

## Assignment 7

Deadline: 22<sup>nd</sup> February, 11:59pm

### Instructions:

- 1) This assignment consists of 2 problems. All problems are compulsory.
- 2) Mention all assumptions while answering the questions.
- 3) Be clear in your arguments. Vague arguments shall not be given full credit.
- 4) Only Handwritten Submissions are allowed. Scan and upload it on moodle.

### Problems:

1. Let  $V$  and  $W$  be vector spaces over the field  $F$  and let  $T$  be a linear transformation from  $V$  into  $W$ . Suppose that  $V$  is finite-dimensional. Then prove that:

$$\text{rank}(T) + \text{nullity}(T) = \dim V.$$

2. Let  $V$  be an  $n$ -dimensional vector space over the field  $F$  and let  $W$  be an  $m$ -dimensional vectorspace over  $F$ . Then the space  $L(V, W)$  is finite-dimensional and has dimension  $mn$ . Formally, prove that:

$$\dim L(V, W) = \dim(V) \times \dim(W)$$