# Contents



# 1 Learning Goals

By the end of the exercise, you should be able to

- Formulate a real-world problem as a constraint satisfaction problem by defining variables, domains, and constraints.
- Trace the execution of the backtracking search algorithm with the full AC-3 arc consistency algorithm on the 4-queens problem.
- Trace the execution of the backtracking search algorithm with forward checking on the 4-queens problem.

# 2 The 4-Queens Problem

The 4-queens problem consists of a 4x4 chessboard with 4 queens. The goal is to place the 4 queens on the chessboard such that no two queens can attack each other. Each queen attacks anything in the same row, in the same column, or in the same diagonal.

## 2.1 The CSP formulation

Formulate the state of the 4-queens problem below.

- Assume that exactly one queen is in each column. Given this, we only need to keep track of the row position of each queen.
- Variables: $x_0, x_1, x_2, x_3$ where $x_i$ is the row position of the queen in column $i$, where $i \in \{0, 1, 2, 3\}$.
- Domains: $dom(x_i) = \{0, 1, 2, 3\}$ for all $x_i$.
- Constraints: No pair of queens are in the same row or diagonal.

  $(\forall i(\forall j((i \neq j) \rightarrow ((x_i \neq x_j) \wedge (|x_i - x_j| \neq |i - j|)))))$

  All the constraints are explicitly given below.

  $((x_0 \neq x_1) \wedge (|x_0 - x_1| \neq 1) \wedge (x_0 \neq x_2) \wedge (|x_0 - x_2| \neq 2) \wedge (x_0 \neq x_3) \wedge (|x_0 - x_3| \neq 3) \wedge (x_1 \neq x_2) \wedge (|x_1 - x_2| \neq 1) \wedge (x_1 \neq x_3) \wedge (|x_1 - x_3| \neq 2) \wedge (x_2 \neq x_3) \wedge (|x_2 - x_3| \neq 1))$

Formulate the 4-queens problem as a CSP below.

- State: one queen per column in the leftmost $k$ columns with no pair of queens attacking each other.
- Initial state: no queens on the board.
- Goal state: 4 queens on the board. No pair of queens are attacking each other.
- Successor function: add a queen to the leftmost empty column such that it is not attacked by any other existing queen.

## 2.2 The AC-3 Arc Consistency Algorithm

**Algorithm 1** Revise($X_i, C$)

revised ← false
**for** $x$ in $dom(X_i)$ **do**
  **if** $\neg\exists\, y \in dom(X_j)$ s.t. $(x, y)$ satisfies the constraint $C$ **then**
    remove $x$ from $dom(X_i)$
    revised ← true
  **end if**
**end for**
**return** revised

**Algorithm 2** The AC-3 Algorithm

Put $(v, C)$ in the set $S$ for every variable $v$ and every constraint involving $v$.
**while** $S$ is not empty **do**
  remove $(X_i, C_{ij})$ from $S$ ($C_{ij}$ is a constraint between $X_i$ and $X_j$.)
  **if** Revise($X_i, C_{ij}$) **then**
    **if** $dom(X_i)$ is empty **then return** false
    **for** $X_k$ where $C_{ki}$ is a constraint between $X_k$ and $X_i$ **do**
      add $(X_k, C_{ki})$ to $S$
    **end for**
  **end if**
**end while**
**return** true

## 2.3 Backtracking Search with Arc Consistency

**Algorithm 3** BACKTRACK-INFERENCES(assignment, csp)

**if** assignment is complete **then return** true
var ← SELECT-UNASSIGNED-VARIABLE(csp)
**for all** value in ORDER-DOMAIN-VALUES(var, assignment, csp) **do**
  **if** adding {var = value} satisfies every constraint **then**
    add {var = value} to assignment
    inf-result ← INFERENCES(assignment, csp)
    **if** inf-result is true **then**
      add the inference results to assignment
      result ← BACKTRACK(assignment, csp)
      **if** result is true **then return** result
    **end if**
  **end if**
  remove {var = value} and the inference results from assignment
**end for**
**return** false

# 3 Practice Questions

## 3.1 Arc Consistency with an Initial Assignment

Start with an initial assignment of $x_0 = 0$ for the 4-queens problem. Let's execute the AC-3 algorithm.

