



GPU Teaching Kit
Accelerated Computing



Lecture 15: CUDA Memories



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Part I: Memory and Data Locality

CUDA Memories

Objective

- To learn to effectively use the CUDA memory types in a parallel program
 - Importance of memory access efficiency
 - Registers, shared memory, global memory
 - Scope and lifetime

3



Review: Image Blur Kernel.

```
// Get the average of the surrounding 2xBLUR_SIZE x 2xBLUR_SIZE box
for(int blurRow = -BLUR_SIZE; blurRow < BLUR_SIZE+1; ++blurRow) {
    for(int blurCol = -BLUR_SIZE; blurCol < BLUR_SIZE+1; ++blurCol) {

        int curRow = Row + blurRow;
        int curCol = Col + blurCol;
        // Verify we have a valid image pixel
        if(curRow > -1 && curRow < h && curCol > -1 && curCol < w) {
            ➡ pixVal += in[curRow * w + curCol];
            pixels++; // Keep track of number of pixels in the accumulated total
        }
    }
}

// Write our new pixel value out
out[Row * w + Col] = (unsigned char)(pixVal / pixels);
```

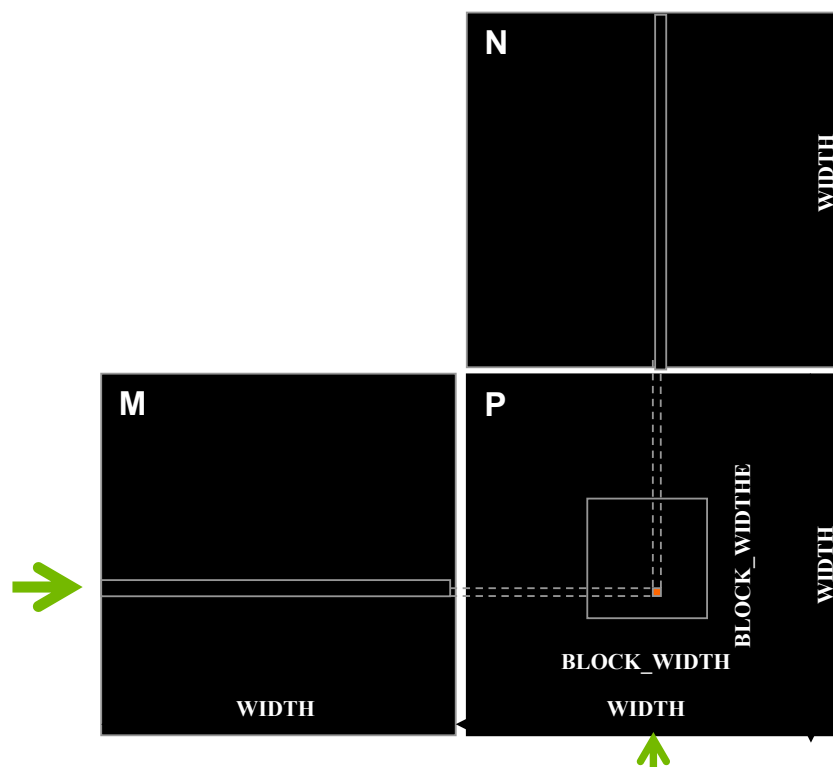
4



How about performance on a GPU

- All threads access global memory for their input matrix elements
 - One memory accesses (4 bytes) per floating-point addition
 - 4B/s of memory bandwidth/FLOPS
- Assume a GPU with
 - Peak floating-point rate 1,500 GFLOPS with 200 GB/s DRAM bandwidth
 - $4 \times 1,500 = 6,000$ GB/s required to achieve peak FLOPS rating
 - The 200 GB/s memory bandwidth limits the execution at $200/4 = 50$ GFLOPS
- This limits the execution rate to 3.3% ($50/1500$) of the peak floating-point execution rate of the device!
- Need to drastically cut down memory accesses to get close to the 1,500 GFLOPS

Example – Matrix Multiplication



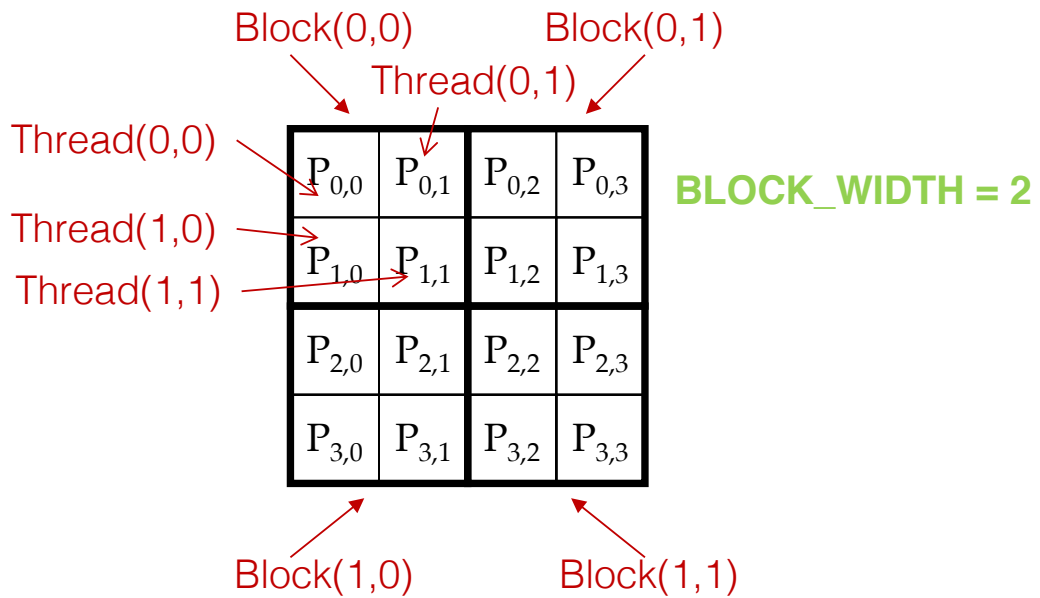
A Basic Matrix Multiplication

```
__global__ void MatrixMulKernel(float* M, float* N, float* P, int Width) {  
    // Calculate the row index of the P element and M  
    int Row = blockIdx.y*blockDim.y+threadIdx.y;  
  
    // Calculate the column index of P and N  
    int Col = blockIdx.x*blockDim.x+threadIdx.x;  
  
    if ((Row < Width) && (Col < Width)) {  
        float Pvalue = 0;  
        // each thread computes one element of the block sub-matrix  
        for (int k = 0; k < Width; ++k) {  
            Pvalue += M[Row*Width+k]*N[k*Width+Col];  
        }  
        P[Row*Width+Col] = Pvalue;  
    }  
}
```

Example – Matrix Multiplication

```
__global__ void MatrixMulKernel(float* M, float* N, float* P, int Width) {  
  
    // Calculate the row index of the P element and M  
    int Row = blockIdx.y*blockDim.y+threadIdx.y;  
  
    // Calculate the column index of P and N  
    int Col = blockIdx.x*blockDim.x+threadIdx.x;  
  
    if ((Row < Width) && (Col < Width)) {  
        float Pvalue = 0;  
        // each thread computes one element of the block sub-matrix  
        for (int k = 0; k < Width; ++k) {  
            Pvalue += M[Row*Width+k]*N[k*Width+Col];  
        }  
        P[Row*Width+Col] = Pvalue;  
    }  
}
```

A Toy Example: Thread to P Data Mapping

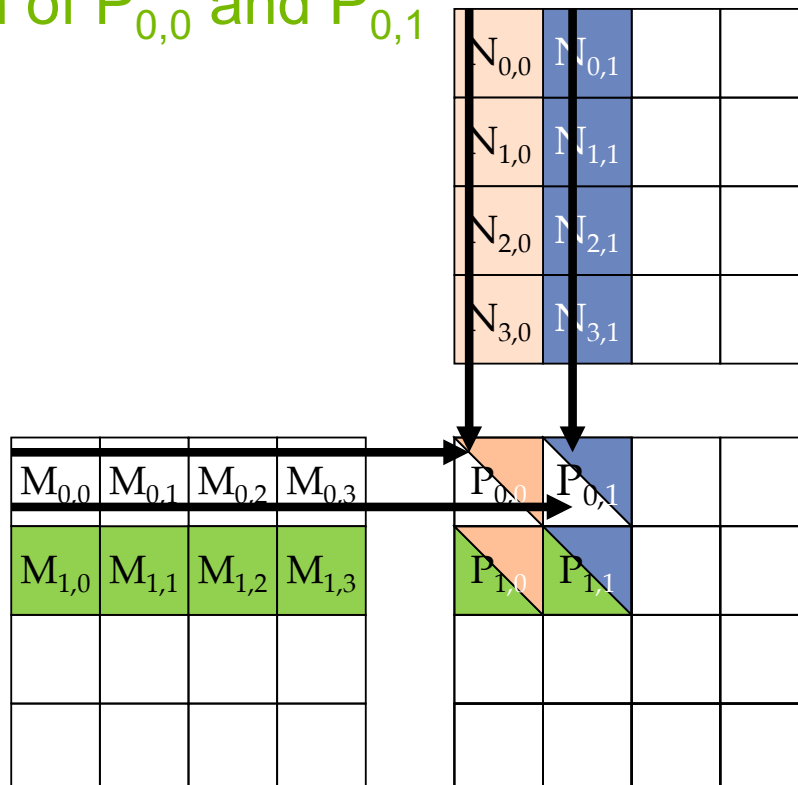


9

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Calculation of $P_{0,0}$ and $P_{0,1}$

