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1 Learning Goals

By the end of the exercise, you should be able to

- Formulate a real-world problem as a constraint satisfaction problem by defining variables, domains, and constraints.
- Trace the execution of the backtracking search algorithm with the full AC-3 arc consistency algorithm on the 4-queens problem.
- Trace the execution of the backtracking search algorithm with forward checking on the 4-queens problem.

2 The 4-Queens Problem

The 4-queens problem consists of a 4x4 chessboard with 4 queens. The goal is to place the 4 queens on the chessboard such that no two queens can attack each other. Each queen attacks anything in the same row, in the same column, or in the same diagonal.

2.1 The CSP formulation

Formulate the state of the 4-queens problem below.

- Assume that exactly one queen is in each column. Given this, we only need to keep track of the row position of each queen.
- Variables: x_0, x_1, x_2, x_3 where x_i is the row position of the queen in column i , where $i \in \{0, 1, 2, 3\}$.
- Domains: $dom(x_i) = \{0, 1, 2, 3\}$ for all x_i .
- Constraints: No pair of queens are in the same row or diagonal.

$$(\forall i(\forall j((i \neq j) \rightarrow ((x_i \neq x_j) \wedge (|x_i - x_j| \neq |i - j|)))))$$

All the constraints are explicitly given below.

$$((x_0 \neq x_1) \wedge (|x_0 - x_1| \neq 1) \wedge (x_0 \neq x_2) \wedge (|x_0 - x_2| \neq 2) \wedge (x_0 \neq x_3) \wedge (|x_0 - x_3| \neq 3) \wedge (x_1 \neq x_2) \wedge (|x_1 - x_2| \neq 1) \wedge (x_1 \neq x_3) \wedge (|x_1 - x_3| \neq 2) \wedge (x_2 \neq x_3) \wedge (|x_2 - x_3| \neq 1))$$

Formulate the 4-queens problem as a CSP below.

- State: one queen per column in the leftmost k columns with no pair of queens attacking each other.
- Initial state: no queens on the board.
- Goal state: 4 queens on the board. No pair of queens are attacking each other.
- Successor function: add a queen to the leftmost empty column such that it is not attacked by any other existing queen.

2.2 The AC-3 Arc Consistency Algorithm

Algorithm 1 Revise(X_i, C)

```

1: revised  $\leftarrow$  false
2: for  $x$  in  $\text{dom}(X_i)$  do
3:   if  $\neg \exists y \in \text{dom}(X_j)$  s.t.  $(x, y)$  satisfies the constraint  $C$  then
4:     remove  $x$  from  $\text{dom}(X_i)$ 
5:     revised  $\leftarrow$  true
6:   end if
7: end for
8: return revised

```

Algorithm 2 The AC-3 Algorithm

```

1: Put  $(v, C)$  in the set  $S$  for every variable  $v$  and every constraint involving  $v$ .
2: while  $S$  is not empty do
3:   remove  $(X_i, C_{ij})$  from  $S$  ( $C_{ij}$  is a constraint between  $X_i$  and  $X_j$ .)
4:   if Revise( $X_i, C_{ij}$ ) then
5:     if  $\text{dom}(X_i)$  is empty then return false
6:     for  $X_k$  where  $C_{ki}$  is a constraint between  $X_k$  and  $X_i$  do
7:       add  $(X_k, C_{ki})$  to  $S$ 
8:     end for
9:   end if
10: end while
11: return true

