CSC 447: Parallel Programming for Multi-Core and Cluster Systems

Performance Analysis

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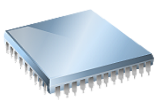
Outline

- Performance scalability
- Analytical performance measures
- Amdahl's law and Gustafson-Barsis' law

Performance

- In computing, performance is defined by 2 factors
  - Computational requirements (what needs to be done)
  - Computing resources (what it costs to do it)
- Computational problems translate to requirements
- Computing resources interplay and tradeoff

$$\textit{Performance} \sim \frac{1}{\textit{Resources for solution}}$$



Hardware

Time

Energy

… and ultimately

Money

Measuring Performance

- Performance itself is a measure of how well the computational requirements can be satisfied
- We evaluate performance to understand the relationships between requirements and resources
  - Decide how to change "solutions" to target objectives
- Performance measures reflect decisions about how and how well "solutions" are able to satisfy the computational requirements
- When measuring performance, it is important to understand exactly what you are measuring and how you are measuring it

Scalability

- A program can scale up to use many processors
  - What does that mean?
- How do you evaluate scalability?
- How do you evaluate scalability goodness?
- Comparative evaluation
  - If double the number of processors, what to expect?
  - Is scalability linear?
- Use parallel efficiency measure
  - Is efficiency retained as problem size increases?
- Apply performance metrics

Performance and Scalability

- Evaluation
  - Sequential runtime (T_{seq}) is a function of
    - problem size and architecture
  - Parallel runtime (T_{par}) is a function of
    - problem size and parallel architecture
    - # processors used in the execution
  - Parallel performance affected by
    - algorithm + architecture
- Scalability
  - Ability of parallel algorithm to achieve performance gains proportional to the number of processors and the size of the problem

Performance Metrics and Formulas

- T_1 is the execution time on a single processor
- T_p is the execution time on a p processor system
- S(p) (S_p) is the speedup $S(p) = \frac{T_1}{T_p}$
- E(p) (E_p) is the efficiency $Efficiency = \frac{S_p}{p}$
- Cost(p) (C_p) is the cost $Cost = p \times T_p$

- Parallel algorithm is cost-optimal
  - Parallel time = sequential time ($C_p = T_1$, Ep = 100%)

Speed-Up

- Provides a measure of application performance with respect to a given program platform
  - Speedup can also be cast in terms of computational steps
    - Can extend time complexity to parallel computations
- Use the fastest known sequential algorithm for running on a single processor

What is a "good" speedup?

- Hopefully, S(n) > 1
- Linear speedup:
  - S(n) = n
  - Parallel program considered perfectly scalable
- *Super*linear speedup:
  - S(n) > n
  - Can this happen?

Defining Speed-Up

- We need more information to evaluate speedup:
  - What problem size? Worst case time? Average case time?
  - What do we count as work?
    - Parallel computation, communication, overhead?
  - What serial algorithm and what machine should we use for the numerator?
    - Can the algorithms used for the numerator and the denominator be different?

Common Definitions of Speed-Up

- Common definitions of Speedup:
  - Serial machine is one processor of parallel machine and serial algorithm is interleaved version of parallel algorithm

$$S(n) = \frac{T(1)}{T(n)}$$

  - Serial algorithm is fastest known serial algorithm for running on a serial processor

$$S(n) = \frac{T_s}{T(n)}$$

  - Serial algorithm is fastest known serial algorithm running on a one processor of the parallel machine

$$S(n) = \frac{T'(1)}{T(n)}$$

Can speedup be superlinear?

- Speedup CANNOT be *superlinear*:
  - Let M be a parallel machine with n processors
  - Let T(X) be the time it takes to solve a problem on M with X processors
  - Speedup definition: $S(n) = \frac{T(1)}{T(n)}$

$$S(n) = \frac{T(1)}{T(n)} \leq \frac{nt}{t} = n$$

    - Suppose a parallel algorithm A solves an instance I of a problem in t time units
      - Then A can solve the same problem in *n x t* units of time on M through time slicing
      - The best serial time for I will be no bigger than *n x t*
      - Hence speedup cannot be greater than n.

Can speedup be superlinear?

- Speedup CAN be superlinear:
  - Let M be a parallel machine with n processors
  - Let T(X) be the time it takes to solve a problem on M with X processors
  - Speedup definition: $S(n) = \frac{T_s}{T(n)}$
  - Serial version of the algorithm may involve more overhead than the parallel version of the algorithm
    - E.g. A=B+C on a SIMD machine with A,B,C matrices vs. loop overhead on a serial machine
  - Hardware characteristics may favor parallel algorithm
    - E.g. if all data can be decomposed in main memories of parallel processors vs. needing secondary storage on serial processor to retain all data
  - “work” may be counted differently in serial and parallel algorithms

Speedup Factor

- Maximum speedup is usually n with n processors (linear speedup).
- Possible to get *superlinear* speedup (greater than n) but usually a specific reason such as:
  - Extra memory in multiprocessor system
  - Nondeterministic algorithm

