their state and the state of node i when changing their connection with respect to homophily to node i. Node j also focuses on the dimension with the smallest difference in their state and the perceived weighted average state of node i when changing their connection with respect to attention to novelty to node i.

All policies are implemented for 1, 2, 5 and 10 dimensional node states for a network of $n = 100$ nodes. For each policy and dimensionality (e.g., MAX MIN principle with a 2 dimensional node state), we have five independent simulation runs for all combination of the parameters $c, h, a, \theta_h, \theta_a \in \{0.01, 0.03, 0.1, 0.3\}$ (as we did in the base model). The results of these implementations are shown in figure 6.1, figure 6.2, figure 6.3 and figure 6.4. The results are plotted in the same format as (Sayama, 2020) wherein each dot in each graph is a simulation run. Each graph in the figures shows the relationship between a parameter and an outcome measure for dimensions 1,2,5 and 10 (wherein blue, green, orange and red dots are 1, 2, 5 and 10 dimensional simulation runs respectively).

To simplify the results observed in all figures, 5 graphs are produced for each policy which are shown in figure 6.5, figure 6.6, figure 6.7 and figure 6.8. Each graph calculates for a given dimension, the average of a given outcome measure across all simulation runs of that dimension and plots this average against the corresponding dimension.

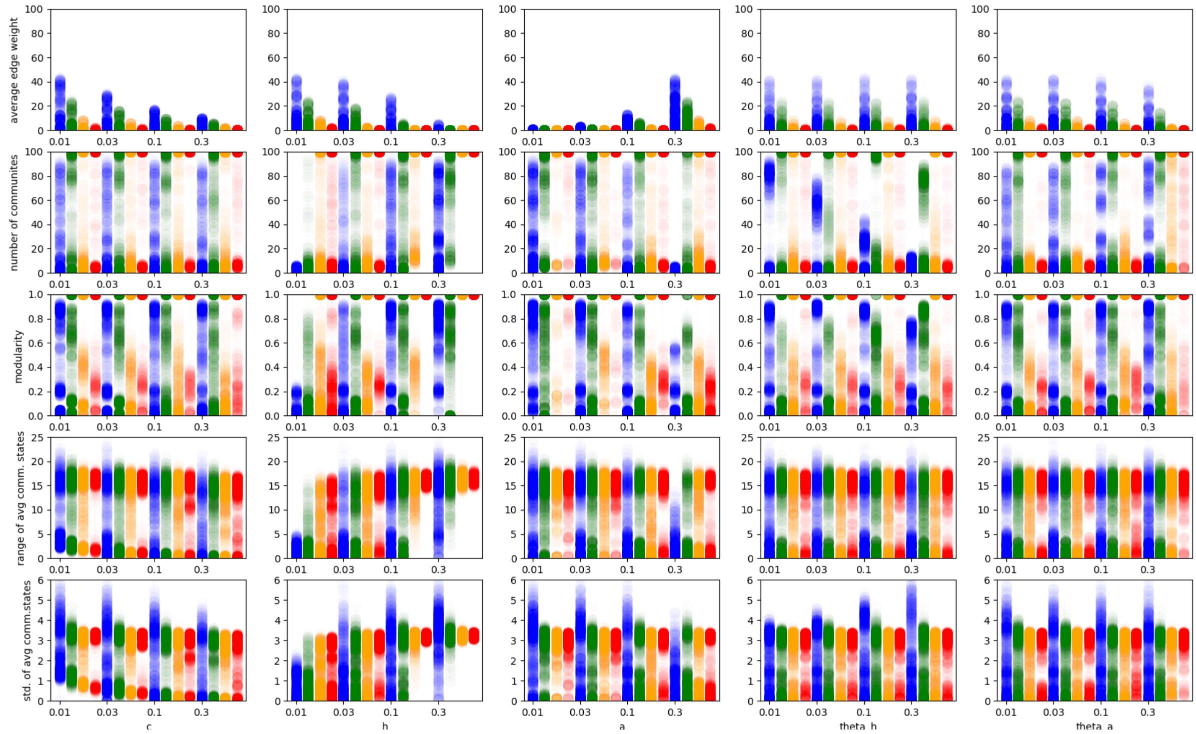

Figure 6.1: Relationships between model parameters and outcome measures for networks of $n = 100$ abiding by the H:MAX, A:MIN policy