```

2.3 Backtracking Search with Arc Consistency

Algorithm 3 BACKTRACK-INFERENCES($\text{assignment}, \text{csp}$)

```

1: if  $\text{assignment}$  is complete then return true
2:  $\text{var} \leftarrow \text{SELECT-UNASSIGNED-VARIABLE}(\text{csp})$ 
3: for all  $\text{value}$  in  $\text{ORDER-DOMAIN-VALUES}(\text{var}, \text{assignment}, \text{csp})$  do
4:   if adding  $\{\text{var} = \text{value}\}$  satisfies every constraint then
5:     add  $\{\text{var} = \text{value}\}$  to  $\text{assignment}$ 
6:      $\text{inf-result} \leftarrow \text{INFERENCES}(\text{assignment}, \text{csp})$ 
7:     if  $\text{inf-result}$  is true then
8:       add the inference results to  $\text{assignment}$ 
9:        $\text{result} \leftarrow \text{BACKTRACK}(\text{assignment}, \text{csp})$ 
10:      if  $\text{result}$  is true then return  $\text{result}$ 
11:    end if
12:  end if
13:  remove  $\{\text{var} = \text{value}\}$  and the inference results from  $\text{assignment}$ 
14: end for
15: return false

```

3 Practice Questions

3.1 Arc Consistency with an Initial Assignment

Start with an initial assignment of $x_0 = 0$ for the 4-queens problem. Let's execute the AC-3 algorithm.

The starting domains and assignment:

$$x_0 = 0, \text{ dom}(x_1) \in \{0, 1, 2, 3\}, \text{ dom}(x_2) \in \{0, 1, 2, 3\}, \text{ and } \text{dom}(x_3) \in \{0, 1, 2, 3\}$$

The set of variable-constraint pairs:

$$\begin{aligned} &(x_0, x_0 \neq x_1), (x_1, x_0 \neq x_1), (x_0, x_0 \neq x_2), (x_2, x_0 \neq x_2), (x_0, x_0 \neq x_3), (x_3, x_0 \neq x_3), (x_1, x_1 \neq x_2), \\ &(x_2, x_1 \neq x_2), (x_1, x_1 \neq x_3), (x_3, x_1 \neq x_3), (x_2, x_2 \neq x_3), (x_3, x_2 \neq x_3), (x_0, |x_0 - x_1| \neq 1), \\ &(x_1, |x_0 - x_1| \neq 1), (x_0, |x_0 - x_2| \neq 2), (x_2, |x_0 - x_2| \neq 2), (x_0, |x_0 - x_3| \neq 3), (x_3, |x_0 - x_3| \neq 3), \\ &(x_1, |x_1 - x_2| \neq 1), (x_2, |x_1 - x_2| \neq 1), (x_1, |x_1 - x_3| \neq 2), (x_3, |x_1 - x_3| \neq 2), (x_2, |x_2 - x_3| \neq 1), \\ &(x_3, |x_2 - x_3| \neq 1). \end{aligned}$$

Note that every constraint appears in exactly two pairs, one for each variable in the constraint.

Below, we will write out the details of a few steps. Executing the AC-3 algorithm from start to finish will take roughly 24 steps. I encourage you to trace through all the execution steps on your own.

Answer the three questions below.

Question 1:

Let the starting domains and assignment be

$$x_0 = 0, \text{ dom}(x_1) \in \{0, 1, 2, 3\}, \text{ dom}(x_2) \in \{0, 1, 2, 3\}, \text{ and } \text{dom}(x_3) \in \{0, 1, 2, 3\}$$

Suppose that we remove the pair $(x_0, x_0 \neq x_1)$ from the set.

Describe any change to the domain of the variable.

Solution: x_0 is arc-consistent with respect to x_1 for the constraint $x_0 \neq x_1$. We do not need to change the domain of x_0 .

Describe any variable-constraint pairs that we need to add back to the set.

Solution: We do not need to add any pairs back because we did not change the domain of x_0 .

Question 2:

Let the starting domains and assignment be

$x_0 = 0$, $dom(x_1) \in \{0, 1, 2, 3\}$, $dom(x_2) \in \{0, 1, 2, 3\}$, and $dom(x_3) \in \{0, 1, 2, 3\}$

Suppose that we remove the pair $(x_1, x_0 \neq x_1)$ from the set.