Maximum Speedup: Amdahl's law

- f = fraction of program (algorithm) that is serial and cannot be parallelized
  - Data setup
  - Reading/writing to a single disk file
- Speedup factor is given by:

$$T_s = fT_s + (1-f)T_s$$

$$T_p = fT_s + \frac{(1-f)T_s}{n}$$

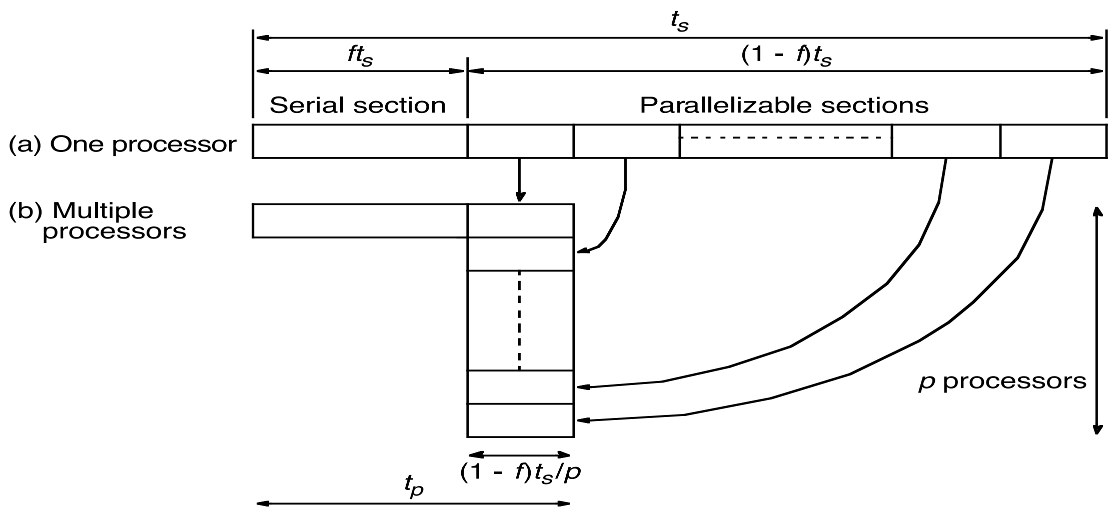
$$S(n) = \frac{T_s}{fT_s + \frac{(1-f)T_s}{n}} = \frac{n}{1+(n-1)f}$$

$$\lim_{n \to \infty} = \frac{1}{f}$$

The above equation is known as Amdahl's Law

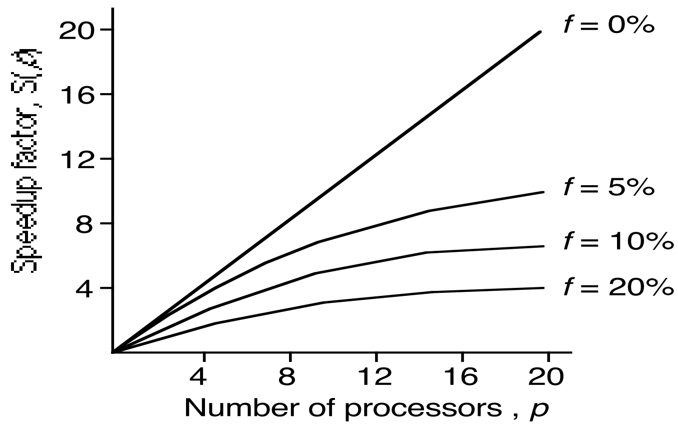
Note that as n $\rightarrow \infty$, the maximum speedup is limited to 1/f.

Bounds on Speedup



Speedup Against Number of Processors

- Even with infinite number of processors, maximum speedup limited to 1/f .
- Example: With only 5% of computation being serial, maximum speedup is 20, irrespective of number of processors.



Example of Amdahl’s Law (1)

- Suppose that a calculation has a 4% serial portion, what is the limit of speedup on 16 processors?
  - 16/(1 + (16 – 1)*.04) = 10
  - What is the maximum speedup?
    - 1/0.04 = 25

Example of Amdahl's Law (2)

- 95% of a program's execution time occurs inside a loop that can be executed in parallel. What is the maximum speedup we should expect from a parallel version of the program executing on 8 CPUs?

