Fleury's Algorithm for printing Eulerian Path or Circuit

Eulerian Path is a path in graph that visits every edge exactly once. Eulerian Circuit is an Eulerian Path which starts and ends on the same vertex.

We strongly recommend to first read the following post on Euler Path and Circuit.
https://www.geeksforgeeks.org/eulerian-path-and-circuit/

In the above mentioned post, we discussed the problem of finding out whether a given graph is Eulerian or not. In this post, an algorithm to print Eulerian trail or circuit is discussed.

Following is Fleury's Algorithm for printing Eulerian trail or cycle (Source Ref1).

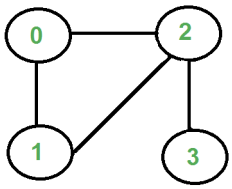
1. Make sure the graph has either 0 or 2 odd vertices.

2. If there are 0 odd vertices, start anywhere. If there are 2 odd vertices, start at one of them.

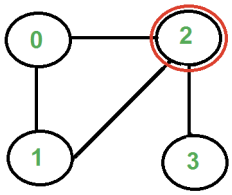
3. Follow edges one at a time. If you have a choice between a bridge and a non-bridge, *always choose the non-bridge*.

4. Stop when you run out of edges.

The idea is, ***"don't burn bridges"*** so that we can come back to a vertex and traverse remaining edges. For example let us consider the following graph.

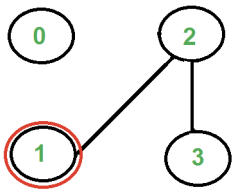


There are two vertices with odd degree, '2' and '3', we can start path from any of them. Let us start tour from vertex '2'.

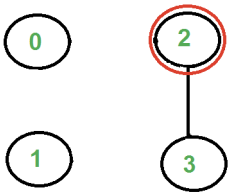


There are three edges going out from vertex '2', which one to pick? We don't pick the edge '2-3' because that is a bridge (we won't be able to come back to '3'). We can pick any of the remaining two edge. Let us say we pick '2-0'. We remove this edge and move to vertex '0'.

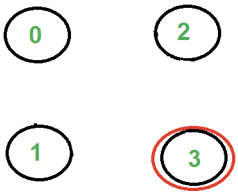
There is only one edge from vertex '0', so we pick it, remove it and move to vertex '1'. Euler tour becomes '2-0 0-1'.



There is only one edge from vertex '1', so we pick it, remove it and move to vertex '2'. Euler tour becomes '2-0 0-1 1-2'



Again there is only one edge from vertex 2, so we pick it, remove it and move to vertex 3. Euler tour becomes '2-0 0-1 1-2 2-3'



There are no more edges left, so we stop here. Final tour is '2-0 0-1 1-2 2-3'.

See this for and this fore more examples.

Following is C++ implementation of above algorithm. In the following code, it is assumed that the given graph has an Eulerian trail or Circuit. The main focus is to print an Eulerian trail or circuit. We can use isEulerian() to first check whether there is an Eulerian Trail or Circuit in the given graph.