Describe any change to the domain of the variable.

Solution: If $x_1 = 0$, there no value for x_0 such that $x_0 \neq x_1$. Therefore, we need to remove 0 from $dom(x_1)$.

Describe any variable-constraint pairs that we need to add back to the set.

Solution: We will need to add back the following pairs if they are not in the set already.

$(x_0, x_0 \neq x_1)$, $(x_2, x_1 \neq x_2)$, $(x_3, x_1 \neq x_3)$, $(x_0, |x_0 - x_1| \neq 1)$, $(x_2, |x_1 - x_2| \neq 1)$, $(x_3, |x_1 - x_3| \neq 2)$.

Question 3:

Let the starting domains and assignment be

$x_0 = 0$, $dom(x_1) \in \{2, 3\}$, $dom(x_2) \in \{1, 3\}$, and $dom(x_3) \in \{1, 2\}$

Suppose that we remove the pair $(x_3, |x_2 - x_3| \neq 1)$ from the set.

Describe any change to the domain of the variable.

Solution: If $x_3 = 2$, there is no value for x_2 such that $|x_2 - x_3| \neq 1$. Therefore, we need to remove 2 from $dom(x_3)$.

Describe any variable-constraint pairs that we need to add back to the set.

Solution: We will need to add back the following pairs if they are not in the set already.

$(x_2, x_2 \neq x_3)$, $(x_0, x_0 \neq x_3)$, $(x_1, x_1 \neq x_3)$, $(x_0, |x_0 - x_3| \neq 3)$, $(x_1, |x_1 - x_3| \neq 2)$, $(x_2, |x_2 - x_3| \neq 1)$

Solution:

Start with an initial assignment of $x_0 = 0$ for the 4-queens problem. Execute the AC-3 algorithm.

1. Remove $(x_0, x_0 \neq x_1)$

No change to domains

2. Remove $(x_1, x_0 \neq x_1)$

Remove 0 from $dom(x_1)$.

Domains: $x_0 = 0$, $dom(x_1) = \{1, 2, 3\}$, $dom(x_2) = \{0, 1, 2, 3\}$, and $dom(x_3) = \{0, 1, 2, 3\}$

Add back constraints $(x_0, x_0 \neq x_1)$, $(x_2, x_1 \neq x_2)$, $(x_3, x_1 \neq x_3)$, $(x_0, |x_0 - x_1| \neq 1)$, $(x_2, |x_1 - x_2| \neq 1)$, $(x_3, |x_1 - x_3| \neq 2)$.

Constraints: $(x_0, x_0 \neq x_2)$, $(x_2, x_0 \neq x_2)$, $(x_0, x_0 \neq x_3)$, $(x_3, x_0 \neq x_3)$, $(x_1, x_1 \neq x_2)$, $(x_2, x_1 \neq x_2)$, $(x_1, x_1 \neq x_3)$, $(x_3, x_1 \neq x_3)$, $(x_2, x_2 \neq x_3)$, $(x_3, x_2 \neq x_3)$, $(x_0, |x_0 - x_1| \neq 1)$, $(x_1, |x_0 - x_1| \neq 1)$, $(x_0, |x_0 - x_2| \neq 2)$, $(x_2, |x_0 - x_2| \neq 2)$, $(x_0, |x_0 - x_3| \neq 3)$, $(x_3, |x_0 - x_3| \neq 3)$, $(x_1, |x_1 - x_2| \neq 1)$, $(x_2, |x_1 - x_2| \neq 1)$, $(x_1, |x_1 - x_3| \neq 2)$, $(x_3, |x_1 - x_3| \neq 2)$, $(x_2, |x_2 - x_3| \neq 1)$, $(x_3, |x_2 - x_3| \neq 1)$, $(x_0, x_0 \neq x_1)$.