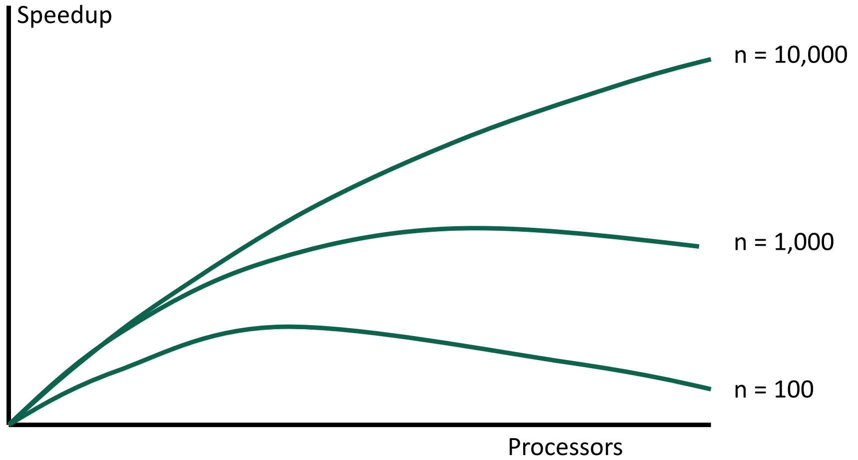
$$\psi \leq \frac{1}{0.05 + (1 - 0.05)/8} \cong 5.9$$

Example of Amdahl's Law (3)

- 20% of a program's execution time is spent within inherently sequential code. What is the limit to the speedup achievable by a parallel version of the program?

$$\lim_{p \to \infty} \frac{1}{0.2 + (1 - 0.2)/p} = \frac{1}{0.2} = 5$$

Illustration of Amdahl Effect



Amdahl's Law and Scalability

- Scalability
  – Ability of parallel algorithm to achieve performance gains proportional to the number of processors and the size of the problem
- When does Amdahl's Law apply?
  – When the problem size is fixed
  – *Strong scaling* ($p \rightarrow \infty$, $S_p = S_\infty \rightarrow 1/f$)
  – Speedup bound is determined by the degree of sequential execution time in the computation, not # processors!!!
  – Perfect efficiency is hard to achieve
- See original paper by Amdahl on course webpage

Variants of Speedup: Efficiency

- Efficiency: E(n) = S(n)/n * 100%
- Efficiency measures the fraction of time that processors are being used on the computation.
  - A program with linear speedup is 100% efficient.
- Using efficiency:
  - A program attains 89% efficiency with a serial fraction of 2%. Approximately how many processors are being used according to Amdahl's law?

Efficiency

$$\text{Efficiency} = \frac{\text{Sequential execution time}}{\text{Processors used} \times \text{Parallel execution time}}$$

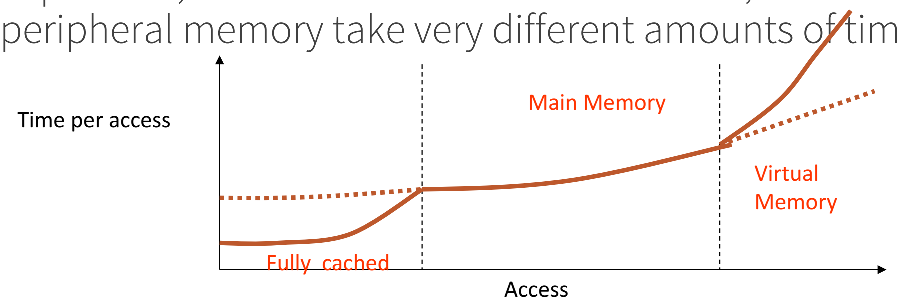
$$\text{Efficiency} = \frac{\text{Speedup}}{\text{Processors used}}$$

Limitations of Speedup

- Conventional notions of speedup don't always provide a reasonable measure of performance
- Questionable assumptions:
  - "work" in conventional definitions of speedup is defined by operation count
    - communication more expensive than computation on current high-performance computers
  - best serial algorithm defines the least work necessary
    - for some languages on some machines, serial algorithm may do more work -- (loop operations vs. data parallel for example)
  - good performance for many users involves fast time on a sufficiently large problem; faster time on a smaller problem (better speedup) is less interesting
  - traditional speedup measures assume a "flat memory approximation", i.e. all memory accesses take the same amount of time

"Flat Memory Approximation"

- "Flat memory Approximation" – all accesses to memory take the same amount of time
  - in practice, accesses to information in cache, main memory and peripheral memory take very different amounts of time.



Another Perspective

- We often use faster computers to solve larger problem instances
- Let's treat time as a constant and allow problem size to increase with number of processors

Limitations of Speedup

- Gustafson challenged Amdahl's assumption that the proportion of a program given to serial computations (f) and the proportion of a program given to parallel computations remains the same over all problem sizes.
  - For example, if the serial part is a loop initialization and it can be executed in parallel over the size of the input list, then the serial initialization becomes a smaller proportion of the overall calculation as the problem size grows larger.
- Gustafson defined two "more relevant" notions of speedup
  - Scaled speedup
  - Fixed-time speedup
    - (usual version he called fixed-size speedup)

Gustafson-Barsis's Law

- Begin with parallel execution time
- Estimate sequential execution time to solve same problem
- Problem size is an increasing function of p
- Predicts scaled speedup