The starting domains and assignment:

$x_0 = 0$, $dom(x_1) \in \{0, 1, 2, 3\}$, $dom(x_2) \in \{0, 1, 2, 3\}$, and $dom(x_3) \in \{0, 1, 2, 3\}$

The set of variable-constraint pairs:

$(x_0, x_0 \neq x_1)$, $(x_1, x_0 \neq x_1)$, $(x_0, x_0 \neq x_2)$, $(x_2, x_0 \neq x_2)$, $(x_0, x_0 \neq x_3)$, $(x_3, x_0 \neq x_3)$, $(x_1, x_1 \neq x_2)$, $(x_2, x_1 \neq x_2)$, $(x_1, x_1 \neq x_3)$, $(x_3, x_1 \neq x_3)$, $(x_2, x_2 \neq x_3)$, $(x_3, x_2 \neq x_3)$, $(x_0, |x_0 - x_1| \neq 1)$, $(x_1, |x_0 - x_1| \neq 1)$, $(x_0, |x_0 - x_2| \neq 2)$, $(x_2, |x_0 - x_2| \neq 2)$, $(x_0, |x_0 - x_3| \neq 3)$, $(x_3, |x_0 - x_3| \neq 3)$, $(x_1, |x_1 - x_2| \neq 1)$, $(x_2, |x_1 - x_2| \neq 1)$, $(x_1, |x_1 - x_3| \neq 2)$, $(x_3, |x_1 - x_3| \neq 2)$, $(x_2, |x_2 - x_3| \neq 1)$, $(x_3, |x_2 - x_3| \neq 1)$.

Note that every constraint appears in exactly two pairs, one for each variable in the constraint.

Below, we will write out the details of a few steps. Executing the AC-3 algorithm from start to finish will take roughly 24 steps. I encourage you to trace through all the execution steps on your own.

Answer the three questions below.

**Question 1:**

Let the starting domains and assignment be

$x_0 = 0$, $dom(x_1) \in \{0, 1, 2, 3\}$, $dom(x_2) \in \{0, 1, 2, 3\}$, and $dom(x_3) \in \{0, 1, 2, 3\}$

Suppose that we remove the pair $(x_0, x_0 \neq x_1)$ from the set.

Describe any change to the domain of the variable.

**Solution:** $x_0$ is arc-consistent with respect to $x_1$ for the constraint $x_0 \neq x_1$. We do not need to change the domain of $x_0$.

Describe any variable-constraint pairs that we need to add back to the set.

**Solution:** We do not need to add any pairs back because we did not change the domain of $x_0$.

**Question 2:**

Let the starting domains and assignment be

$x_0 = 0$, $dom(x_1) \in \{0, 1, 2, 3\}$, $dom(x_2) \in \{0, 1, 2, 3\}$, and $dom(x_3) \in \{0, 1, 2, 3\}$

Suppose that we remove the pair $(x_1, x_0 \neq x_1)$ from the set.

Describe any change to the domain of the variable.

**Solution:** If $x_1 = 0$, there no value for $x_0$ such that $x_0 \neq x_1$. Therefore, we need to remove 0 from $dom(x_1)$.

Describe any variable-constraint pairs that we need to add back to the set.

**Solution:** We will need to add back the following pairs if they are not in the set already.

$(x_0, x_0 \neq x_1)$, $(x_2, x_1 \neq x_2)$, $(x_3, x_1 \neq x_3)$, $(x_0, |x_0 - x_1| \neq 1)$, $(x_2, |x_1 - x_2| \neq 1)$, $(x_3, |x_1 - x_3| \neq 2)$.

**Question 3:**

Let the starting domains and assignment be

$x_0 = 0$, $dom(x_1) \in \{2, 3\}$, $dom(x_2) \in \{1, 3\}$, and $dom(x_3) \in \{1, 2\}$

Suppose that we remove the pair $(x_3, |x_2 - x_3| \neq 1)$ from the set.

Describe any change to the domain of the variable.

**Solution:** If $x_3 = 2$, there is no value for $x_2$ such that $|x_2 - x_3| \neq 1$. Therefore, we need to remove 2 from $dom(x_3)$.

Describe any variable-constraint pairs that we need to add back to the set.

**Solution:** We will need to add back the following pairs if they are not in the set already.

$(x_2, x_2 \neq x_3)$, $(x_0, x_0 \neq x_3)$, $(x_1, x_1 \neq x_3)$, $(x_0, |x_0 - x_3| \neq 3)$, $(x_1, |x_1 - x_3| \neq 2)$, $(x_2, |x_2 - x_3| \neq 1)$

**Solution:**

Start with an initial assignment of $x_0 = 0$ for the 4-queens problem. Execute the AC-3 algorithm.

