Transient Behaviour of a Forced Mechanical Oscillator with Eddy Damping

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Investigated the transient and resonance behaviour of a non-ideal harmonic oscillator. After adding a periodic external force to the oscillator, the data was modelled to Newtonian theory[1]. Free and forced oscillation trials were conducted at various frequencies to analyze behavior near resonance frequency. The transient and resonance frequency behaviours in the mechanical oscillator were analyzed with three different scenarios of damping. The goal was to gather optimal parameters for each case of damping and compare the accuracy of our findings to the theoretical model of a simple harmonic oscillator. The behaviour of the mechanical oscillator was observed at various points around resonance frequency to understand the limitations of the theoretical model.

The experimental apparatus consisted of a hacksaw blade fixed onto a wooden block. The end of the blade was extended and a vibration driver[2] was placed perpendicular to the blade. An accelerometer[3] and a magnet were attached to the free end of the hacksaw blade as a makeshift mass. The oscillations of this setup were measured by an oscilloscope and wired onto Labview through the DAQ. Before proceeding with data collection, a fixed gravitational calibration was calculated for the accelerometer. The acceleromter was placed pointing up and calibrated to scale our measurements in terms of gravity. The positioning of the driver perpendicular to the hacksaw blade was shifted until the oscilloscope showed frequency readings close to 12 Hz, an initial estimate of resonance frequency. This gave a damped sine wave with a decaying amplitude. The vibration driver, oscillator and battery setup were clamped to a firm foundation to avoid any error occurring during the experiment. Any accidental moving of the apparatus could significantly alter the results of the experiment as the accelerometer was very sensitive to external force.

To induce eddy damping, a copper block was placed over the magnet. By shifting the position of the copper block, trials for low, medium and high cases of damping were analyzed. For each measurement, iterations were conducted across several frequencies to gather a range of data points. When deviating away from the approximate resonance frequency of 12 Hz, the amplitude of the sinusoidal wave approached zero. Our iterations ranged from 5Hz to 20Hz, with the amplitude increasing as we approached resonance frequency from both ends. Close to resonance, we observed a spike in the amplitude of the driving frequency while the amplitude of voltage supplied remained constant. A phase shift of $\frac{\pi}{2}$ was observed at resonance frequency. During each change of frequency, transient behaviour of the sinusoidal wave was noticed on the oscilloscope. To reduce the uncertainty of error from the transient behaviour, a time delay was incorporated to let the transients die down before proceeding with each measurement. This behaviour could affect the final results as several different cases of resonant frequency were observed alongside the transient behaviour. This uncertainty was seen while shifting the frequency by very small amounts on the function generator as several frequencies gave the same resonance amplitude. To analyze the transient oscillations, the oscillator mass was moved away from equilibrium by hand and data was collected while it decayed.

Data analysis was done through curve fitting. Using the driven harmonic oscillator model (1), the experimental values were compared to theoretical expectations[1]. Here, a_0 is the acceleration, ω is the natural oscillating frequency, ω_0 is the driving frequency, F_0 is the magnitude of the driving force, m is the mass of the accelerometer and γ is the damping coefficient. By varying the natural oscillation frequency at points away and near resonance, best fit parameters were compiled by curve fitting the range of data.

$$a_0(\omega) = \frac{\omega^2(F_0/m)}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\gamma^2\omega^2}} \tag{1}$$

$$a_0(\omega) = \omega_0^2 - \omega^2 + 2\gamma\omega \tag{2}$$

$$\theta = \arctan \frac{Y}{X} \tag{3}$$

Phase shift curve fitting was achieved through (2) in conjunction with (3), where $\omega_0^2 - \omega^2$ was denoted as X and $2\gamma\omega$ as Y.

Figure 1 shows the relationship between the acceleration and the driving frequency. By covering a larger area of the magnet with the copper block, the effects of eddy damping were increased. As damping increased, the accelerometer amplitude decreased. At frequencies away from resonance, the acceleration amplitude was close to zero. A similar pattern was noticed with the plot residuals. Optimal parameters are shown in Figure 2a. At points close to resonance frequency, the residuals increased in both the positive and negative direction. Away from resonance, the residuals were close to zero.

