CSC 447: Parallel Programming for Multi-Core and Cluster Systems

Introduction to Parallel Algorithms

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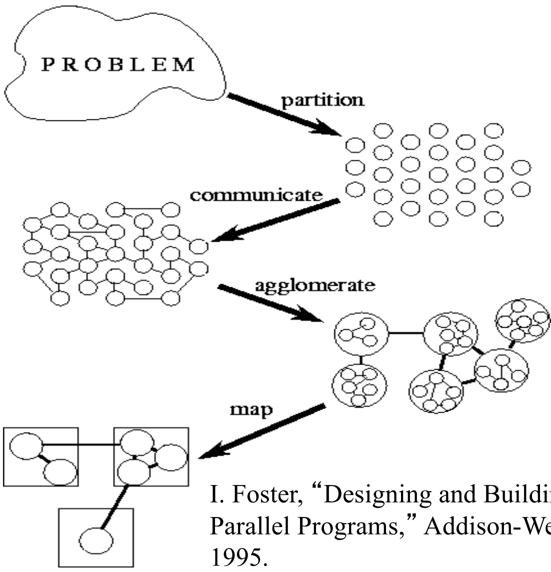
Spring 2016

Methodological Design

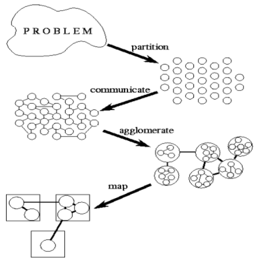
- Partition
  - Task/data decomposition
- Communication
  - Task execution coordination
- Agglomeration
  - Evaluation of the structure
- Mapping
  - Resource assignment



I. Foster, "Designing and Building Parallel Programs," Addison-Wesley, 1995.

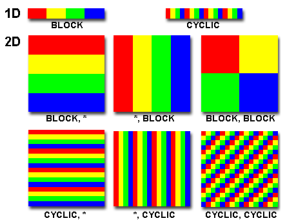
Partitioning

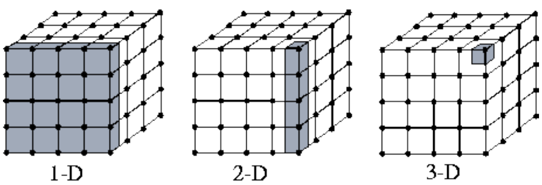
- Partitioning stage is intended to expose opportunities for parallel execution
- Focus on defining large number of small task to yield a fine-grained decomposition of the problem
- A good partition divides into small pieces both the computational tasks associated with a problem and the data on which the tasks operates
- Domain decomposition focuses on computation data
- Functional decomposition focuses on computation tasks
- Mixing domain/functional decomposition is possible



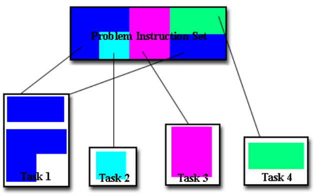
Domain and Functional Decomposition

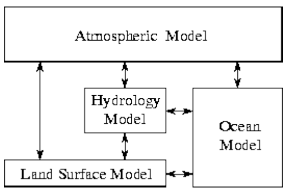
- Domain decomposition of 2D / 3D grid





- Functional decomposition of a climate model





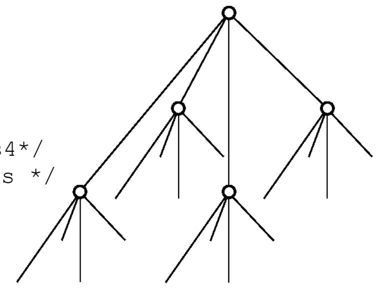
Divide and Conquer

- Divide a problem into sub-problems that are of the same form as the larger problem
- Further divisions into still smaller sub-problems are usually done by recursion

M-ary Divide and Conquer

- Divide and conquer can also be applied where a task is divided into more than two parts at each stage
- For example, if the task is broken into four parts, the sequential recursive definition would be

```
int add(int *s)      /* add list of numbers, s */
{
    if (number(s) =< 4) return(n1 + n2 + n3 + n4);
    else {
        Divide (s,s1,s2,s3,s4);      /* divide s into s1,s2,s3,s4*/
        part_sum1 = add(s1);   /*recursive calls to add sublists */
        part_sum2 = add(s2);
        part_sum3 = add(s3);
        part_sum4 = add(s4);
        return (part_sum1 + part_sum2 + part_sum3 + part_sum4);
    }
}
```



M-ary Divide and Conquer

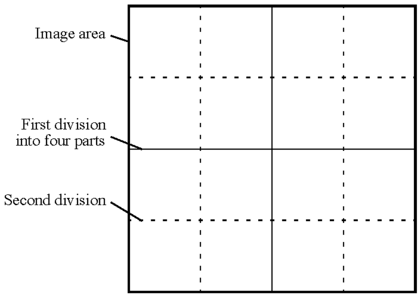


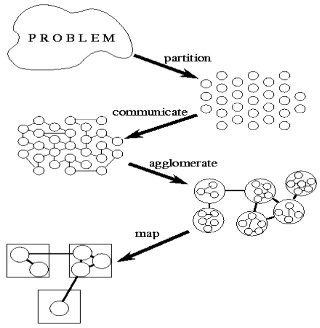
Figure 4.7 Dividing an image.

Partitioning Checklist

- Does your partition define at least an order of magnitude more tasks than there are processors in your target computer? If not, may lose design flexibility.
- Does your partition avoid redundant computation and storage requirements? If not, may not be scalable.
- Are tasks of comparable size? If not, it may be hard to allocate each processor equal amounts of work.
- Does the number of tasks scale with problem size? If not may not be able to solve larger problems with more processors
- Have you identified several alternative partitions?

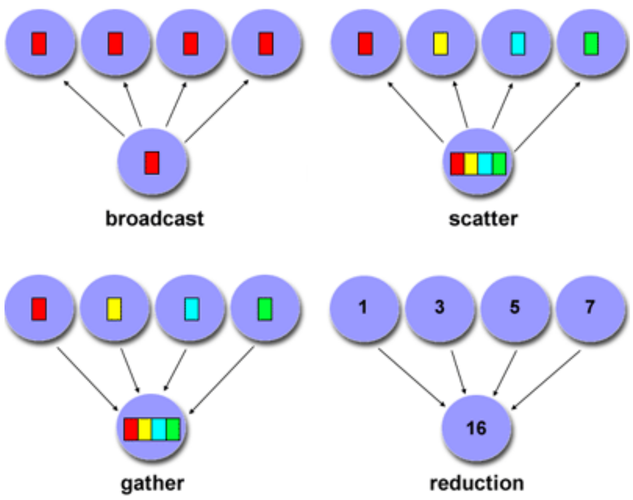
Communication (Interaction)

- Tasks generated by a partition must interact to allow the computation to proceed
  - Information flow: data and control
- Types of communication
  - Local vs. Global: locality of communication
  - Structured vs. Unstructured: communication patterns
  - Static vs. Dynamic: determined by runtime conditions
  - Synchronous vs. Asynchronous: coordination degree
- Granularity and frequency of communication
  - Size of data exchange
- Think of communication as interaction and control
  - Applicable to both shared and distributed memory parallelism