1. Remove $(x_0, x_0 \neq x_1)$

   No change to domains

2. Remove $(x_1, x_0 \neq x_1)$

   Remove 0 from $dom(x_1)$.

   Domains: $x_0 = 0$, $dom(x_1) = \{1, 2, 3\}$, $dom(x_2) = \{0, 1, 2, 3\}$, and $dom(x_3) = \{0, 1, 2, 3\}$

   Add back constraints $(x_0, x_0 \neq x_1)$, $(x_2, x_1 \neq x_2)$, $(x_3, x_1 \neq x_3)$, $(x_0, |x_0 - x_1| \neq 1)$, $(x_2, |x_1 - x_2| \neq 1)$, $(x_3, |x_1 - x_3| \neq 2)$.

   Constraints: $(x_0, x_0 \neq x_2)$, $(x_2, x_0 \neq x_2)$, $(x_0, x_0 \neq x_3)$, $(x_3, x_0 \neq x_3)$, $(x_1, x_1 \neq x_2)$, $(x_2, x_1 \neq x_2)$, $(x_1, x_1 \neq x_3)$, $(x_3, x_1 \neq x_3)$, $(x_2, x_2 \neq x_3)$, $(x_3, x_2 \neq x_3)$, $(x_0, |x_0 - x_1| \neq 1)$, $(x_1, |x_0 - x_1| \neq 1)$, $(x_0, |x_0 - x_2| \neq 2)$, $(x_2, |x_0 - x_2| \neq 2)$, $(x_0, |x_0 - x_3| \neq 3)$, $(x_3, |x_0 - x_3| \neq 3)$, $(x_1, |x_1 - x_2| \neq 1)$, $(x_2, |x_1 - x_2| \neq 1)$, $(x_1, |x_1 - x_3| \neq 2)$, $(x_3, |x_1 - x_3| \neq 2)$, $(x_2, |x_2 - x_3| \neq 1)$, $(x_3, |x_2 - x_3| \neq 1)$, $(x_0, x_0 \neq x_1)$.

3. Remove $(x_0, x_0 \neq x_2)$

   No change to domains

4. Remove $(x_2, x_0 \neq x_2)$

   Remove 0 from $dom(x_2)$

   Domains: $x_0 = 0$, $dom(x_1) = \{1, 2, 3\}$, $dom(x_2) = \{1, 2, 3\}$, and $dom(x_3) = \{0, 1, 2, 3\}$

   Add back constraints $(x_0, x_0 \neq x_2)$, $(x_1, x_1 \neq x_2)$, $(x_3, x_2 \neq x_3)$, $(x_0, |x_0 - x_2| \neq 2)$, $(x_1, |x_1 - x_2| \neq 1)$, $(x_3, |x_2 - x_3| \neq 1)$.

   Constraints: $(x_0, x_0 \neq x_3)$, $(x_3, x_0 \neq x_3)$, $(x_1, x_1 \neq x_2)$, $(x_2, x_1 \neq x_2)$, $(x_1, x_1 \neq x_3)$, $(x_3, x_1 \neq x_3)$, $(x_2, x_2 \neq x_3)$, $(x_3, x_2 \neq x_3)$, $(x_0, |x_0 - x_1| \neq 1)$, $(x_1, |x_0 - x_1| \neq 1)$, $(x_0, |x_0 - x_2| \neq 2)$, $(x_2, |x_0 - x_2| \neq 2)$, $(x_0, |x_0 - x_3| \neq 3)$, $(x_3, |x_0 - x_3| \neq 3)$, $(x_1, |x_1 - x_2| \neq 1)$, $(x_2, |x_1 - x_2| \neq 1)$, $(x_1, |x_1 - x_3| \neq 2)$, $(x_3, |x_1 - x_3| \neq 2)$, $(x_2, |x_2 - x_3| \neq 1)$, $(x_3, |x_2 - x_3| \neq 1)$, $(x_0, x_0 \neq x_1)$, $(x_0, x_0 \neq x_2)$.

