Matrix Chain Multiplication

$M_1 \; M_2 \; \ldots \; M_t \; \} \; n \times n$

M_{12} - - — .

$\overline{M_1 M_2}$

$k \times l \quad l \times s$

$O(kls)$

$M_i \leftarrow n_i \times n_{i+1}$

A	B	C
6×2	2×4	4×3

$(AB) \; C$

$(6 \times 2 \times 4) + (6 \times 4 \times 3)$

$48 + 72$

$A \; (BC)$

$6 \times 2 \times 3 + 2 \times 4 \times 3$

6×10

$M_1 \quad M_2 \quad \ldots \quad M_t$

$n_1 \times n_2 \quad n_2 \times n_3 \quad n_t \times n_{t+1}$.

$M_k \quad M_{k+1}$

MCM: Matrix Chain Mult.

MCP: Matrix Chain Product.

$MCM(1, n) \xrightarrow[k \in [1, t-1]]{} MCM(1, k) \cdot \text{merge} \cdot MCM(k+1, t)$

$\boxed{MCP(1, k) \cdot MCP(k+1, t)}$

$\downarrow$

$T(1, k) + T(k+1, t) + n_1 \times n_{k+1} \times n_{t+1}$.

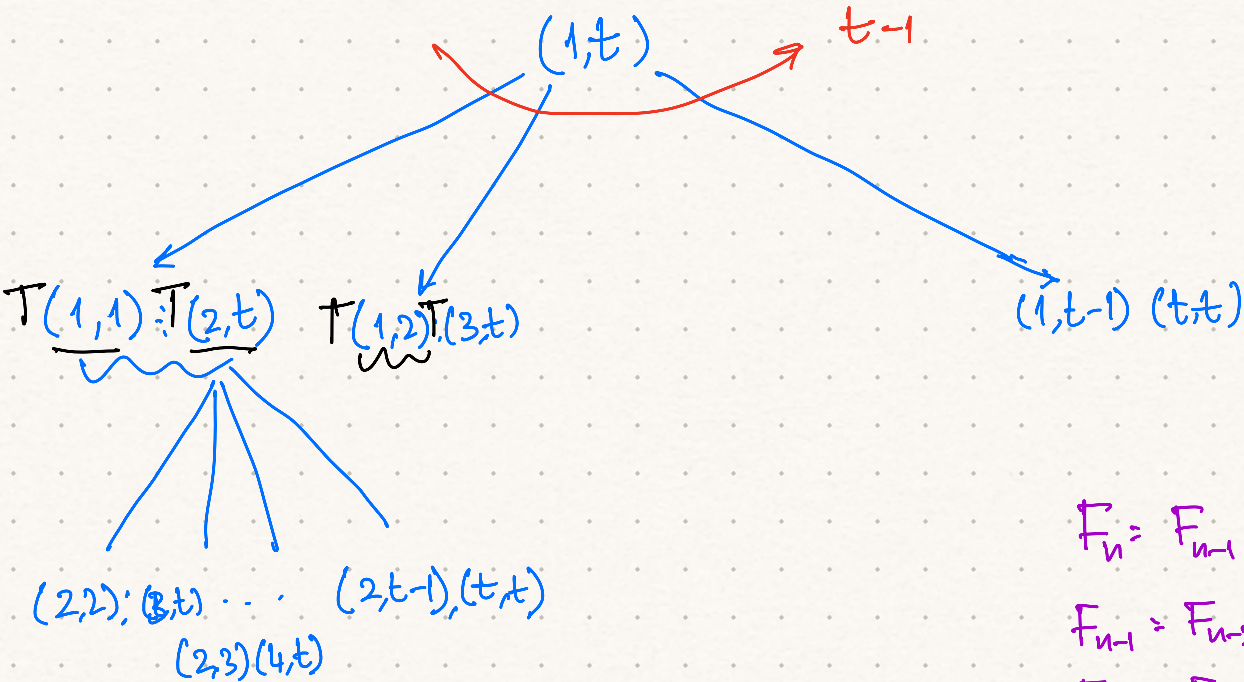
$$T(1, t) = \min_{k \in [1, t-1]} \{ T(1, k) + T(k+1, t) + n_1 \times n_{k+1} \times n_{t+1} \}.$$

A B C
1 2 3

$n_1 \times n_2$
$n_2 \times n_3$
$n_3 \times n_4$

$n_1 = 6,\ n_2 = 2,\ n_3 = 4,\ n_4 = 3.$

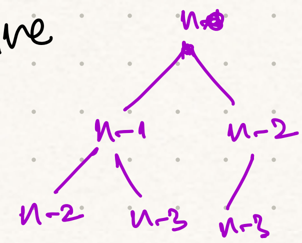
$$T(1,3) = \min \{ T(1,1) + T(2,3) + n_1 \times n_2 \times n_4 ;$$
$$T(1,2) + T(3,3) + n_1 \times n_3 \times n_4 \}$$



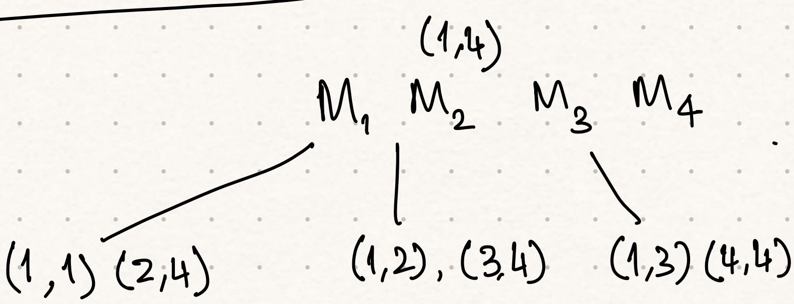
$F_1 = F_0 = 1$

$F_n = F_{n-1} + F_{n-2}$
$F_{n-1} = F_{n-2} + F_{n-3}$
$F_{n-2} = F_{n-3}$

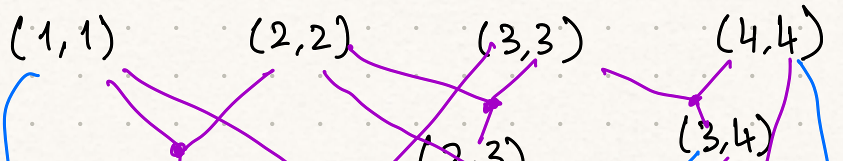
"Make good use of space to save time".