For each damping case, the transient behaviour of free oscillations were analyzed. Figure 3 shows the change in transient amplitude as the damping was increased. The optimal parameters of the transient data are shown in Figure 2b. The error estimates in the data were taken from the forced oscillations trial and were too small to be seen. The results depicted an inverse relationship for damping and amplitude as expected.

The fitted parameters had an associated error, however, it was not exact due to experimental uncertainties. The transient behaviour contributed uncertainty in the accelerometer amplitude and phase angles at resonance frequency. The shifting of the copper block for each case trial could also lead to some error in the eddy damping. The vertical distance from the magnet and copper block was constant throughout the experiment as the apparatus was clamped. The surface area coverage was not constant as the positioning of the copper block was changed to evaluate different eddy damping values. The error uncertainty in the experiment gave a wider range of possible parameters. This effect was largely due to the transient behaviour noticed near the resonance frequency. χ^2 values were very high for each case of damping and this was inferred from Figure 1. In each trial, many of the data points were not aligned with the model curve fit. The error uncertainty in the damping values was also very high due to the transient effects of the system. In the free oscillation cases, the reduced χ^2 was less than one. This signifies that the error estimates for curve fitting were too large. Compared to the error estimates of the curve fits which were taken from the forced oscillation data, the free oscillation error was expected to be larger. The driven oscillations were much more controlled and precise as compared to the manual free oscillations. From equation (4) the quality factor was calculated using optimal parameters.

$$Q = \frac{\omega_0}{2\gamma} \tag{4}$$

Each data set gave a consistent pattern for the quality factor. As γ increased, Q decreased. From the forced oscillation data, quality factor values were 14.9 for minimal damping, 8.7 for moderate and 3.85 for the highest eddy damping case.

The simple harmonic oscillator model supported the data but the system deviated away from the model at certain limits. Conclusively, a noticeable difference existed between the forced and free oscillation data. This was evident in the large variations between χ^2 for the data set as well as the error propagation. It is evident from the optimal parameters in Figure 2 that χ^2 was largely dependent on error estimates and larger estimates gave a very small reduced χ^2. The data successfully supported the inverse relationship of γ and the quality factor Q. This pattern was expected from the simple harmonic oscillations. The model started to fail when the driving amplitude was increased by a threshold above and below resonant frequency. At this deviation the acccelerometer amplitude went to zero. This anharmonic behaviour was noticed at very high and low frequencies. This points to ambiguity in the sinusoidal force of the vibration driver. At points very close to resonance, the data became non linear and the system deviated away from the model. It was evident that the frequency was amplitude dependent and the model failed to support the data at non-linear limits. The damping constant, γ, was constant throughout each trial, hence leading to possible error in the model at high amplitudes. The transient behaviour of our system deviated from the model due to the visibility of several resonant frequencies. If one successfully let the transients die down, they would be able to remove some of the error uncertainty in the simple harmonic model. To reduce the error uncertainty, one could repeat the experiment by slightly changing the position of vibration driver and averaging the data over multiple experiments. This could reduce the uncertainty around multiple resonant frequencies noted during the transient behaviour. Overall, the optimal parameters showcased a reasonable relation to the Newtonian model of a simple harmonic oscillator, however, at certain limits, the data deviated from theoretical expectations.

REFERENCES

[1]Smith, W. (2010). Waves and Oscillations: A prelude to Quantum Mechanics, (pp. 85-104). *Oxford University Press.*

[2]PASCO, Mechanical Wave Driver, SF-9234 Manual, www.pasco.com.

[3]Analog Devices, Three-Axis Accelerometer Evaluation Board, EVAL-ADXL335Z Manual, www.analog.com.