Types of Communication

- Point-to-point
- Group-based
- Hierachical
- Collective

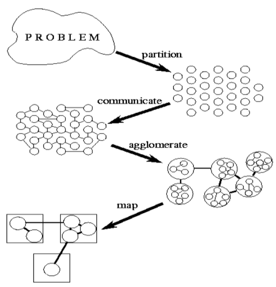


Communication Design Checklist

- Is the distribution of communications equal?
  - Unbalanced communication may limit scalability
- What is the communication locality?
  - Wider communication locales are more expensive
- What is the degree of communication concurrency?
  - Communication operations may be parallelized
- Is computation associated with different tasks able to proceed concurrently? Can communication be overlapped with computation?
  - Try to reorder computation and communication to expose opportunities for parallelism

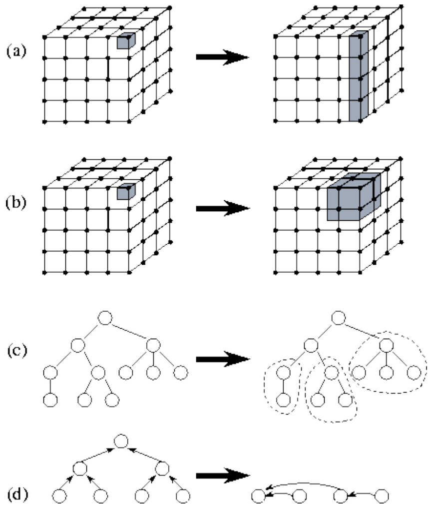
Agglomeration

- Move from parallel abstractions to real implementation
- Revisit partitioning and communication
  - View to efficient algorithm execution
- Is it useful to agglomerate?
  - What happens when tasks are combined?
- Is it useful to replicate data and/or computation?
- Changes important algorithm and performance ratios
  - Surface-to-volume: reduction in communication at the expense of decreasing parallelism
  - Communication/computation: which cost dominates
- Replication may allow reduction in communication
- Maintain flexibility to allow overlap



Types of Agglomeration

- Element to column
- Element to block
  - Better surface to volume
- Task merging
- Task reduction
  - Reduces communication

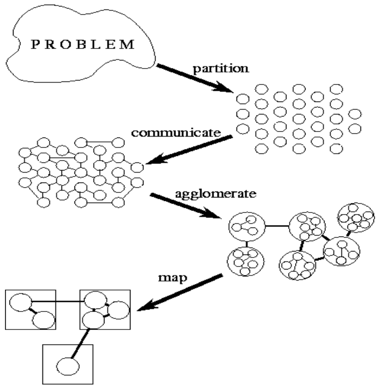


Agglomeration Design Checklist

- Has increased locality reduced communication costs?
- Is replicated computation worth it?
- Does data replication compromise scalability?
- Is the computation still balanced?
- Is scalability in problem size still possible?
- Is there still sufficient concurrency?
- Is there room for more agglomeration?
- Fine-grained vs. coarse-grained?

Mapping

- Specify where each task is to execute
  - Less of a concern on shared-memory systems
- Attempt to minimize execution time
  - Place concurrent tasks on different processors to enhance physical concurrency
  - Place communicating tasks on same processor, or on processors close to each other, to increase locality
  - Strategies can conflict!
- Mapping problem is NP-complete
  - Use problem classifications and heuristics
- Static and dynamic load balancing



Mapping Algorithms

- Load balancing (partitioning) algorithms
- Data-based algorithms
  - Think of computational load with respect to amount of data being operated on
  - Assign data (i.e., work) in some known manner to balance
  - Take into account data interactions
- Task-based (task scheduling) algorithms
  - Used when functional decomposition yields many tasks with weak locality requirements
  - Use task assignment to keep processors busy computing
  - Consider centralized and decentralize schemes

Mapping Design Checklist

- Is static mapping too restrictive and non-responsive?
- Is dynamic mapping too costly in overhead?
- Does centralized scheduling lead to bottlenecks?
- Do dynamic load-balancing schemes require too much coordination to re-balance the load?
- What is the tradeoff of dynamic scheduling complexity versus performance improvement?
- Are there enough tasks to achieve high levels of concurrency? If not, processors may idle.

Types of Parallel Programs

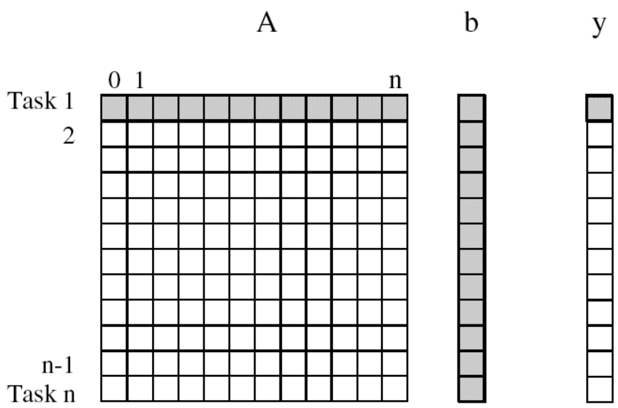
- Flavors of parallelism
  - Data parallelism
    - all processors do same thing on different data
  - Task parallelism
    - processors are assigned tasks that do different things
- Parallel execution models
  - Data parallel
  - Pipelining (Producer-Consumer)
  - Task graph
  - Work pool
  - Master-Worker

Data Parallel

- Data is decomposed (mapped) onto processors
- Processors performance similar (identical) tasks on data
- Tasks are applied concurrently
- Load balance is obtained through data partitioning
  - Equal amounts of work assigned
- Certainly may have interactions between processors
- Data parallelism scalability
  - Degree of parallelism tends to increase with problem size
  - Makes data parallel algorithms more efficient
- Single Program Multiple Data (SPMD)
  - Convenient way to implement data parallel computation
  - More associated with distributed memory parallel execution

Matrix - Vector Multiplication

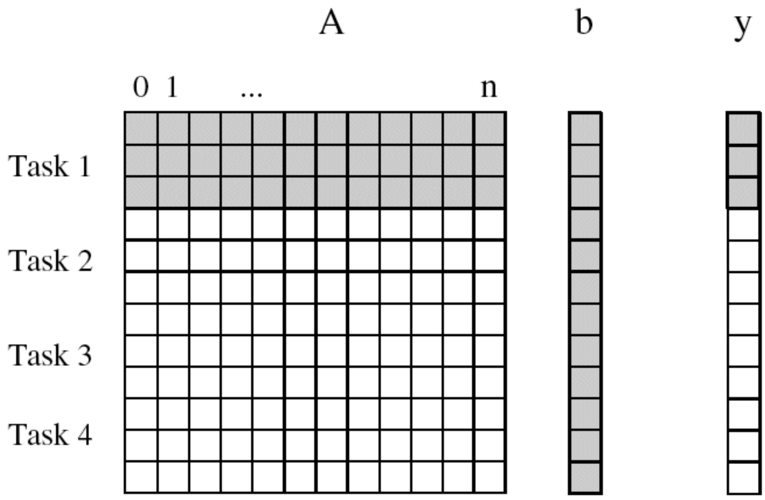
- A x b = y
- Allocate tasks to rows of A
  - $y[i] = \sum_j A[i,j]*b[j]$



- Dependencies?
- Speedup?
- Computing each element of y can be done independently

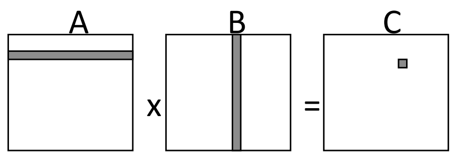
Matrix-Vector Multiplication (Limited Tasks)

- Suppose we only have 4 tasks
- Dependencies?
- Speedup?

