Probability and Statistics: MA6.101

Tutorial 9

Topics Covered: CLT, Random Vectors

Q1: During each day, the probability that an athlete misses training due to illness is 5%, independent of every other day. Find the probability that the athlete will attend training on at least 45 out of the next 50 days using the Central Limit Theorem (CLT) and otherwise.

Q2: If 10 fair dice are rolled, find the approximate probability that the sum obtained is between 30 and 40. (Given $\Phi(\sqrt{\frac{6}{7}}) = 0.82$

Q3: The quadratic form of a random vector X is given by $X^T AX$. Find the expectation of the quadratic form.

Q4: Using the central limit theorem show that

$$\lim_{n\to\infty} e^{-n} \sum_{k=0}^{n} \frac{n^k}{k!} = \frac{1}{2}$$

Hint: Let S_n be Poisson with mean n. Use the central limit theorem to show that $P\{S_n \leq n\} \to \frac{1}{2}$.

Q5: Let $\mathbf{X} = [X_1, X_2]^T$ be a two-dimensional zero-mean Gaussian random vector with covariance matrix $\mathbf{C}$ given by:

$$\mathbf{C} = \begin{bmatrix} 1 & r \\ r & 2 \end{bmatrix}$$

1. Give an expression for $f_{X_2}(x_2)$. 2. Determine the conditional pdf, conditional mean, and conditional variance of X_1 given $X_2 = x_2$.

Q6: You are given the random vector $Y' = [Y_1, Y_2, Y_3, Y_4]$ with mean vector

$$\mu_Y = [5, -1, 4, -3]$$

and variance-covariance matrix

$$\Sigma_Y = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}.$$

Let

$$B = \begin{bmatrix} 2 & -1 & 0 & 1 \\ -1 & 2 & -1 & 0 \\ 0 & 1 & 1 & -2 \end{bmatrix}.$$

(a) Find $E(BY)$, the mean of BY.

(b) Find $\text{Cov}(BY)$, the variances and covariances of BY.

(c) Which pairs of linear combinations have zero covariances?