Contents

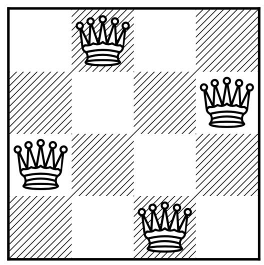
1 Learning Goals

By the end of the exercise, you should be able to

- Formulate a real-world problem as a local search problem.
- Design multiple neighbour relations for a given problem.
- Trace the execution of the hill climbing algorithm with a given neighbour relation.
- Trace the execution of the simulated annealing algorithm with a given neighbour relation.

2 The 4-Queens Problem

The 4-queens problem consists of a 4 x 4 chessboard with 4 queens. The goal is to place the 4 queens on the chessboard such that no two queens can attack each other. Each queen attacks anything in the same row, in the same column, or in the same diagonal. The image below gives one solution to this problem.



2.1 Local Search Formulation

Recall that we've formulated the 4-queens problem as a local search problem.

- State: 4 queens on the board. One queen per column.
  - Variables: x_0, x_1, x_2, x_3 where x_i is the row position of the queen in column i. Assume that there is one queen per column.
  - Domain for each variable: $x_i \in \{0, 1, 2, 3\}, \forall i$.
- Initial state: a random state.
- Goal state: 4 queens on the board. No pair of queens are attacking each other.
- Neighbour relation:
  - Version A: Move a single queen to another square in the same column.
  - Version B: Swap the row positions of two queens.
- Cost function: The number of pairs of queens attacking each other, directly or indirectly.

2.2 Hill Climbing

Algorithm 1 Hill Climbing

current $\leftarrow$ a random state
while true **do**
  next $\leftarrow$ get-best-neighbour(current)
  if cost(current) $\leq$ cost(next) **then**
    break
  end if
  current $\leftarrow$ next
end while
return current

2.3 Simulated Annealing

Algorithm 2 Simulated Annealing

current $\leftarrow$ initial state
T $\leftarrow$ a large positive value
while T $>$ 0 **do**
  next $\leftarrow$ a random neighbour of current
  $\Delta E \leftarrow$ current.cost - next.cost
  if $\Delta E > 0$ **then**
    current $\leftarrow$ next
  else
    current $\leftarrow$ next with probablity $p = e^{\frac{\Delta E}{T}}$
  end if
  decrease T
end while
return current

3 Practice Questions

3.1 Hill Climbing

For questions 1 to 4, consider version B of the neighbour relation: swap the row positions of two queens.

Question 1:

How many neighbours are there for a state?

Question 2:

Start with the initial state $x_0 = 3, x_1 = 1, x_2 = 2, x_3 = 0$. Show the steps of executing the hill climbing algorithm until it terminates.

			Q
	Q		
		Q	
Q			

If multiple neighbours have the same cost, choose the neighbour where the pair of queens swapped has the smallest subscript/column number. For example, when we can swap either (x_0, x_4) or (x_2, x_3), we will swap (x_0, x_4). When we can swap either (x_2, x_3) or (x_2, x_4) we will swap (x_2, x_3).

Question 3:

Suppose that we are executing the hill climbing algorithm. Let the current state be $x_0 = 3, x_1 = 2, x_2 = 0, x_3 = 1$.

		Q	
			Q
	Q		
Q			

What is the cost of the current state? Is this state a local optimum? If not, give an example of a neighbour with a lower cost. If yes, is this state a global optimum?

Question 4:

Suppose that we are executing the hill climbing algorithm. Let the current state be $x_0 = 0, x_1 = 0, x_2 = 0, x_3 = 0$.

Q	Q	Q	Q

What is the cost of the current state? Is this state a local optimum? If no, give an example of a neighbour with a lower cost. If yes, is this state a global optimum?

For questions 5 and 6, consider version A of the neighbour relation: move any queen to another square in the same column.

Question 5:

How many neighbours are there for a state?

Question 6:

Suppose that we are executing the hill climbing algorithm. Let the current state be $x_0 = 3, x_1 = 2, x_2 = 0, x_3 = 1$.

		Q	
			Q
	Q		
Q			

What is the cost of the current state? Is this state a local optimum? If not, give an example of a neighbour with a lower cost. If yes, is this state a global optimum?

3.2 Simulated Annealing

Suppose that we are running the simulated annealing algorithm. In the following, fill in the values of ΔE and the probabilities of moving to the neighbour under different temperatures. For probabilities, keep three significant digits.

current.cost	next.cost	ΔE	p(T=100)	p(T=50)	p(T=10)
50	40				
50	100				
50	200				

Based on your calculations, summarize your observations below.

- What is the probability of moving to a neighbour with a lower cost (i.e. the neighbour is better than the current state)?

- Consider a neighbour with a higher cost (i.e. the neighbour is worse than the current state). As the temperature decreases, how does such a worse neighbour change?

- Consider a neighbour with a higher cost (i.e. the neighbour is worse than the current state). As the difference between the neighbour's cost and the current state's cost increases (i.e. as the neighbour becomes worse), how does the probability of moving to such a worse neighbour change?