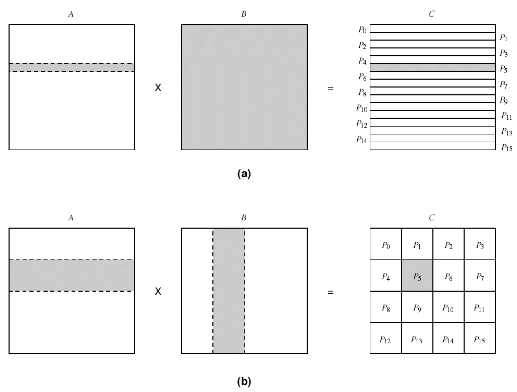


Matrix Multiplication

- A x B = C
- A[i,:] • B[:,j] = C[i,j]



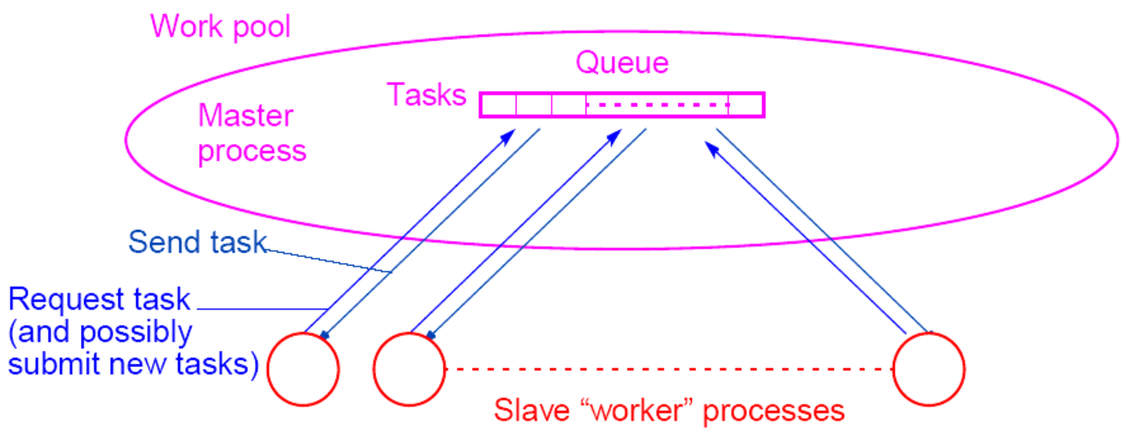
- Row partitioning
  – N tasks
- Block partitioning
  – N*N/B tasks
- Shading shows data sharing in B matrix



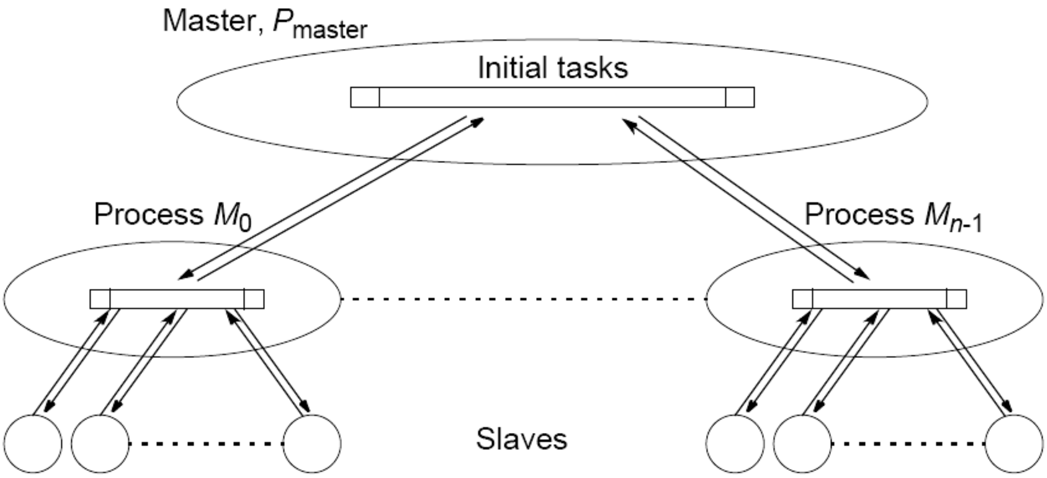
Granularity of Task and Data Decompositions

- Granularity can be with respect to tasks and data
- Task granularity
  - Equivalent to choosing the number of tasks
  - Fine-grained decomposition results in large # tasks
  - Large-grained decomposition has smaller # tasks
  - Translates to data granularity after # tasks chosen
    - consider matrix multiplication
- Data granularity
  - Think of in terms of amount of data needed in operation
  - Relative to data as a whole
  - Decomposition decisions based on input, output, input-output, or intermediate data

Centralized work pool

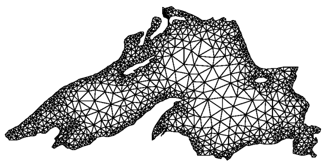


Decentralized Dynamic Load Balancing Distributed Work Pool



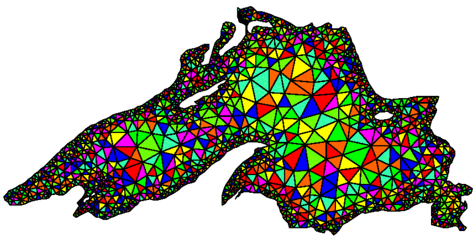
Mesh Allocation to Processors

- Mesh model of Lake Superior
- How to assign mesh elements to processors

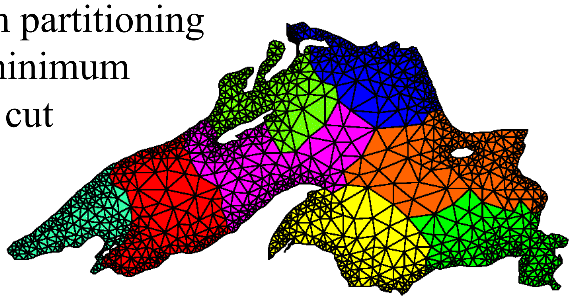


- Distribute onto 8 processors

randomly

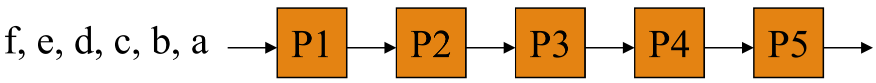


graph partitioning for minimum edge cut



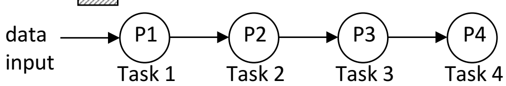
Pipelined Computations

- Pipelined program divided into a series of tasks that have to be completed one after the other.
- Each task executed by a separate pipeline stage
- Data streamed from stage to stage to form computation



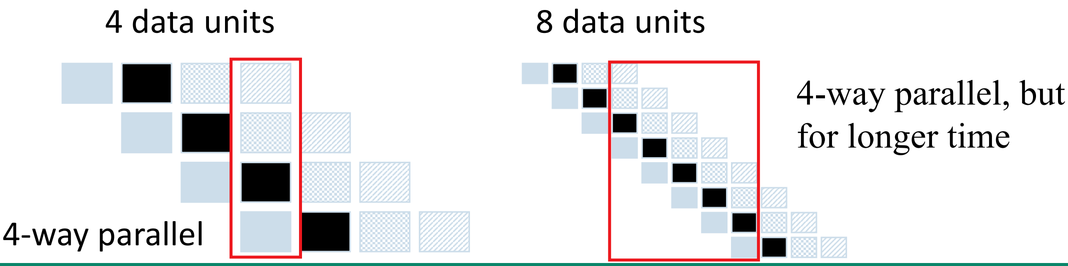
Pipeline Model

- Stream of data operated on by succession of tasks
  - Task 1 Task 2 Task 3 Task 4
  - Tasks are assigned to processors