We first find the starting point which must be an odd vertex (if there are odd vertices) and store it in variable 'u'. If there are zero odd vertices, we start from vertex '0'. We call printEulerUtil() to print Euler tour starting with u. We traverse all adjacent vertices of u, if there is only one adjacent vertex, we immediately consider it. If there are more than one adjacent vertices, we consider an adjacent v only if edge u-v is not a bridge. How to find if a given is edge is bridge? We count number of vertices reachable from u. We remove edge u-v and again count number of reachable vertices from u. If number of reachable vertices are reduced, then edge u-v is a bridge. To count reachable vertices, we can either use BFS or DFS, we have used DFS in the above code. The function DFSCount(u) returns number of vertices reachable from u.
Once an edge is processed (included in Euler tour), we remove it from the graph. To remove the edge, we replace the vertex entry with -1 in adjacency list. Note that simply deleting the node may not work as the code is recursive and a parent call may be in middle of adjacency list.

```
// A C++ program print Eulerian Trail in a given Eulerian or Semi-Eulerian Graph
#include <iostream>
#include <string.h>
#include <algorithm>
#include <list>
using namespace std;

// A class that represents an undirected graph
class Graph
{
  int V;    // No. of vertices
  list<int> *adj;    // A dynamic array of adjacency lists
public:
    // Constructor and destructor
  Graph(int V)  { this->V = V;  adj = new list<int>[V];  }
  ~Graph()      { delete [] adj;  }

  // functions to add and remove edge
  void addEdge(int u, int v) {  adj[u].push_back(v); adj[v].push_back(u); }
  void rmvEdge(int u, int v);

  // Methods to print Eulerian tour
  void printEulerTour();
  void printEulerUtil(int s);

  // This function returns count of vertices reachable from v. It does DFS
  int DFSCount(int v, bool visited[]);

  // Utility function to check if edge u-v is a valid next edge in
  // Eulerian trail or circuit
  bool isValidNextEdge(int u, int v);
};

/* The main function that print Eulerian Trail. It first finds an odd
   degree vertex (if there is any) and then calls printEulerUtil()
   to print the path */
void Graph::printEulerTour()
{
  // Find a vertex with odd degree
  int u = 0;
  for (int i = 0; i < V; i++)
      if (adj[i].size() & 1)
        {   u = i; break;  }

  // Print tour starting from oddv
  printEulerUtil(u);
  cout << endl;
}

// Print Euler tour starting from vertex u
void Graph::printEulerUtil(int u)
{
  // Recur for all the vertices adjacent to this vertex
  list<int>::iterator i;
  for (i = adj[u].begin(); i != adj[u].end(); ++i)
  {
      int v = *i;

      // If edge u-v is not removed and it's a a valid next edge
      if (v != -1 && isValidNextEdge(u, v))
      {
          cout << u << "-" << v << "  ";
          rmvEdge(u, v);
          printEulerUtil(v);
      }
  }
```

```
bool Graph::isValidNextEdge(int u, int v)
{
  // The edge u-v is valid in one of the following two cases:

  // 1) If v is the only adjacent vertex of u
  int count = 0;  // To store count of adjacent vertices
  list<int>::iterator i;
  for (i = adj[u].begin(); i != adj[u].end(); ++i)
     if (*i != -1)
        count++;
  if (count == 1)
    return true;


  // 2) If there are multiple adjacents, then u-v is not a bridge
  // Do following steps to check if u-v is a bridge

  // 2.a) count of vertices reachable from u
  bool visited[V];
  memset(visited, false, V);
  int count1 = DFSCount(u, visited);

  // 2.b) Remove edge (u, v) and after removing the edge, count
  // vertices reachable from u
  rmvEdge(u, v);
  memset(visited, false, V);
  int count2 = DFSCount(u, visited);

  // 2.c) Add the edge back to the graph
  addEdge(u, v);

  // 2.d) If count1 is greater, then edge (u, v) is a bridge
  return (count1 > count2)? false: true;
}

// This function removes edge u-v from graph.  It removes the edge by
// replacing adjcent vertex value with -1.
void Graph::rmvEdge(int u, int v)
{
  // Find v in adjacency list of u and replace it with -1
  list<int>::iterator iv = find(adj[u].begin(), adj[u].end(), v);
  *iv = -1;

  // Find u in adjacency list of v and replace it with -1
  list<int>::iterator iu = find(adj[v].begin(), adj[v].end(), u);
  *iu = -1;
}

// A DFS based function to count reachable vertices from v
int Graph::DFSCount(int v, bool visited[])
{
  // Mark the current node as visited
  visited[v] = true;
  int count = 1;

  // Recur for all vertices adjacent to this vertex
  list<int>::iterator i;
  for (i = adj[v].begin(); i != adj[v].end(); ++i)
      if (*i != -1 && !visited[*i])
          count += DFSCount(*i, visited);

  return count;
}

// Driver program to test above function
int main()
```

```
  g1.addEdge(0, 2);
  g1.addEdge(1, 2);
  g1.addEdge(2, 3);
  g1.printEulerTour();

  Graph g2(3);
  g2.addEdge(0, 1);
  g2.addEdge(1, 2);
  g2.addEdge(2, 0);
  g2.printEulerTour();

  Graph g3(5);
  g3.addEdge(1, 0);
  g3.addEdge(0, 2);
  g3.addEdge(2, 1);
  g3.addEdge(0, 3);
  g3.addEdge(3, 4);
  g3.addEdge(3, 2);
  g3.addEdge(3, 1);
  g3.addEdge(2, 4);
  g3.printEulerTour();

  return 0;
}
```