Gustafson's Law

Fix execution time on a **single processor**

- $s + p$ = serial part + parallelizable part = **1** (normalized serial time)
- (s = same as f previously)
- Assume problem fits in memory of serial computer
- **Fixed-size speedup**

$$S_{fixed\_size} = \frac{s+p}{s+\frac{p}{n}} = \frac{1}{s+\frac{1-s}{n}}$$

Amdahl's law

Fix execution time on a **parallel computer (multiple processors)**

- $s + p$ = serial part + parallelizable part = **1** (normalized parallel time)
- $s + np$ = serial time on a single processor
- Assume problem fits in memory of parallel computer
- **Scaled Speedup**

$$S_{scaled} = \frac{s+np}{s+p} = n + (1-n)s$$

Gustafson's Law

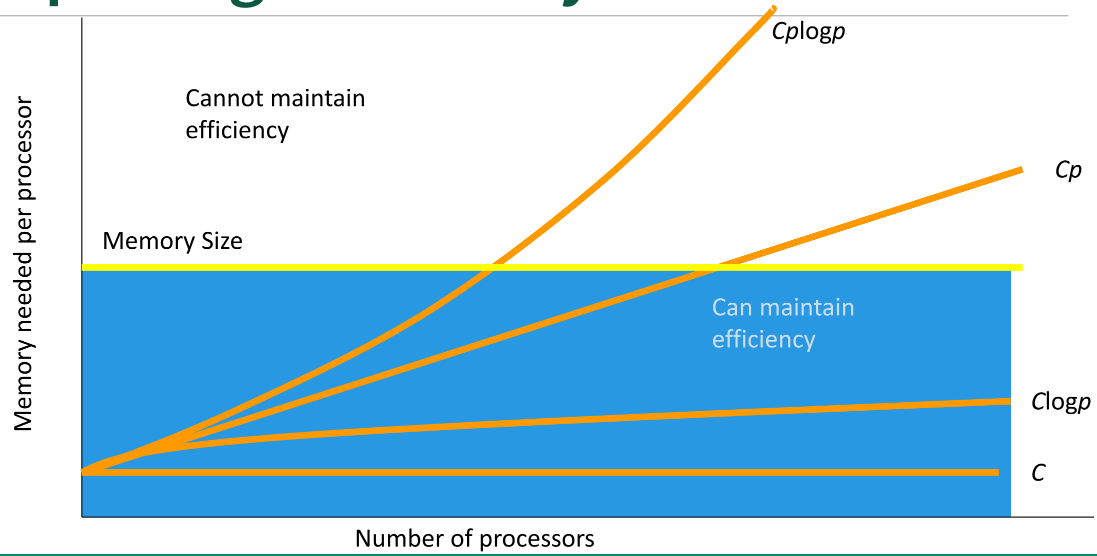
Scaled Speedup

- Scaling implies that problem size can increase with number of processors
  - Gustafson's law gives measure of how much
- Scaled Speedup derived by fixing the parallel execution time (Amdahl fixed the problem size → fixes serial execution time)
  - Amdahl's law may be too conservative for high-performance computing.
- Interesting consequence of scaled speedup: no bound to speedup as n→ infinity, speedup can easily become superlinear!
- In practice, unbounded scalability is unrealistic as quality of answer will reach a point where no further increase in problem size may be justified

Meaning of Scalability Function

- To maintain efficiency when increasing p, we must increase n
- Maximum problem size limited by available memory, which is linear in p
- Scalability function shows how memory usage per processor must grow to maintain efficiency
- Scalability function a constant means parallel system is perfectly scalable