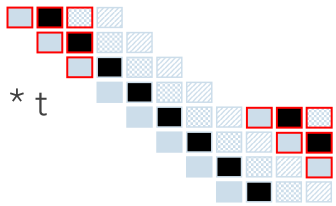
- Consider N data units
  - Sequential

  - Parallel (each task assigned to a processor)



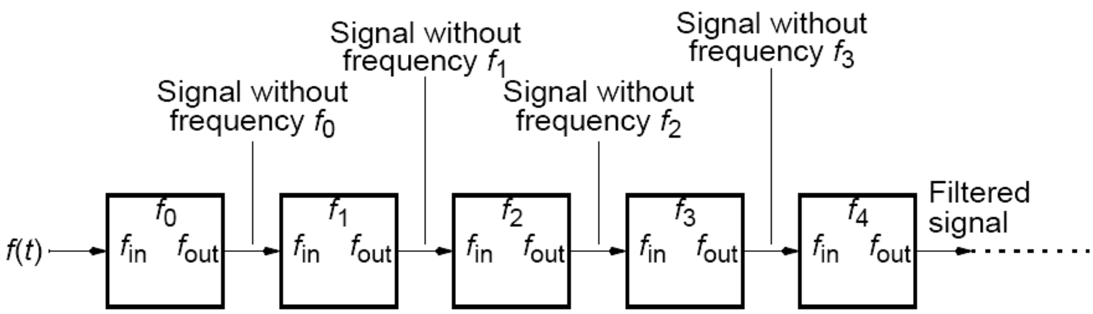
Pipeline Performance

- N data and T tasks
- Each task takes unit time t
- Sequential time = N*T*t
- Parallel pipeline time = start + finish + (N-2T)/T * t
  = O(N/T) (for N>>T)
- Try to find a lot of data to pipeline
- Try to divide computation in a lot of pipeline tasks
  - More tasks to do (longer pipelines)
  - Shorter tasks to do
- Pipeline computation is a special form of producer-consumer parallelism
  - Producer tasks output data input by consumer tasks



Example

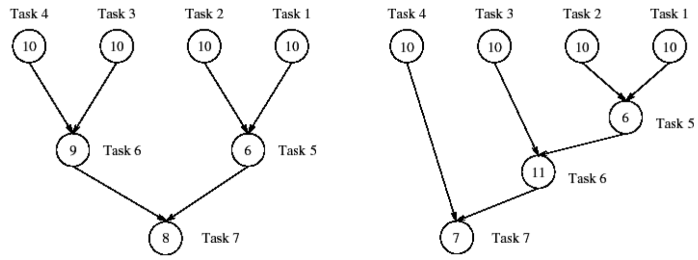
- Frequency filter - Objective to remove specific frequencies (f0, f1, f2,f3, etc.) from a digitized signal, f(t).
- Signal enters pipeline from left:



Tasks Graphs

- Computations in any parallel algorithms can be viewed as a task dependency graph
- Task dependency graphs can be non-trivial
  - Pipeline 

  - Arbitrary (represents the algorithm dependencies)

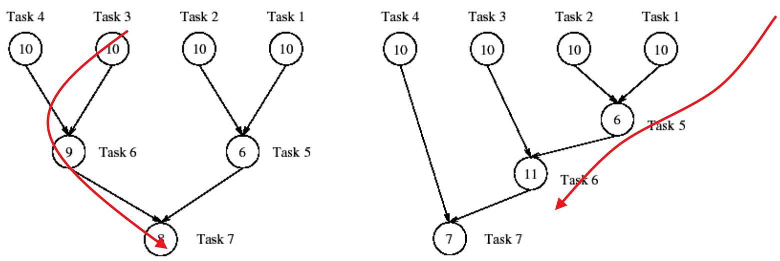


(a) (b)

Numbers are time taken to perform task

Task Graph Performance

- Determined by the critical path (span)
  - Sequence of dependent tasks that takes the longest time



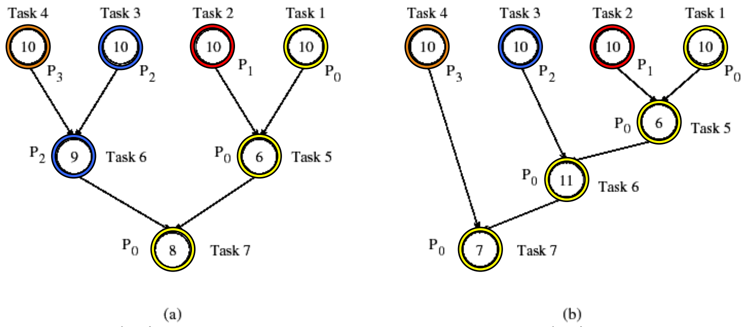
(a) (b)

Min time = 27 Min time = 34

  - Critical path length bounds parallel execution time

Task Assignment (Mapping) to Processors

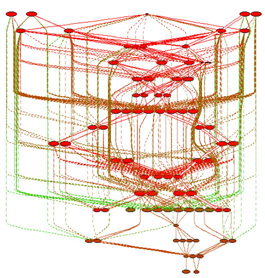
- Given a set of tasks and number of processors
- How to assign tasks to processors?
- Should take dependencies into account
- Task mapping will determine execution time



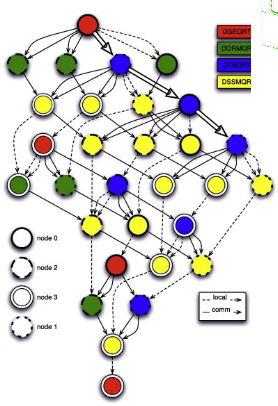
Total time = ?   Total time = ?

Task Graphs in Action

- Uintah task graph scheduler
  - C-SAFE: Center for Simulation of Accidental Fires and Explosions, University of Utah
  - Large granularity tasks
- PLASMA
  - DAG-based parallel linear algebra
  - DAGuE: A generic distributed DAG engine for HPC



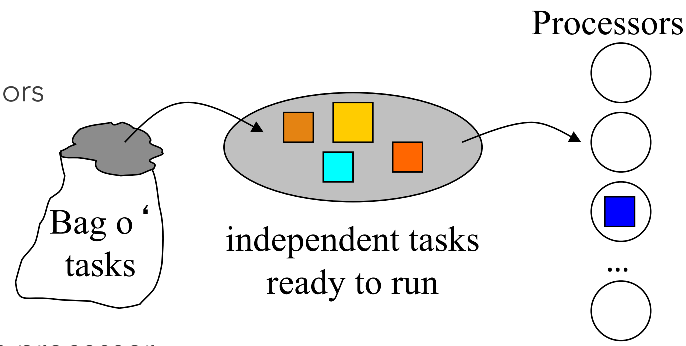
Task graph for PDE solver



DAG of QR for a 4 × 4 tiles matrix on a 2 × 2 grid of processors.

