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Abstract A stochastic chartist–fundamentalist model of speculative asset dynamics in financial markets is developed. The model is represented by a stochastic delay-differential equation (SDDE). The SDDE is then solved using approximation and numerical Monte Carlo methods. The results show that for large time delays, the SDDE generates market-like stock price dynamics that reflect the memory effects of the time delay. The resultant dynamics agree with the empirical observation of the tendency of stock markets to deviate from pure random walk.

Keywords Speculative models · Stochastic models · Delay-differential equations

1 Introduction

Financial markets exhibit persistent volatility where speculative booms are at times followed by severe crashes as the financial market crashes of 1929, 1987, 2000, and 2007. Market dynamics are typically interpreted by two main theories. The *efficient market hypothesis* (EMH) argues that asset prices follow a random walk motion that represents the fundamental value of the asset price. This typically implies that there are no profitable opportunities or forecasting in markets. Other theories argue that asset prices contain a deterministic element that is generated by herd behavior (Lux 1995), chartist speculators (Sethi 1996), or market psychology (Ohanian 1996). In such

G. Dibeh
Department of Economics, Lebanese American University, Byblos 1401 2010, Lebanon

H. M. Harmanani (✉)
Department of Computer Science and Mathematics, Lebanese American University,
Byblos 1401 2010, Lebanon
e-mail: haidar@lau.edu.lb

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models, asset prices can follow a deterministic path that deviates from fundamental values generating what is called a *speculative bubble* in asset markets. In chartist–fundamentalists models of speculative markets the market is populated by two types of speculators: the *fundamentalists* who follow the EMH theory and base their decisions on the demand of the asset on the fundamental price of the asset (Chiarella et al. 2002; Sethi 1996). *Chartists*, on the other hand, attempt to forecast future price levels based on past patterns of price dynamics. In the chartist/fundamentalists models the interaction of these two types of speculators cause the market to exhibit speculative bubbles. These models of speculative markets have been of the continuous-time type modeled as differential equations systems (Sethi 1996) or discrete-time type modeled as difference equations models (Franke and Sethi 1998).

This paper develops a stochastic chartist–fundamentalist model of stock markets with time delays where the time delay represents memory in financial markets. The resultant stochastic model is represented by a stochastic delay-differential equation (SDDE) which is the stochastic equivalent of delay-differential equations (DDEs). The importance of time delays in modeling memory in financial markets and their effect on the qualitative behavior of asset prices in SDDEs have been discussed lately by Frank (2006).

Recently, chartist–fundamentalist models of stock markets have been extended from modeling deterministic systems to the modeling of stochastic systems that can capture the effects of noise fluctuations present in financial markets on the dynamics of asset prices (Anufriev 2007; Chiarella 2007; Westerhoff 2003). It is well known that the presence of noise in physical systems has a fundamental effect on the dynamics of such systems. For example, noise can cause synchronization between otherwise autonomous oscillators (Pikovsky and Kurths 1997). In financial systems, noise and stochastic disturbances play a large role in price dynamics and in the pricing of financial instruments such as options. Hence, stochastic models are prevalent in the econophysics and financial engineering modeling. In this respect, stochastic models can be also applied to modeling contagion and synchronization phenomena that are prevalent in financial markets.

The paper starts with the deterministic delayed chartist–fundamentalist model developed in Dibeh (2005, 2007) and extends it to produce a stochastic variant of the model. The resultant stochastic model is represented by a SDDE which is the stochastic equivalent of DDEs. DDEs are a perfect tool to add temporal richness to mathematical models of economic dynamics. The temporal richness of the model derives from the assumptions made by the theorist concerning the frequency of occurrence of economic phenomena in a certain interval of time. For example, if the modeler assumes that economic phenomena take place only once in a certain interval of time then the resultant dynamical model will be of the type $x_{t+1} = f(x_t)$ where x_t is a vector of the relevant economic variables. If however, the theorist assumes that economic phenomena are densely occurring in all intervals of time, then the resultant dynamical model will be of the type. Mathematically, when discrete time delays are considered in an otherwise continuous dynamical system, the model describing the system becomes a DDE of the type. To the authors best knowledge, SDDE models have not been used previously in modeling of speculative markets and asset price dynamics. The noise effects are introduced into the delayed-chartist–fundamentalist model through excitation with additive noise. The resultant Langevin equation hence models the dynamics of stock prices under chartist–fundamentalist assumptions when external noise is present. The additive noise model is then solved numerically for stock price dynamics using approximation methods (Guillouzic et al. 1999, 2000).