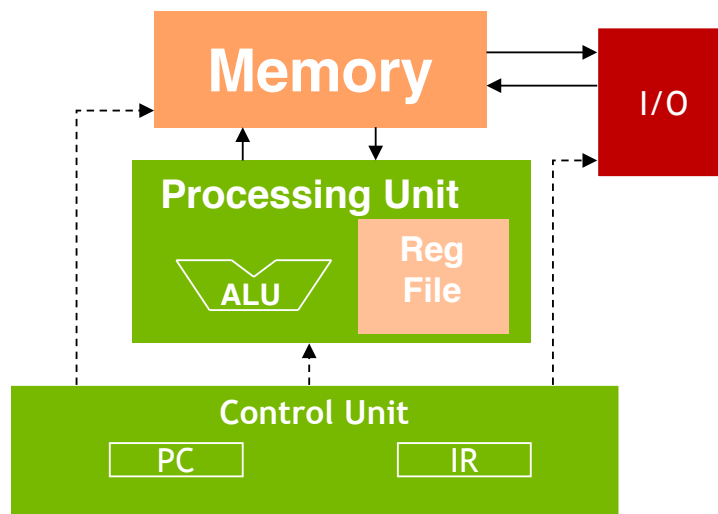


10

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Memory and Registers in the Von-Neumann Model

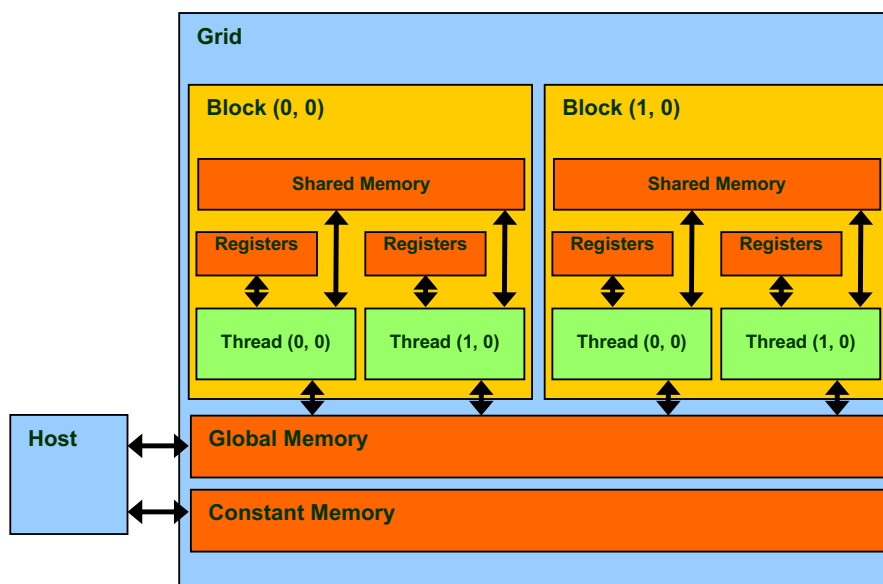


11

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Programmer View of CUDA Memories



12

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Declaring CUDA Variables

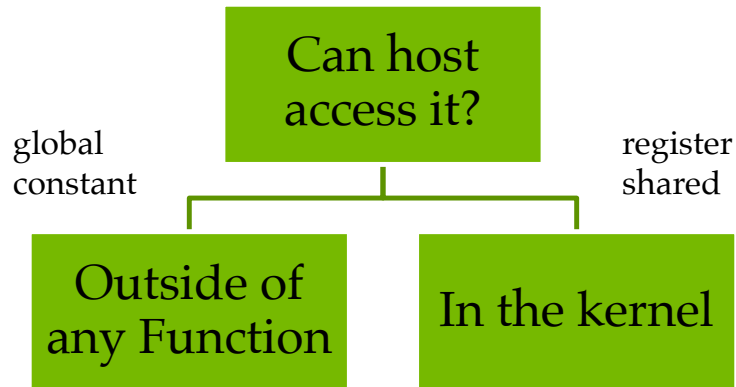
Variable declaration	Memory	Scope	Lifetime
int LocalVar;	register	thread	thread
__device__ __shared__ int SharedVar;	shared	block	block
__device__ int GlobalVar;	global	grid	application
__device__ __constant__ int ConstantVar;	constant	grid	application

- **__device__** is optional when used with **__shared__**, or **__constant__**
- Automatic variables reside in a **register**
 - Except **per-thread arrays** that reside in global memory

Example: Shared Memory Variable Declaration

```
void blurKernel(unsigned char * in, unsigned char * out, int w, int h)
{
    __shared__ float ds_in[TILE_WIDTH][TILE_WIDTH];
    ...
}
```

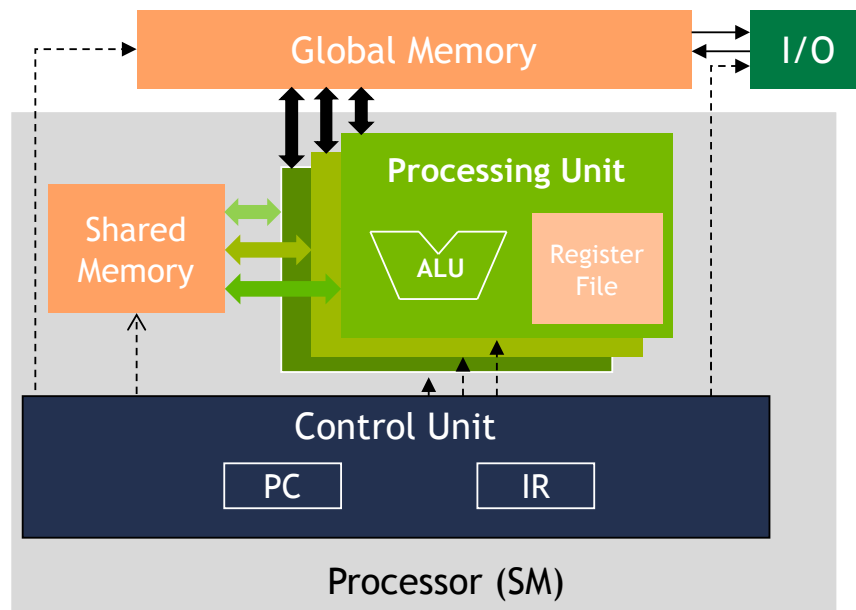
Where to Declare Variables?



Shared Memory in CUDA

- A special type of memory whose contents are explicitly defined and used in the kernel source code
 - One in each SM
 - Accessed at much higher speed (in both latency and throughput) than global memory
 - Scope of access and sharing - thread blocks
 - Lifetime – thread block, contents will disappear after the corresponding thread finishes terminates execution
 - Accessed by memory load/store instructions
 - A form of scratchpad memory in computer architecture

Hardware View of CUDA Memories



17

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Part II: Memory and Data Locality

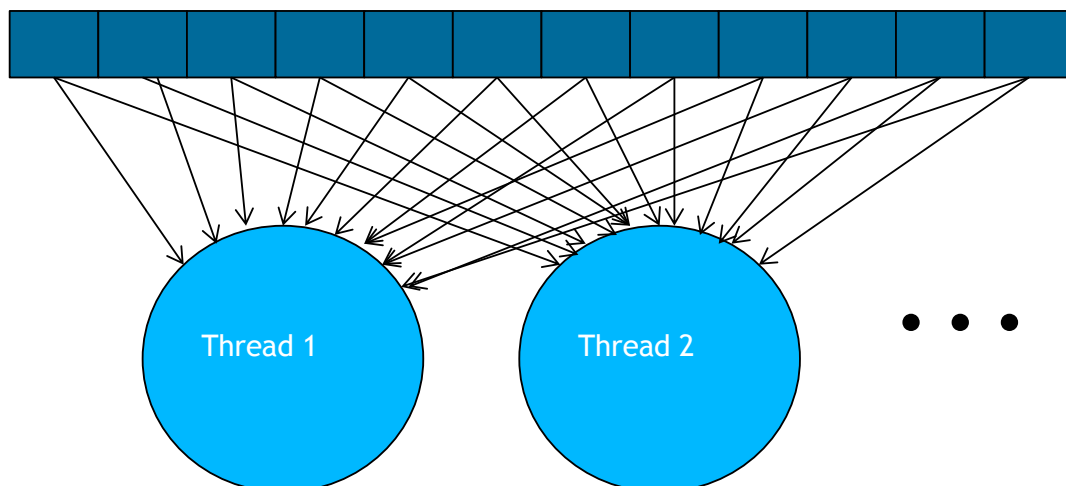
Tiled Parallel Algorithms

Objective

- To understand the motivation and ideas for tiled parallel algorithms
 - Reducing the limiting effect of memory bandwidth on parallel kernel performance
 - Tiled algorithms and barrier synchronization