5. Remove $(x_0, x_0 \neq x_3)$

   No change to domains

6. Remove $(x_3, x_0 \neq x_3)$

   Remove 0 from $dom(x_3)$

   Domains: $x_0 = 0$, $dom(x_1) = \{1, 2, 3\}$, $dom(x_2) = \{1, 2, 3\}$, and $dom(x_3) = \{1, 2, 3\}$

   Add back constraints $(x_0, x_0 \neq x_3)$, $(x_1, x_1 \neq x_3)$, $(x_2, x_2 \neq x_3)$, $(x_0, |x_0 - x_3| \neq 3)$, $(x_1, |x_1 - x_3| \neq 2)$, $(x_2, |x_2 - x_3| \neq 1)$.

   Constraints: $(x_1, x_1 \neq x_2)$, $(x_2, x_1 \neq x_2)$, $(x_1, x_1 \neq x_3)$, $(x_3, x_1 \neq x_3)$, $(x_2, x_2 \neq x_3)$, $(x_3, x_2 \neq x_3)$, $(x_0, |x_0 - x_1| \neq 1)$, $(x_1, |x_0 - x_1| \neq 1)$, $(x_0, |x_0 - x_2| \neq 2)$, $(x_2, |x_0 - x_2| \neq 2)$, $(x_0, |x_0 - x_3| \neq 3)$, $(x_3, |x_0 - x_3| \neq 3)$, $(x_1, |x_1 - x_2| \neq 1)$, $(x_2, |x_1 - x_2| \neq 1)$, $(x_1, |x_1 - x_3| \neq 2)$, $(x_3, |x_1 - x_3| \neq 2)$, $(x_2, |x_2 - x_3| \neq 1)$, $(x_3, |x_2 - x_3| \neq 1)$, $(x_0, x_0 \neq x_1)$, $(x_0, x_0 \neq x_2)$, $(x_0, x_0 \neq x_3)$.

7. Remove $(x_1, x_1 \neq x_2)$

   No change to domains

8. Remove $(x_2, x_1 \neq x_2)$

   No change to domains

9. Remove $(x_1, x_1 \neq x_3)$

   No change to domains

10. Remove $(x_3, x_1 \neq x_3)$

    No change to domains

11. Remove $(x_2, x_2 \neq x_3)$

    No change to domains

12. Remove $(x_3, x_2 \neq x_3)$

    No change to domains

13. Remove $(x_0, |x_0 - x_1| \neq 1)$

    No change to domains

14. Remove $(x_1, |x_0 - x_1| \neq 1)$

    Remove 1 from $dom(x_1)$

    Add back constraints $(x_0, x_0 \neq x_1)$, $(x_2, x_1 \neq x_2)$, $(x_3, x_1 \neq x_3)$, $(x_0, |x_0 - x_1| \neq 1)$, $(x_2, |x_1 - x_2| \neq 1)$, $(x_3, |x_1 - x_3| \neq 2)$.

    Domains: $x_0 = 0$, $dom(x_1) = \{2, 3\}$, $dom(x_2) = \{1, 2, 3\}$, and $dom(x_3) = \{1, 2, 3\}$

    Constraints: $(x_0, |x_0 - x_2| \neq 2)$, $(x_2, |x_0 - x_2| \neq 2)$, $(x_0, |x_0 - x_3| \neq 3)$, $(x_3, |x_0 - x_3| \neq 3)$, $(x_1, |x_1 - x_2| \neq 1)$, $(x_2, |x_1 - x_2| \neq 1)$, $(x_1, |x_1 - x_3| \neq 2)$, $(x_3, |x_1 - x_3| \neq 2)$, $(x_2, |x_2 - x_3| \neq 1)$, $(x_3, |x_2 - x_3| \neq 1)$, $(x_0, x_0 \neq x_1)$, $(x_0, x_0 \neq x_2)$, $(x_0, x_0 \neq x_3)$, $(x_2, x_1 \neq x_2)$, $(x_3, x_1 \neq x_3)$.

15. Remove $(x_0, |x_0 - x_2| \neq 2)$

    No change to domains

16. Remove $(x_2, |x_0 - x_2| \neq 2)$

    Remove 2 from $dom(x_2)$.

    Add back constraints $(x_0, x_0 \neq x_2)$, $(x_1, x_1 \neq x_2)$, $(x_3, x_2 \neq x_3)$, $(x_0, |x_0 - x_2| \neq 2)$, $(x_1, |x_1 - x_2| \neq 1)$, $(x_3, |x_2 - x_3| \neq 1)$.

