

The second Renyi entropy of a state  $\rho$  is

$$S_2(\rho) = -\ln \text{tr}[\rho^2] = -\ln \text{tr} \left[ \left( \sum_u W_u(\rho) A_u \right)^2 \right] = -\ln \left( d^n \sum_u W_u(\rho)^2 \right) \quad (1)$$

$$= -n \ln d - \ln \sum_u W_u(\rho)^2 \quad (2)$$

Jensen's inequality for  $f(x) = x^2$  gives us:

$$\left( \sum_u |W_u(\rho)| \right)^2 \leq \sum_u W_u(\rho)^2 \quad (3)$$

Since  $\ln x$  is an increasing function, this means that we can find a bound on the mana

$$\mathcal{M}(\rho) = \ln \sum_u |W_u(\rho)| \quad (4)$$

like so:

$$n \ln d - S_2(\rho) = \ln \sum_u W_u(\rho)^2 \geq \ln (\mathcal{M}(\rho)^2) = 2 \ln \mathcal{M}(\rho) \quad (5)$$

Which gives:

$$\mathcal{M}(\rho) \leq \frac{1}{2} (n \ln d - S_2(\rho)) \leq \frac{n}{2} \ln d \quad (6)$$

Where we also used the fact that  $S_2(\rho) < 0$ .