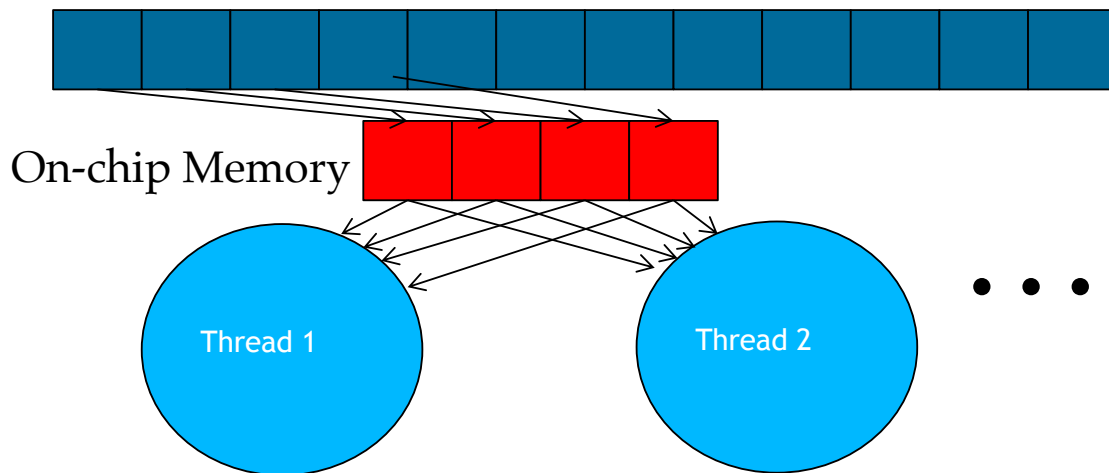
Global Memory Access Pattern of the Basic Matrix Multiplication Kernel

Global Memory



Tiling/Blocking - Basic Idea

Global Memory

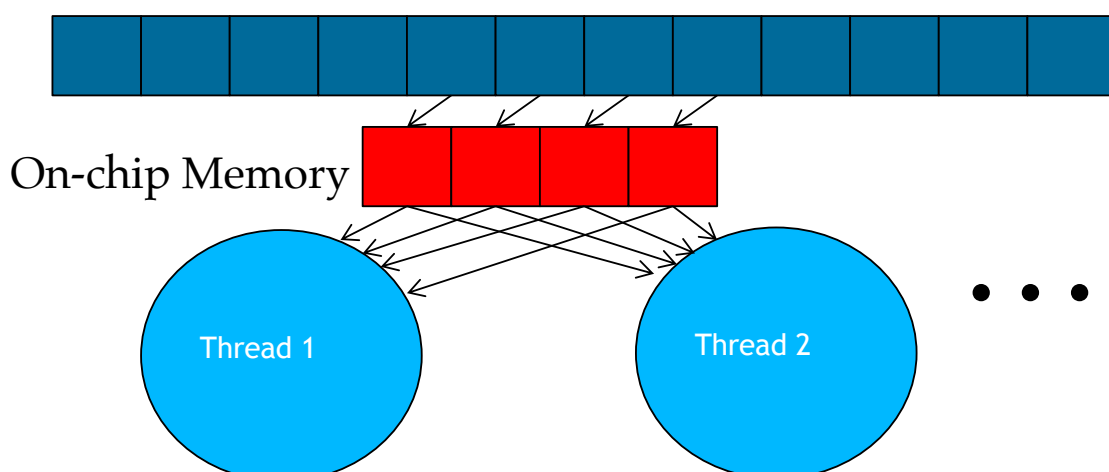


Divide the global memory content into tiles

Focus the computation of threads on one or a small number of tiles at each point in time

Tiling/Blocking - Basic Idea

Global Memory



Basic Concept of Tiling

- In a congested traffic system, significant reduction of vehicles can greatly improve the delay seen by all vehicles
 - Carpooling for commuters
 - Tiling for global memory accesses
 - drivers = threads accessing their memory data operands
 - cars = memory access requests



23

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Some Computations are More Challenging to Tile

- Some carpools may be easier than others
 - Car pool participants need to have similar work schedule
 - Some vehicles may be more suitable for carpooling
- Similar challenges exist in tiling



24

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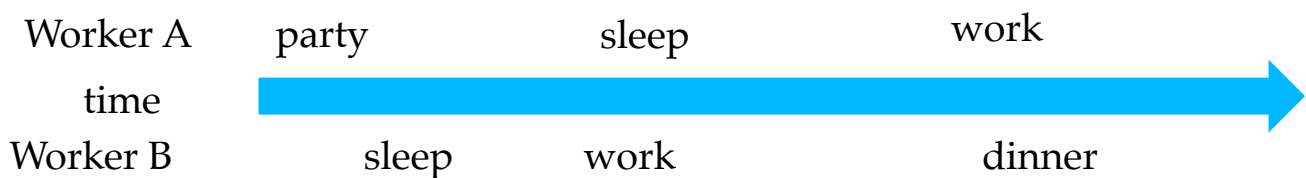
Carpools need synchronization.

- Good: when people have similar schedule



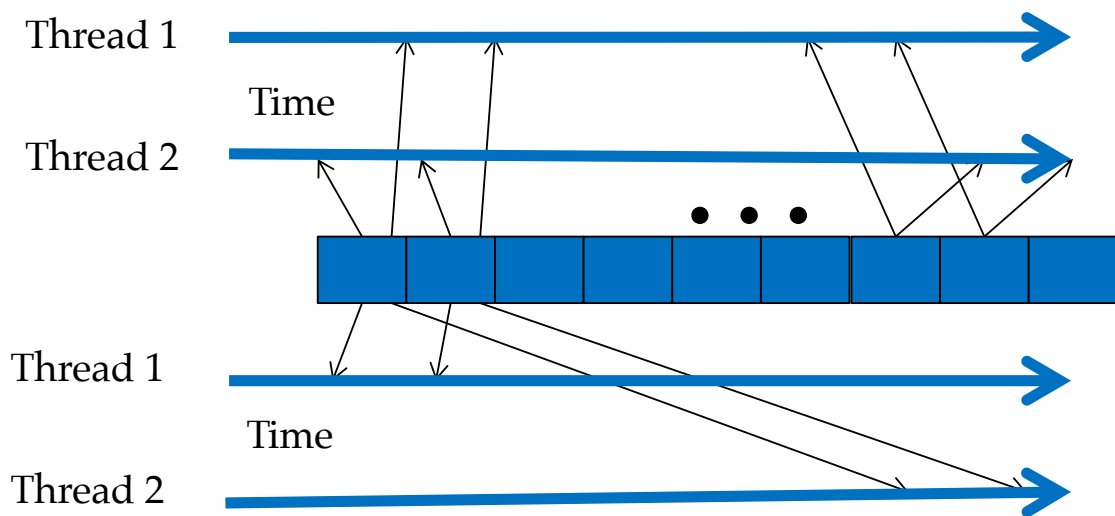
Carpools need synchronization.

- Bad: when people have very different schedule



Same with Tiling

- Good: when threads have similar access timing



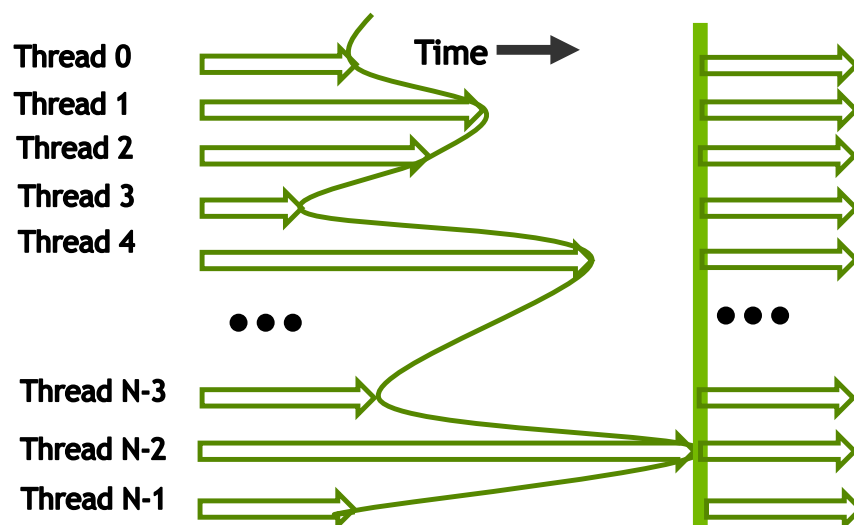
- Bad: when threads have very different timing

27

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Barrier Synchronization for Tiling