    Domains: $x_0 = 0$, $dom(x_1) = \{2, 3\}$, $dom(x_2) = \{1, 3\}$, and $dom(x_3) = \{1, 2, 3\}$

    Constraints: $(x_0, |x_0 - x_3| \neq 3)$, $(x_3, |x_0 - x_3| \neq 3)$, $(x_1, |x_1 - x_2| \neq 1)$, $(x_2, |x_1 - x_2| \neq 1)$, $(x_1, |x_1 - x_3| \neq 2)$, $(x_3, |x_1 - x_3| \neq 2)$, $(x_2, |x_2 - x_3| \neq 1)$, $(x_3, |x_2 - x_3| \neq 1)$, $(x_0, x_0 \neq x_1)$, $(x_0, x_0 \neq x_2)$, $(x_0, x_0 \neq x_3)$, $(x_2, x_1 \neq x_2)$, $(x_3, x_1 \neq x_3)$, $(x_1, x_1 \neq x_2)$, $(x_3, x_2 \neq x_3)$, $(x_0, |x_0 - x_2| \neq 2)$.

17. Remove $(x_0, |x_0 - x_3| \neq 3)$

    No change to domains

18. Remove $(x_3, |x_0 - x_3| \neq 3)$

    Remove 3 from $dom(x_3)$.

    Add back constraints $(x_0, x_0 \neq x_3)$, $(x_1, x_1 \neq x_3)$, $(x_2, x_2 \neq x_3)$, $(x_0, |x_0 - x_3| \neq 3)$, $(x_1, |x_1 - x_3| \neq 2)$, $(x_2, |x_2 - x_3| \neq 1)$.

    Domains: $x_0 = 0$, $dom(x_1) = \{2, 3\}$, $dom(x_2) = \{1, 3\}$, and $dom(x_3) = \{1, 2\}$

    Constraints: $(x_1, |x_1 - x_2| \neq 1)$, $(x_2, |x_1 - x_2| \neq 1)$, $(x_1, |x_1 - x_3| \neq 2)$, $(x_3, |x_1 - x_3| \neq 2)$, $(x_2, |x_2 - x_3| \neq 1)$, $(x_3, |x_2 - x_3| \neq 1)$, $(x_0, x_0 \neq x_1)$, $(x_0, x_0 \neq x_2)$, $(x_0, x_0 \neq x_3)$, $(x_2, x_1 \neq x_2)$, $(x_3, x_1 \neq x_3)$, $(x_1, x_1 \neq x_2)$, $(x_3, x_2 \neq x_3)$, $(x_0, |x_0 - x_2| \neq 2)$, $(x_1, x_1 \neq x_3)$, $(x_2, x_2 \neq x_3)$, $(x_0, |x_0 - x_3| \neq 3)$.

19. Remove $(x_1, |x_1 - x_2| \neq 1)$

    Remove 2 from $dom(x_1)$.

    Add back constraints $(x_0, x_0 \neq x_1)$, $(x_2, x_1 \neq x_2)$, $(x_3, x_1 \neq x_3)$, $(x_0, |x_0 - x_1| \neq 1)$, $(x_2, |x_1 - x_2| \neq 1)$, $(x_3, |x_1 - x_3| \neq 2)$.

    Domains: $x_0 = 0$, $dom(x_1) = \{3\}$, $dom(x_2) = \{1, 3\}$, and $dom(x_3) = \{1, 2\}$

    Constraints: $(x_2, |x_1 - x_2| \neq 1)$, $(x_1, |x_1 - x_3| \neq 2)$, $(x_3, |x_1 - x_3| \neq 2)$, $(x_2, |x_2 - x_3| \neq 1)$, $(x_3, |x_2 - x_3| \neq 1)$, $(x_0, x_0 \neq x_1)$, $(x_0, x_0 \neq x_2)$, $(x_0, x_0 \neq x_3)$, $(x_2, x_1 \neq x_2)$, $(x_3, x_1 \neq x_3)$, $(x_1, x_1 \neq x_2)$, $(x_3, x_2 \neq x_3)$, $(x_0, |x_0 - x_2| \neq 2)$, $(x_1, x_1 \neq x_3)$, $(x_2, x_2 \neq x_3)$, $(x_0, |x_0 - x_3| \neq 3)$, $(x_0, |x_0 - x_1| \neq 1)$.