Bag o' Tasks Model and Worker Pool

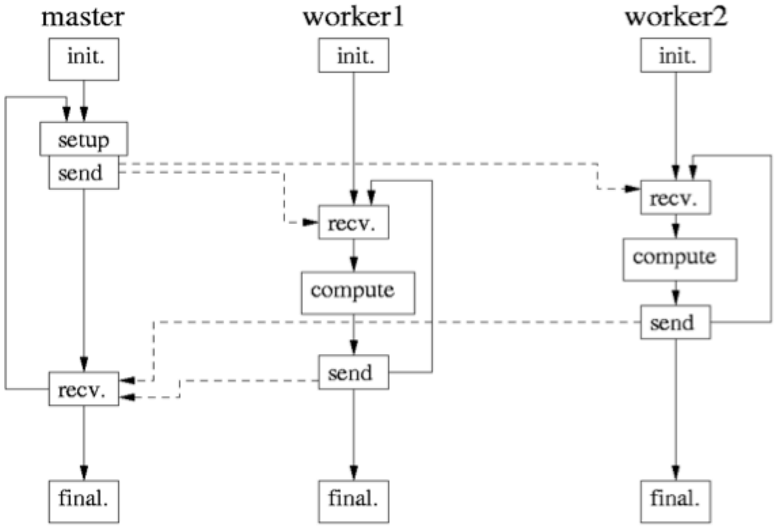
- Set of tasks to be performed
- How do we schedule them?
  - Find independent tasks
  - Assign tasks to available processors
- Bag o' Tasks approach
  - Tasks are stored in a bag waiting to run
  - If all dependencies are satisified, it is moved to a ready to run queue
  - Scheduler assigns a task to a free processor
- Dynamic approach that is effective for load balancing



Master-Worker Parallelism

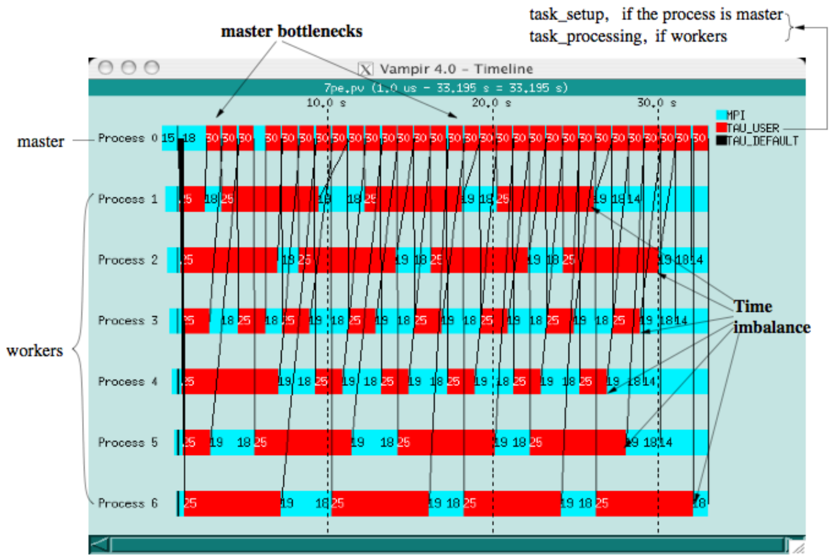
- One or more master processes generate work
- Masters allocate work to worker processes
- Workers idle if have nothing to do
- Workers are mostly stupid and must be told what to do
  - Execute independently
  - May need to synchronize, but most be told to do so
- Master may become the bottleneck if not careful
- What are the performance factors and expected performance behavior
  - Consider task granularity and asynchrony
  - How do they interact?

Master-Worker Execution Model (Li Li)



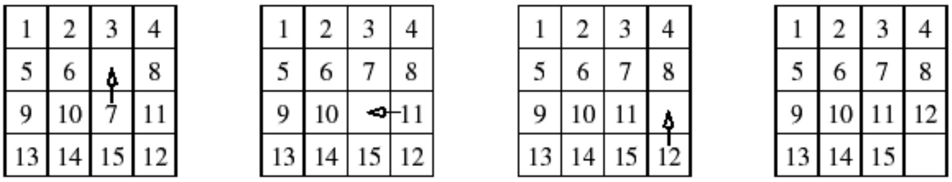
Li Li, "Model-based Automatics Performance Diagnosis of Parallel Computations," Ph.D. thesis, 2007.

M-W Execution Trace (Li Li)



Search-Based (Exploratory) Decomposition

- 15-puzzle problem
- 15 tiles numbered 1 through 15 placed in 4x4 grid
  - Blank tile located somewhere in grid
  - Initial configuration is out of order
  - Find shortest sequence of moves to put in order

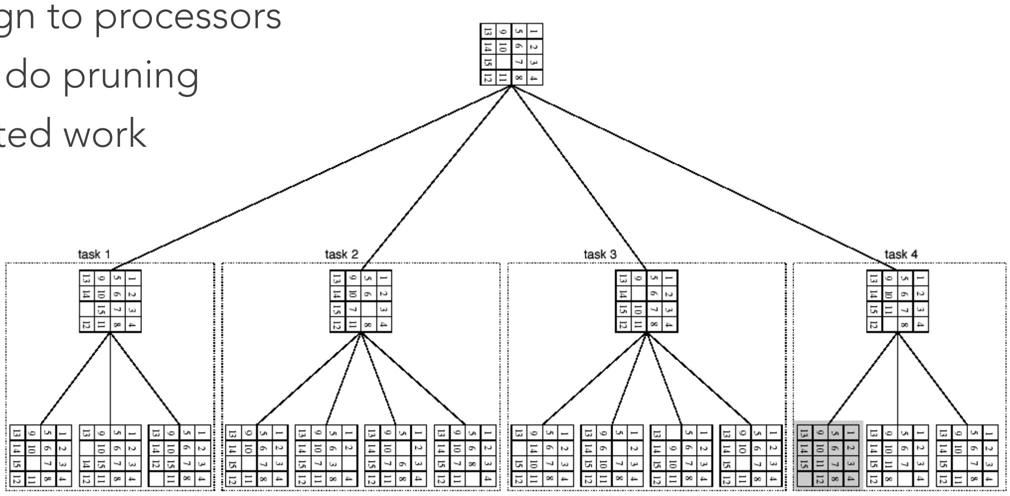


(a) (b) (c) (d)

- Seq
  - May involve some heuristics

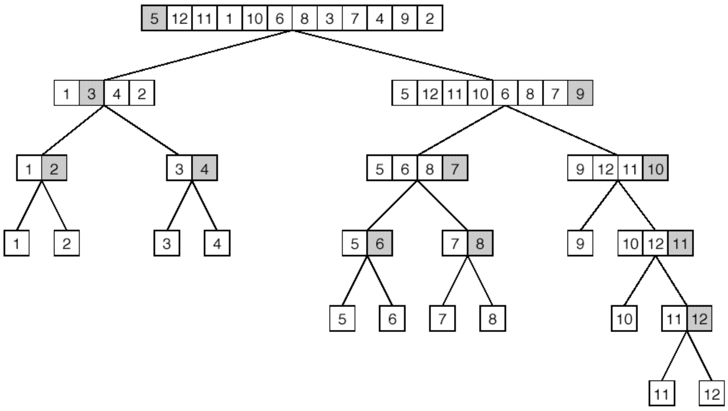
Parallelizing the 15-Puzzle Problem

- Enumerate move choices at each stage
- Assign to processors
- May do pruning
- Wasted work



