Math 308 L Week 2 Conceptual Problems

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TOTAL POINTS

20 / 20

QUESTION 1

1 no answer needed **5 / 5**

  ✓ **+ 5 pts** Correct

QUESTION 2

2 5 / 5

  ✓ **+ 5 pts** Correct

QUESTION 3

3 5 / 5

  ✓ **+ 5 pts** Correct

QUESTION 4

4 5 / 5

  ✓ **+ 5 pts** Correct

2)

$$x = \begin{bmatrix} -1 \\ 1 \\ 0 \\ 6 \end{bmatrix} + S \begin{bmatrix} -1 \\ 2 \\ 1 \\ 0 \end{bmatrix}$$

$x_1 = -S - 1$ $x_1 + x_3 = -1$ $\begin{bmatrix} 1 & 0 & 1 & 0 & | & -1 \\ 0 & 1 & -2 & 0 & | & 1 \\ 0 & 0 & 0 & 1 & | & 6 \end{bmatrix}$

$x_2 = 2S + 1$ $x_2 - 12 x_3 = 1$

$x_3 = S$ $x_3 - x_3 = 0$

$x_4 = 6$ $x_4 = 6$

1 no answer needed **5 / 5**

✓ **+ 5 pts** Correct

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2)

$$x = \begin{bmatrix} -1 \\ 1 \\ 0 \\ 6 \end{bmatrix} + S \begin{bmatrix} -1 \\ 2 \\ 1 \\ 0 \end{bmatrix}$$

$x_1 = -S - 1$
$x_2 = 2S + 1$
$x_3 = S$
$x_4 = 6$

$x_1 + x_3 = -1$
$x_2 - 2x_3 = 1$
$x_3 - x_3 = 0$
$x_4 = 6$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & | & -1 \\ 0 & 1 & -2 & 0 & | & 1 \\ 0 & 0 & 0 & 1 & | & 6 \end{bmatrix}$$

✓ **+ 5 pts Correct**

3)

$$\begin{bmatrix} 2 & 11 & -4 & | & b_1 \\ -1 & 2 & z_1 & | & b_2 \\ 3 & 2 & z_2 & | & b_3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & \frac{1}{2} & -2 & | & b_1/2 \\ 0 & \frac{5}{2} & z_1-2 & | & b_2+\frac{b_1}{2} \\ 0 & -\frac{1}{6} & -2-\frac{z_2}{3} & | & \frac{b_1}{2}-\frac{b_3}{3} \end{bmatrix}$$

$$\begin{bmatrix} 1 & \frac{1}{2} & -2 & | & \frac{b_1}{2} \\ 0 & 5 & 2z_1-4 & | & 2b_2+b_1 \\ 0 & -1 & -12-2z_2 & | & 3b_1-2b_3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & \frac{1}{2} & -2 & | & \frac{b_1}{2} \\ 0 & 1 & 12+2z_2 & | & 2b_3-3b_1 \\ 0 & 5 & 2z_1-4 & | & 2b_2+b_1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & \frac{1}{2} & -2 & | & \frac{b_1}{2} \\ 0 & 1 & 12+2z_2 & | & 2b_3-3b_1 \\ 0 & 0 & & | & \end{bmatrix}$$

$x_3(64+10z_2-2z_1)=10b_3-2b_2-16b_1$

$x_3 = \dfrac{10b_3-2b_2-16b_1}{64+10z_2-2z_1}$

$0 = 64+10z_2-2z_1$

$-64 = 10z_2-2z_1$

$-32 = 5z_2-z_1 \qquad z_1 = S$

$-32+S = 5z_2$

$z_2 = \dfrac{S-32}{5}$

$\boxed{(z_1,z_2)=\left\{\left(s,\ \dfrac{S-32}{5}\right)\right\}}$

4

$-\dfrac{2}{3} \qquad -\dfrac{z_2}{3} \qquad -\dfrac{b_3}{3}$

$\dfrac{1}{2} - \dfrac{2}{3} = 3-4 = -\dfrac{1}{6}$

$-\dfrac{z_2}{3}$

$-\dfrac{1}{6} \qquad \dfrac{-6-z_2}{3}$

$12+2z_2 - \left(\dfrac{2z_1-4}{5}\right)$

$\dfrac{60+10z_2-2z_1+4}{5}$

$\dfrac{64+10z_2-2z_1}{5}$

$2b_3-3b_1 - \left(\dfrac{2b_2+b_1}{5}\right)$

$\dfrac{10b_3-15b_1-2b_2+b_1}{5}$

$\dfrac{10b_3-2b_2-16b_1}{5}$

3 5 / 5

✓ **+ 5 pts** Correct

4)

a)

$$P = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} ; \ 2x_1 - x_2 + 4x_3 = 0 \right\} \qquad x_2 = 2x_1 + 4x_3$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ 2x_1 + 4x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ 2x_1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 4x_3 \\ x_3 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix}$$

$$u_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \qquad u_2 \begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix} \qquad P = \text{span}\{u_1, u_2\} \ \text{where } x_1, x_3 \text{ are scalars}$$

b)

$$\begin{bmatrix} 2 & 3 & -5 & | & b_1 \\ 7 & 2 & 8 & | & b_2 \\ -1 & 1 & -5 & | & b_3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -1 & 5 & | & -b_3 \\ 7 & 2 & 8 & | & b_2 \\ 2 & 3 & -5 & | & b_1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -1 & 5 & | & -b_3 \\ 0 & 9 & -27 & | & b_2 + 7b_3 \\ 0 & 5 & -15 & | & b_1 + 2b_3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 5 & | & -b_3 \\ 0 & 1 & -3 & | & (b_2 + 7b_3)/9 \\ 0 & 0 & 0 & | & \dfrac{9b_1 - 5b_2 - 17b_3}{9} \end{bmatrix} \qquad \dfrac{9b_1 - 5b_2 - 17b_3}{9} = 0 \Rightarrow 9b_1 - 5b_2 - 17b_3 = 0$$

$\mathbb{R}^3 \qquad ax_1 + bx_2 + cx_3 = d$

$$\left. \begin{array}{l} 2x_1 + 7x_2 - x_3 = d \\ 3x_1 + 2x_2 + x_3 = d \\ -5x_1 + 8x_2 - 5x_3 = d \end{array} \right\} \to \begin{array}{l} 5x_1 + 9x_2 = 2d \\ 10x_1 + 18x_2 = 6d \end{array} \Rightarrow 10x_1 + 18x_2 = 4d$$

$$d = 0 \qquad b = \tfrac{-5}{9} d$$

$$2a + 7\left(\tfrac{-5}{9}a\right) - c = 0 \qquad\qquad ax_1 + \left(\tfrac{-5}{9}a\right)x_2 + \left(\tfrac{-17}{9}a\right)x_3 = 0$$

$$c = \dfrac{18a - 35a}{9} = \dfrac{-17a}{9} \qquad\qquad \tfrac{a}{9}\left(9x_1 - 5x_2 - 17x_3\right) = 0$$

$$\boxed{9x_1 - 5x_2 - 17x_3 = 0}$$

4 5 / 5

✓ **+ 5 pts** Correct