20. Remove $(x_2, |x_1 - x_2| \neq 1)$

    No change to domains

21. Remove $(x_1, |x_1 - x_3| \neq 2)$

    No change to domains

22. Remove $(x_3, |x_1 - x_3| \neq 2)$

    Remove 1 from $dom(x_3)$.

    Add back constraints $(x_0, x_0 \neq x_3)$, $(x_1, x_1 \neq x_3)$, $(x_2, x_2 \neq x_3)$, $(x_0, |x_0 - x_3| \neq 3)$, $(x_1, |x_1 - x_3| \neq 2)$, $(x_2, |x_2 - x_3| \neq 1)$.

    Domains: $x_0 = 0$, $dom(x_1) = \{3\}$, $dom(x_2) = \{1, 3\}$, and $dom(x_3) = \{2\}$

    Constraints: $(x_2, |x_2 - x_3| \neq 1)$, $(x_3, |x_2 - x_3| \neq 1)$, $(x_0, x_0 \neq x_1)$, $(x_0, x_0 \neq x_2)$, $(x_0, x_0 \neq x_3)$, $(x_2, x_1 \neq x_2)$, $(x_3, x_1 \neq x_3)$, $(x_1, x_1 \neq x_2)$, $(x_3, x_2 \neq x_3)$, $(x_0, |x_0 - x_2| \neq 2)$, $(x_1, x_1 \neq x_3)$, $(x_2, x_2 \neq x_3)$, $(x_0, |x_0 - x_3| \neq 3)$, $(x_0, |x_0 - x_1| \neq 1)$, $(x_1, |x_1 - x_3| \neq 2)$.

23. Remove $(x_2, |x_2 - x_3| \neq 1)$

    No change to domains

24. Remove $(x_3, |x_2 - x_3| \neq 1)$

    Remove 2 from $dom(x_3)$.

    Add back constraints $(x_0, x_0 \neq x_3)$, $(x_1, x_1 \neq x_3)$, $(x_2, x_2 \neq x_3)$, $(x_0, |x_0 - x_3| \neq 3)$, $(x_1, |x_1 - x_3| \neq 2)$, $(x_2, |x_2 - x_3| \neq 1)$.

    Domains: $x_0 = 0$, $dom(x_1) = \{3\}$, $dom(x_2) = \{1, 3\}$, and $dom(x_3) = \{\}$

    Constraints: $(x_0, x_0 \neq x_1)$, $(x_0, x_0 \neq x_2)$, $(x_0, x_0 \neq x_3)$, $(x_2, x_1 \neq x_2)$, $(x_3, x_1 \neq x_3)$, $(x_1, x_1 \neq x_2)$, $(x_3, x_2 \neq x_3)$, $(x_0, |x_0 - x_2| \neq 2)$, $(x_1, x_1 \neq x_3)$, $(x_2, x_2 \neq x_3)$, $(x_0, |x_0 - x_3| \neq 3)$, $(x_0, |x_0 - x_1| \neq 1)$, $(x_1, |x_1 - x_3| \neq 2)$, $(x_2, |x_2 - x_3| \neq 1)$.

    No solution since the domain of $x_3$ is empty.

## 3.2 Backtracking Search with Forward Checking

Start with an initial assignment of $x_0 = 0$ for the 4-queens problem. Execute the backtracking search algorithm with forward checking until a solution is reached.

The starting domains and assignment:

$x_0 = 0$, $dom(x_1) = \{0, 1, 2, 3\}$, $dom(x_2) = \{0, 1, 2, 3\}$, and $dom(x_3) = \{0, 1, 2, 3\}$

Choose variables and values using the following conventions.

- When choosing which variable to assign value to, always choose the left most unassigned variable.
- When choosing which value to assign to a variable, always choose the top unassigned value.

Show the steps of backtracking search with forward checking below.