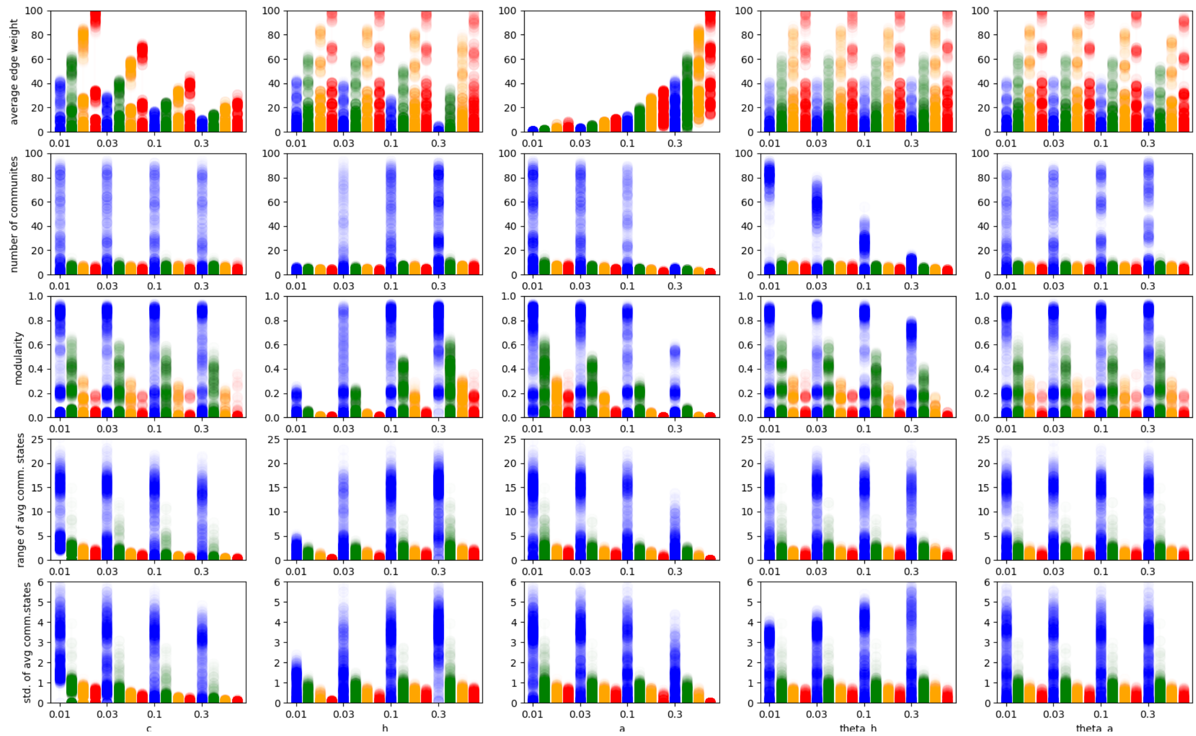

Figure 6.2: Relationships between model parameters and outcome measures for networks of $n = 100$ abiding by the H:MIN, A:MAX policy

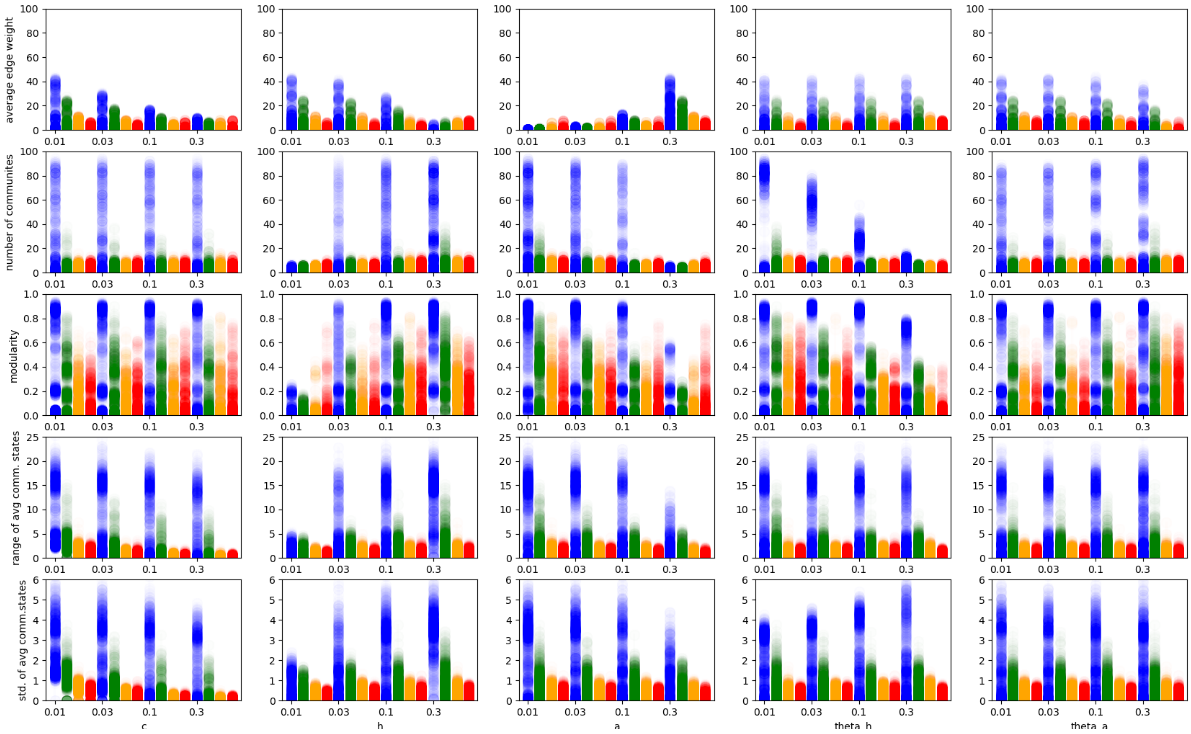

Figure 6.3: Relationships between model parameters and outcome measures for networks of $n = 100$ abiding by the H:MIN, A:MIN policy