28

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Outline of Tiling Technique

- Identify a tile of global memory contents that are accessed by multiple threads
- Load the tile from global memory into on-chip memory
- Use barrier synchronization to make sure that all threads are ready to start the phase
- Have the multiple threads to access their data from the on-chip memory
- Use barrier synchronization to make sure that all threads have completed the current phase
- Move on to the next tile



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Part III: Memory Model and Locality

Tiled Matrix Multiplication

Objective

- To understand the design of a tiled parallel algorithm for matrix multiplication
 - Loading a tile
 - Phased execution
 - Barrier Synchronization

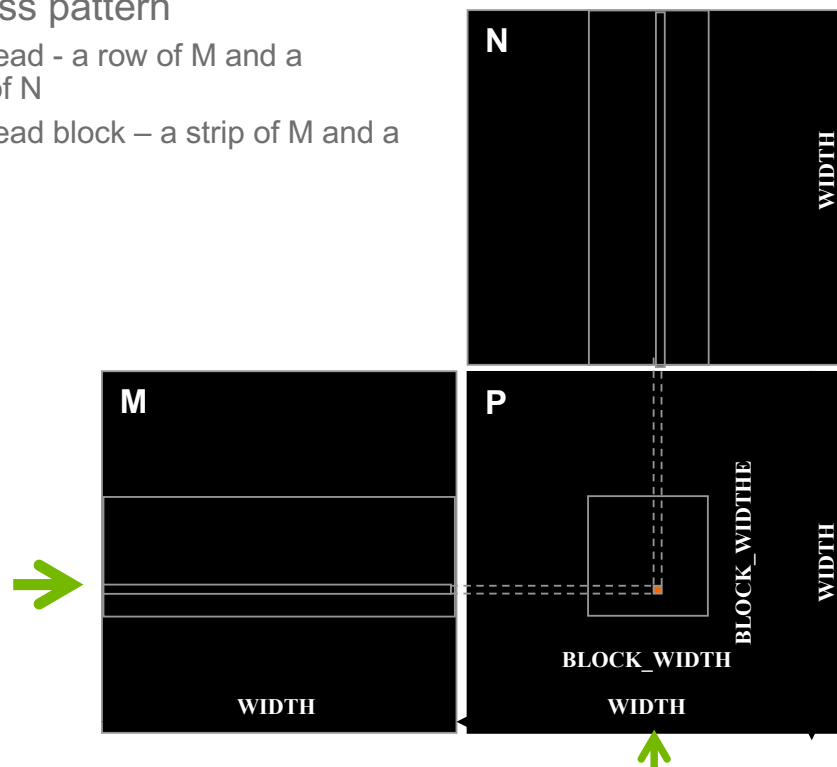
31

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Matrix Multiplication

- Data access pattern
 - Each thread - a row of M and a column of N
 - Each thread block – a strip of M and a strip of N



32

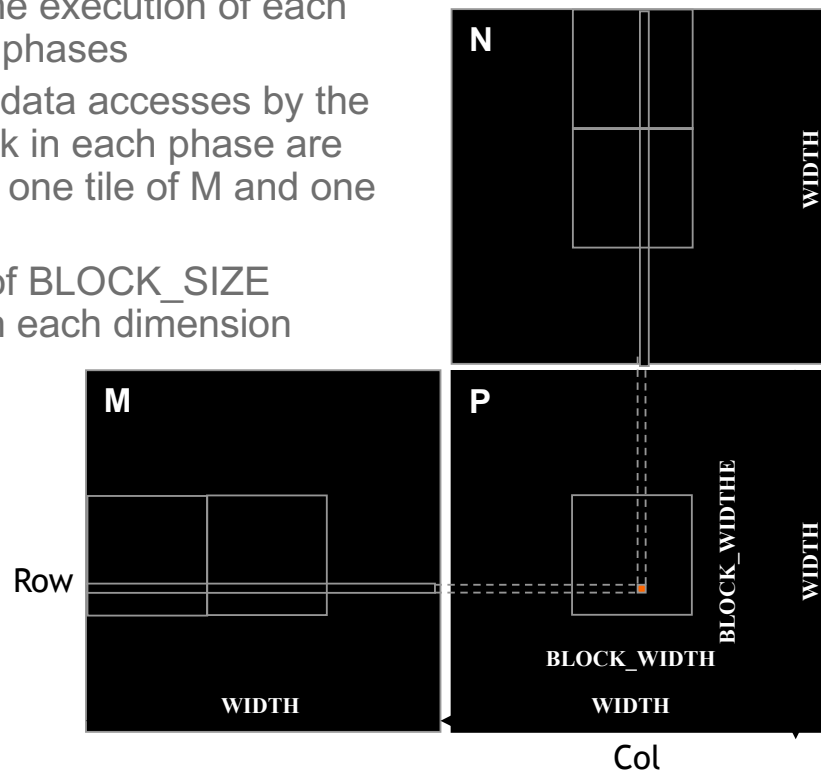
Col

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Tiled Matrix Multiplication

- Break up the execution of each thread into phases
- so that the data accesses by the thread block in each phase are focused on one tile of M and one tile of N
- The tile is of BLOCK_SIZE elements in each dimension



33

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Loading a Tile

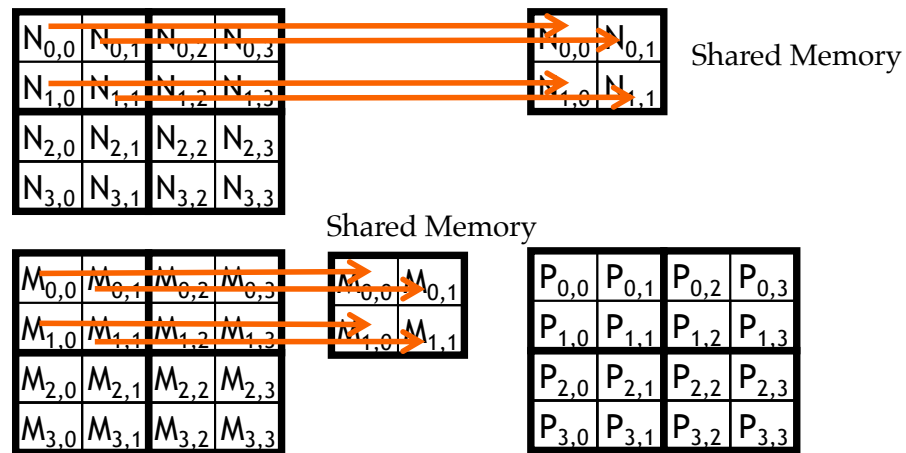
- All threads in a block participate
 - Each thread loads one M element and one N element in tiled code

34

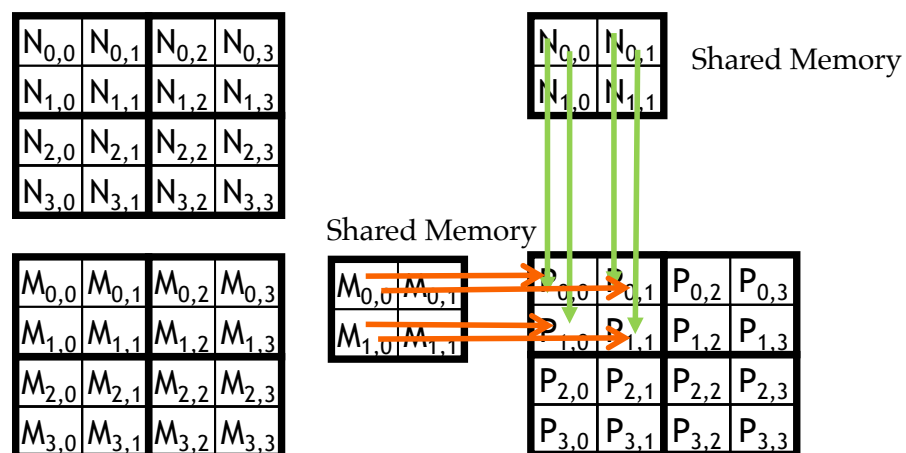
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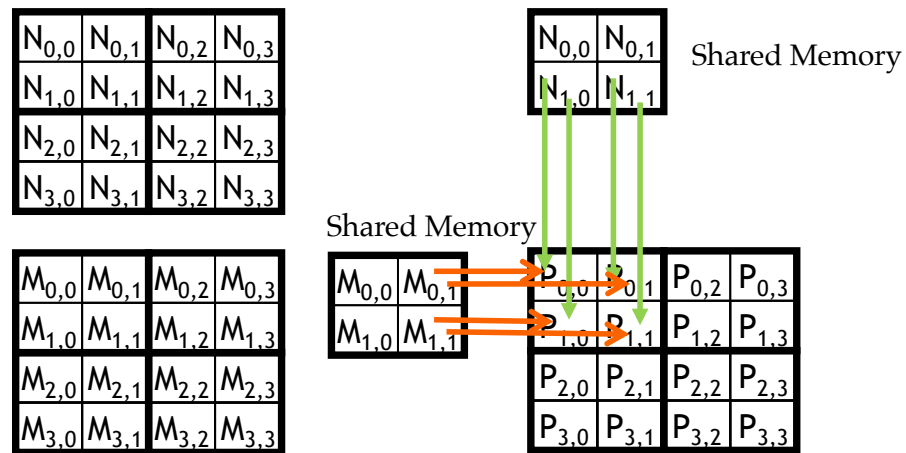
Phase 0 Load for Block (0,0)