3. Remove $(x_0, x_0 \neq x_2)$

No change to domains

4. Remove $(x_2, x_0 \neq x_2)$

Remove 0 from $dom(x_2)$

Domains: $x_0 = 0$, $dom(x_1) = \{1, 2, 3\}$, $dom(x_2) = \{1, 2, 3\}$, and $dom(x_3) = \{0, 1, 2, 3\}$

Add back constraints $(x_0, x_0 \neq x_2)$, $(x_1, x_1 \neq x_2)$, $(x_3, x_2 \neq x_3)$, $(x_0, |x_0 - x_2| \neq 2)$, $(x_1, |x_1 - x_2| \neq 1)$, $(x_3, |x_2 - x_3| \neq 1)$.

Constraints: $(x_0, x_0 \neq x_3)$, $(x_3, x_0 \neq x_3)$, $(x_1, x_1 \neq x_2)$, $(x_2, x_1 \neq x_2)$, $(x_1, x_1 \neq x_3)$, $(x_3, x_1 \neq x_3)$, $(x_2, x_2 \neq x_3)$, $(x_3, x_2 \neq x_3)$, $(x_0, |x_0 - x_1| \neq 1)$, $(x_1, |x_0 - x_1| \neq 1)$, $(x_0, |x_0 - x_2| \neq 2)$, $(x_2, |x_0 - x_2| \neq 2)$, $(x_0, |x_0 - x_3| \neq 3)$, $(x_3, |x_0 - x_3| \neq 3)$, $(x_1, |x_1 - x_2| \neq 1)$, $(x_2, |x_1 - x_2| \neq 1)$, $(x_1, |x_1 - x_3| \neq 2)$, $(x_3, |x_1 - x_3| \neq 2)$, $(x_2, |x_2 - x_3| \neq 1)$, $(x_3, |x_2 - x_3| \neq 1)$, $(x_0, x_0 \neq x_1)$, $(x_0, x_0 \neq x_2)$.

5. Remove $(x_0, x_0 \neq x_3)$

No change to domains

6. Remove $(x_3, x_0 \neq x_3)$

Remove 0 from $dom(x_3)$

Domains: $x_0 = 0$, $dom(x_1) = \{1, 2, 3\}$, $dom(x_2) = \{1, 2, 3\}$, and $dom(x_3) = \{1, 2, 3\}$

Add back constraints $(x_0, x_0 \neq x_3)$, $(x_1, x_1 \neq x_3)$, $(x_2, x_2 \neq x_3)$, $(x_0, |x_0 - x_3| \neq 3)$, $(x_1, |x_1 - x_3| \neq 2)$, $(x_2, |x_2 - x_3| \neq 1)$.

Constraints: $(x_1, x_1 \neq x_2)$, $(x_2, x_1 \neq x_2)$, $(x_1, x_1 \neq x_3)$, $(x_3, x_1 \neq x_3)$, $(x_2, x_2 \neq x_3)$, $(x_3, x_2 \neq x_3)$, $(x_0, |x_0 - x_1| \neq 1)$, $(x_1, |x_0 - x_1| \neq 1)$, $(x_0, |x_0 - x_2| \neq 2)$, $(x_2, |x_0 - x_2| \neq 2)$, $(x_0, |x_0 - x_3| \neq 3)$, $(x_3, |x_0 - x_3| \neq 3)$, $(x_1, |x_1 - x_2| \neq 1)$, $(x_2, |x_1 - x_2| \neq 1)$, $(x_1, |x_1 - x_3| \neq 2)$, $(x_3, |x_1 - x_3| \neq 2)$, $(x_2, |x_2 - x_3| \neq 1)$, $(x_3, |x_2 - x_3| \neq 1)$, $(x_0, x_0 \neq x_1)$, $(x_0, x_0 \neq x_2)$, $(x_0, x_0 \neq x_3)$.

