Probability and Statistics: MA6.101

Homework 2

Topics Covered: Conditional Probability, Mutual Independence and Exclusion, Principles of Counting (Permutations and Combinations)

Q1: A person has n boxes, where $n \geq 2$. We assume that either a box can have one or zero items (both cases have equal probability). What is the probability that all of the boxes contain an item for the following cases:

   (a) We pick one box at random and find that it contains an item.

   (b) We ask the person, "Do you have at least one box with a special item?" They respond, "Yes!" (Here, assume that if a box contains an item, it has a special item with probability $\alpha \ll 1$, independently of other boxes. If the box is empty, it does not have a special item.)

Q2: A coin is tossed n times, what is the probability of getting an odd number of heads if the probability of getting heads is p.

Q3: Harry Potter's closet contains 12 numbered brooms, of which 8 are Comet Two Sixty's (numbered 1 - 8) and 4 are Nimbus Two Thousand's (Numbered 9-12). Harry, Ron, George and Fred want to sneak out for a game of Quidditch in the middle of the night. They don't want to turn on the light in case Snape catches them. They reach in the closet and pull out a sample of 4 brooms. Give answers for both unordered and ordered samples.

   (a) How many different samples are possible?

   (b) How many samples have exactly one Comet Two Sixty in them?

   (c) How many samples have at least 3 Comet Two Sixty's?

   (d) Now, Ginny and Demelza wanna join so they grab 6 brooms, but Demelza believes that the both teams should have equal number of each type of broom for the game to be balanced. So, for any broom for a team we would need another broom of the same type for the other team. How many samples will keep Demelza happy?

Q4: The Celtics and the Lakers are set to play a playoff series of n basketball games, where n is odd. The Celtics have a probability p of winning any one game, independent of other games.

   (a) Find the values of p for which $n = 5$ is better for the Celtics than $n = 3$.

   (b) Generalize part (a). For any $k > 0$, find the values for p for which $n = 2k + 1$ is better for the Celtics than $n = 2k - 1$.

Q5: In a company, there are three teams: Team A, Team B, and Team C. Each team has different success probabilities for their projects:

   • Team A has a 70% chance of completing a project successfully.

- Team B has a 60% chance of completing a project successfully.
- Team C has a 50% chance of completing a project successfully.

Projects are assigned to these teams with the following probabilities:

- 40% of projects are assigned to Team A
- 35% of projects are assigned to Team B
- 25% of projects are assigned to Team C

Consider a project is completed successfully.

(a) What is the probability that the project was assigned to Team A given that it was completed successfully?

(b) If you have two projects, what is the probability that both projects are completed successfully?

(c) Calculate the probability that a project assigned to Team B is not completed successfully.

(d) Suppose the company has 12 projects to assign, and you want to assign exactly 5 projects to Team A, 4 to Team B, and 3 to Team C. How many different ways can you assign these projects?

(e) Out of these 12 projects, how many ways can you select 6 projects such that exactly 3 are from Team A and 3 are from Team B?