2 The Chartist–Fundamentalist Model with Noise

The basic DDE chartist–fundamentalist model was developed in Ref Dibeh (2005). In chartist–fundamentalist models, the asset price is driven by excess demand $D(s, p)$ according to the following excess demand equation

$$\frac{\dot{p}}{p} = \sigma D(s, p) \tag{1}$$

where p is the asset price and s is a measure of price trend. The total market demand is equal to the sum of demands by the two market participants: the fundamentalists and the chartists

$$D(s, p) = D^c(s) + D^f(p) \tag{2}$$

The demand function of the fundamentalists can be written then as

$$D^f(p) = -m(p - \nu) \tag{3}$$

where ν is equal to the fundamental price of the asset, and m is equal to the fundamentalists share of wealth. Since fundamentalists believe that asset prices must converge instantaneously to ν then they will sell (buy) the asset when $p > (<)\nu$. On the other hand, the chartists observe the price trend of the asset and use the information of past price movements to make their decisions of purchasing or selling the asset. If chartists presently observe that prices in the past rose (fell), then their demand for the asset will increase (decrease). This reflects the belief of the chartists that future prices depend on past movement of the asset price. In this model, the expectations function of the chartists is nonlinear which represents the chartists' belief that prices cannot increase indefinitely and hence introduces a "saturation effect" into the demand function of the chartists producing an S-shaped nonlinearity (Chiarella et al. 2002). The nonlinearity is modeled using the hyperbolic tangent function of the trend in the price of the asset represented by $p(t) - p(t-\tau)$. The demand function of the chartist can then be written as:

$$D^c(s) = (1 - m)\tanh(p(t) - p(t - \tau)) \tag{4}$$

Combining (1–4) and assuming for simplicity that $\sigma = 1$, the model then becomes

$$\dot{p} = (1 - m)\tanh(p(t) - p(t - \tau))p(t) - m(p - \upsilon)p(t) \tag{5}$$

Solutions to the DDE in (5) (see Ref. Dibeh 2005) show a variety of attractor types including limit cycles. Limit cycle solutions indicate that in the presence of a bound on chartists expectations, persistent fluctuations in asset prices are possible. The deterministic model hence forms an adequate model of speculative markets in terms of generation of persistent market oscillations produced endogenously by the behavior of market participants. However, the deterministic model does not capture some of the key stylized facts of financial markets namely the probabilistic properties of price time series that may include fat tails. The speculative model is hence extended to include external noise which is done through additive noise, a typical procedure when investigating the effects of noise on the natural dynamics of systems (Tambe et al. 1995). Suppose that the DDE in (5) is perturbed with additive noise where $E = noise$. The noise represents the random arrival of information in financial markets affecting asset prices as typically modeled in the literature. In the context of the chartist–fundamentalist model, the additive noise represents the noisy environment that perturbs the chartist–fundamentalist excess demand. The model with the additive noise becomes

$$\dot{p} = (1-m)\tanh(p(t) - p(t-\tau))p(t) - m(p-\nu)p(t) + \sigma\epsilon_t \tag{6}$$

or

$$dp = ((1-m)\tanh(p(t) - p(t-\tau))p(t) - m(p-\nu)p(t))dt + \sigma dW \tag{7}$$

where W_t = Wiener process.

3 Numerical Solution of the Approximate Model

Empirical evidence has shown that financial markets have the following "stylized facts" (Johnson et al. 2002): fat-tailed PDF for price changes, slow decay of the autocorrelations of absolute returns, volatility clustering and fast decay of autocorrelations of returns. The SDDE (7) has no known analytical solutions. Numerical methods are used to generate solutions for such models. The applications of such methods impose various restrictions of the type of models that can be solved given their degree of complexity and nonlinearity and stability issues associated with such numerical procedures (Kuchler and Platen 2002; Bukwar 2000). In this paper we apply the approximation methods for SDDEs developed in Refs (Guillouzic et al. 1999, 2000). Using Taylor expansions the SDDE is transformed into an SDE that forms an approximated model to the original SDDE. The resultants SDE is then solved through standard Euler numerical schemes. The approximation method is summarized as follows. Let the SDDE model be

$$dx = f(x(t), x(t-\tau))dt + \sigma g(x(t))dW \tag{8}$$

where τ = time delay. The SDDE in Eq. (8) is approximated given Taylor expansion of $f(x(t), x(t-\tau))$ around $x(t)$ in powers of τ to produce the following SDE

$$dx = f_a(x_0)dt + \sigma g_a(x_0)dW \tag{9}$$

where

$$f_a(x_0) = f(x_0, x_0)(1 - \tau \frac{\partial}{\partial x_\tau} f(x_0, x_0)) \tag{10}$$

$$g_a(x_0) = g(x_0)(1 - \tau \frac{\partial}{\partial x_\tau} f(x_0, x_0)) \tag{11}$$

