

# Probability and Statistics: MA6.101

## Tutorial 3

Topics Covered: Random Variables, Expectation, Functions on Random Variables, Discrete Random Variables.

Q1: Let  $X$  be a discrete random variable with the following PMF:

$$p_X(x) = \begin{cases} 0.25 & \text{for } x = 2, \\ 0.15 & \text{for } x = 4, \\ 0.35 & \text{for } x = 7, \\ 0.25 & \text{for } x = 9, \\ 0 & \text{otherwise.} \end{cases}$$

Find and plot the CDF of  $X$ .

Q2: The median of a random variable  $X$  is defined as any number  $m$  that satisfies both of the following conditions:

$$P(X \geq m) \geq \frac{1}{2} \quad \text{and} \quad P(X \leq m) \geq \frac{1}{2}.$$

Note that the median of  $X$  is not necessarily unique. Find the median of  $X$  if

(a) The PMF of  $X$  is given by

$$p_X(k) = \begin{cases} 0.4 & \text{for } k = 1, \\ 0.3 & \text{for } k = 2, \\ 0.3 & \text{for } k = 3, \\ 0 & \text{otherwise.} \end{cases}$$

(b)  $X$  is the result of rolling a fair die. (try this in HW)

(c)  $X \sim \text{Geometric}(p)$ , where  $0 < p < 1$ .

Q3: Consider a random variable  $X$  and another random variable  $Y$  defined as a function of  $X$ :

$$Y = \begin{cases} 2 & \text{if } X < 2, \\ X & \text{if } X \geq 2. \end{cases}$$

Express  $Y$  using the indicator functions of the events  $\{X < 2\}$  and  $\{X \geq 2\}$ .

Q4: We roll  $n$  dice and sum the highest 3. What is the probability that the sum is 18? Compute formula for general  $n$ , and give the value for  $n=5$ .

Q5: Two coins are simultaneously tossed until one of them comes up a head and the other a tail. The first coin comes up a head with probability  $p$  and the second with probability  $q$ . All tosses are assumed independent.

(a) Find the PMF, the expected value, and the variance of the number of tosses.

(b) What is the probability that the last toss of the first coin is a head?

Q6: Let  $X$  be a random variable that takes values from 0 to 9 with equal probability  $\frac{1}{10}$ .

(a) Find the PMF of the random variable  $Y = X \bmod 3$ .

(b) Find the PMF of the random variable  $Y = 5 \bmod (X + 1)$ .