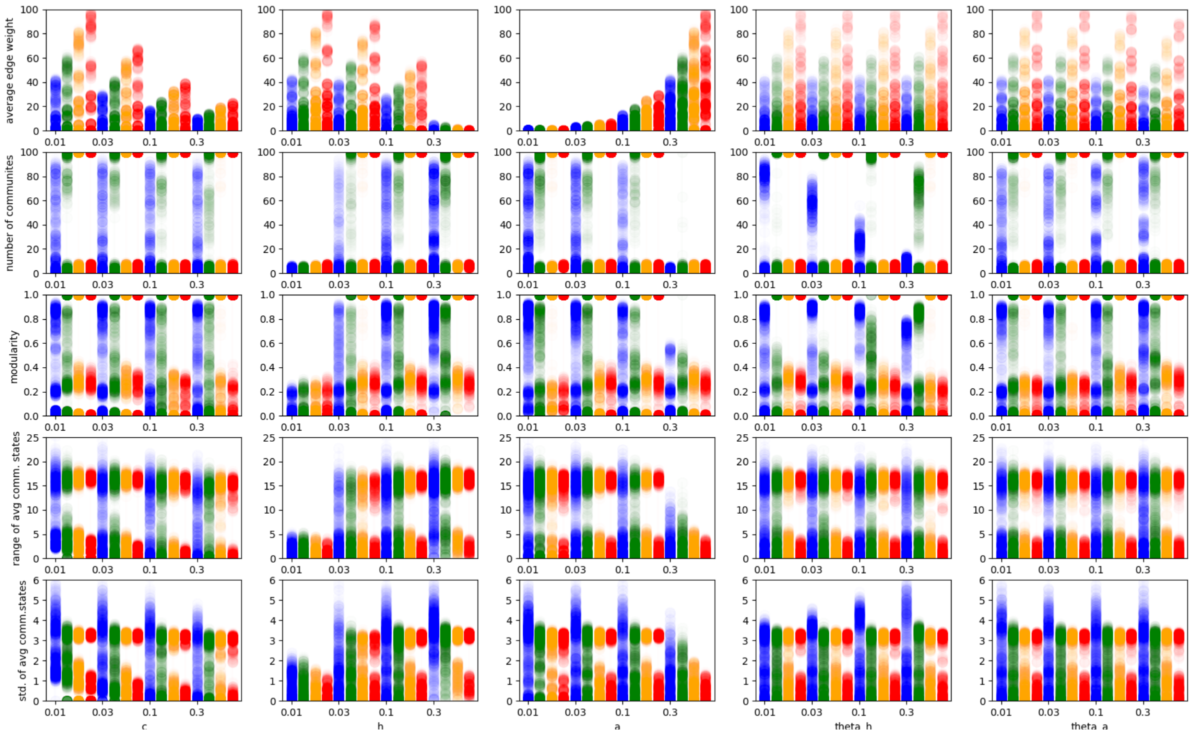

Figure 6.4: Relationships between model parameters and outcome measures for networks of $n = 100$ abiding by the H:MAX, A:MAX policy

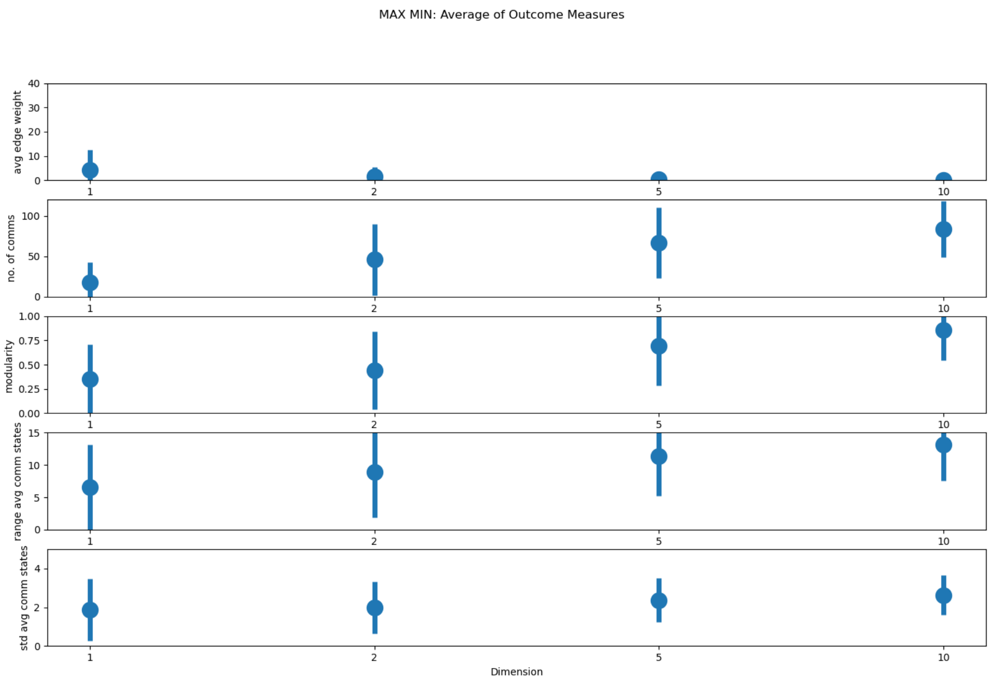

Figure 6.5: Averages of outcome measures for networks of $n = 100$ abiding by the H:MAX, A:MIN policy

