# Probability and Statistics: MA6.101

# Homework 7

Topics Covered: Moment Generating Functions and Stochastic Simulation

Q1: Let $X_1, X_2, \ldots, X_n$ be $n$ i.i.d exponential random variables with parameter $\lambda$. Let $Z_{\min} = \min(X_1, X_2, \ldots, X_n)$ and $Z_{\max} = \max(X_1, X_2, \ldots, X_n)$.

Generate $Z_{\min}$ and $Z_{\max}$.

Q2: Let $X$ be a discrete random variable with the following moment-generating function:

$$M(t) = \frac{1}{10}e^{t} + \frac{2}{10}e^{2t} + \frac{3}{10}e^{3t} + \frac{4}{10}e^{4t}$$

for all $t$.

Determine the p.m.f of $X$.

Q3: Let $X$ and $Y$ be two independent random variables with respective moment generating functions

$$m_X(t) = \frac{1}{1-5t}, \quad \text{if } t < \frac{1}{5}, \quad m_Y(t) = \frac{1}{(1-5t)^2}, \quad \text{if } t < \frac{1}{5}.$$

Find $\mathbb{E}[(X+Y)^2]$.

Q4: True or False? If $X \sim \text{Exp}(\lambda_x)$ and $Y \sim \text{Exp}(\lambda_y)$, then $X + Y \sim \text{Exp}(\lambda_x + \lambda_y)$. Justify your answer.

Q5: Given an integral $I = \int_0^{2\pi} f(x)dx$, solve the following:

(a) Write the Monte Carlo Estimate, assuming $X_i'$s are sampled uniformly over the domain.

(b) Write the Monte Carlo Estimate, assuming $X_i'$s are sampled according to some PDF $g(X_i)$.

(c) Prove that the Monte Carlo Estimates from the previous questions compute the right answer on average.

Q6: Let $X \sim \mathit{Gamma}(\alpha, \beta)$. Find the MGF of $X$ and its region of convergence.
**Note:** The pdf of $\mathit{Gamma}(\alpha, \beta)$ is given by $f_X(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$ for $x \geq 0$ and $\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}dt$.