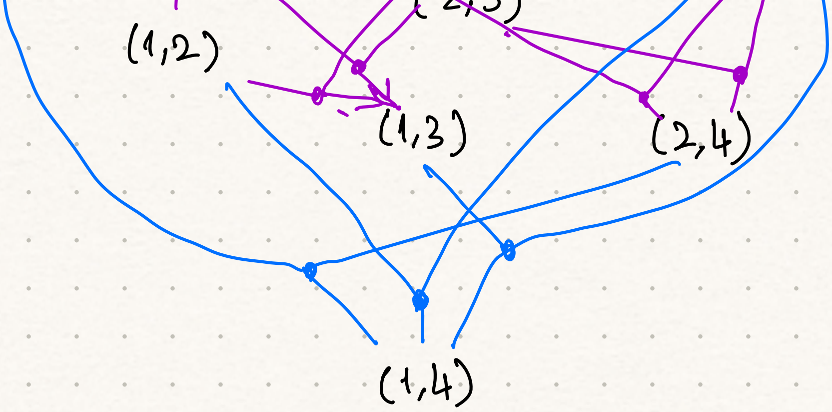


$\boxed{T(i,j) \qquad i \le j}$ # of such cases : $\binom{t}{2} + t$.



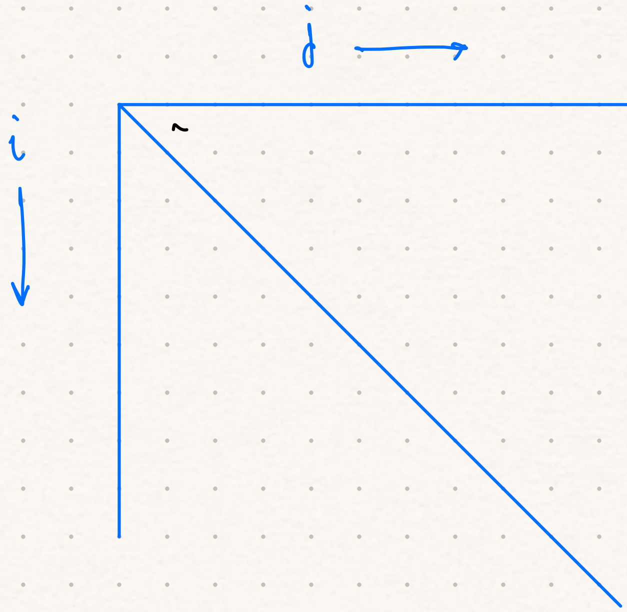
(1,1)
(1,2)
(1,3)
(2,2)
(2,3)
(2,4)
(3,3)
(3,4)
(4,4)





"We are building a look up table for computations for efficient reuse"

- Memoization.



$\left(\binom{t}{2} + t\right)$ entries

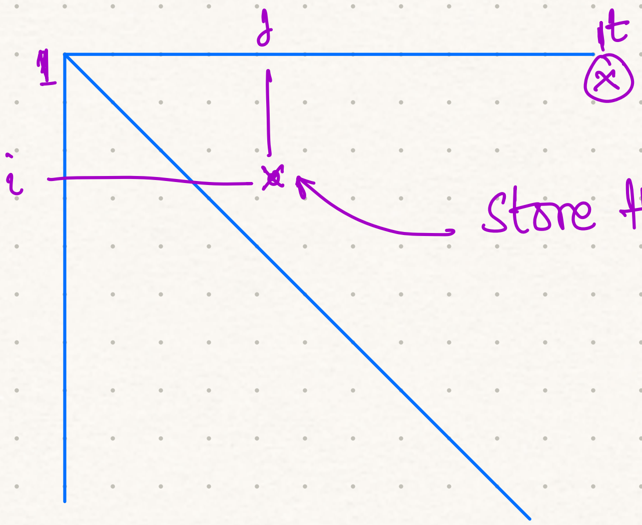
and for each entry we make at most $\underline{2t}$ lookups and $\leq t$ arithmetic computations

$\leq 2(j-i) \quad + \quad (j-i)$

$$T(i,j) = \min_{k \in [i,j-1]} \left\{ T(i,k) + T(k+1,j) + n_i \times n_{k+1} \times n_{j+1} \right\}.$$

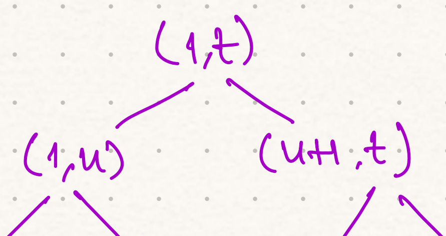
<u>Ref:</u> Aho, Hopcraft, Ullman.

Another matrix for "storing" the nesting structure.



(1,u) (u+1, t)

Store the $\underline{k}$ that gives us the min (i,k) & $(k+1, j)$



$(1, k_1)$ (k_1+1, n)