Divide-and-Conquer Parallelism

- Break problem up in orderly manner into smaller, more manageable chunks and solve
- Quicksort example



Dense Matrix Algorithms

- Great deal of activity in algorithms and software for solving linear algebra problems
  - Solution of linear systems (Ax = b)
  - Least-squares solution of over- or under-determined systems
    - (min ||Ax-b||)
  - Computation of eigenvalues and eigenvectors (Ax= λ x)
  - Driven by numerical problem solving in scientific computation
- Solutions involves various forms of matrix computations
- Focus on high-performance matrix algorithms
  - Key insight is to maximize computation to communication

Solving a System of Linear Equations

- $Ax=b$

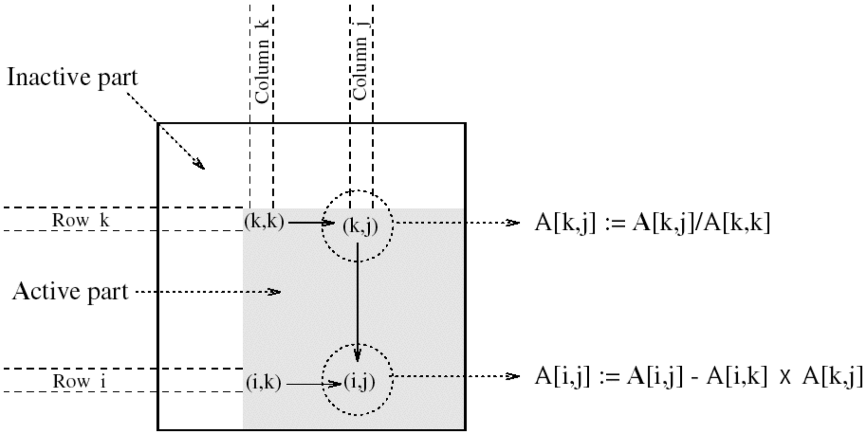
$$a_{0,0}x_0 + a_{0,1}x_1 + \dots + a_{0,n-1}x_{n-1} = b_0$$
$$a_{1,0}x_0 + a_{1,1}x_1 + \dots + a_{1,n-1}x_{n-1} = b_1$$
$$\dots$$
$$A_{n-1,0}x_0 + a_{n-1,1}x_1 + \dots + a_{n-1,n-1}x_{n-1} = b_{n-1}$$

- Gaussian elimination (classic algorithm)
  - Forward elimination to $Ux=y$ (U is upper triangular)
    - without or with partial pivoting
  - Back substitution to solve for x
  - Parallel algorithms based on partitioning of A

Sequential Gaussian Elimination

```
1.  procedure GAUSSIAN ELIMINATION (A, b, y)
2.  Begin
3.      for k := 0 to n - 1 do /* Outer loop */
4.      begin
5.          for j := k + 1 to n - 1 do
6.              A[k, j] := A[k, j ]/A[k, k]; /* Division step */
7.          y[k] := b[k]/A[k, k];
8.          A[k, k] := 1;
9.          for i := k + 1 to n - 1 do
10.         begin
11.             for j := k + 1 to n - 1 do
12.                 A[i, j] := A[i, j] - A[i, k] x A[k, j ]; /* Elimination step */
13.             b[i] := b[i] - A[i, k] x y[k];
14.             A[i, k] := 0;
15.         endfor;     /*Line9*/
16.     endfor;         /*Line3*/
17. end GAUSSIAN ELIMINATION
```

Computation Step in Gaussian Elimination



5x + 3y = 22
8x + 2y = 13

x = (22 - 3y) / 5
8(22 - 3y)/5 + 2y = 13

x = (22 - 3y) / 5
y = (13 - 176/5) / (24/5 + 2)

Rowwise Partitioning on Eight Processes

P_0	1	(0,1)	(0,2)	(0,3)	(0,4)	(0,5)	(0,6)	(0,7)
P_1	0	1	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)	(1,7)
P_2	0	0	1	(2,3)	(2,4)	(2,5)	(2,6)	(2,7)
P_3	0	0	0	(3,3)	(3,4)	(3,5)	(3,6)	(3,7)
P_4	0	0	0	(4,3)	(4,4)	(4,5)	(4,6)	(4,7)
P_5	0	0	0	(5,3)	(5,4)	(5,5)	(5,6)	(5,7)
P_6	0	0	0	(6,3)	(6,4)	(6,5)	(6,6)	(6,7)
P_7	0	0	0	(7,3)	(7,4)	(7,5)	(7,6)	(7,7)

n

(a) Computation:

(i) A[k,j] := A[k,j]/A[k,k] for k < j <

(ii) A[k,k] := 1

P_0	1	(0,1)	(0,2)	(0,3)	(0,4)	(0,5)	(0,6)	(0,7)
P_1	0	1	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)	(1,7)
P_2	0	0	1	(2,3)	(2,4)	(2,5)	(2,6)	(2,7)
P_3	0	0	0	1	(3,4)	(3,5)	(3,6)	(3,7)
P_4	0	0	0	(4,3)	(4,4)	(4,5)	(4,6)	(4,7)
P_5	0	0	0	(5,3)	(5,4)	(5,5)	(5,6)	(5,7)
P_6	0	0	0	(6,3)	(6,4)	(6,5)	(6,6)	(6,7)
P_7	0	0	0	(7,3)	(7,4)	(7,5)	(7,6)	(7,7)

(b) Communication:

One−to−all broadcast of row A[k,*]

Rowwise Partitioning on Eight Processes

P_0	1	(0,1)	(0,2)	(0,3)	(0,4)	(0,5)	(0,6)	(0,7)
P_1	0	1	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)	(1,7)
P_2	0	0	1	(2,3)	(2,4)	(2,5)	(2,6)	(2,7)
P_3	0	0	0	1	(3,4)	(3,5)	(3,6)	(3,7)
P_4	0	0	0	(4,3)	(4,4)	(4,5)	(4,6)	(4,7)
P_5	0	0	0	(5,3)	(5,4)	(5,5)	(5,6)	(5,7)
P_6	0	0	0	(6,3)	(6,4)	(6,5)	(6,6)	(6,7)
P_7	0	0	0	(7,3)	(7,4)	(7,5)	(7,6)	(7,7)

(c) Computation:

(i) A[i,j] := A[i,j] – A[i,k]x A[k,j]
for k < i < n and k < j < n

(ii) A[i,k] := 0 for k < i < n

2D Mesh Partitioning on 64 Processes

1	(0,1)	(0,2)	(0,3)	(0,4)	(0,5)	(0,6)	(0,7)
0	1	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)	(1,7)
0	0	1	(2,3)	(2,4)	(2,5)	(2,6)	(2,7)
0	0	0	(3,3)	(3,4)	(3,5)	(3,6)	(3,7)
0	0	0	(4,3)	(4,4)	(4,5)	(4,6)	(4,7)
0	0	0	(5,3)	(5,4)	(5,5)	(5,6)	(5,7)
0	0	0	(6,3)	(6,4)	(6,5)	(6,6)	(6,7)
0	0	0	(7,3)	(7,4)	(7,5)	(7,6)	(7,7)

