## **Heuristic Analysis**

The three heuristics built upon the improved heuristic (score = # of open spaces - # of legal moves for opponent) by first rewarding the center square as the opening move, with a score of +infinity (custom\_score). The second heuristic (custom\_score\_2) took custom\_score's schema and added a reward of +infinity for reflecting the opponent about the center position on the third move, assuming the player had the center square on the first move. Lastly, custom\_score\_3, the third heuristic, added to custom\_score\_2. In addition to the reward schema of custom\_score\_2, the third heuristic rewarded +inifinity for choosing a position that could not be reflected by the opponent, assuming the player had turn two and the opponent grabbed the center square on the first move.

The tournament script was run three separate times to evaluate the three heuristics. In the first tournament, the results were as shown in Figure 1 below:

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Match #  1 2 3 4 5 6 7	Opponent  Random  MM_Open  MM_Center  MM_Improved  AB_Open  AB_Center  AB_Improved	AB_Improved Won   Lost 8	AB_Custom Won   Lost 7   3 8   2 7   3 7   3 3   7 4   6 3   7	AB_Custom_2 Won   Lost 8	AB_Custom_3 Won   Lost 9   1 7   3 9   1 6   4 7   3 3   7 8   2				
	Win Rate:	58.6%	55.7%	72.9%	70.0%				

The second and third heuristics performed decently, with a 72.9% win rate for custom\_score\_2 and a 70.0% win rate for custom\_score3. However, it was noted that the agent forfeited 248 matches despite having further legal moves possible. While it is possible that the forfeitures were due to projecting losses, the high number suggested that forfeitures due to timeouts were more likely. So two separate strategies were used to address the handing of timeouts by the agent.

First, the agent was modified to set its default move to the first random legal move, assuming there was one, before searching. This way, a move would be returned in the case of a timeout, instead of a forfeiture. With the first random legal move method in place, the results were as depicted in Figure 2 below:

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Match #	Opponent	AB_Improved Won   Lost	AB_Custom Won   Lost	AB_Custom_2 Won   Lost	AB_Custom_3 Won   Lost				
1 2 3 4 5 6 7	Random MM_Open MM_Center MM_Improved AB_Open AB_Center AB_Improved	9   1 10   0 10   0 6   4 6   4 9   1 5   5	10   0 6   4 8   2 6   4 8   2 6   4 4   6	10   0 8   2 9   1 6   4 5   5 6   4 3   7	9   1 5   5 10   0 6   4 4   6 7   3 6   4				
	Win Rate:	78.6%	68.6%	67.1%	67.1%				

Here, the three heuristics all performed more poorly than the AB\_Improved agent, with heuristics one, two, and three obtaining winning rates of 68.6%, 67.1%, and 67.1%, respectively, compared to AB\_Improved's rate of 78.6%.

Since the modification made such a substantial difference in the outcomes, it was inferred that many of the forfeitures in the first tournament had indeed been due to timeouts. Since the search space is largest in the beginning, because there are more legal moves to evaluate, it was then decided to change the default move assignment from random to the center position, assuming that it was a legal move and available, and a random available move otherwise. With this modification in place, the third tournament was run.

The results of the third tournament are shown in Figure 3 below:

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Match # 1 2 3 4 5 6 7	Opponent  Random  MM_Open  MM_Center  MM_Improved  AB_Open  AB_Center  AB_Improved	AB_Improved Won   Lost 9	AB_Custom Won   Lost 9   1 6   4 8   2 9   1 7   3 5   5 4   6	AB_Custom_2 Won   Lost 10   0 6   4 9   1 8   2 4   6 8   2 5   5	AB_Custom_3 Won   Lost 8			
	Win Rate:	67.1%	68.6%	71.4%	61 . 4%			

In the third tournament, the first and second heuristic outperformed AB\_Improved, with win rates of 68.6% and 71.4%, surpassing AB\_Improved's rate of 67.1%. The third heuristic underperformed, though, with a win rate of just 61.4%. Seeing as the third heuristic is merely the second heuristic with one additional reward returned for choosing a second move that cannot be reflected about the center square on the third move, there are two main possibilities that would lead to the third heuristic underperforming. Either the additional reward for the second move is misguiding the agent, or the additional check in the score function is causing more timeouts, leading to random choices instead of optimal moves. Considering the frequency of timeouts in the first tournament, the latter explanation seems more likely. Having the score function compute quickly enough to avoid timing out is just as important as having some opening strategies. From the results of tournament three, the second heuristic seems to strike this balance the most effectively of the three, so the submitted algorithm will use custom\_score\_2.