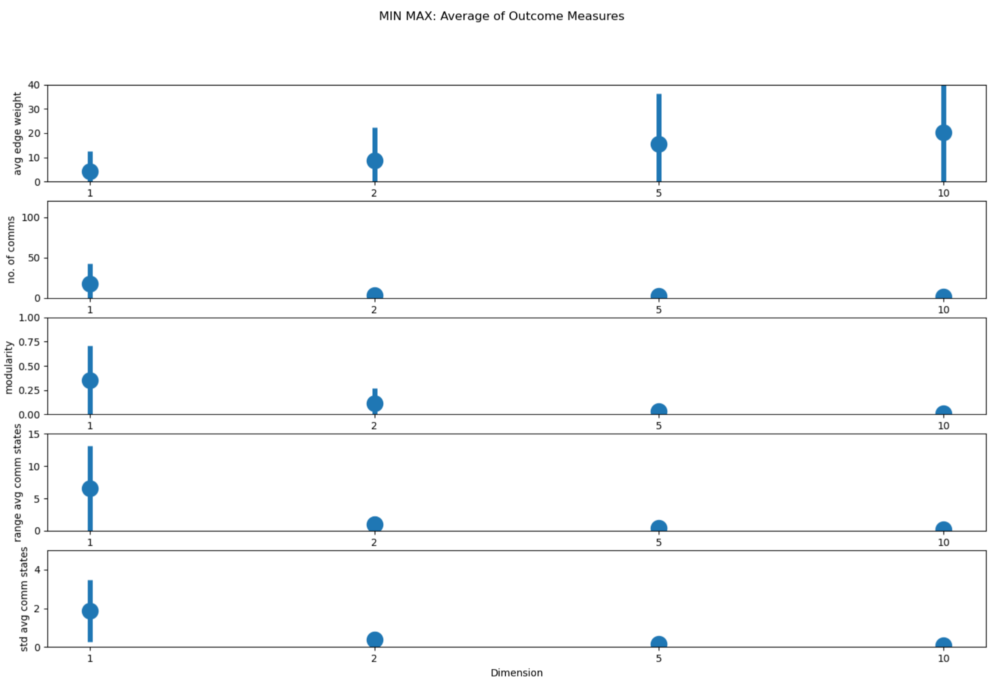

Figure 6.6: Averages of outcome measures for networks of $n = 100$ abiding by the H:MIN, A:MAX policy

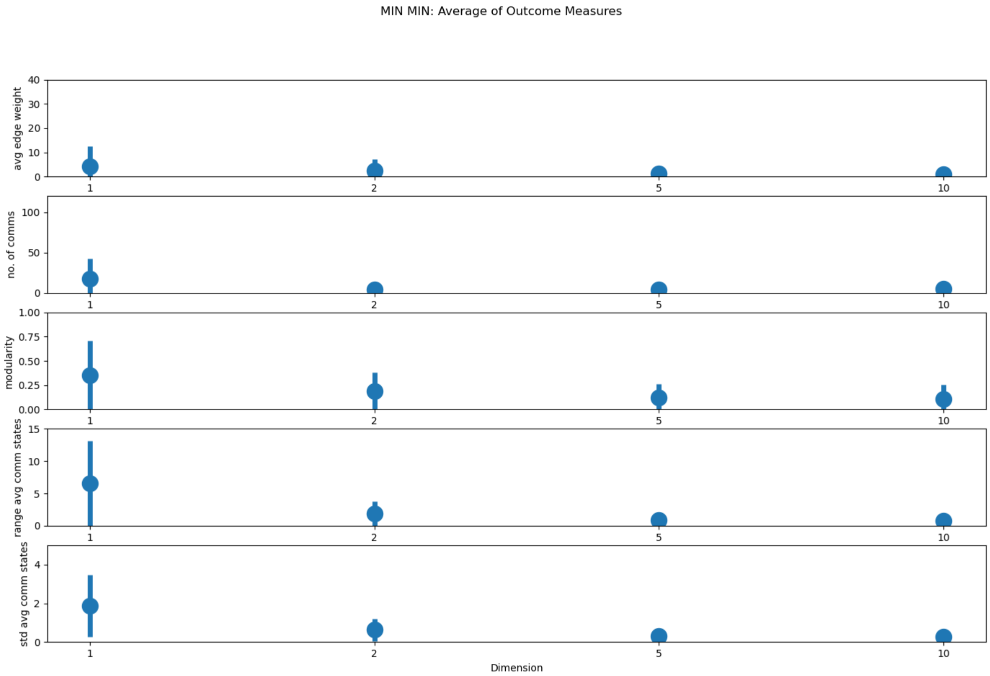

Figure 6.7: Averages of outcome measures for networks of $n = 100$ abiding by the H:MIN, A:MIN policy