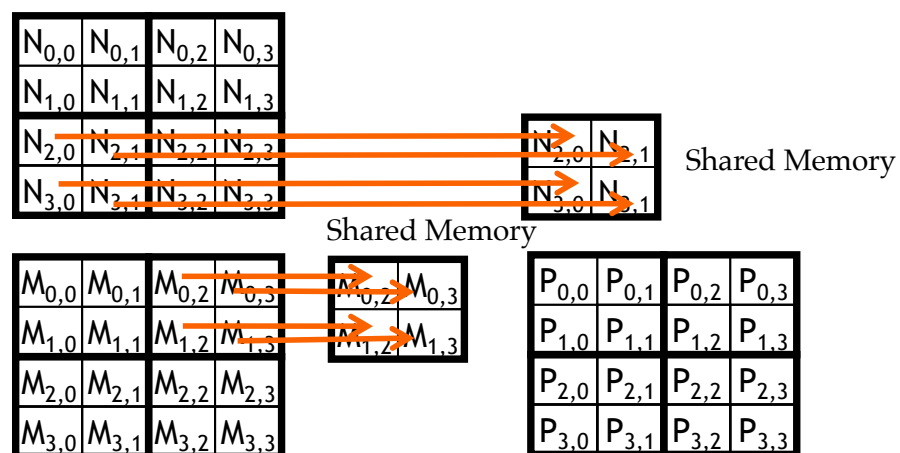
Phase 0 Use for Block (0,0) (iteration 0)



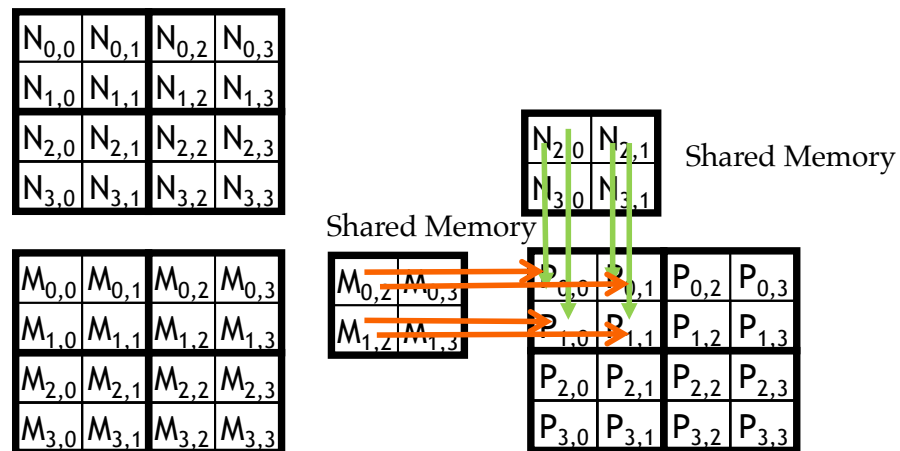
Phase 0 Use for Block (0,0) (iteration 1)



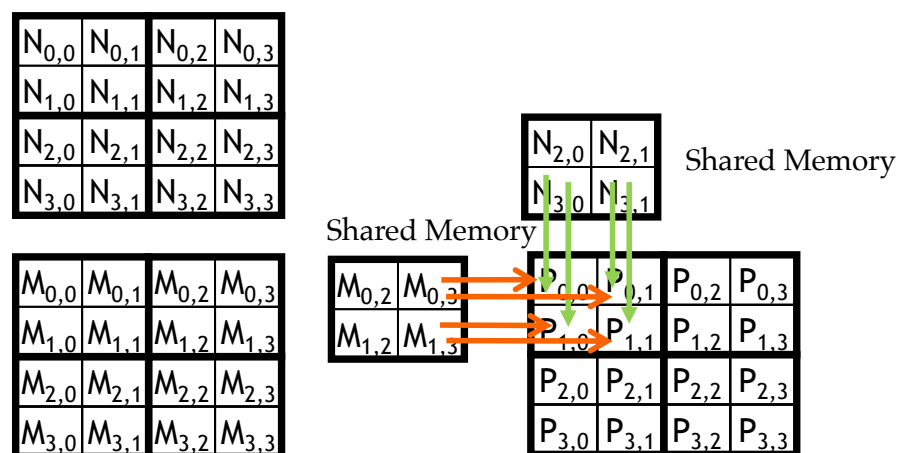
Phase 1 Load for Block (0,0)



Phase 1 Use for Block (0,0) (iteration 0)



Phase 1 Use for Block (0,0) (iteration 1)



Execution Phases of Toy Example

	Phase 0			Phase 1		
thread _{0,0}	M_{0,0} ↓ Mds _{0,0}	N_{0,0} ↓ Nds _{0,0}	PValue _{0,0} += Mds _{0,0} *Nds _{0,0} + Mds _{0,1} *Nds _{1,0}	M_{0,2} ↓ Mds _{0,0}	N_{2,0} ↓ Nds _{0,0}	PValue _{0,0} += Mds _{0,0} *Nds _{0,0} + Mds _{0,1} *Nds _{1,0}
thread _{0,1}	M_{0,1} ↓ Mds _{0,1}	N_{0,1} ↓ Nds _{1,0}	PValue _{0,1} += Mds _{0,0} *Nds _{0,1} + Mds _{0,1} *Nds _{1,1}	M_{0,3} ↓ Mds _{0,1}	N_{2,1} ↓ Nds _{0,1}	PValue _{0,1} += Mds _{0,0} *Nds _{0,1} + Mds _{0,1} *Nds _{1,1}
thread _{1,0}	M_{1,0} ↓ Mds _{1,0}	N_{1,0} ↓ Nds _{1,0}	PValue _{1,0} += Mds _{1,0} *Nds _{0,0} + Mds _{1,1} *Nds _{1,0}	M_{1,2} ↓ Mds _{1,0}	N_{3,0} ↓ Nds _{1,0}	PValue _{1,0} += Mds _{1,0} *Nds _{0,0} + Mds _{1,1} *Nds _{1,0}
thread _{1,1}	M_{1,1} ↓ Mds _{1,1}	N_{1,1} ↓ Nds _{1,1}	PValue _{1,1} += Mds _{1,0} *Nds _{0,1} + Mds _{1,1} *Nds _{1,1}	M_{1,3} ↓ Mds _{1,1}	N_{3,1} ↓ Nds _{1,1}	PValue _{1,1} += Mds _{1,0} *Nds _{0,1} + Mds _{1,1} *Nds _{1,1}

time →

Execution Phases of Toy Example (cont.)

	Phase 0			Phase 1		
thread _{0,0}	M_{0,0} ↓ Mds _{0,0}	N_{0,0} ↓ Nds _{0,0}	PValue _{0,0} += Mds_{0,0} *Nds _{0,0} + Mds _{0,1} *Nds _{1,0}	M_{0,2} ↓ Mds _{0,0}	N_{2,0} ↓ Nds _{0,0}	PValue _{0,0} += Mds _{0,0} *Nds _{0,0} + Mds _{0,1} *Nds _{1,0}
thread _{0,1}	M_{0,1} ↓ Mds _{0,1}	N_{0,1} ↓ Nds _{1,0}	PValue _{0,1} += Mds_{0,0} *Nds _{0,1} + Mds _{0,1} *Nds _{1,1}	M_{0,3} ↓ Mds _{0,1}	N_{2,1} ↓ Nds _{0,1}	PValue _{0,1} += Mds _{0,0} *Nds _{0,1} + Mds _{0,1} *Nds _{1,1}
thread _{1,0}	M_{1,0} ↓ Mds _{1,0}	N_{1,0} ↓ Nds _{1,0}	PValue _{1,0} += Mds _{1,0} *Nds _{0,0} + Mds _{1,1} *Nds _{1,0}	M_{1,2} ↓ Mds _{1,0}	N_{3,0} ↓ Nds _{1,0}	PValue _{1,0} += Mds _{1,0} *Nds _{0,0} + Mds _{1,1} *Nds _{1,0}
thread _{1,1}	M_{1,1} ↓ Mds _{1,1}	N_{1,1} ↓ Nds _{1,1}	PValue _{1,1} += Mds _{1,0} *Nds _{0,1} + Mds _{1,1} *Nds _{1,1}	M_{1,3} ↓ Mds _{1,1}	N_{3,1} ↓ Nds _{1,1}	PValue _{1,1} += Mds _{1,0} *Nds _{0,1} + Mds _{1,1} *Nds _{1,1}

time →

Shared memory allows each value to be accessed by multiple threads

Barrier Synchronization

- Synchronize all threads in a block
 - `__syncthreads()`
- All threads in the same block must reach the `__syncthreads()` before any of them can move on
- Best used to coordinate the phased execution tiled algorithms
 - To ensure that all elements of a tile are loaded at the beginning of a phase
 - To ensure that all elements of a tile are consumed at the end of a phase