7. Remove $(x_1, x_1 \neq x_2)$
No change to domains
8. Remove $(x_2, x_1 \neq x_2)$
No change to domains
9. Remove $(x_1, x_1 \neq x_3)$
No change to domains
10. Remove $(x_3, x_1 \neq x_3)$
No change to domains
11. Remove $(x_2, x_2 \neq x_3)$
No change to domains
12. Remove $(x_3, x_2 \neq x_3)$
No change to domains
13. Remove $(x_0, |x_0 - x_1| \neq 1)$
No change to domains
14. Remove $(x_1, |x_0 - x_1| \neq 1)$
Remove 1 from $dom(x_1)$
Add back constraints $(x_0, x_0 \neq x_1), (x_2, x_1 \neq x_2), (x_3, x_1 \neq x_3), (x_0, |x_0 - x_1| \neq 1), (x_2, |x_1 - x_2| \neq 1), (x_3, |x_1 - x_3| \neq 2)$.
Domains: $x_0 = 0, dom(x_1) = \{2, 3\}, dom(x_2) = \{1, 2, 3\},$ and $dom(x_3) = \{1, 2, 3\}$
Constraints: $(x_0, |x_0 - x_2| \neq 2), (x_2, |x_0 - x_2| \neq 2), (x_0, |x_0 - x_3| \neq 3), (x_3, |x_0 - x_3| \neq 3),$
 $(x_1, |x_1 - x_2| \neq 1), (x_2, |x_1 - x_2| \neq 1), (x_1, |x_1 - x_3| \neq 2), (x_3, |x_1 - x_3| \neq 2), (x_2, |x_2 - x_3| \neq 1),$
 $(x_3, |x_2 - x_3| \neq 1), (x_0, x_0 \neq x_1), (x_0, x_0 \neq x_2), (x_0, x_0 \neq x_3), (x_2, x_1 \neq x_2), (x_3, x_1 \neq x_3).$
15. Remove $(x_0, |x_0 - x_2| \neq 2)$
No change to domains
16. Remove $(x_2, |x_0 - x_2| \neq 2)$
Remove 2 from $dom(x_2)$.
Add back constraints $(x_0, x_0 \neq x_2), (x_1, x_1 \neq x_2), (x_3, x_2 \neq x_3), (x_0, |x_0 - x_2| \neq 2), (x_1, |x_1 - x_2| \neq 1), (x_3, |x_2 - x_3| \neq 1)$.
Domains: $x_0 = 0, dom(x_1) = \{2, 3\}, dom(x_2) = \{1, 3\},$ and $dom(x_3) = \{1, 2, 3\}$
Constraints: $(x_0, |x_0 - x_3| \neq 3), (x_3, |x_0 - x_3| \neq 3), (x_1, |x_1 - x_2| \neq 1), (x_2, |x_1 - x_2| \neq 1),$
 $(x_1, |x_1 - x_3| \neq 2), (x_3, |x_1 - x_3| \neq 2), (x_2, |x_2 - x_3| \neq 1), (x_3, |x_2 - x_3| \neq 1), (x_0, x_0 \neq x_1),$
 $(x_0, x_0 \neq x_2), (x_0, x_0 \neq x_3), (x_2, x_1 \neq x_2), (x_3, x_1 \neq x_3), (x_1, x_1 \neq x_2), (x_3, x_2 \neq x_3),$
 $(x_0, |x_0 - x_2| \neq 2).$

17. Remove $(x_0, |x_0 - x_3| \neq 3)$

No change to domains

18. Remove $(x_3, |x_0 - x_3| \neq 3)$

Remove 3 from $dom(x_3)$.

Add back constraints $(x_0, x_0 \neq x_3)$, $(x_1, x_1 \neq x_3)$, $(x_2, x_2 \neq x_3)$, $(x_0, |x_0 - x_3| \neq 3)$, $(x_1, |x_1 - x_3| \neq 2)$, $(x_2, |x_2 - x_3| \neq 1)$.