Java

```
// A Java program print Eulerian Trail
// in a given Eulerian or Semi-Eulerian Graph
import java.util.ArrayList;

// An Undirected graph using
// adjacency list representation
public class Graph {

    private int vertices; // No. of vertices
    private ArrayList<Integer>[] adj; // adjacency list

    // Constructor
    Graph(int numOfVertices)
    {
        // initialise vertex count
        this.vertices = numOfVertices;

        // initialise adjacency list
        initGraph();
    }

    // utility method to initialise adjacency list
    @SuppressWarnings("unchecked")
    private void initGraph()
    {
        adj = new ArrayList[vertices];
        for (int i = 0; i < vertices; i++)
        {
            adj[i] = new ArrayList<>();
        }
    }

    // add edge u-v
    private void addEdge(Integer u, Integer v)
    {
        adj[u].add(v);
```

```
    private void removeEdge(Integer u, Integer v)
    {
        adj[u].remove(v);
        adj[v].remove(u);
    }

    /* The main function that print Eulerian Trail.
       It first finds an odd degree vertex (if there
       is any) and then calls printEulerUtil() to
       print the path */
    private void printEulerTour()
    {
        // Find a vertex with odd degree
        Integer u = 0;
        for (int i = 0; i < vertices; i++)
        {
            if (adj[i].size() % 2 == 1)
            {
                u = i;
                break;
            }
        }

        // Print tour starting from oddv
        printEulerUtil(u);
        System.out.println();
    }

    // Print Euler tour starting from vertex u
    private void printEulerUtil(Integer u)
    {
        // Recur for all the vertices adjacent to this vertex
        for (int i = 0; i < adj[u].size(); i++)
        {
            Integer v = adj[u].get(i);
            // If edge u-v is a valid next edge
            if (isValidNextEdge(u, v))
            {
                System.out.print(u + "-" + v + " ");

                // This edge is used so remove it now
                removeEdge(u, v);
                printEulerUtil(v);
            }
        }
    }

    // The function to check if edge u-v can be
    // considered as next edge in Euler Tout
    private boolean isValidNextEdge(Integer u, Integer v)
    {
        // The edge u-v is valid in one of the
        // following two cases:

        // 1) If v is the only adjacent vertex of u
        // ie size of adjacent vertex list is 1
        if (adj[u].size() == 1) {
            return true;
        }

        // 2) If there are multiple adjacents, then
        // u-v is not a bridge Do following steps
        // to check if u-v is a bridge
        // 2.a) count of vertices reachable from u
        boolean[] isVisited = new boolean[this.vertices];
        int count1 = dfsCount(u, isVisited);
```

```
        isVisited = new boolean[this.vertices];
        int count2 = dfsCount(u, isVisited);

        // 2.c) Add the edge back to the graph
        addEdge(u, v);
        return (count1 > count2) ? false : true;
    }

    // A DFS based function to count reachable
    // vertices from v
    private int dfsCount(Integer v, boolean[] isVisited)
    {
        // Mark the current node as visited
        isVisited[v] = true;
        int count = 1;
        // Recur for all vertices adjacent to this vertex
        for (int adj : adj[v])
        {
            if (!isVisited[adj])
            {
                count = count + dfsCount(adj, isVisited);
            }
        }
        return count;
    }

    // Driver program to test above function
    public static void main(String a[])
    {
        // Let us first create and test
        // graphs shown in above figure
        Graph g1 = new Graph(4);
        g1.addEdge(0, 1);
        g1.addEdge(0, 2);
        g1.addEdge(1, 2);
        g1.addEdge(2, 3);
        g1.printEulerTour();

        Graph g2 = new Graph(3);
        g2.addEdge(0, 1);
        g2.addEdge(1, 2);
        g2.addEdge(2, 0);
        g2.printEulerTour();

        Graph g3 = new Graph(5);
        g3.addEdge(1, 0);
        g3.addEdge(0, 2);
        g3.addEdge(2, 1);
        g3.addEdge(0, 3);
        g3.addEdge(3, 4);
        g3.addEdge(3, 2);
        g3.addEdge(3, 1);
        g3.addEdge(2, 4);
        g3.printEulerTour();
    }
}

// This code is contributed by Himanshu Shekhar
```