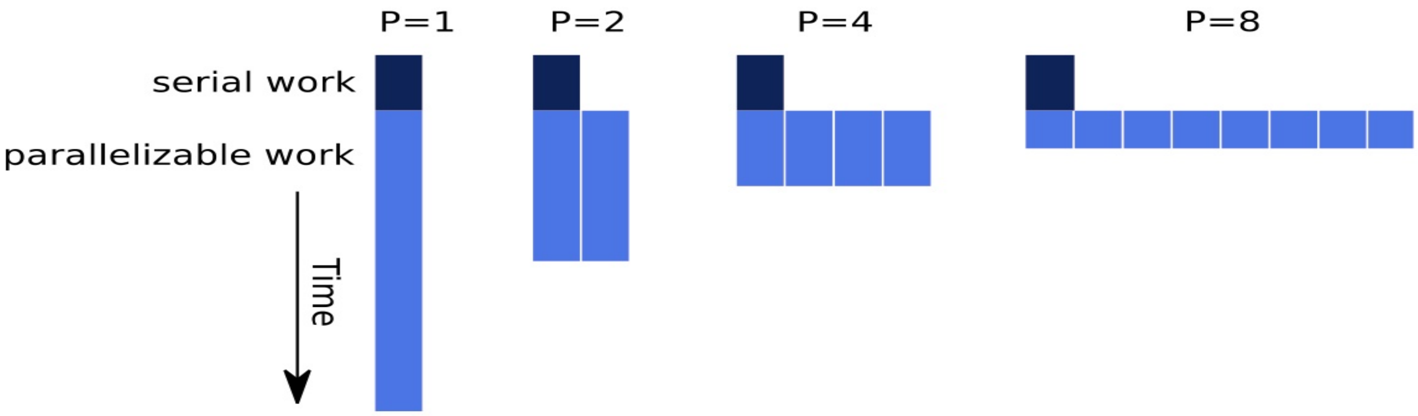
Interpreting Scalability Function



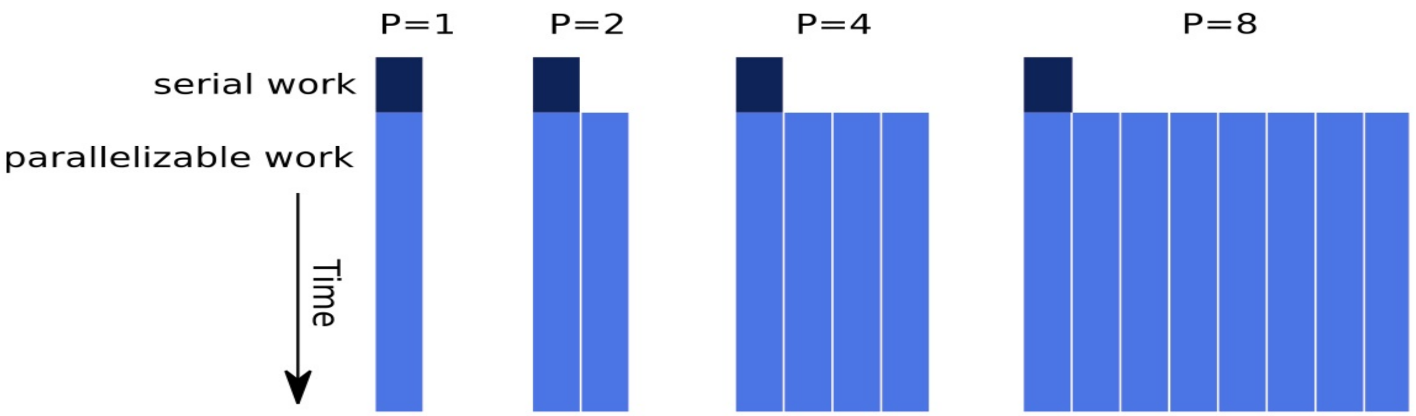
Gustafson-Barsis' Law and Scalability

- Scalability
  - Ability of parallel algorithm to achieve performance gains proportional to the number of processors and the size of the problem
- When does Gustafson's Law apply?
  - When the problem size can increase as the number of processors increases
  - *Weak scaling* ($S_p = 1 + (p-1)f_{par}$)
  - Speedup function includes the number of processors!!!
  - Can maintain or increase parallel efficiency as the problem scales
- See original paper by Gustafson on course webpage

Amdahl



Gustafson-Baris



Using Gustafson's Law

- Given a scaled speedup of 20 on 32 processors, what is the serial fraction from Amdahl's law? What is the serial fraction from Gustafson's Law?

$$S_{scaled} = \frac{s + np}{s + p}$$
$$= n + (1 - n)s$$

Example 1

- An application running on 10 processors spends 3% of its time in serial code. What is the scaled speedup of the application?

$$\psi = 10 + (1 - 10)(0.03) = 10 - 0.27 = 9.73$$

…except 9 do not have to execute serial code

Execution on 1 CPU takes 10 times as long…

Example 2

- What is the maximum fraction of a program's parallel execution time that can be spent in serial code if it is to achieve a scaled speedup of 7 on 8 processors?

$$7 = 8 + (1-8)s \Rightarrow s \approx 0.14$$

Why Are not Parallel Applications Scalable?

Critical Paths
- Dependencies between computations spread across processors

Bottlenecks
- One processor holds things up

Algorithmic overhead
- Some things just take more effort to do in parallel

Communication overhead
- Spending increasing proportion of time on communication

Load Imbalance
- Makes all processor wait for the "slowest" one
- Dynamic behavior

Speculative loss
- Do A and B in parallel, but B is ultimately not needed

Critical Paths

- Long chain of dependence
  - Main limitation on performance
  - Resistance to performance improvement
- Diagnostic
  - Performance stagnates to a (relatively) fixed value
  - Critical path analysis
- Solution
  - Eliminate long chains if possible
  - Shorten chains by removing work from critical path

Bottlenecks

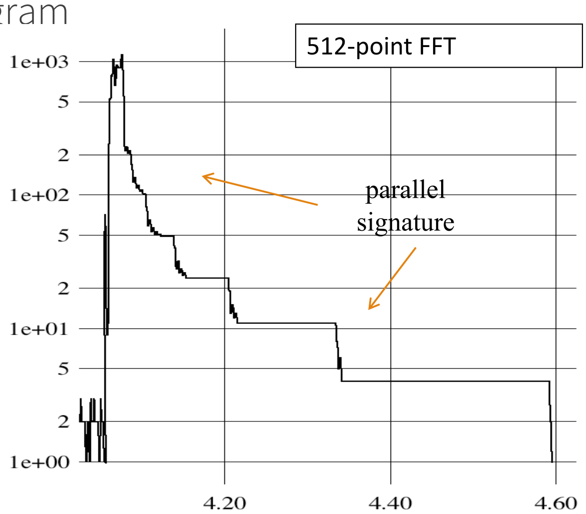
- How to detect?
  - One processor A is busy while others wait
  - Data dependency on the result produced by A
- Typical situations:
  - N-to-1 reduction / computation / 1-to-N broadcast
  - One processor assigning job in response to requests
- Solution techniques:
  - More efficient communication
  - Hierarchical schemes for master slave
- Program may not show ill effects for a long time
- Shows up when scaling