Domains: $x_0 = 0$, $dom(x_1) = \{2, 3\}$, $dom(x_2) = \{1, 3\}$, and $dom(x_3) = \{1, 2\}$

Constraints: $(x_1, |x_1 - x_2| \neq 1)$, $(x_2, |x_1 - x_2| \neq 1)$, $(x_1, |x_1 - x_3| \neq 2)$, $(x_3, |x_1 - x_3| \neq 2)$, $(x_2, |x_2 - x_3| \neq 1)$, $(x_3, |x_2 - x_3| \neq 1)$, $(x_0, x_0 \neq x_1)$, $(x_0, x_0 \neq x_2)$, $(x_0, x_0 \neq x_3)$, $(x_2, x_1 \neq x_2)$, $(x_3, x_1 \neq x_3)$, $(x_1, x_1 \neq x_2)$, $(x_3, x_2 \neq x_3)$, $(x_0, |x_0 - x_2| \neq 2)$, $(x_1, x_1 \neq x_3)$, $(x_2, x_2 \neq x_3)$, $(x_0, |x_0 - x_3| \neq 3)$.

19. Remove $(x_1, |x_1 - x_2| \neq 1)$

Remove 2 from $dom(x_1)$.

Add back constraints $(x_0, x_0 \neq x_1)$, $(x_2, x_1 \neq x_2)$, $(x_3, x_1 \neq x_3)$, $(x_0, |x_0 - x_1| \neq 1)$, $(x_2, |x_1 - x_2| \neq 1)$, $(x_3, |x_1 - x_3| \neq 2)$.

Domains: $x_0 = 0$, $dom(x_1) = \{3\}$, $dom(x_2) = \{1, 3\}$, and $dom(x_3) = \{1, 2\}$

Constraints: $(x_2, |x_1 - x_2| \neq 1)$, $(x_1, |x_1 - x_3| \neq 2)$, $(x_3, |x_1 - x_3| \neq 2)$, $(x_2, |x_2 - x_3| \neq 1)$, $(x_3, |x_2 - x_3| \neq 1)$, $(x_0, x_0 \neq x_1)$, $(x_0, x_0 \neq x_2)$, $(x_0, x_0 \neq x_3)$, $(x_2, x_1 \neq x_2)$, $(x_3, x_1 \neq x_3)$, $(x_1, x_1 \neq x_2)$, $(x_3, x_2 \neq x_3)$, $(x_0, |x_0 - x_2| \neq 2)$, $(x_1, x_1 \neq x_3)$, $(x_2, x_2 \neq x_3)$, $(x_0, |x_0 - x_3| \neq 3)$, $(x_0, |x_0 - x_1| \neq 1)$.

20. Remove $(x_2, |x_1 - x_2| \neq 1)$

No change to domains

21. Remove $(x_1, |x_1 - x_3| \neq 2)$

No change to domains

22. Remove $(x_3, |x_1 - x_3| \neq 2)$

Remove 1 from $dom(x_3)$.

Add back constraints $(x_0, x_0 \neq x_3)$, $(x_1, x_1 \neq x_3)$, $(x_2, x_2 \neq x_3)$, $(x_0, |x_0 - x_3| \neq 3)$, $(x_1, |x_1 - x_3| \neq 2)$, $(x_2, |x_2 - x_3| \neq 1)$.

Domains: $x_0 = 0$, $dom(x_1) = \{3\}$, $dom(x_2) = \{1, 3\}$, and $dom(x_3) = \{2\}$

Constraints: $(x_2, |x_2 - x_3| \neq 1)$, $(x_3, |x_2 - x_3| \neq 1)$, $(x_0, x_0 \neq x_1)$, $(x_0, x_0 \neq x_2)$, $(x_0, x_0 \neq x_3)$, $(x_2, x_1 \neq x_2)$, $(x_3, x_1 \neq x_3)$, $(x_1, x_1 \neq x_2)$, $(x_3, x_2 \neq x_3)$, $(x_0, |x_0 - x_2| \neq 2)$, $(x_1, x_1 \neq x_3)$, $(x_2, x_2 \neq x_3)$, $(x_0, |x_0 - x_3| \neq 3)$, $(x_0, |x_0 - x_1| \neq 1)$, $(x_1, |x_1 - x_3| \neq 2)$.