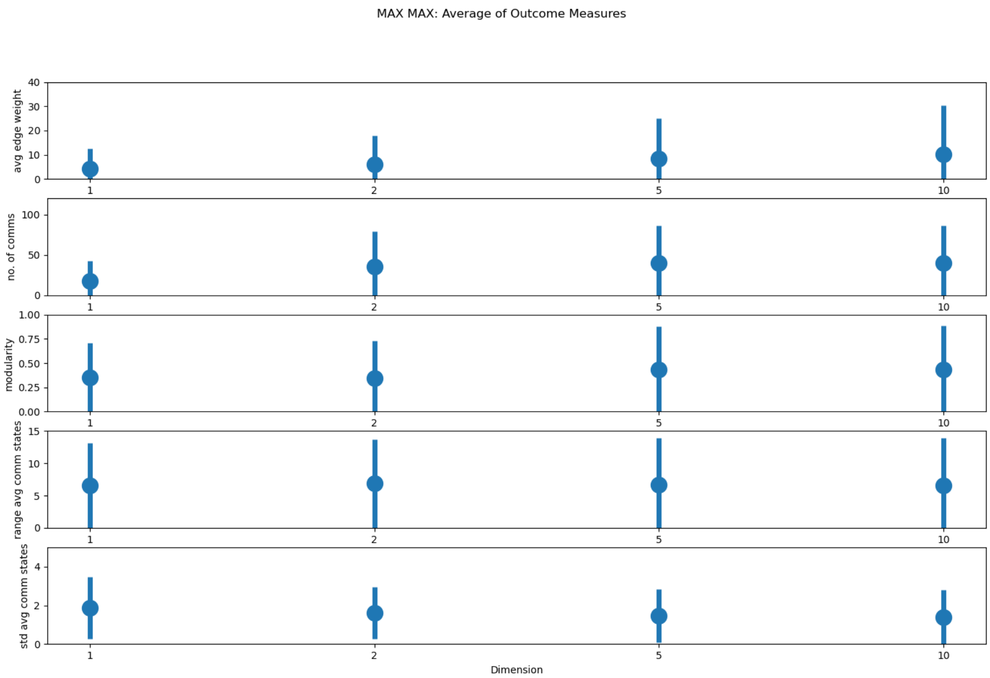

Figure 6.8: Averages of outcome measures for networks of $n = 100$ abiding by the H:MAX, A:MAX policy

Chapter 7

Critical Evaluation

7.1 Parameter Effects on Social Outcomes

Previously, we had two models: the base model and the extended model. In this chapter we now denote a model to be an implementation of a D dimensional node state with policy P (so for example, a 2 dimensional node state with a MAX MIN policy is a model we have implemented and likewise a 10 dimensional node state with a MIN MAX policy is a model we have implemented etc). Here D = [1,2,5,10] and P = [MAX MIN, MIN MAX, MIN MIN, MAX MAX].

Thus, regardless of whichever model has been implemented, figures 5.2, 6.1, 6.2, 6.3 and 6.4 show the general pattern of each parameter on the outcome measures to remain the same. For example, as parameter c increases, then for each model it can be observed the average edge weight decreases and the range and standard deviation of average community states slightly decreases. This shows, for any model, the average connection between any two nodes has decreased suggesting parameter c tends to increase fragmentation in the network. The decrease in range and standard deviation of average community states suggests parameter c to decrease extremism to some extent. Parameter c is a measure of the conformity of individuals, and thus the greater conformity individuals hold, the quicker they adopt the views they are surrounded by. This suggests clusters of individuals become formed sharing similar opinions, thus the extremism in opinions somewhat reduces (individuals adopt similar opinions) and fragmentation increases (individuals adopt opinions they are exposed to and thus as a result only connect to opinions similar to theirs).

As parameter h increases, all models show a decrease in the average edge weight and an increase in the number of communities and modularity. This suggests parameter h is responsible for fragmentation and extremism within the network. Parameter h encodes homophily, and the larger the homophily of individuals, the greater similarity in opinion individuals require to form stronger bonds with other individuals. Thus, individuals only connect to those most similar to them, resulting in clusters of groups with differing ideas (hence an increase in extremism) and less strong bonds in societies overall due to the greater requirement in similarity for a connection to be strengthened (hence increased fragmentation).

As parameter a increases, we observe all models to show the opposite effect to what we had observed for parameter h. An increase in parameter a results in an increase in average edge weight and a decrease in the number of communities and modularity; the range and standard deviation also decreases. Thus, parameter a reduces fragmentation and extremism. Parameter a encodes attention to novelty, thus the larger the attention to novelty of individuals the more they seek individuals with a larger exposure to differing opinions. Thus, more connections are formed between individuals hence fragmentation within societies is reduced; individuals find themselves tolerant of individuals with an exposure to differing views, hence extremism also finds itself lessening.

For all models, θ_h and θ_a show little effect on the social outcome measures; as stated and suggested by (Sayama, 2020) this could be due to 'their values becoming less significant as ideas are diversified within the network and it would be worthwhile to test more values of θ_h and θ_a to capture the effects they have on the network'.