Applying the approximation method to the additive noise model (7) where

$$f(p(t), p(t-\tau)) = (1-m)\tanh(p(t)-p(t-\tau))p(t) - m(p(t)-\nu)p(t) \tag{12}$$

$$g(p(t)) = 1 \tag{13}$$

we get the following approximate SDE for the additive noise model

$$dp = -m(p-\nu)(1-\tau(1-m)p)dt + \sigma(1-\tau(1-m)p)dW \tag{14}$$

where

$$f_a(x_0) = -m(p-\nu)p(1-\tau(1-m))p \tag{15}$$

$$g_a(x_0) = 1-\tau(1-m)p \tag{16}$$

Numerical Monte Carlo methods are applied to the approximate SDE model in 14.

4 Numerical Solution

The model in Eq. (14) was discretized and solved using the *Monte-Carlo* method. In order to estimate the price using Monte Carlo, the method approximates the stochastic part of the equation by generating twelve uniformly random numbers, ψ_j, between 0 and 1. The random numbers are next added and the resulting sum has a mean of six and a standard deviation of 1.0. The mean is next adjusted to $zero$ by subtracting six. The numerical integration for the SDE is next carried out using Euler's method, which is known to converge in expectation to the solution with an error that is equal approximately to the size of the step (Johnson and Shanno 1987; Rumelin 1982). In order to increase the solution accuracy, the time steps were set to $\delta_t = 0.0001$. In order to guarantee random sampling, the random deviates were generated using a pseudo Data Encryption Standard that acts on 64-bits of input by iteratively applying a highly non-linear bit-mixing cipher function. The algorithm was very fast and all results were reported in a maximum of 1 CPU minute. The implementation of the algorithm goes thorough the following steps:

- *Step 1* Accept the model's input parameters: *initial price*, τ,m, ν, and σ
- *Step 2* Initialize the seed. Set $t = 0$ and $\delta_t = 0.0001$.
- *Step 3* Use Euler's Method to compute the next P based on the following:

$$p_{t+1} = -m(p_t-\nu)(1-\tau(1-m)p_t)\delta_t + \sigma(1-\tau(1-m)p_t)W_t \tag{17}$$

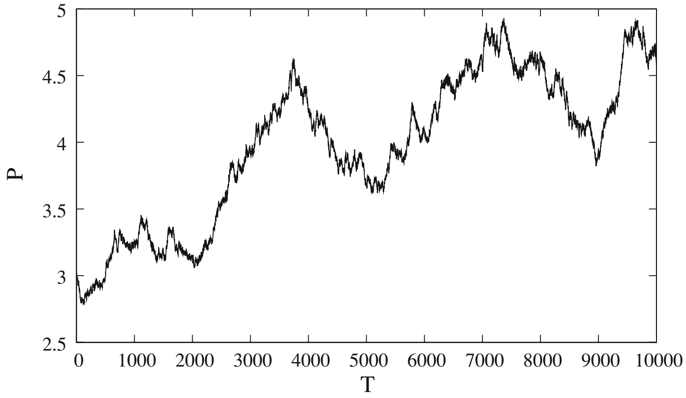

Fig. 1 Random walk dynamics of the asset price for $[p(0) = 3, \nu = 0.5, \sigma = 0.3, \tau = 0, m = 0.6]$

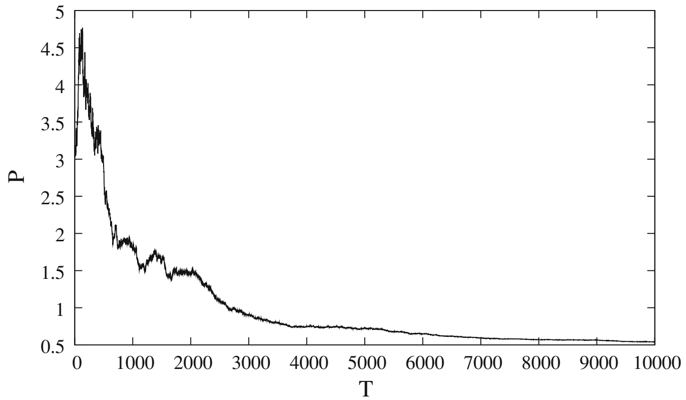

Fig. 2 The dynamics of the asset price for $[p(0) = 3, \nu = 0.5, \sigma = 0.3, \tau = 5, m = 0.6]$

where

$$W_t = \left(\sum_{i=1}^{12} \psi_i\right) - 6$$

and ψ_i's are independent random variables.

- *Step 4* $t \leftarrow t + \delta_t$. Repeat *step 3* until t=1.