Python

```
# Python program print Eulerian Trail in a given Eulerian or Semi-Eulerian Graph
```

```
    def __init__(self,vertices):
        self.V= vertices #No. of vertices
        self.graph = defaultdict(list) # default dictionary to store graph
        self.Time = 0

    # function to add an edge to graph
    def addEdge(self,u,v):
        self.graph[u].append(v)
        self.graph[v].append(u)

    # This function removes edge u-v from graph
    def rmvEdge(self, u, v):
        for index, key in enumerate(self.graph[u]):
            if key == v:
                self.graph[u].pop(index)
        for index, key in enumerate(self.graph[v]):
            if key == u:
                self.graph[v].pop(index)

    # A DFS based function to count reachable vertices from v
    def DFSCount(self, v, visited):
        count = 1
        visited[v] = True
        for i in self.graph[v]:
            if visited[i] == False:
                count = count + self.DFSCount(i, visited)
        return count

    # The function to check if edge u-v can be considered as next edge in
    # Euler Tour
    def isValidNextEdge(self, u, v):
        # The edge u-v is valid in one of the following two cases:

          #  1) If v is the only adjacent vertex of u
        if len(self.graph[u]) == 1:
            return True
        else:
            '''
             2) If there are multiple adjacents, then u-v is not a bridge
                 Do following steps to check if u-v is a bridge

            2.a) count of vertices reachable from u'''
            visited =[False]*(self.V)
            count1 = self.DFSCount(u, visited)

            '''2.b) Remove edge (u, v) and after removing the edge, count
                vertices reachable from u'''
            self.rmvEdge(u, v)
            visited =[False]*(self.V)
            count2 = self.DFSCount(u, visited)

            #2.c) Add the edge back to the graph
            self.addEdge(u,v)

            # 2.d) If count1 is greater, then edge (u, v) is a bridge
            return False if count1 > count2 else True


    # Print Euler tour starting from vertex u
    def printEulerUtil(self, u):
        #Recur for all the vertices adjacent to this vertex
        for v in self.graph[u]:
            #If edge u-v is not removed and it's a a valid next edge
            if self.isValidNextEdge(u, v):
                print("%d-%d " %(u,v)),
```

```
    '''The main function that print Eulerian Trail. It first finds an odd
    degree vertex (if there is any) and then calls printEulerUtil()
    to print the path '''
    def printEulerTour(self):
        #Find a vertex with odd degree
        u = 0
        for i in range(self.V):
            if len(self.graph[i]) %2 != 0 :
                u = i
                break
        # Print tour starting from odd vertex
        print ("\n")
        self.printEulerUtil(u)

# Create a graph given in the above diagram

g1 = Graph(4)
g1.addEdge(0, 1)
g1.addEdge(0, 2)
g1.addEdge(1, 2)
g1.addEdge(2, 3)
g1.printEulerTour()


g2 = Graph(3)
g2.addEdge(0, 1)
g2.addEdge(1, 2)
g2.addEdge(2, 0)
g2.printEulerTour()

g3 = Graph (5)
g3.addEdge(1, 0)
g3.addEdge(0, 2)
g3.addEdge(2, 1)
g3.addEdge(0, 3)
g3.addEdge(3, 4)
g3.addEdge(3, 2)
g3.addEdge(3, 1)
g3.addEdge(2, 4)
g3.printEulerTour()


#This code is contributed by Neelam Yadav
```

Output:

```
 2-0  0-1  1-2  2-3
 0-1  1-2  2-0
 0-1  1-2  2-0  0-3  3-4  4-2  2-3  3-1
```

Note that the above code modifies given graph, we can create a copy of graph if we don't want the given graph to be modified.

Time Complexity: Time complexity of the above implementation is O $((V+E)^2)$. The function printEulerUtil() is like DFS and it calls isValidNextEdge() which also does DFS two times. Time complexity of DFS for adjacency list representation is O(V+E). Therefore overall time complexity is O((V+E)*(V+E)) which can be written as $O(E^2)$ for a connected graph.

There are better algorithms to print Euler tour, Hierholzer's Algorithm finds in O(V+E) time.

http://en.wikipedia.org/wiki/Eulerian_path#Fleury.27s_algorithm

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above

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3

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