23. Remove $(x_2, |x_2 - x_3| \neq 1)$

No change to domains

24. Remove $(x_3, |x_2 - x_3| \neq 1)$

Remove 2 from $dom(x_3)$.

Add back constraints $(x_0, x_0 \neq x_3)$, $(x_1, x_1 \neq x_3)$, $(x_2, x_2 \neq x_3)$, $(x_0, |x_0 - x_3| \neq 3)$, $(x_1, |x_1 - x_3| \neq 2)$, $(x_2, |x_2 - x_3| \neq 1)$.

Domains: $x_0 = 0$, $dom(x_1) = \{3\}$, $dom(x_2) = \{1, 3\}$, and $dom(x_3) = \{\}$

Constraints: $(x_0, x_0 \neq x_1)$, $(x_0, x_0 \neq x_2)$, $(x_0, x_0 \neq x_3)$, $(x_2, x_1 \neq x_2)$, $(x_3, x_1 \neq x_3)$, $(x_1, x_1 \neq x_2)$, $(x_3, x_2 \neq x_3)$, $(x_0, |x_0 - x_2| \neq 2)$, $(x_1, x_1 \neq x_3)$, $(x_2, x_2 \neq x_3)$, $(x_0, |x_0 - x_3| \neq 3)$, $(x_0, |x_0 - x_1| \neq 1)$, $(x_1, |x_1 - x_3| \neq 2)$, $(x_2, |x_2 - x_3| \neq 1)$.

No solution since the domain of x_3 is empty.

3.2 Backtracking Search with Forward Checking

Start with an initial assignment of $x_0 = 0$ for the 4-queens problem. Execute the backtracking search algorithm with forward checking until a solution is reached.

The starting domains and assignment:

$$x_0 = 0, \text{ dom}(x_1) = \{0, 1, 2, 3\}, \text{ dom}(x_2) = \{0, 1, 2, 3\}, \text{ and } \text{ dom}(x_3) = \{0, 1, 2, 3\}$$

Choose variables and values using the following conventions.

- When choosing which variable to assign value to, always choose the left most unassigned variable.
- When choosing which value to assign to a variable, always choose the top unassigned value.

Show the steps of backtracking search with forward checking below.

Solution:

1. Assign $x_0 = 0$.

Domains and assignments: $x_0 = 0$, $\text{dom}(x_1) = \{0, 1, 2, 3\}$, $\text{dom}(x_2) = \{0, 1, 2, 3\}$, and $\text{dom}(x_3) = \{0, 1, 2, 3\}$

Forward checking:

- Remove 0 from $\text{dom}(x_1)$ since $x_0 \neq x_1$
- Remove 1 from $\text{dom}(x_1)$ since $|x_0 - x_1| \neq 1$
- Remove 0 from $\text{dom}(x_2)$ since $x_0 \neq x_2$
- Remove 2 from $\text{dom}(x_2)$ since $|x_0 - x_2| \neq 2$
- Remove 0 from $\text{dom}(x_3)$ since $x_0 \neq x_3$
- Remove 3 from $\text{dom}(x_3)$ since $|x_0 - x_3| \neq 3$

Updated domains and assignment: $x_0 = 0$, $\text{dom}(x_1) = \{2, 3\}$, $\text{dom}(x_2) = \{1, 3\}$, and $\text{dom}(x_3) = \{1, 2\}$

2. Assign $x_1 = 2$.

Domains and assignments: $x_0 = 0$, $x_1 = 2$, $\text{dom}(x_2) = \{1, 3\}$, and $\text{dom}(x_3) = \{1, 2\}$

Forward Checking:

- Remove 1 from $\text{dom}(x_2)$ since $|x_1 - x_2| \neq 1$
- Remove 3 from $\text{dom}(x_2)$ since $|x_1 - x_2| \neq 1$

Updated domains and assignments: $x_0 = 0$, $x_1 = 2$, $dom(x_2) = \{\}$, and $dom(x_3) = \{1, 2\}$

This attempt yields **no solution** since $dom(x_2)$ is empty.

