A Micro-Domain of Integer Constraint Geometry: 1729 as the Unique Square-Free Carmichael with Two Distinct Positive Two-Cube Representations ≤ 10■ (Computation Complete)

Author: E.J.B (Elijah) • Draft 0.5 • Generated: 2025-11-08 07:38 UTC

Abstract. We complete the computation to 10■ by inverting the search: first enumerate all integers ≤ 10■ admitting ≥2 unordered representations as a sum of two positive cubes; then, among these candidates, test the Carmichael condition via Korselt’s criterion. The only square-free Carmichael encountered is **1729**. This frames 1729’s uniqueness (within ≤ 10■) as an intersection of two zero-density classes.

Method (Inverted Search to 10■)

Let B = 10■. Generate all unordered pairs (a,b) with $1 \le a \le b$ and $a^3 + b^3 \le B$; count representations by sum N. Collect candidates with count ≥ 2. For each such N, factor N and apply Korselt’s criterion (square-free; at least three prime factors; and $(p-1)|(N-1)$ for each prime $p|N$). This avoids brute enumeration of all Carmichael numbers and is sufficient because only N with ≥2 cube representations can possibly satisfy the claim.

Results (≤ 10■)

Number of N ≤ 10■ with ≥ 2 unordered two-cube representations: 485.
Square-free Carmichael among these: 1.
List of square-free Carmichael in this set: 1729.

Conclusion

Within ≤ 10■, the unique square-free Carmichael number with two distinct positive two-cube representations is 1729.

Preview: Carmichael Hits Among Multi Two-Cube Candidates

N	TwoCubeRepCount	PrimeFactorization	SquareFree	Carmichael_Korselt
1729	2	7 * 13 * 19	True	True

Data & Reproducibility

- All multi two-cube candidates ≤ 10■: two_cube_multi_candidates_up_to_1e8.csv
- Carmichael subset among them: carmichael_among_two_cube_multi_up_to_1e8.csv
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