# Probability and Statistics: MA6.101

# Tutorial 4

Topics Covered: Continuous Random Variables, PDF, CDF, Joint random variables

Q1: Let $X$ be a continuous random variable with PDF

$$f_X(x) = \begin{cases} x^2\left(2x + \frac{3}{2}\right) & \text{for } 0 < x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

If $Y = \frac{2}{X} + 3$, find $\mathrm{Var}(Y)$.

Q2: An absent-minded professor schedules two student appointments for the same time. The appointment durations are independent and exponentially distributed with mean thirty minutes. The first student arrives on time, but the second student arrives five minutes late. What is the expected value of the time between the arrival of the first student and the departure of the second student?

Q3: Let $X$ be a random variable uniformly distributed in $[0, \frac{\pi}{2}]$. Let $Y = \sin(X)$. Calculate the probability density function (PDF) of $Y$.

Also, calculate the PDF of $Y$ if $X$ is uniformly distributed in $\left[-\frac{\pi}{2}, \pi\right]$.

Q4: Let $X_1, X_2, \ldots, X_n$ be $n$ independent exponential random variables with the same parameter $\lambda$. Let $Z_{\min} = \min(X_1, X_2, \ldots, X_n)$ and $Z_{\max} = \max(X_1, X_2, \ldots, X_n)$. Calculate the probability density functions of $Z_{\min}$ and $Z_{\max}$.

Q5: Let $X$ be a non-negative continuous random variable. Show that

$$E[X^2] = \int_{x=0}^{\infty} 2x\mathbb{P}(X > x)\, dx$$

Q6: Let $Y$ be Geometric($p$) where $p = \lambda h$. Define $X = Yh$ where $\lambda, h > 0$. Prove that for any $x \in (0, \infty)$, we have

$$\lim_{h \to 0} F_X(x) = 1 - e^{-\lambda x}$$