Also observed for the one dimensional base model, figure 5.4 captures h and a having the most impact on social outcome measures. Thus, since these parameters have the largest effect, the effects of θ_h and θ_a may be subsided as a result. We also observe in figure 5.4 the interactions of parameters having an effect on the social outcome measures thus indicating non-linearity in the relationship between the social outcome measures and the parameters. It would be thus worth running non-linear models of parameters and social outcome measures (e.g., regression splines (Marsh, 2011)) to better capture the relationships between parameters and outcome measures.

7.2 Policies Evaluated

All policies show a change in the network with increasing dimensionality thus implying the importance multiple opinions play in the evolution of a network with time.

7.2.1 H:MAX, A:MIN Critical Evaluation

For the H:MAX, A:MIN principle, we observe from figure 6.1 the average edge weight decreasing, modularity, number of communities and range and standard deviation of average community states increasing with the number of dimensions. This suggests the more dimensions we have in the H:MAX, A:MIN setting, the more fragmented and extreme the network becomes.

Thus, for a setting where individuals have multiple opinions (such as politics, religion, food and more) and only focus on the topic wherein other individuals are most different when connecting with respect to homophily and only focus on the topic wherein other individuals are most similar to those they are surrounded by when connecting with respect to attention to novelty, we observe great fragmentation and extremism as the number of topics considered increases.

Suppose individuals have two topics in consideration (e.g., politics and food preference). The introduction of a third topic (such as religion) may imply the introduction of a topic where individuals disagree the most and thus the connection between individuals is now weaker (since they focus on topics wherein they disagree the most when connecting with respect to homophily) than it would have been without this third topic being introduced. Adding more topics reinforces this idea of likely observing more areas of greater disagreement and thus greater fragmentation.

Likewise, with the introduction of more topics, there are likely more areas where individuals are similar to those they are surrounded by and thus the connection between individuals with respect to attention to novelty may not be as strong as it could have with less topics. Hence, individuals are exposed and connected to less individuals of differing opinions, thus suggesting a rise in extremism with a rise in the number of topics being considered.

This is expected, as both updaters are set to the 'worst case scenario' (so adding more topics increases the chances of observing a new topic with a greater disagreement or less attention to novelty and thus resulting in connections not being as strong as they could have been otherwise without the introduction of these topics).

7.2.2 H:MIN ,A:MAX Critical Evaluation

For the H:MIN, A:MAX principle, figure 6.2 captures the opposite effect to figure 6.1; average edge weight increases and all other social outcome measures decrease thus suggesting a strongly connected network with little extremism. This is expected, as now both updaters are set to their best case scenario so introducing more dimensions can never worsen the connection between two individuals (either a new dimension with have a greater difference in state between nodes than existing dimensions and not be considered when updating with respect to homophily or will have a smaller difference in state between nodes than existing dimensions, in which case the connection will be stronger between two nodes than it was previously). A similar explanation holds for the neophily updater = MAX being the ideal case for the neophily updater for increasing dimensionality (i.e., the case where adding more dimensions does not make connections weaker, but rather has the potential to make connections stronger).

Thus, for a setting where individuals have multiple topics and focus on topics where individuals are most similar to them when connecting with respect to homophily, and focus on topics where individuals are exposed to a greater diversity of opinions when connecting with respect to attention to novelty, we observe a better connected network as the number of topics considered increases.

7.2.3 H:MIN ,A:MIN Critical Evaluation

Figure 6.7 shows under the H:MIN, A:MIN principle, average edge weight decreases with dimensionality suggesting fragmentation of the network. However, the number of communities and modularity also decreases which implies a better connected network with few independent clusters of nodes tolerant to differing views (e.g., 10 dimensional node states have a modularity close to 0). The range and standard deviation of average community states also decreases, hence suggesting under the H:MIN, A:MIN principle, more dimensions reduce the level of extremity observed. Thus, under this principle the network has weaker connections between nodes however overall is well connected.

The H:MIN, A:MIN principle focused on dimensions with states most similar when connecting with respect to homophily and on dimensions where the perceived weighted average state of one node and the state of another node was most similar when connecting with respect to attention to novelty. Thus, with respect to homophily, the more dimensions added would suggest more dimensions where nodes are likely to be more similar (e.g., if two individuals consider 3 topics, then introducing 3 more topics might give them a topic which they agree upon better than the previous three topics). Thus, the more dimensions introduced, the stronger the connections would be formed with respect to homophily. However, the more dimensions introduced, the more dimensions for nodes to be similar to their surroundings and thus the connections formed with respect to attention to novelty become less strong than if less dimensions were in place. This paints a paradoxical picture as adding more dimensions does not make connections worst under homophily, but may make connections weaker under attention to novelty.

Perhaps the MIN property of connecting under homophily thus drives less fragmentation, as it is likely individuals form better connections to other individuals if focusing on a topic with similar views than if they had focused on a topic where their views differed the most (focusing on a differing view could also imply connections breaking with time, hence the MIN principle for homophily gives a chance for individuals to connect to other individuals). Hence, individuals are connected to more individuals indicating less communities. Individuals also conform to the opinions they are surrounded to with respect to time (as was the assumption of all models); being connected to many individual suggest all individuals eventually adopt similar beliefs thus reducing levels of extremism observed in the network. However, the low average edge weight with dimensionality suggests though individuals are overall well-connected, these connections are weak. This could be a result of the MIN property of attention to novelty which limits how well connected two individuals could be and is only exacerbated with the number of opinions (as there are more topics for individuals to be similar to their surroundings, thus reducing the strength of the connection which could have been observed otherwise).