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Part IV: Memory and Data Locality

Tiled Matrix Multiplication Kernel

Objective

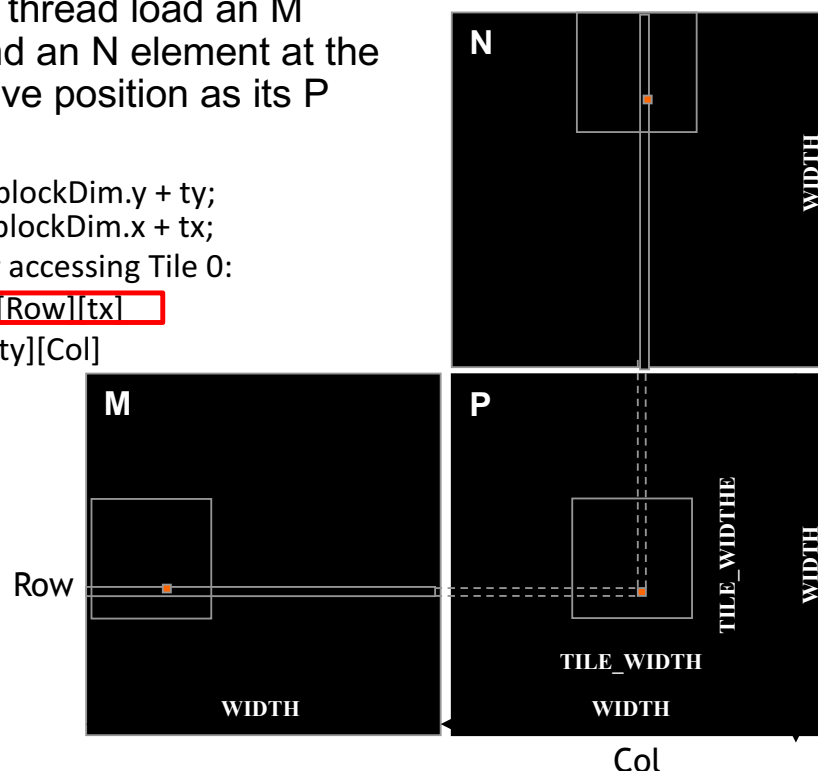
- To learn to write a tiled matrix-multiplication kernel
 - Loading and using tiles for matrix multiplication
 - Barrier synchronization, shared memory
 - Resource Considerations
 - Assume that Width is a multiple of tile size for simplicity

Loading Input Tile 0 of M (Phase 0)

- Have each thread load an M element and an N element at the same relative position as its P element.

```
int Row = by * blockDim.y + ty;  
int Col = bx * blockDim.x + tx;  
2D indexing for accessing Tile 0:
```

`M[Row][tx]`
`N[ty][Col]`



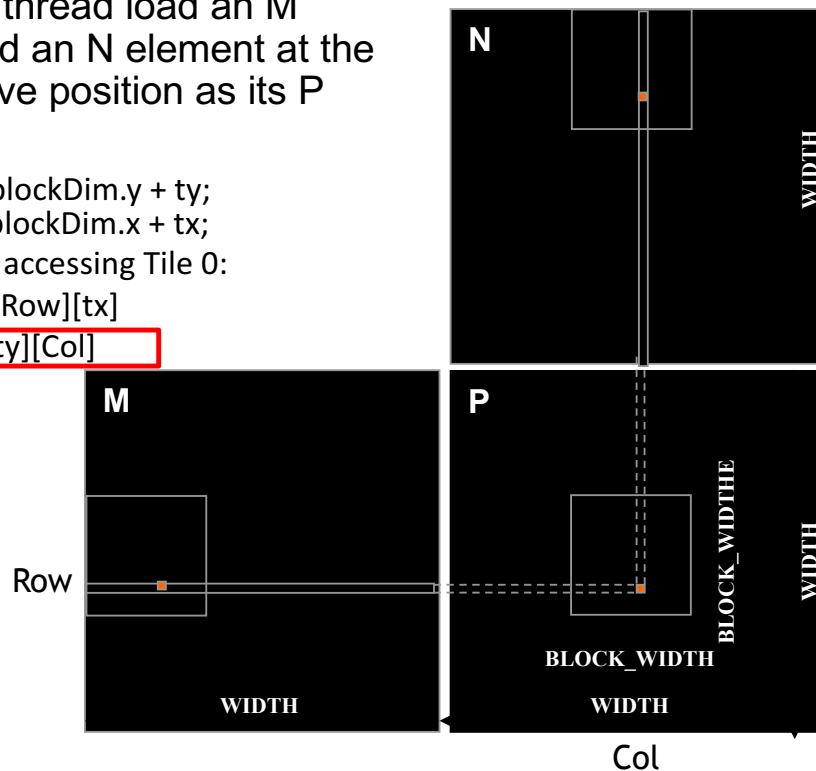
Loading Input Tile 0 of N (Phase 0)

- Have each thread load an M element and an N element at the same relative position as its P element.

```
int Row = by * blockDim.y + ty;
int Col = bx * blockDim.x + tx;
2D indexing for accessing Tile 0:
```

M[Row][tx]

N[ty][Col]



Loading Input Tile 1 of M (Phase 1)

2D indexing for accessing Tile 1:

M[Row][1*TILE_WIDTH + tx]

N[1*TILE*WIDTH + ty][Col]

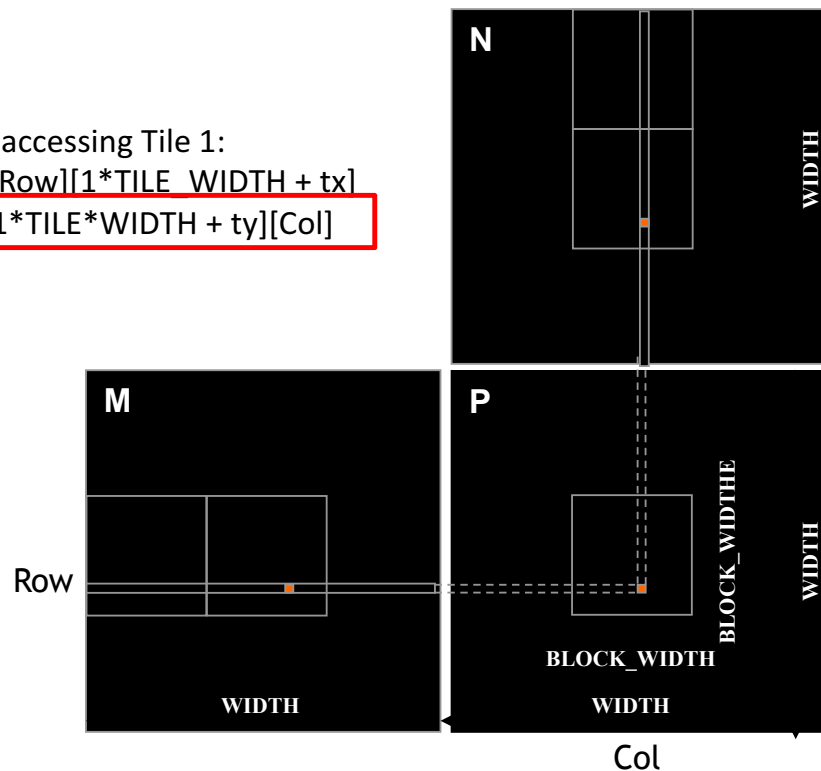


Loading Input Tile 1 of N (Phase 1)

2D indexing for accessing Tile 1:

$M[\text{Row}][1 * \text{TILE_WIDTH} + \text{tx}]$

$N[1 * \text{TILE} * \text{WIDTH} + \text{ty}][\text{Col}]$



M and N are dynamically allocated - use 1D indexing

➡ $M[\text{Row}][p * \text{TILE_WIDTH} + \text{tx}]$
➡ $M[\text{Row} * \text{Width} + p * \text{TILE_WIDTH} + \text{tx}]$

➡ $N[p * \text{TILE_WIDTH} + \text{ty}][\text{Col}]$
➡ $N[(p * \text{TILE_WIDTH} + \text{ty}) * \text{Width} + \text{Col}]$

where p is the sequence number of the current phase

Tiled Matrix Multiplication Kernel

```
__global__ void MatrixMulKernel(float* M, float* N, float* P, Int Width)
{
    __shared__ float ds_M[TILE_WIDTH][TILE_WIDTH];
    __shared__ float ds_N[TILE_WIDTH][TILE_WIDTH];

    int bx = blockIdx.x; int by = blockIdx.y;
    int tx = threadIdx.x; int ty = threadIdx.y;

    int Row = by * blockDim.y + ty;
    int Col = bx * blockDim.x + tx;
    float Pvalue = 0;

    // Loop over the M and N tiles required to compute the P element
    for (int p = 0; p < n/TILE_WIDTH; ++p) {
        // Collaborative loading of M and N tiles into shared memory
        ds_M[ty][tx] = M[Row*Width + p*TILE_WIDTH+tx];
        ds_N[ty][tx] = N[(t*TILE_WIDTH+ty)*Width + Col];
        __syncthreads();

        for (int i = 0; i < TILE_WIDTH; ++i) Pvalue += ds_M[ty][i] * ds_N[i][tx];
        __syncthreads();
    }
    P[Row*Width+Col] = Pvalue;
}
```