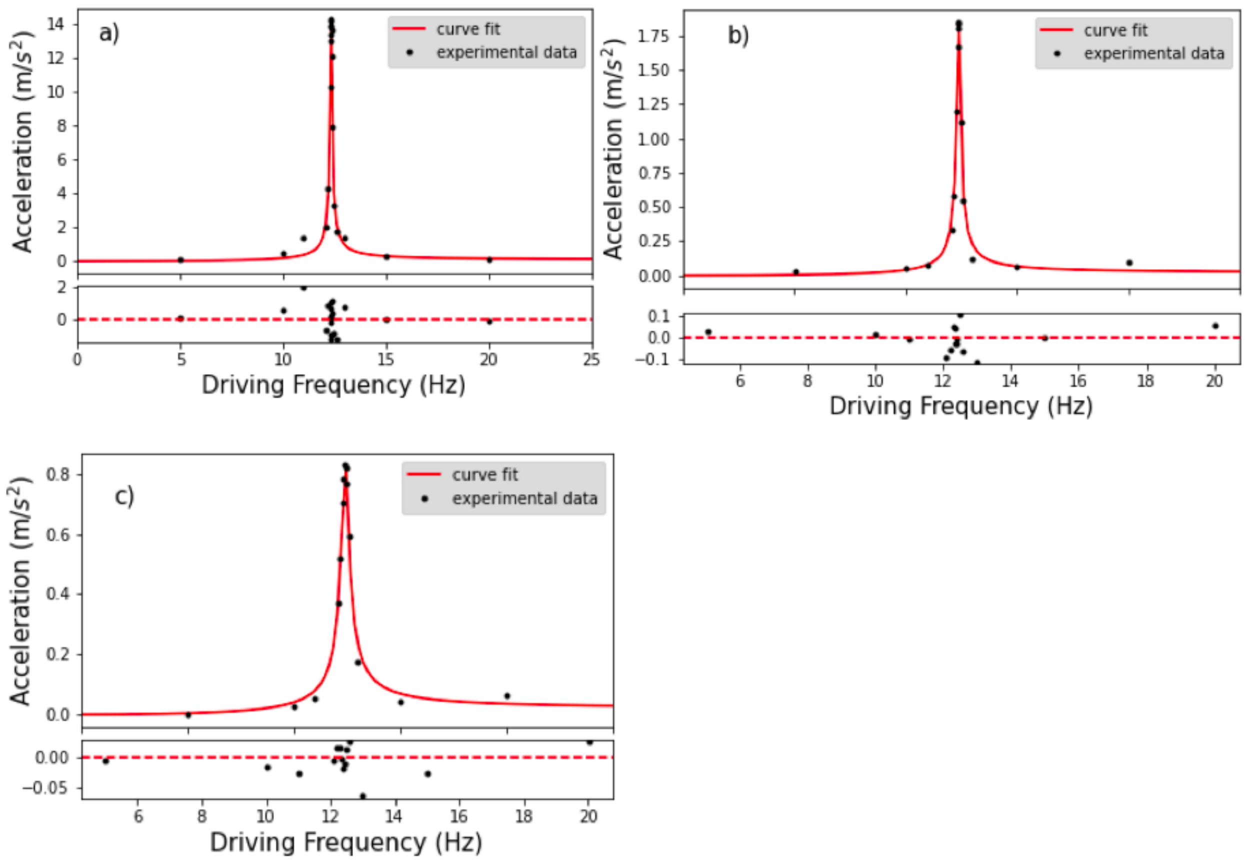


FIG. 1: a) Forced oscillations of the mechanical oscillator show acceleration over time for the minimally damped case. Resonance frequency behaviour models the relationship shown in equation (1). Bottom plot shows the normalized residuals of data over time. b) Moderately damped case. c) High damped case.