Nonetheless, though the MIN property of attention to novelty suggests weaker connections as dimensionality increases (as well as perhaps increased extremism since nodes are less well connected to other nodes with access to a greater diversity of states), the MIN property of homophily seems to balance out these suggestions and results in a network being less extreme and more well-connected with time, despite these connections becoming overall weaker.

7.2.4 H:MAX, A:MAX Critical Evaluation

Figure 6.8 shows under the H:MAX, A:MAX policy, as the dimensionality increases the average edge weight, number of communities and modularity also increase. The range stays roughly the same and the standard deviation dips.

Thus, under this policy on average individuals become better connected with more opinions considered, however the increase in number of communities and modularity suggest an increase in fragmentation in the network under the consideration of further opinions (i.e., more independent clusters with less communication between clusters); the standard deviation indicates a slight reduction in the extremism of differing clusters.

The homophily updater is set to MAX, hence nodes focus on the dimension most different when connecting with respect to homophily. Thus, the more dimensions are added, the more chance there is for a greater difference in one of the dimensions than was previously witnessed and hence the weaker connections can become. This property perhaps results in greater fragmentation which results in the number of communities and modularity increasing with dimensionality.

In contrast, the attention to novelty updater being set to MIN implies nodes focus on dimensions most different. Thus adding more dimensions increases the chance of observing a greater difference between the perceived weighted average state of one node to the state of another node. Thus, adding more dimensions leads to stronger connections for the attention to novelty updater being possible. This, as in the MIN MIN case, paints again a paradoxical picture wherein the homophily updater can result in weaker connections but the attention to novelty updater can result in stronger connections as the dimensionality increases. Perhaps, this is why the standard deviation of average community states decreases slightly, since nodes are more willing to connect to nodes exposed to a wider range of states hence nodes have stronger connections overall.

7.2.5 Policies Summarised

The H:MIN,A:MAX and H:MAX,A:MIN principles operate as expected. Both cases can be thought of as the 'best' and 'worst' cases for the homophily and neophily updaters. We observe the first case leads to a well-connected network with little fragmentation and extremism and the second case yields exactly the opposite. The other two cases follow a less intuitive lead, and below we summarise their findings and conclusions.

What can be observed is under H:MIN,A:MIN, all social outcome measures decrease whereas in H:MAX,A:MAX only the standard deviation of average community state decreases, that too slightly. Hence, these two principles exhibit mostly opposite behaviours, despite both painting a paradoxical picture.

The H:MIN, A:MIN principle shows with the increase of dimensionality, the fragmentation decreases (from the decrease in the number of communities and modularity) whereas the H:MAX,A:MAX principle shows the fragmentation to increase.

Intuitively discussed before, connections are strengthened with greater dimensions for homophily updater = MIN and attention to novelty updater = MAX. The H:MIN,A:MIN case has a 'good' homophily updater (connections strengthened with dimensions) and this case has reduced fragmentation with dimensionality. In contrast, the H:MAX,A:MAX case has a 'bad' homophily updater and this case has increased fragmentation with dimensionality, thus pointing towards the idea that the homophily updater has a greater role to play in the fragmentation of the network.

Though the H:MAX,A:MAX case has increased fragmentation with the dimensionality, the average edge weight increases with the dimensionality thus suggesting on average nodes are well-connected within the network. In contrast, the H:MIN,A:MIN case with reduced fragmentation as dimensionality increases has the average edge weight decreasing with dimensionality and almost approximating zero when D=10. This thus suggests for a large number of dimensions under the H:MIN,A:MIN principle, the connections between nodes become very weak though the network is still well connected (as seen by the number of communities and modularity). Thus, a 'good' neophily updater (i.e. MAX) seems to influence the average connection between two nodes whereas a 'bad' neophily updater results in weaker connections between two nodes. Hence, though the homophily updater influences the fragmentation of the network, the strength of the connection between any two nodes is influenced by the neophily updater.

Both H:MIN,A:MIN and H:MAX, A:MAX show a reduction in extremism, with MIN MIN showing a greater reduction with dimensionality. Hence, the 'good' elements of both these principles help in reducing extremism however it seems (since MIN MIN shows a greater reduction in extremism) the homophily updater has a greater hand to play in reducing extremism with an increasing number of dimensions.

7.3 Linking to the real-world

We discussed the benefit of opinion dynamics in understanding the birth of extreme ideas and opinions in a society where individuals simply try to conform to the social norms within their neighbourhood (Sayama, 2020). To best model the spread of opinions in a society with time, we focused on a multi-dimensional model which captured individuals holding multiple opinions in the real-world. This model showed increasing dimensionality has an impact on the evolution of the network, with different results being shown for different policies implemented.