Tiled Matrix Multiplication Kernel

```
__global__ void MatrixMulKernel(float* M, float* N, float* P, Int Width)
{
    __shared__ float ds_M[TILE_WIDTH][TILE_WIDTH];
    __shared__ float ds_N[TILE_WIDTH][TILE_WIDTH];

    int bx = blockIdx.x; int by = blockIdx.y;
    int tx = threadIdx.x; int ty = threadIdx.y;

    int Row = by * blockDim.y + ty;
    int Col = bx * blockDim.x + tx;
    float Pvalue = 0;

    // Loop over the M and N tiles required to compute the P element
    for (int p = 0; p < n/TILE_WIDTH; ++p) {
        // Collaborative loading of M and N tiles into shared memory
        ds_M[ty][tx] = M[Row*Width + p*TILE_WIDTH+tx];
        ds_N[ty][tx] = N[(t*TILE_WIDTH+ty)*Width + Col];
        __syncthreads();

        for (int i = 0; i < TILE_WIDTH; ++i) Pvalue += ds_M[ty][i] * ds_N[i][tx];
        __syncthreads();
    }
    P[Row*Width+Col] = Pvalue;
}
```

Tiled Matrix Multiplication Kernel

```
__global__ void MatrixMulKernel(float* M, float* N, float* P, int Width)
{
    __shared__ float ds_M[TILE_WIDTH][TILE_WIDTH];
    __shared__ float ds_N[TILE_WIDTH][TILE_WIDTH];

    int bx = blockIdx.x; int by = blockIdx.y;
    int tx = threadIdx.x; int ty = threadIdx.y;

    int Row = by * blockDim.y + ty;
    int Col = bx * blockDim.x + tx;
    float Pvalue = 0;

    // Loop over the M and N tiles required to compute the P element
    for (int p = 0; p < n/TILE_WIDTH; ++p) {
        // Collaborative loading of M and N tiles into shared memory
        ds_M[ty][tx] = M[Row*Width + p*TILE_WIDTH+tx];
        ds_N[ty][tx] = N[(t*TILE_WIDTH+ty)*Width + Col];
        __syncthreads();

        for (int i = 0; i < TILE_WIDTH; ++i) Pvalue += ds_M[ty][i] * ds_N[i][tx];
        __syncthreads();
    }
    P[Row*Width+Col] = Pvalue;
}
```

53



Tile (Thread Block) Size Considerations

- Each **thread block** should have many threads
 - TILE_WIDTH of 16 gives $16*16 = 256$ threads
 - TILE_WIDTH of 32 gives $32*32 = 1024$ threads
- For 16, in each phase, each block performs $2*256 = 512$ float loads from global memory for $256 * (2*16) = 8,192$ mul/add operations. (16 floating-point operations for each memory load)
- For 32, in each phase, each block performs $2*1024 = 2048$ float loads from global memory for $1024 * (2*32) = 65,536$ mul/add operations. (32 floating-point operation for each memory load)

54



Shared Memory and Threading

- For an SM with 16KB shared memory
 - Shared memory size is implementation dependent!
 - For `TILE_WIDTH = 16`, each thread block uses $2 \times 256 \times 4B = 2KB$ of shared memory.
 - For 16KB shared memory, one can potentially have up to 8 thread blocks executing
 - This allows up to $8 \times 512 = 4,096$ pending loads. (2 per thread, 256 threads per block)
 - The next `TILE_WIDTH 32` would lead to $2 \times 32 \times 32 \times 4 \text{ Byte} = 8K \text{ Byte}$ shared memory usage per thread block, allowing 2 thread blocks active at the same time
 - However, the thread count limitation of 1536 threads per SM in current generation GPUs will reduce the number of blocks per SM to one!
- Each `__syncthread()` can reduce the number of active threads for a block
 - More thread blocks can be advantageous



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Part V: Memory and Data Locality

Handling Arbitrary Matrix Sizes in Tiled Algorithms

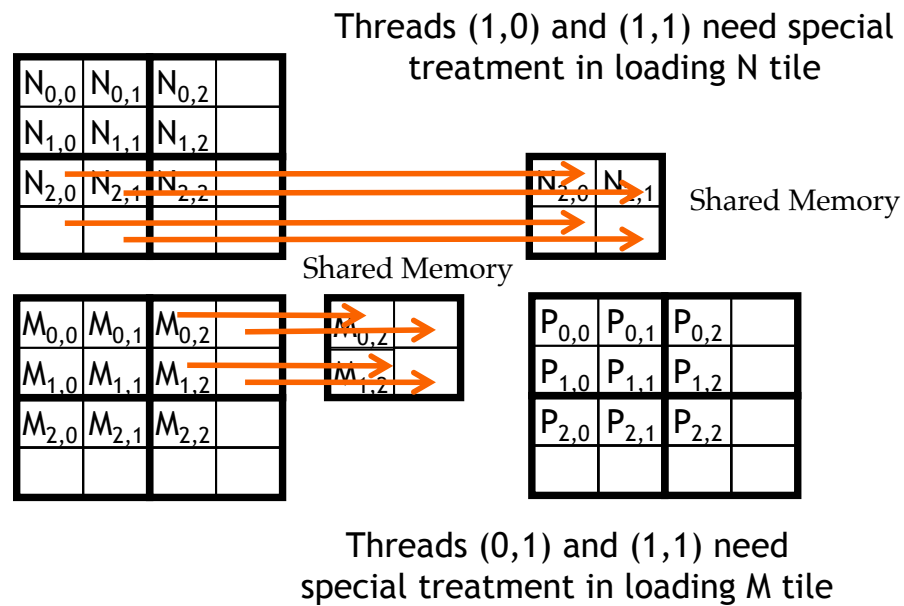
Objective

- To learn to handle arbitrary matrix sizes in tiled matrix multiplication
 - Boundary condition checking
 - Regularizing tile contents
 - Rectangular matrices

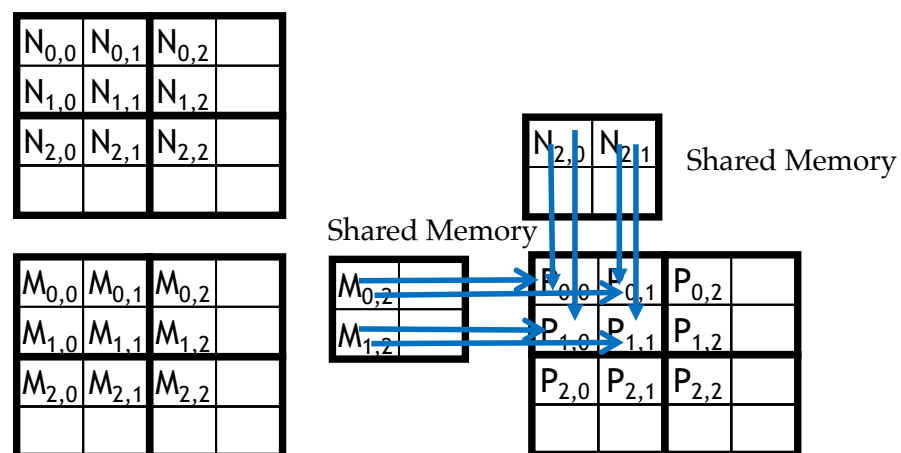
Handling Matrix of Arbitrary Size

- The tiled matrix multiplication kernel we presented so far can handle only square matrices whose dimensions (Width) are multiples of the tile width (TILE_WIDTH)
 - However, real applications need to handle arbitrary sized matrices.
 - One could pad (add elements to) the rows and columns into multiples of the tile size, but would have significant space and data transfer time overhead.
- We will take a different approach.

