10. Probability distributions

These notes consider the Chapter 10 of the handbook on various probability distributions.

Discrete distributions

For random variables with countable number of possible values.

Binomial distribution

See page 200 for proper definition.

Useful when a random variable X has exactly two exclusive possible outcomes (e.g. success/fail) with known probabilities $p \in [0, 1]$ (success) and $q = 1 - p$ (fail). For $n \in \mathbb{N}$ trials with $k = 0, 1, 2, \ldots, n$ successes

$$X \sim Bin(n, p)$$

$$\mathrm{P}(X = k) = \binom{n}{k} p^k q^{n-k} \tag{1}$$

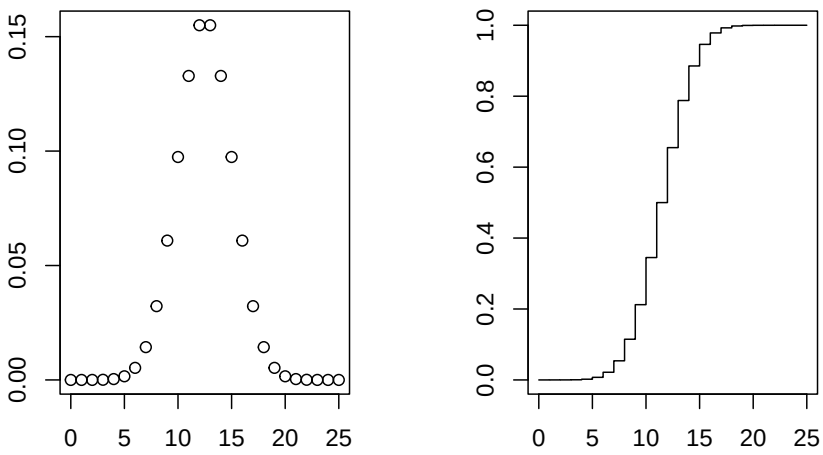
$$\mathrm{F}(k) = \mathrm{P}(X \leqslant k) = \sum_{i=0}^{k} \binom{n}{i} p^i q^{n-i} \tag{2}$$

In the following code examples, equal probabilities are assumed, $n =$ **size**, $p =$ **prob** $= 0.5$.

```
# Sequence for visualization
seq <- seq(0, 25, by=1)

# Functions
binomial_density_function <- dbinom(x = seq, size = 25, prob = 0.5)
binomial_distribution_function <- pbinom(q = seq, size = 25, prob = 0.5)

# Plot
par(mfrow=c(1,2))
plot(seq, binomial_density_function, ann = FALSE)
plot(seq, binomial_distribution_function, type = "S", ann = FALSE)
```




```r
# Probability for exactly 3 successes out of 10 trials
dbinom(x = 3, size = 10, prob = 0.5)
```

```
## [1] 0.1171875
```

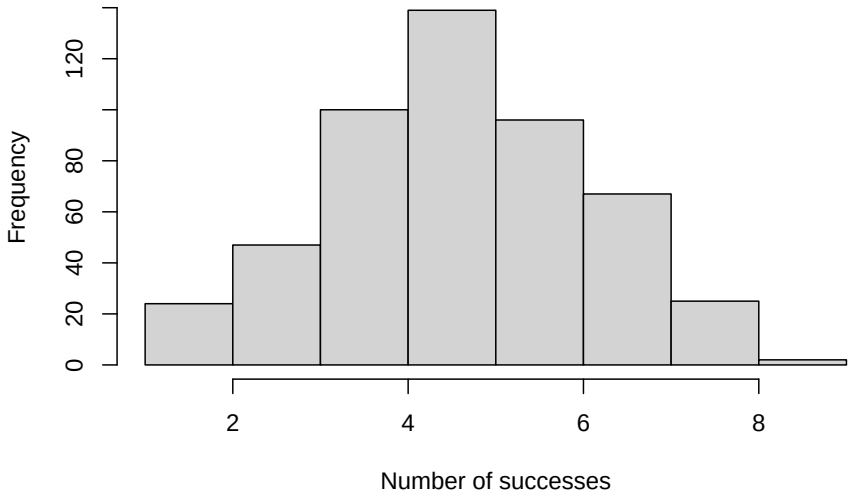
```r
# Probability for up to 3 successes out of 10 trials
pbinom(q = 3, size = 10, prob = 0.5)
```

```
## [1] 0.171875
```

```r
# Simulate 10 times how many successes there is using random numbers
rbinom(n = 10, size = 10, prob = 0.5)
```

```
##  [1] 6 4 2 6 5 7 6 2 4 4
```

```r
# With large enough n, expected value (np = 5) should become visible
rbinom500 <- rbinom(n = 500, size = 10, prob = 0.5)
hist(rbinom500, xlab = "Number of successes", ylab = "Frequency", main = NULL)
```




```
summary(rbinom500)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   1.000   4.000   5.000   5.084   6.000   9.000
```