Solutions are found for various parameters of the model. The effect of the time delay on stock price dynamics is investigated. Figures 1 and 2 show price realizations for the model for $\tau = 0$ and $\tau = 5$ respectively with the same initial price $p_0 = 3.0$. Figure 3 further shows the histograms for the stock returns for various time delays. Although, the accuracy of the numerical solution is reduced as the time delay gets larger, the solutions shows that the higher the time delays, the more the asset price

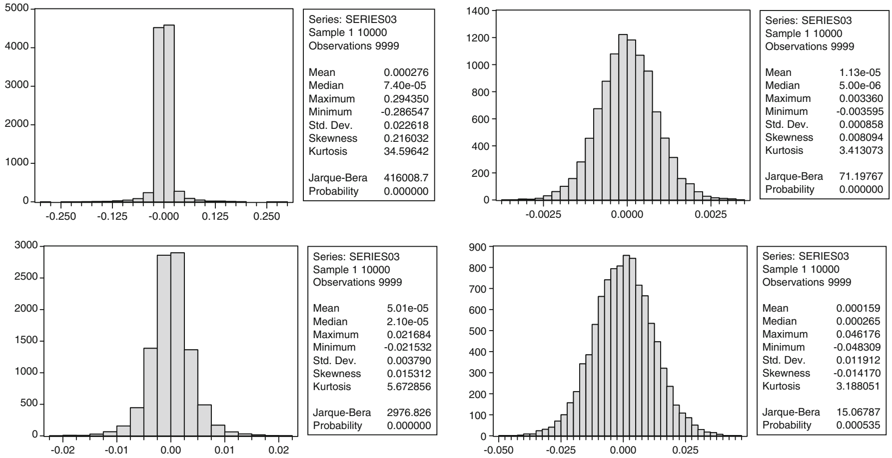

Fig. 3 Histograms of returns generated by SDDE model for different time delays [*upper left* ($\tau = 5$), *lower left* ($\tau = 4$), *upper right* ($\tau = 3$) and *lower right* ($\tau = 0$)]

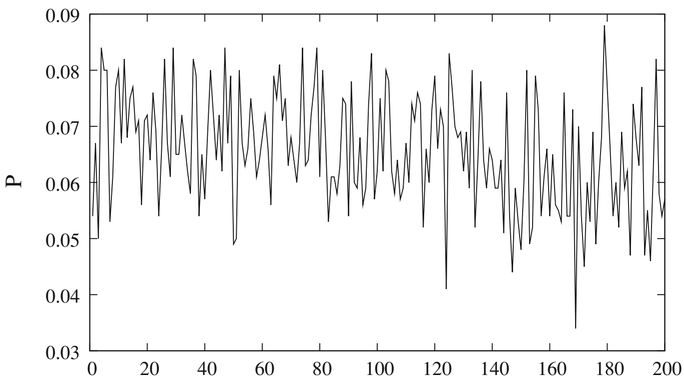

Fig. 4 Autocorrelation of the absolute returns for $\tau = 4$

exhibits the stylized facts of financial markets memory effects that represent deviation from random walk. Figure 1 shows the typical random walk dynamics when $\tau = 0$. The asset price wanders randomly driven by the Weiner process. In contrast, Fig. 2 shows the possibility of persistent dynamics in asset prices that are the result of memory effects of the time delay ($\tau = 5$). The asset price exhibits initially a boom rising to $p = 4.5$, then exhibiting persistent downfall reaching the equilibrium (fundamental) value of $p = \nu = 0.5$. This boom-bust dynamics reflect long-term deviation of the price from its fundamental value. Moreover, the histogram from $\tau = 0$ indicates a normal distribution. The histograms show clearly fat tail PDFs as τ gets larger. The results also show that the autocorrelation of the absolute returns decay slowly (Fig. 4).

5 Conclusions

In this paper an SDDE model of speculative market dynamics was developed based on a chartist–fundamentalist framework. The additive noise SDDE model was solved using Monte Carlo methods with Euler schemes for the approximate SDE model. The observed simulations of price dynamics show that the price of the stock follows paths that are dependent on the strength of the time-delay. These results provide a glimpse on possible market dynamics generated by SDDEs that represent persistent dynamics generated by the memory effects of time delays. The model can also be extended to include multiplicative noise effects in financial markets in addition to periodic inputs. Such extensions pave the way for the generation of a class of SDDE chartist–fundamentalist models that may exhibit more complex dynamics such as random attractors and noisy limit cycles. Finally, the proposed model can also be solved using various classes of *Runge-Kutta methods*, including *A-stable theta methods*, *Gauss methods* and *Radau methods*.

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