**Solution:**

1. Assign $x_0 = 0$.

   Domains and assignments: $x_0 = 0$, $dom(x_1) = \{0, 1, 2, 3\}$, $dom(x_2) = \{0, 1, 2, 3\}$, and $dom(x_3) = \{0, 1, 2, 3\}$

   Forward checking:
   - Remove 0 from $dom(x_1)$ since $x_0 \neq x_1$
   - Remove 1 from $dom(x_1)$ since $|x_0 - x_1| \neq 1$
   - Remove 0 from $dom(x_2)$ since $x_0 \neq x_2$
   - Remove 2 from $dom(x_2)$ since $|x_0 - x_2| \neq 2$
   - Remove 0 from $dom(x_3)$ since $x_0 \neq x_3$
   - Remove 3 from $dom(x_3)$ since $|x_0 - x_3| \neq 3$

   Updated domains and assignment: $x_0 = 0$, $dom(x_1) = \{2, 3\}$, $dom(x_2) = \{1, 3\}$, and $dom(x_3) = \{1, 2\}$

2. Assign $x_1 = 2$.

   Domains and assignments: $x_0 = 0$, $x_1 = 2$, $dom(x_2) = \{1, 3\}$, and $dom(x_3) = \{1, 2\}$

   Forward Checking:
   - Remove 1 from $dom(x_2)$ since $|x_1 - x_2| \neq 1$
   - Remove 3 from $dom(x_2)$ since $|x_1 - x_2| \neq 1$

Updated domains and assignments: $x_0 = 0$, $x_1 = 2$, $dom(x_2) = \{\}$, and $dom(x_3) = \{1, 2\}$

This attempt yields **no solution** since $dom(x_2)$ is empty.

**Backtrack!**

3. Assign $x_1 = 3$.

   Domains and assignments: $x_0 = 0$, $x_1 = 3$, $dom(x_2) = \{1, 3\}$, and $dom(x_3) = \{1, 2\}$

   Forward Checking:
   - Remove 3 from $dom(x_2)$ since $x_1 \neq x_2$
   - Remove 1 from $dom(x_3)$ since $|x_1 - x_3| \neq 2$

   Updated domains and assignments: $x_0 = 0$, $x_1 = 3$, $dom(x_2) = \{1\}$, and $dom(x_3) = \{2\}$

4. Assign $x_2 = 1$.

   Domains and assignments: $x_0 = 0$, $x_1 = 3$, $x_2 = 1$, and $dom(x_3) = \{2\}$

   Forward Checking:
   - Remove 2 from $dom(x_3)$ since $|x_2 - x_3| \neq 1$

   Updated domains and assignments: $x_0 = 0$, $x_1 = 3$, $x_2 = 1$, and $dom(x_3) = \{\}$

   This attempt yields **no solution** since $dom(x_3)$ is empty.

   **Backtrack!**

5. Assign $x_0 = 1$.

   Domains and assignments: $x_0 = 1$, $dom(x_1) = \{0, 1, 2, 3\}$, $dom(x_2) = \{0, 1, 2, 3\}$, and $dom(x_3) = \{0, 1, 2, 3\}$

   Forward Checking:
   - Remove 0 from $dom(x_1)$ since $|x_0 - x_1| \neq 1$
   - Remove 1 from $dom(x_1)$ since $x_0 \neq x_1$
   - Remove 2 from $dom(x_1)$ since $|x_0 - x_1| \neq 1$
   - Remove 1 from $dom(x_2)$ since $x_0 \neq x_2$
   - Remove 3 from $dom(x_2)$ since $|x_0 - x_2| \neq 2$
   - Remove 1 from $dom(x_3)$ since $x_0 \neq x_3$

   Updated domains and assignments: $x_0 = 1$, $dom(x_1) = \{3\}$, $dom(x_2) = \{0, 2\}$, and $dom(x_3) = \{0, 2, 3\}$

6. Assign $x_1 = 3$.

   Domains and assignments: $x_0 = 1$, $x_1 = 3$, $dom(x_2) = \{0, 2\}$, and $dom(x_3) = \{0, 2, 3\}$

   Forward Checking:
   - Remove 2 from $dom(x_2)$ since $|x_1 - x_2| \neq 1$
   - Remove 3 from $dom(x_3)$ since $x_1 \neq x_3$

Updated domains and assignments: $x_0 = 1$, $x_1 = 3$, $dom(x_2) = \{0\}$, and $dom(x_3) = \{0, 2\}$

7. Assign $x_2 = 0$.

   Domains and assignments: $x_0 = 1$, $x_1 = 3$, $x_2 = 0$, and $dom(x_3) = \{0, 2\}$