Algorithmic Overhead

- Different sequential algorithms to solve the same problem
- All parallel algorithms are sequential when run on 1 processor
- All parallel algorithms introduce addition operations
  - Parallel overhead
- Where should be the starting point for a parallel algorithm?
  - Best sequential algorithm might not parallelize at all
  - Or, it does not parallelize well (e.g., not scalable)
- What to do?
  - Choose algorithmic variants that minimize overhead
  - Use two level algorithms
- Performance is the rub
  - Are you achieving better parallel performance?
  - Must compare with the best sequential algorithm

What is the maximum parallelism possible?

- Depends on application, algorithm, program
  - Data dependencies in execution
- Remember MaxPar
  - Analyzes the earliest possible "time" any data can be computed
  - Assumes a simple model for time it takes to execute instruction or go to memory
  - Result is the maximum parallelism available
- Parallelism varies!



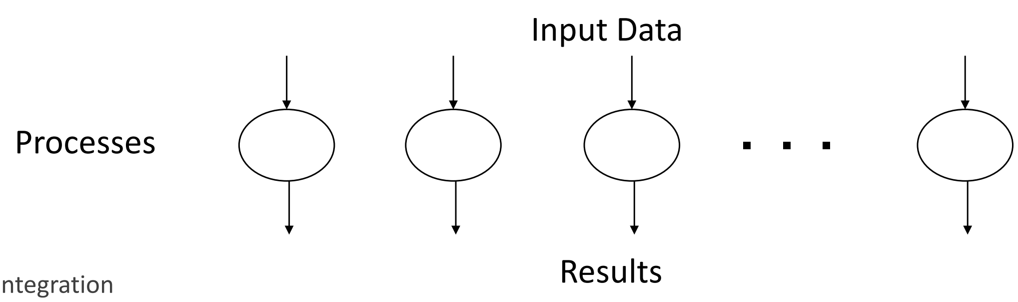
Embarrassingly Parallel Computations

- An embarrassingly parallel computation is one that can be obviously divided into completely independent parts that can be executed simultaneously
  - In a truly embarrassingly parallel computation there is no interaction between separate processes
  - In a nearly embarrassingly parallel computation results must be distributed and collected/combined in some way
- Embarrassingly parallel computations have potential to achieve maximal speedup on parallel platforms
  - If it takes T time sequentially, there is the potential to achieve T/P time running in parallel with P processors
  - What would cause this not to be the case always?

Embarrassingly Parallel Computations

No or very little communication between processes

Each process can do its tasks without any interaction with other processes

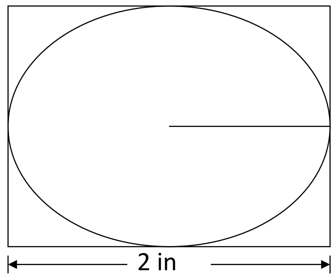


Examples
- Numerical integration
- Mandelbrot set
- Monte Carlo methods

Calculating π with Monte Carlo

Consider a circle of unit radius

Place circle inside a square box with side of 2 in

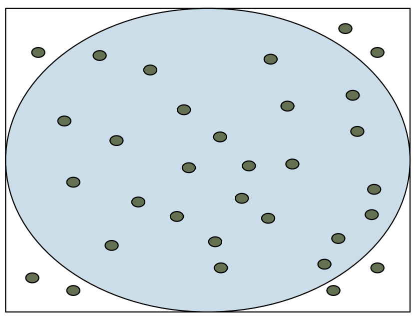


The ratio of the circle area to the square area is:

$$\frac{\pi * 1 * 1}{2 * 2} = \frac{\pi}{4}$$

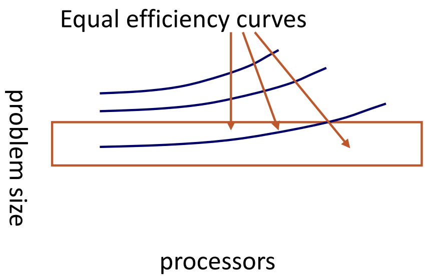
Monte Carlo Calculation of π

- Randomly choose a number of points in the square
- For each point p, determine if p is inside the circle
- The ratio of points in the circle to points in the square will give an approximation of $\pi/4$



