

Probability and Statistics: MA6.101

Tutorial 2

Topics Covered: Conditional Probability, Mutual Independence and Exclusion,
Principles of Counting (Permutations and Combinations)

- Q1: Suppose a student scores exactly 100 marks across three subjects, with the marks in each subject represented by s_1 , s_2 , and s_3 . Find the number of distinct ways the student can achieve this, given that at least one subject has a passing mark, defined as more than 40 marks ($s_i > 40$ for at least one i). The marks in each subject are non-negative integers.
- Q2: Consider a tetrahedral die with 3 of its sides painted red, blue and green and the fourth side has all the three colours. The colour of the side on which the die lands is to be considered. When the die lands on the fourth side, all 3 colours need to be considered.
- (a) Find the probability of getting red $P(R)$, green $P(G)$ and blue $P(B)$.
 - (b) Find the probability of getting red and blue $P(R \cap B)$, red and green $P(R \cap G)$ and blue and green $P(B \cap G)$.
 - (c) Is the collection of these 3 events (getting red, blue and green) pairwise independent? Mutually independent?
- Q3: A box contains 3 blue balls and some red balls. Your friend adds a blue ball or a red ball to the box with equal probability. After, your friend adds the ball, you randomly pick a ball from the box, and it is blue. What is the probability that the ball that your friend added was blue?
- Q4: Percy Jackson and his two close friends, Annabeth Chase and Grover Underwood, are part of a group of ten demigods preparing for a quest. To decide who will go on the first mission, Chiron randomly splits the group into two teams of five. What is the probability that Percy, Annabeth, and Grover will end up on the same team for this mission? (Assume, teams are not labeled)
- Q5: Consider two fair six-sided dice. Let event A be the first die showing a 2 or 3. Let event B be the sum of the dice being less than equal to 3. Are these events independent? Now, you are given the event C that the first dice does not show a 4. Are the events A and B conditionally independent?
- Q6: Kushal and Medha play a game where they toss coins and compete to see who can get the most heads. Kushal gets to toss 100 coins and Medha gets to toss 101 coins, but if they get the same number of heads, Kushal wins the game. What is the probability that Kushal wins?
- Q7: Consider a machine that produces a defective item with probability p and a non-defective item with probability $1 - p$. Suppose that items produced by the machine are selected at random and inspected one at a time until exactly five defective items have been obtained. Determine the probability p that exactly n items must be selected to obtain the five defectives.