However, there are limitations within our model that in a sense refrain from its extension from the real world. For example, each node is assumed to hold the same values of $c, h, a, \theta_h, \theta_a$ as well as abide by the same policy. Indeed, this is not the case in the real-world (some individuals may want greater similarity when connecting to other individuals and thus h may be larger for some individuals, likewise some individuals may be 'lenient' in their connections and thus adopt a H:MIN,A:MAX policy). Alongside this, there are policies which exist beyond the ones mentioned. For example, if two topics such as politics and sport are considered, then an individual may be greater influenced by the political opinion of another individual rather than their opinion in sport in determining a connection (so here nodes are not looking for a 'MAX' or 'MIN' dimension, but simply hold certain dimensions to a greater degree than others).

In the next chapter, we discuss possible future work to tackle these limitations within the existing model we have implemented.

Chapter 8

Further Work

It would be worth adapting the model by randomly assigning each node a $c, h, a, \theta_h, \theta_a$ from a set of values [0.01, 0.03, 0.1,0.3] to better capture the aspect of individuals being different from one another when deciding how to change their opinions and connections. Alongside this, a random choice for each node for [MAX MAX, MAX MIN, MIN MAX, MIN MIN] would capture individuality of the real-world better than the existing model. Assigning each dimension a 'priority' and then having a weighted formula for changing connections with respect to homophily and neophily would model certain topics given a greater weightage than others for individuals connecting (e.g. for two dimensions d_1 and d_2 if d_1 has a higher 'priority' and individual i has state $x_i = [x_{i1}, x_{i2}]$ and individual j has state $x_j = [x_{j1}, x_{j2}]$ then a weighted difference could be calculated such as $D = w_1 * |x_{i1} - x_{j1}| + w_2 * |x_{i2} - x_{j2}|$ where w_1 is larger than w_2). Then, this difference can be used to update connections with respect to homophily (so $hF_h = h(\theta_h - D)$) and a similar suggestion can be applied to neophily.

Chapter 9

Conclusion

Our discussion began with highlighting the importance of understanding the spread of opinions with time (Brede, 2019), (Javarone, 2014; Hegselmann et al., 2014), (Laciana and Rovere, 2011), (Easley and Kleinberg, 2010), (Sayama, 2020). We moved onto looking at an existing but simple model which focused on N agents, each with a continuous opinion; two agent randomly meet and, if their opinions lie within a given threshold, their opinions move towards one another (Deffuant et al., 2000). This model showed opinions converge with time (as was visualised in Figure 2.1) (Deffuant et al., 2000). We then moved onto models in an adaptive network setting wherein the general intuition of each model in this setting was the same where nodes represented individuals and their states the opinion of individuals (which was captured as a real-number) and edges connections between individuals (Das et al., 2014), (Rosvall and Sneppen, 2006), (Brede, 2019).

We understood the real-world had to account for individuals interacting in settings of multiple opinions and thus we focused on models which captured multiple topics being taken into consideration. The opinion state of nodes was now a D dimensional vector wherein each dimension represented a different topic (such as sports, food preferences). Models of this form showed a consensus with increasing dimensions (i.e., a well-connected network) under given conditions holding true for the model (e.g., (Lorenz, 2007) 'fosters a consensus when inter-related issues are bought into discussion and prevents a consensus when independent issues are brought into discussion' and (Lorenz, 2008) 'fosters consensus if issues are under a budget constraint but diminishes consensus otherwise'.

Our multi-dimensional extension of (Sayama, 2020) followed the same pattern of a well-connected network with increasing dimensions under certain conditions holding true. For example, we observed for the H:MIN, A:MAX case, the network became a unity, with a small number of communities and well connected nodes. This was the ideal case, with the worst case being the H:MAX, A:MIN case which led to the most fragmented network with increasing dimensionality. The H:MIN, A:MIN and H:MAX,A:MAX cases were a lot less intuitive, however both suggested a decrease in the levels of extremism observed implying having at least one of the updaters being set to their optimal value (e.g., homophily updater = MIN or attention to novelty updater = MAX) helps in reducing the levels of extremism observed with greater dimensionality. These cases also suggested the homophily updater had a greater hand to play in determining the number of independent clusters/communities and the tolerance of different communities to one another in the network and the neophily updater had a greater hand to play in determining on average how well connected two nodes were.

From these results and the knowledge that as individuals we hold multiple opinions, all which guide our connections and opinions with time, we observe the ideal scenario as individuals is to each have our homophily updater = MIN and our neophily updater = MAX. We also observe the greater importance of having the homophily updater as MIN, since in all the cases where this was not the case, fragmentation (independent clusters with little communication between clusters) increased with dimensionality. Hence, as individuals learning to focus on the areas we agree with the most with other individuals and giving less importance to the areas we disagree upon can help our societies be well-connected under the consideration of multiple opinions. For the neophily updater = MAX, we observed an increasing average edge weight thus the neophily updater tends to enable stronger connections between nodes on average. Thus, for a strongly connected society, as individuals we would benefit on having neophily updaters = MAX.

Thus, to conclude we extended an existing opinion dynamics adaptive network model (Sayama, 2020) to model multiple opinions. This extension led us to consider two further parameters, precisely a homophily updater and an neophily updater and we showed as individuals it is best for us to practice focusing on topics where other individuals are similar to us and on topics where other individuals stand out the most from their surroundings to enable well-connected societies. For connections on average to be strong between individuals, it is best to focus on neophily updater = MAX and for less clusters of individuals with low tolerance between different clusters, it is best to focus on homophily updater = MIN.

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