Backtrack!

3. Assign $x_1 = 3$.

Domains and assignments: $x_0 = 0$, $x_1 = 3$, $dom(x_2) = \{1, 3\}$, and $dom(x_3) = \{1, 2\}$

Forward Checking:

- Remove 3 from $dom(x_2)$ since $x_1 \neq x_2$
- Remove 1 from $dom(x_3)$ since $|x_1 - x_3| \neq 2$

Updated domains and assignments: $x_0 = 0$, $x_1 = 3$, $dom(x_2) = \{1\}$, and $dom(x_3) = \{2\}$

4. Assign $x_2 = 1$.

Domains and assignments: $x_0 = 0$, $x_1 = 3$, $x_2 = 1$, and $dom(x_3) = \{2\}$

Forward Checking:

- Remove 2 from $dom(x_3)$ since $|x_2 - x_3| \neq 1$

Updated domains and assignments: $x_0 = 0$, $x_1 = 3$, $x_2 = 1$, and $dom(x_3) = \{\}$

This attempt yields **no solution** since $dom(x_3)$ is empty.

Backtrack!

5. Assign $x_0 = 1$.

Domains and assignments: $x_0 = 1$, $dom(x_1) = \{0, 1, 2, 3\}$, $dom(x_2) = \{0, 1, 2, 3\}$, and $dom(x_3) = \{0, 1, 2, 3\}$

Forward Checking:

- Remove 0 from $dom(x_1)$ since $|x_0 - x_1| \neq 1$
- Remove 1 from $dom(x_1)$ since $x_0 \neq x_1$
- Remove 2 from $dom(x_1)$ since $|x_0 - x_1| \neq 1$
- Remove 1 from $dom(x_2)$ since $x_0 \neq x_2$
- Remove 3 from $dom(x_2)$ since $|x_0 - x_2| \neq 2$
- Remove 1 from $dom(x_3)$ since $x_0 \neq x_3$

Updated domains and assignments: $x_0 = 1$, $dom(x_1) = \{3\}$, $dom(x_2) = \{0, 2\}$, and $dom(x_3) = \{0, 2, 3\}$

6. Assign $x_1 = 3$.

Domains and assignments: $x_0 = 1$, $x_1 = 3$, $dom(x_2) = \{0, 2\}$, and $dom(x_3) = \{0, 2, 3\}$

Forward Checking:

- Remove 2 from $dom(x_2)$ since $|x_1 - x_2| \neq 1$
- Remove 3 from $dom(x_3)$ since $x_1 \neq x_3$

Updated domains and assignments: $x_0 = 1$, $x_1 = 3$, $dom(x_2) = \{0\}$, and $dom(x_3) = \{0, 2\}$

7. Assign $x_2 = 0$.

Domains and assignments: $x_0 = 1$, $x_1 = 3$, $x_2 = 0$, and $dom(x_3) = \{0, 2\}$

Forward Checking:

- Remove 0 from $dom(x_3)$ since $x_2 \neq x_3$

Updated domains and assignments: $x_0 = 1$, $x_1 = 3$, $x_2 = 0$, and $dom(x_3) = \{2\}$

8. Assign $x_3 = 2$.

A solution is reached!

Solution: $x_0 = 1$, $x_1 = 3$, $x_2 = 0$, and $x_3 = 2$.

Solution: See the search tree of backtracking search and forward checking below.