(a) Rowwise broadcast of A[i,k]
for (k - 1) < i < n

1	(0,1)	(0,2)	(0,3)	(0,4)	(0,5)	(0,6)	(0,7)
0	1	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)	(1,7)
0	0	1	(2,3)	(2,4)	(2,5)	(2,6)	(2,7)
0	0	0	(3,3)	(3,4)	(3,5)	(3,6)	(3,7)
0	0	0	(4,3)	(4,4)	(4,5)	(4,6)	(4,7)
0	0	0	(5,3)	(5,4)	(5,5)	(5,6)	(5,7)
0	0	0	(6,3)	(6,4)	(6,5)	(6,6)	(6,7)
0	0	0	(7,3)	(7,4)	(7,5)	(7,6)	(7,7)

(b) A[k,j] := A[k,j]/A[k,k]
for k < j < n

1	(0,1)	(0,2)	(0,3)	(0,4)	(0,5)	(0,6)	(0,7)
0	1	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)	(1,7)
0	0	1	(2,3)	(2,4)	(2,5)	(2,6)	(2,7)
0	0	0	1	(3,4)	(3,5)	(3,6)	(3,7)
0	0	0	(4,3)	(4,4)	(4,5)	(4,6)	(4,7)
0	0	0	(5,3)	(5,4)	(5,5)	(5,6)	(5,7)
0	0	0	(6,3)	(6,4)	(6,5)	(6,6)	(6,7)
0	0	0	(7,3)	(7,4)	(7,5)	(7,6)	(7,7)

(c) Columnwise broadcast of A[k,j]
for k < j < n

1	(0,1)	(0,2)	(0,3)	(0,4)	(0,5)	(0,6)	(0,7)
0	1	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)	(1,7)
0	0	1	(2,3)	(2,4)	(2,5)	(2,6)	(2,7)
0	0	0	1	(3,4)	(3,5)	(3,6)	(3,7)
0	0	0	(4,3)	(4,4)	(4,5)	(4,6)	(4,7)
0	0	0	(5,3)	(5,4)	(5,5)	(5,6)	(5,7)
0	0	0	(6,3)	(6,4)	(6,5)	(6,6)	(6,7)
0	0	0	(7,3)	(7,4)	(7,5)	(7,6)	(7,7)

(d) A[i,j] := A[i,j]-A[i,k] x A[k,j]
for k < i < n and k < j < n

Back Substitution to Find Solution

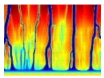
```
procedure BACK SUBSTITUTION (U, x, y)
begin
        for k := n - 1 downto 0 do /* Main loop */
        begin
            x[k] := y[k];
            for i := k - 1 downto 0 do
                y[i] := y[i] - x[k] xU[i, k];
        endfor;
end BACK SUBSTITUTION
```

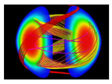

Dense Linear Algebra (www.netlib.gov)

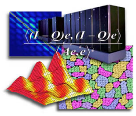
- Basic Linear Algebra Subroutines (BLAS)
  - Level 1 (vector-vector): vectorization
  - Level 2 (matrix-vector): vectorization, parallelization
  - Level 3 (matrix-matrix): parallelization
- LINPACK (Fortran)
  - Linear equations and linear least-squares
- EISPACK (Fortran)
  - Eigenvalues and eigenvectors for matrix classes
- LAPACK (Fortran, C) (LINPACK + EISPACK)
  - Use BLAS internally
- ScaLAPACK (Fortran, C, MPI) (scalable LAPACK)

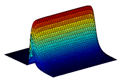
Numerical Libraries

- PETSc
  - Data structures / routines for partial differential equations
  - MPI based
- SuperLU
  - Large sparse nonsymmetric linear systems
- Hypre
  - Large sparse linear systems
- TAO
  - Toolkit for Advanced Optimization
- DOE ACTS
  - Advanced CompuTational Software







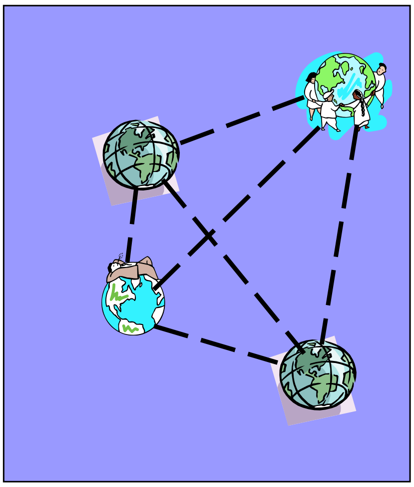


Gravitational N-Body Problem

- The N-body problem: Given n bodies in 3D space, determine the gravitational force *F* between them at any given point in time.

$$F = \frac{G m_a m_b}{r^2}$$

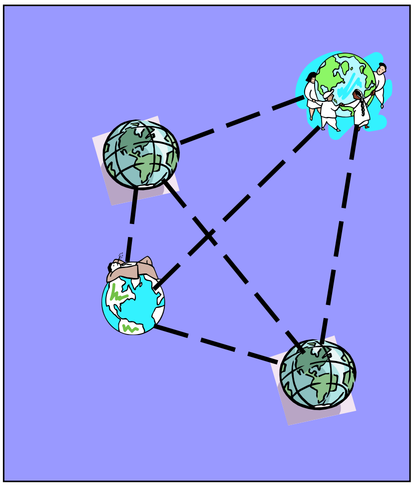
where *G* is the gravitational constant, *r* is the distance between the bodies, m_a and m_b are the masses of the bodies



Exact N-body serial pseudo-code

- At each time t, velocity v and position x of body i may change
- Real problem a bit more complicated than this

```
For (t=0: t<max; t++)
 For (i=0; i<N; i++) {
   F= Force_routine(i);
   v[i]_new = v[i]+F*dt;
   x[i]_new=x[i]+v[i]_new*dt;
 }
For (i=0; i<nmax; i++) {
 x[i] = x[i]_new;
 v[i]=v[i]_new;
 }
```



Parallel Code

- The algorithm is an O(N^2) algorithm (for one iteration) as each of the N bodies is influenced by each of the other N - 1 bodies.
- It is not feasible to use this direct algorithm for most interesting N-body problems where N is very large.
- The time complexity can be reduced using the observation that a cluster of distant bodies can be approximated as a single distant body of the total mass of the cluster sited at the center of mass of the cluster:

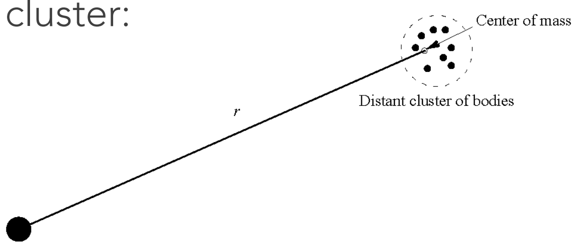
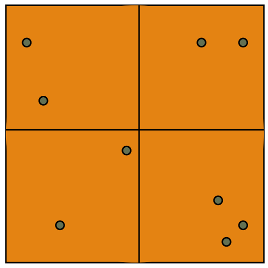
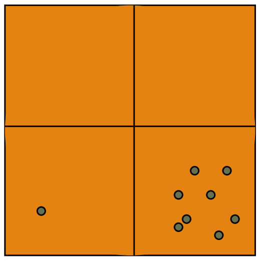


Figure 4.18 Clustering distant bodies.