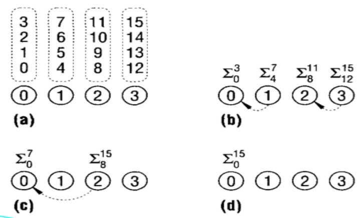
Isoefficiency

- Goal is to quantify scalability
- How much increase in problem size is needed to retain the same efficiency on a larger machine?
- Efficiency
  - $T_1 / (p * T_p)$
  - T_p = computation + communication + idle
- Isoefficiency
  - Equation for equal-efficiency curves
  - If no solution
    - problem is not scalable in the sense defined by isoefficiency
- See original paper by Kumar on webpage



Scalability of Adding n Numbers

- Scalability of a parallel system is a measure of its capacity to increase speedup with more processors
- Adding *n* numbers on *p* processors with strip partition:

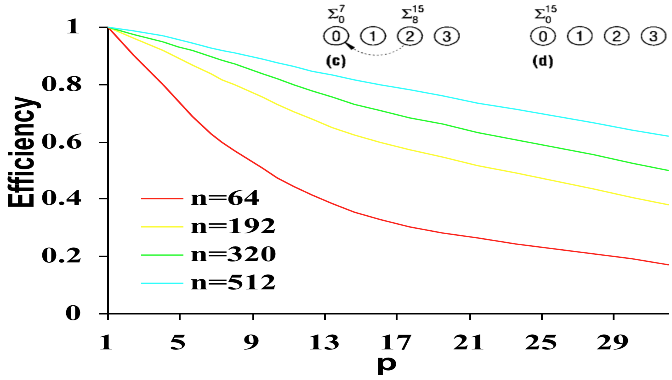


$$T_{par} = \frac{n}{p} - 1 + 2\log p$$

$$Speedup = \frac{n-1}{\frac{n}{p} - 1 + 2\log p}$$

$$\approx \frac{n}{\frac{n}{p} + 2\log p}$$

$$Efficiency = \frac{S}{p} = \frac{n}{n + 2p\log p}$$



Problem Size and Overhead

- Informally, problem size is expressed as a parameter of the input size
- A consistent definition of the size of the problem is the total number of basic operations (T_{seq})
  - Also refer to problem size as "work ($W = T_{seq}$)
- Overhead of a parallel system is defined as the part of the cost not in the best serial algorithm
- Denoted by T_O, it is a function of W and p

  $T_O(W,p) = pT_{par} - W$ (pT_{par} includes overhead)

  $T_O(W,p) + W = pT_{par}$

Isoefficiency Function

- With a fixed efficiency, W is as a function of p

$$T_{par} = \frac{W + T_O(W,p)}{p}$$

$W = T_{seq}$

$$Speedup = \frac{W}{T_{par}} = \frac{Wp}{W + T_O(W,p)}$$

$$Efficiency = \frac{S}{p} = \frac{W}{W + T_O(W,p)} = \frac{1}{1 + \frac{T_O(W,p)}{W}}$$

$$E = \frac{1}{1 + \frac{T_O(W,p)}{W}} \rightarrow \frac{T_O(W,p)}{W} = \frac{1-E}{E}$$

Isoefficiency Function

$$\boxed{W = \frac{E}{1-E} T_O(W,p) = KT_O(W,p)}$$

Scalability Function

- Suppose isoefficiency relation is $n \geq f(p)$
- Let $M(n)$ denote memory required for problem of size n
- $M(f(p))/p$ shows how memory usage per processor must increase to maintain same efficiency
- We call $M(f(p))/p$ the scalability function

Fixed Time Speedup

- Also due to Gustafson
  - In original paper, (http://www.scl.ameslab.gov/Publications/FixedTime/FixedTime.html) Gustafson uses scaled speedup when the memory requirements scale linearly with the number of processors
  - Gustafson uses fixed-time speedup when the work scales linearly with the number of processors, rather than the storage
  - Both measures allow the problem size to scale whereas fixed-size speedup (conventional speedup measure) assumes that the problem size is fixed.

Using Programs to Measure Machine Performance

- Speedup measures performance of an individual program on a particular machine
  - Speedup cannot be used to
    - Compare different algorithms on the same computer
    - Compare the same algorithm on different computers
- Benchmarks are representative programs which can be used to compare performance of machines

Benchmarks used for Parallel Machines

- The Perfect Club
- The Livermore Loops
- The NAS Parallel Benchmarks
- The SPEC Benchmarks
- The “PACKS” (Linpack, LAPACK, ScaLAPACK, etc.)
- ParkBENCH
- SLALOM, HINT

The Perfect Club

- Developed at University of Illinois around 1987
- Set of real applications donated by interested parties organized into a standardized set of benchmarks
  - Originally 13 codes, ~1000 lines of Fortran
  - Full-scale scientific apps rather than kernels or compact apps
  - Floating point-intensive codes usually executed on (vector) supercomputers
- Applications characterized in terms of their algorithmic behavior, allowing users to get meaningful predictions of the performance they could expect for their own applications
- Some codes incorporated into the SPEC benchmarks