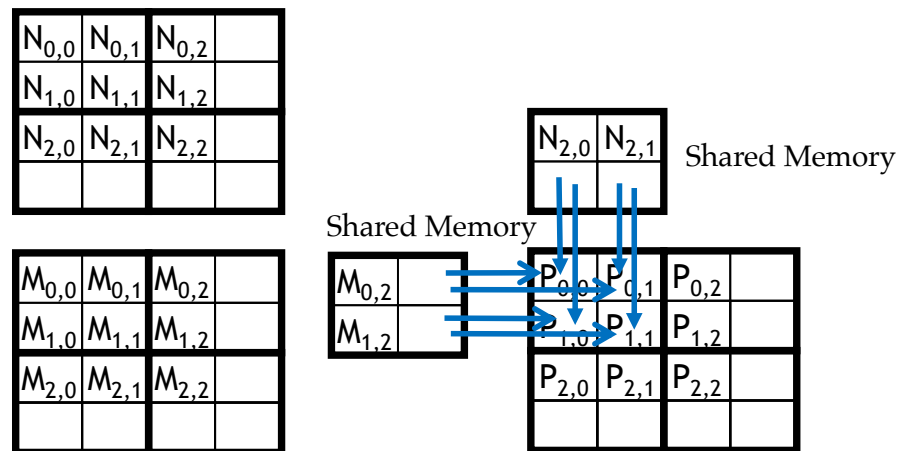
Phase 1 Loads for Block (0,0) for a 3x3 Example



Phase 1 Use for Block (0,0) (iteration 0)



Phase 1 Use for Block (0,0) (iteration 1)



All Threads need special treatment. None of them should introduce invalidate contributions to their P elements.

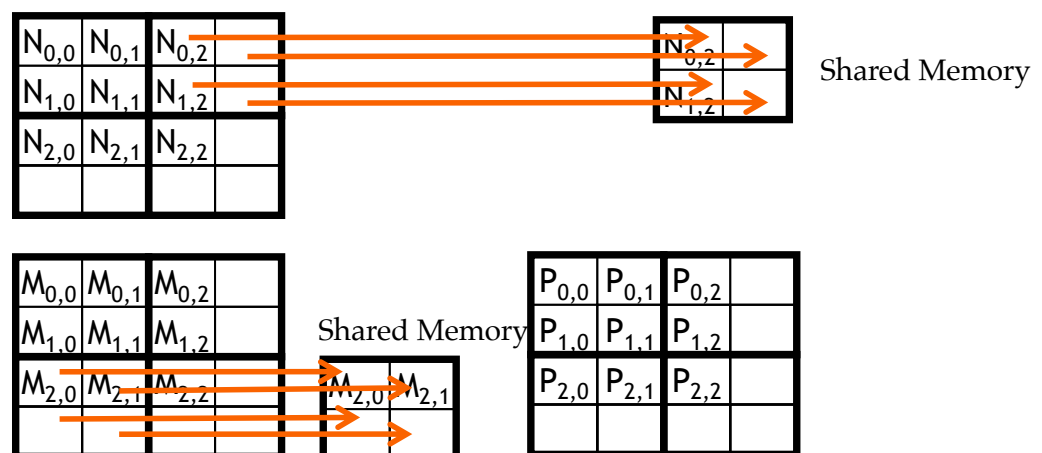
61

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Phase 0 Loads for Block (1,1) for a 3x3 Example

Threads (0,1) and (1,1) need special treatment in loading N tile



Threads (1,0) and (1,1) need special treatment in loading M tile

62

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Major Cases in Toy Example

- Threads that do not calculate valid P elements but still need to participate in loading the input tiles
 - Phase 0 of Block(1,1), Thread(1,0), assigned to calculate non-existent $P[3,2]$ but need to participate in loading tile element $N[1,2]$
- Threads that calculate valid P elements may attempt to load non-existing input elements when loading input tiles
 - Phase 0 of Block(0,0), Thread(1,0), assigned to calculate valid $P[1,0]$ but attempts to load non-existing $N[3,0]$

63

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A “Simple” Solution

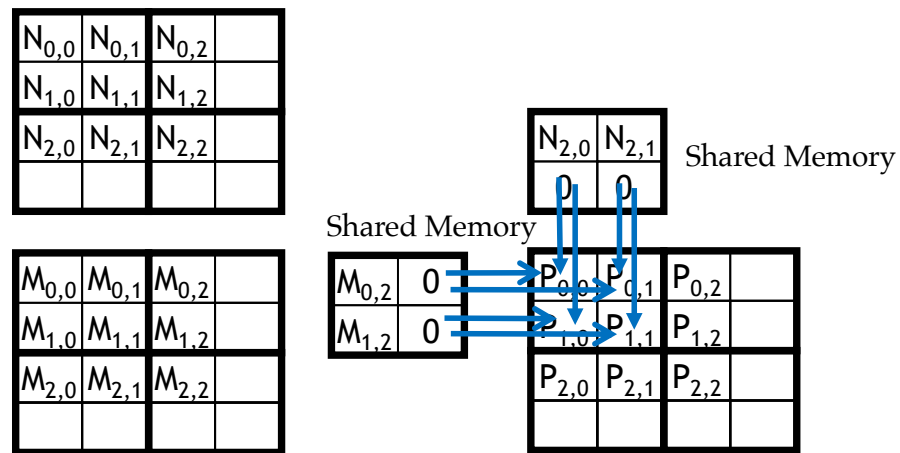
- When a thread is to load any input element, test if it is in the valid index range
 - If valid, proceed to load
 - Else, do not load, just write a 0
- Rationale: a 0 value will ensure that that the multiply-add step does not affect the final value of the output element
- The condition tested for loading input elements is different from the test for calculating output P element
 - A thread that does not calculate valid P element can still participate in loading input tile elements

64

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Phase 1 Use for Block (0,0) (iteration 1)



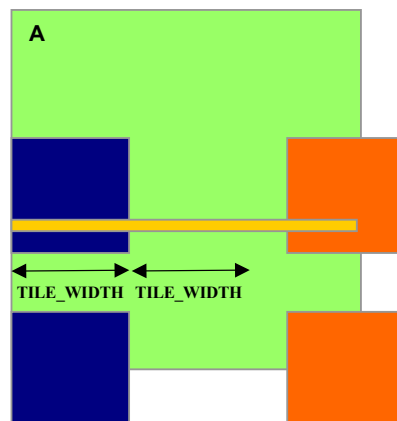
65

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Boundary Condition for Input M Tile

- Each thread loads
 - $M[\text{Row}][p \cdot \text{TILE_WIDTH} + tx]$
 - $M[\text{Row} \cdot \text{Width} + p \cdot \text{TILE_WIDTH} + tx]$
- Need to test
 - $(\text{Row} < \text{Width}) \ \&\& \ (p \cdot \text{TILE_WIDTH} + tx < \text{Width})$
 - If true, load M element
 - Else , load 0



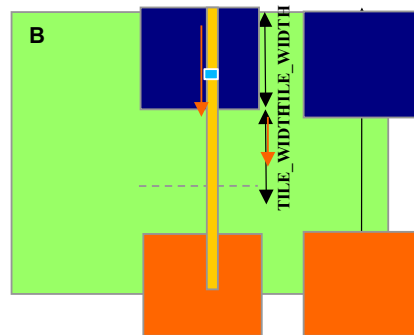
66

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Boundary Condition for Input N Tile

- Each thread loads
 - $N[p * \text{TILE_WIDTH} + ty][\text{Col}]$
 - $N[(p * \text{TILE_WIDTH} + ty) * \text{Width} + \text{Col}]$
- Need to test
 - $(p * \text{TILE_WIDTH} + ty < \text{Width}) \ \&\& \ (\text{Col} < \text{Width})$
 - If true, load N element
 - Else , load 0



67



Loading Elements – with boundary check

```

– 8  for (int p = 0; p < (Width-1) / TILE_WIDTH + 1; ++p) {
–
–  ++    if (Row < Width && t * TILE_WIDTH + tx < Width) {
–  9      ds_M[ty][tx] = M[Row * Width + p * TILE_WIDTH + tx];
–  ++    } else {
–  ++      ds_M[ty][tx] = 0.0;
–  ++    }
–  ++    if (p * TILE_WIDTH + ty < Width && Col < Width) {
–  10     ds_N[ty][tx] = N[(p * TILE_WIDTH + ty) * Width + Col];
–  ++    } else {
–  ++      ds_N[ty][tx] = 0.0;
–  ++    }
–  11    __syncthreads();
–

```

68



Inner Product – Before and After

```
– ++ if(Row < Width && Col < Width) {  
– 12   for (int i = 0; i < TILE_WIDTH; ++i) {  
– 13       Pvalue += ds_M[ty][i] * ds_N[i][tx];  
–       }  
– 14   __syncthreads();  
– 15 } /* end of outer for loop */  
– ++ if (Row < Width && Col < Width)  
– 16     P[Row*Width + Col] = Pvalue;  
– } /* end of kernel */
```

Some Important Points

- For each thread the conditions are different for
 - Loading M element
 - Loading N element
 - Calculating and storing output elements
- The effect of control divergence should be small for large matrices

Handling General Rectangular Matrices

- In general, the matrix multiplication is defined in terms of rectangular matrices
 - A $j \times k$ M matrix multiplied with a $k \times l$ N matrix results in a $j \times l$ P matrix
- We have presented square matrix multiplication, a special case
- The kernel function needs to be generalized to handle general rectangular matrices
 - The Width argument is replaced by three arguments: j, k, l
 - When Width is used to refer to the height of M or height of P, replace it with j
 - When Width is used to refer to the width of M or height of N, replace it with k
 - When Width is used to refer to the width of N or width of P, replace it with l