Probability and Statistics: MA6.101

Homework 1

Q1: Let $\mathcal{F}$ be a σ-algebra of subsets of Ω. Show that $\mathcal{F}$ is closed under countable intersections $\bigcap_n A_n$, under set differences $(A \setminus B)$, under symmetric differences $(A \Delta B)$. Will it be closed under countable union $\bigcup_n A_n$?

Q2: Player X has \$1 and Player Y has \$2. They play a game in which the loser gives \$1 to the winner. Player X is enough better than player Y that he wins $\frac{2}{3}$ of the time. They play until one of them gets bankrupt. What is the probability that Player x wins?

Q3: A wireless sensor grid consists of $21 \times 11 = 231$ sensor nodes that are located at points (i, j) in the plane such that $i \in \{0, 1, \ldots, 20\}$ and $j \in \{0, 1, \ldots, 10\}$ as shown in the figure below. The sensor node located at point $(0, 0)$ needs to send a message to a node located at $(20, 10)$. The messages are sent to the destination by going from each sensor to a neighboring sensor located above or to the right. That is, we assume that each node located at point (i, j) will only send messages to the nodes located at $(i + 1, j)$ or $(i, j + 1)$. How many different paths do exist for sending the message from node $(0, 0)$ to node $(20, 10)$?

BONUS: Give a recursive solution.

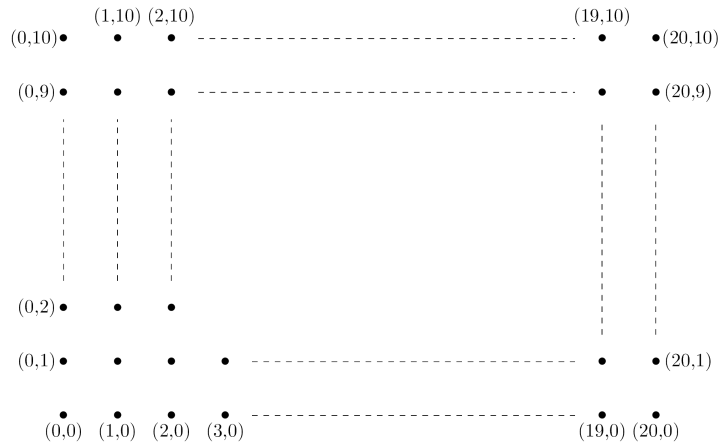

Figure 1: Wireless Sensor Grid

Q4: In Problem 3, assume that all the appropriate paths are equally likely. What is the probability that the sensor located at point (10,5) receives the message? That is, what is the probability that a randomly chosen path from (0,0) to (20,10) goes through the point (10,5)?

Q5: In Problem 3, given that the message has reached the node at $(10, 5)$, find the probability of the message reaching the top-right node passing through the node at $(14, 8)$.

Q6: 4 people are standing in a line, numbered 1,2,3,4 from left to right. A ball is initially given to 3. Each person passes the ball to their left and right neighbours with equal probability, and a person at the end always passes the ball back to their neighbour. A person wins if when they receive the ball for the first time, every other person has already received the ball atleast once. Find probability of winning for every person.