The Livermore Loops

- Set of 24 Fortran DO loops extracted from operational code at LLNL
  - Originated the use of MFLOP/s for performance
  - Performance statistics reported: arithmetic, harmonic, geometric means, . . .
- http://www.netlib.org/benchmark/livermore

NAS Parallel Benchmarks (NPB)

- Benchmarks from CFD (computational fluid dynamics) codes
  - Fortran and C versions available.
  - NPB are kernels and compact pseudo-applications, not full applications
- Algorithmic definition of each program and sequential implementation of each algorithm
- Application can be supported or implemented efficiently in a machine – dependent way
- Users write a set of tuned parallel applications
- Suite gives manufacturers a chance to demonstrate what their machines can do
- NPB are widely used

SPEC Benchmarks

- SPEC = Standard Performance Evaluation Corporation
  - non-profit, industry-sponsored organization
    - Goal is to ensure that the marketplace has a fair and useful metric to differentiate candidate systems
  - SPEC benchmarks are standardized suite of source codes based on existing applications that have already been ported to a variety of architectures
    - E.g. SPECseis is sample code usedc to locate po; amd gas deposits from echo data, SPECchem is GAMESS for drug design and bonding analysis
    - Serial and parallel versions, MP, SM being developed
  - Benchmarker takes the source code, compiles it for target system and can tune system for best results.
  - Focus groups (open system high performance, graphics performance, etc.) develop benchmarks and make them available via a website.
- SPEC benchmark suites include
  - CINT2006 (CPU intensive integer benchmarks
  - CFP20062 (CPU intensive floating point benchmarks)
  - SDM (UNIX Software Development Workloads)
  - SFS (System level file server (NFS) workload)
- http://www.specbench.org/

LinPack

- Linear Algebra routines available in both C and Fortran
- Benchmarks solve a system of equations using Gaussian Elimination
  - MFLOPS reported
- Core of Linpack is subroutine ("saxpy" in the single-precision version, "daxpy" in the double-precision version) doing the inner loop for frequent matrix operations: y(i) = y(i) + a * x(i)
- Standard version operates on 100x100 matrices; there are also versions for sizes 300x300 and 1000x1000, with different optimization rules.
- Optimizations:
  - Linpack is easily vectorizable on many systems.
  - Easy to exploit a multiply-add operation
  - Some compilers have "daxpy recognizers" to substitute hand-optimized code!
- Originator: Jack Dongarra, Univ. of Tennessee,
- netlib@ornl.gov: source, results
- netlib.att.com:/netlib/benchmark/linpack*: source

Other “PACKS”

- LAPACK
  - Subroutine library for solving the most common problems in numerical linear algebra
  - Designed to run efficiently on shared memory vector and parallel processors
- ScaLAPACK
  - Software library for performing dense and band linear algebra computations on a distributed memory MP MIMD computers and networks of workstations supporting PVM and/or MPI
- PARKBENCH
  - Stands for Parallel Kernels and Benchmarks
  - Suite contains sequential codes, communication codes, linear algebra kernels, NPB kernels, NASA compact application codes, parallel spectral transform shallow water model code
  - http://www.netlib.org/parkbench/

SLALOM

- Developed by Gustafson, Rover, Elbert, Carter
- Benchmark computes equilibrium radiation transfer on a closed interior
- Scales in the number of finite elements into which the surface is decomposed
- Benchmark scales automatically to run in one minute on any computer
  - Versions in several languages and for several architecture types (vector, serial, SM, MP, SIMD, MIMD)
  - Memory requirements adjust automatically
- Can be used to compare machines as disparate as laptops and supercomputers
- http://www.scl.ameslab.gov/Publications/FixedTime/FixedTime.html#6.3

HINT

- Developed by Gustafson and Snell
- HINT = Hierarchical Integration benchmark
- Work measure is QUIPS – quality improvement per second – to measure the amount of work a computer can perform over time
- HINT based on successive refinement-style optimization of an integration curve
  - QUIPS computed as a step functions of time whenever an improvement to answer quality is computed
  - Task adjusts to precision available and has unlimited (algorithmic) scalability
- HINT a successor of SLALOM
  - Provides a more precise way of measuring answer quality
  - Reduces the complexity of SLALOM
  - Less difficult to optimize than SLALOM
  - Addresses technical problems of SLALOM wrt memory usage
- http://www.scl.ameslab.gov/Publications/HINT/ComputerPerformance.html#benchmark

Limitations and Pitfalls of Benchmarks

- Benchmarks cannot address questions you did not ask
- Specific application benchmarks will not tell you about the performance of other applications without proper analysis
- General benchmarks will not tell you all the details about the performance of your specific application
- One should understand the benchmark itself to understand what it tells us

Benefits of Benchmarks

- Popular benchmarks keep vendors attuned to applications
- Benchmarks can give useful information about the performance of systems on particular kinds of programs
- Benchmarks help in exposing performance bottlenecks of systems at the technical and applications level