   Forward Checking:
   - Remove 0 from $dom(x_3)$ since $x_2 \neq x_3$

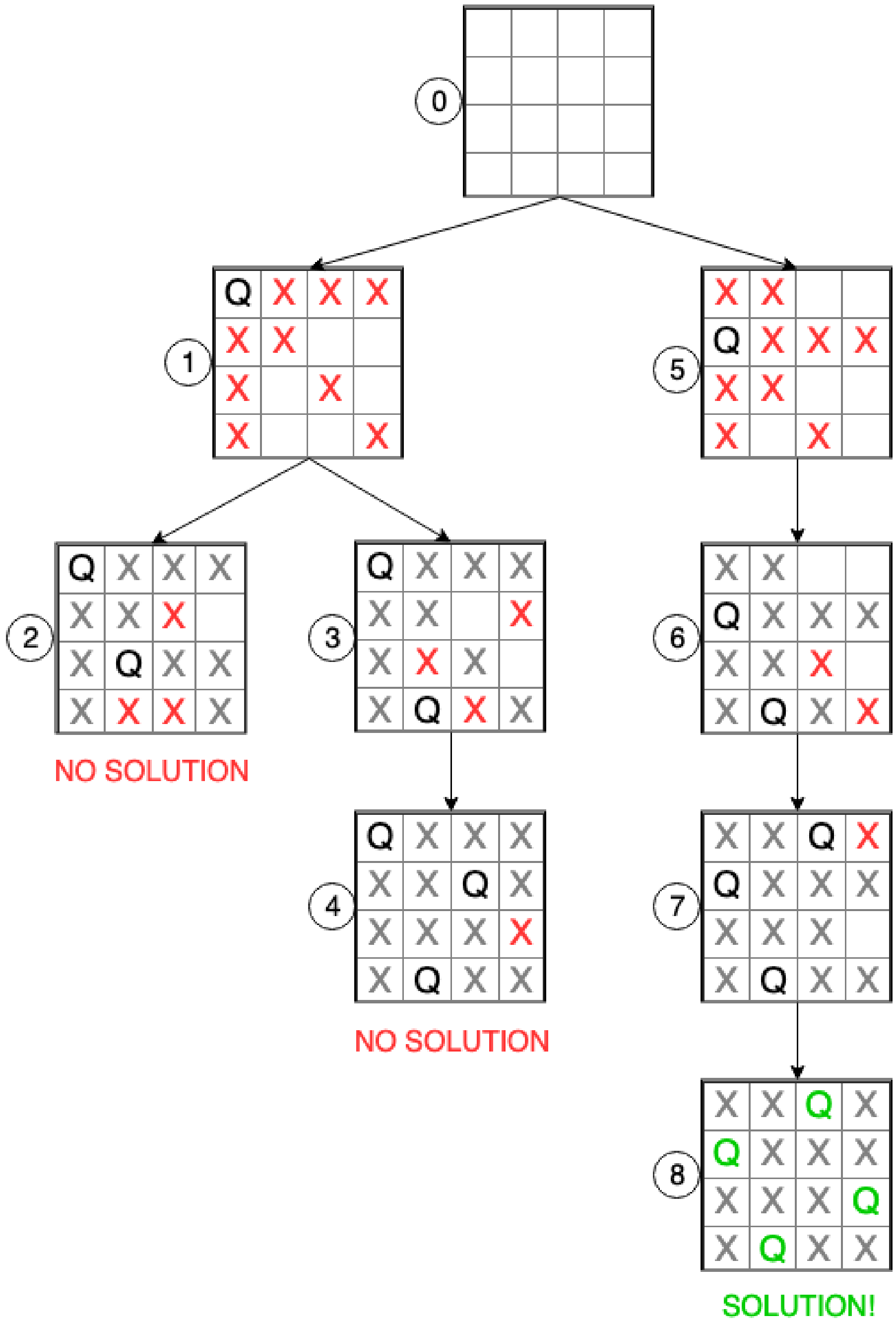
   Updated domains and assignments: $x_0 = 1$, $x_1 = 3$, $x_2 = 0$, and $dom(x_3) = \{2\}$

8. Assign $x_3 = 2$.

   A solution is reached!

   **Solution:** $x_0 = 1$, $x_1 = 3$, $x_2 = 0$, and $x_3 = 2$.

**Solution:** See the search tree of backtracking search and forward checking below.


0
1
5
2
3
6
NO SOLUTION
4
7
NO SOLUTION
8
SOLUTION!