Exact N-body and Static Partitioning

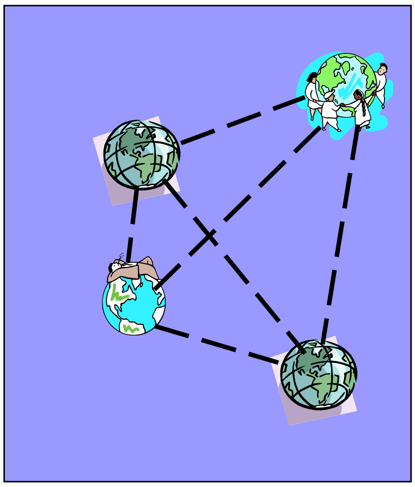
- Can parallelize n-body by tagging velocity and position for each body and updating bodies using correctly tagged information.
- This can be implemented as a data parallel algorithm. What is the worst-case complexity of complexity for a single iteration?
- How should we partition this?
  - Static partitioning can be a bad strategy for n-body problem.
  - Load can be very unbalanced for some configurations





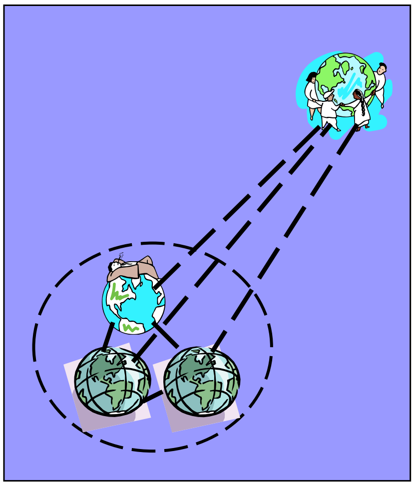
Improving the N-Body Code Complexity

- Complexity of serial n-body algorithm very large: $O(n^2)$ for **each** iteration.
- Communication structure **not** local – each body must gather data from all other bodies.
- Most interesting problems are when n is large – not feasible to use exact method for this
- Barnes-Hut algorithm is well-known approximation to exact n-body problem and can be efficiently parallelized



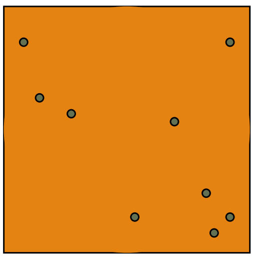
Barnes-Hut Approximation

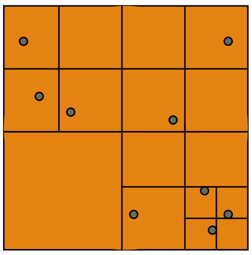
- Barnes-Hut algorithm based on the observation that a cluster of distant bodies can be approximated as a single distant body
  - Total mass = aggregate of bodies in cluster
  - Distance to cluster = distance to center of mass of the cluster
- This clustering idea can be applied recursively



Barnes-Hut Idea

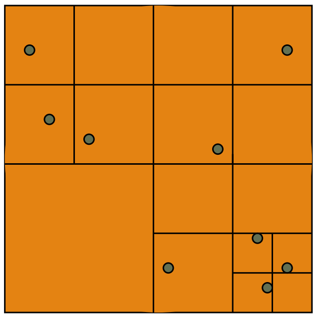
- Dynamic divide and conquer approach:
  - Each region (cube) of space divided into 8 subcubes
  - If subcube contains more than 1 body, it is recursively subdivided
  - If subcube contains no bodies, it is removed from consideration
- 2D example on right – each 2D region divided into 4 subregions

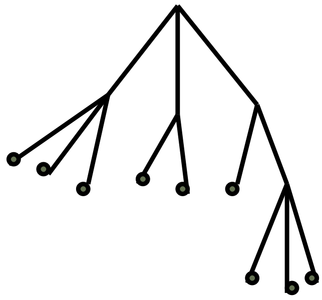




Barnes-Hut idea

- For 2D decomposition, result is a quadtree, pictured below.
- For 3D decomposition, result is an octtree





Barnes Hut 3D Problem Pseudo-code

For (t=0; t< tmax; t++) {
  Build octtree;
  Compute total mass and center;
  Traverse the tree, computing the forces
  Update the position and velocity of all bodies
  }

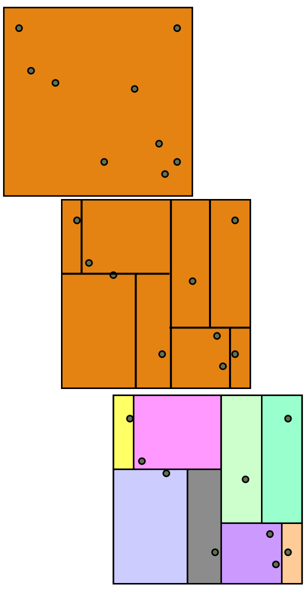
- Notes:
  - Total mass and center of mass of each subcube stored at its root
  - Tree traversal stops at a node when the clustering approximation can be used for a particular body
    - Need criteria for determining when bodies are in the same cluster

Barnes-Hut Complexity

- Partitioning is dynamic: Whole octtree must be reconstructed for each time step because bodies will have moved.
- Constructing tree can be done in O(nlogn)
- Computing forces can be done in O(nlogn)
- Barnes-Hut for one iteration is O(nlogn) [compare to $O(n^2)$ for one iteration with exact solution]

Generalizing the Barnes-Hut approach

- Approach can be used for applications which repeatedly perform some calculation on particles/bodies/data indexed by position.



- Recursive Bisection:
  - Divide region in half so that particles are balanced each time
  - Map rectangular regions onto processors so that load is balanced

Barnes-Hut Algorithm

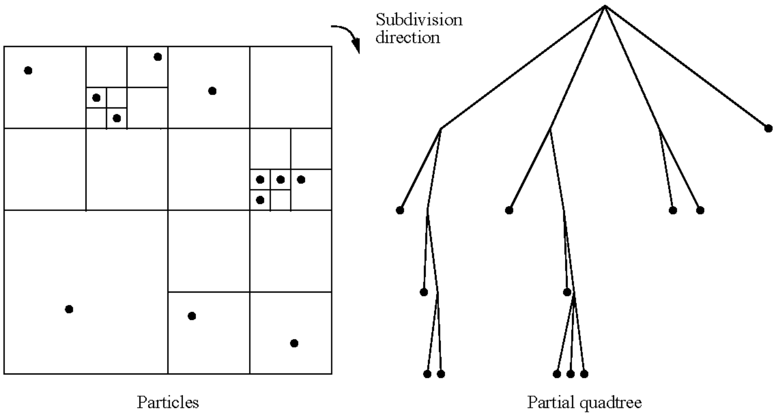


Figure 4.19 Recursive division of two-dimensional space.

Orthogonal Recursive Bisection

- Example for a two-dimensional square area.
  - First, a vertical line is found that divides the area into two areas each with an equal number of bodies.
  - For each area, a horizontal line is found that divides it into two areas each with an equal number of bodies.
  - This is repeated until there are as many areas as processors, and then one processor is assigned to each area.

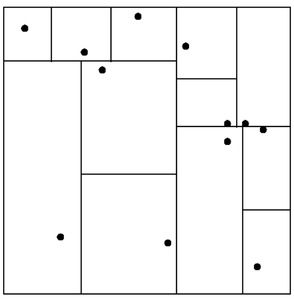


Figure 4.20 Orthogonal recursive bisection method.

Recursive Bisection Programming Issues

- How do we keep track of the regions mapped to each processor?
- What should the density of each region be? [granularity!]
- What is the complexity of performing the partitioning? How often should we repartition to optimize the load balance?
- How can locality of communication or processor configuration be leveraged?