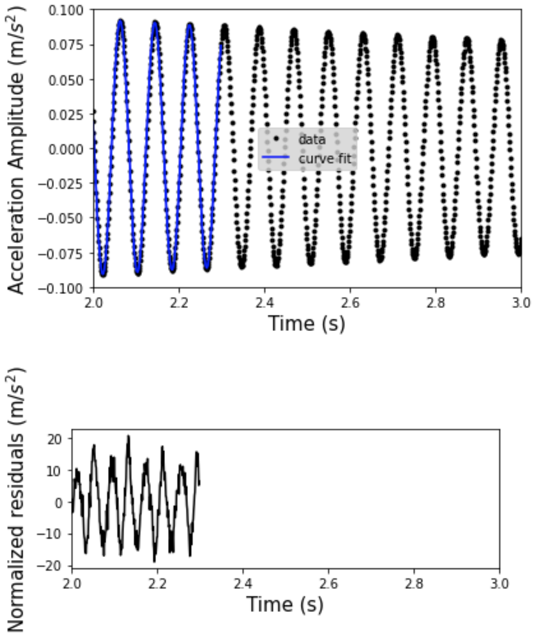
a) Forced Oscillation Data

Low Damping	Mid Damping	High Damping
$\omega = 12.34 \pm 0.0046$ Hz	$\omega = 12.39 \pm 0.0104$ Hz	$\omega = 12.41 \pm 0.0382$ Hz
$\gamma = 0.4141 \pm 0.0006$ s_{-1}	$\gamma = 0.7045 \pm 0.0997$ s_{-1}	$\gamma = 1.611 \pm 0.5528$ s_{-1}
$\chi_2 = 2139993$	$\chi_2 = 20766$	$\chi_2 = 45580$

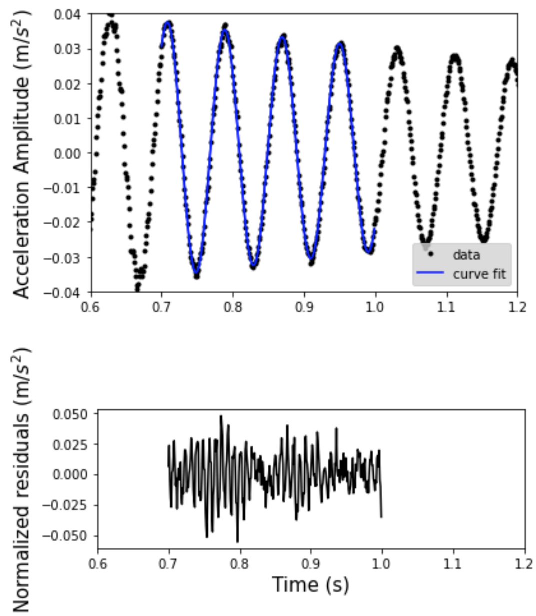
b) Transient Data

Low Damping	Mid Damping	High Damping
$\omega = 12.34 \pm 0.254$ Hz	$\omega = 12.40 \pm 0.2151$ Hz	$\omega = 12.42 \pm 0.2241$ Hz
$\gamma = 0.2099 \pm 0.0734$ s_{-1}	$\gamma = 0.7584 \pm 0.1200$ s_{-1}	$\gamma = 1.712 \pm 0.5291$ s_{-1}
Reduced $\chi_2 = 0.921$	Reduced $\chi_2 = 0.315$	Reduced $\chi_2 = 0.162$

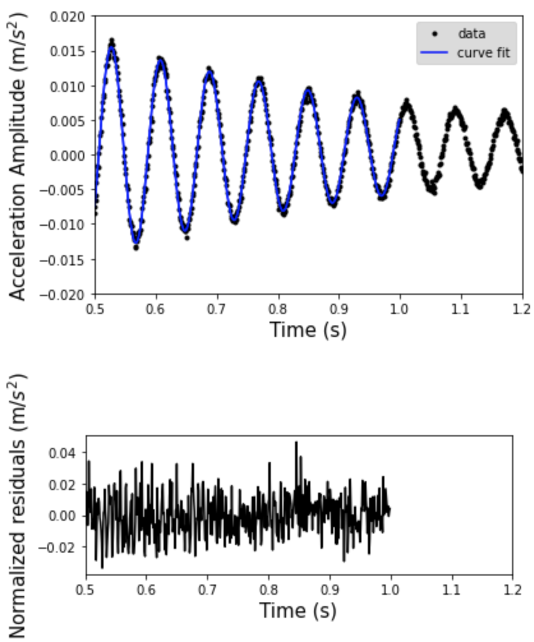
FIG. 2: a) Optimal fitting parameters for forced oscillation data. b) Optimal fitting parameters for transient data



(a) Low damping



(b) Medium damping



(c) High damping

FIG. 3: Free oscillation trials. Blue lines are fits to sinusoidal function and the black dots represent our experimental data. Bottom plot shows the normalized residuals of transient behaviour over time.