

This report provides the investigation results of a Principal Component Analysis (PCA) on a data set regarding the State of the Tropics report. The dataset contains 109 observations and 14 variables where the observations are the value for the countries in the tropics region, and the variables are the social, demographic, and economic measurements of those countries.

In this investigation, PCA was performed to simplify the complexity of high-dimensional data while retaining trends and patterns. Using Singular Value Decomposition, PCA can transform the data into fewer dimensions that serve as feature summaries.

Before performing the SVD, removal of all the string values and missing values is necessary. The dataset was imported as a matrix in MATLAB and then cleaned, with the size of 56 x 13. We will then centre and scale the matrix and perform a correlation calculation between the variables to determine the relationships and their strengths. According to the heatmap in Figure 1., there were some strong negative relationships between life expectancy and mortality under the age of 5, poverty under \$1.25 and poverty under \$2 per day. It is expected for those relationships as life expectancy will decrease when the mortality of young children increases. The poverty rate under the two amounts will be highly positively related as they measure the same rate at a different level only.

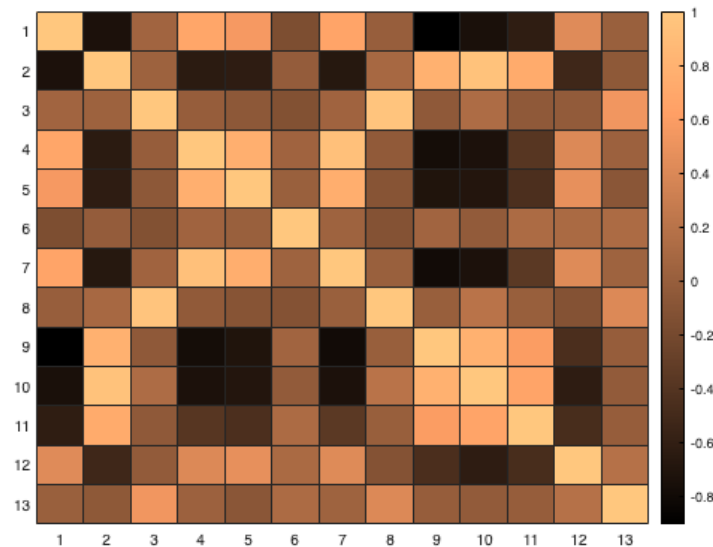


Figure 1 The heatmap that represents the correlation matrix between the variables in the dataset.

After performing the SVD, we will find three matrixes for further calculations. To identify the relationships between the variables described by the first principal component vector, we look at column one of the matrix of eigenvectors (V). We can get the results in table 1. Moreover, to identify relationships between the variables described by the second principal component vector, we look at column two of the matrix of eigenvectors, listed in table 2.

Table 1. The relationships between the variables described by PC1

Youth literacy (% age 15-24)	0.3427
Adult literacy (% above 15)	0.3425
Life Expectancy (years)	0.3421

Mean years of schooling	0.3246
Urban population (%)	0.2493
Area of agricultural land (??)	0.0129
Unemployment (%)	-0.0042
Population under 15	-0.0095
Tuberculosis (cases)	-0.0354
Undernourishment	-0.2771
Poverty under \$1.25 per day	-0.3579
Under 5 mortality (under age 5 per 1000 births)	-0.3672
Poverty under \$2 per day	-0.3734

Table 2. The relationships between the variables described by PC2

Unemployment (%)	0.0839
Mean years of schooling	0.0453
Under 5 mortality (under age 5 per 1000 births)	0.0451
Undernourishment	0.0355
Urban population (%)	0.0046
Poverty under \$1.25 per day	-0.0097
Adult literacy (% above 15)	-0.0146
Youth literacy (% age 15-24)	-0.0494
Life Expectancy (years)	-0.0551
Poverty under \$2 per day	-0.0694
Area of agricultural land	-0.4329
Tuberculosis (cases)	-0.6156
Population under 15	-0.6408

The singular value can be obtained by the diagonal value from the S matrix from SVD, and the proportions of variation will be calculated by the squared values and then computed as the ratio of each component to the total sum of the values. We then calculate the cumulative total of the variations. All three values are displayed in table 3. The proportions have multiplied one hundred to be presented as percentages.

Table 3. The singular values and proportions of variation

PC	Singular Values	Percentage of Variation (%)	Cumulative proportion of variation (%)
1	18.7152	48.1127	48.1127
2	11.4672	18.0626	66.1753
3	8.2715	9.3981	75.5734
4	7.5886	7.9103	83.4837
5	5.8351	4.677	88.1607
6	5.4528	4.0842	92.2448
7	4.5567	2.8522	95.097

8	3.9134	2.1037	97.2007
9	3.3564	1.5475	98.7482
10	2.1671	0.6451	99.3933
11	1.6564	0.3769	99.7702
12	1.0591	0.1541	99.9242
13	0.7427	0.0758	100

After considering the total proportion of variation and the residuals, the cut-off at PC4 with a total of 83.5% of variation would be optimal for this dataset, shown in Figure 2.

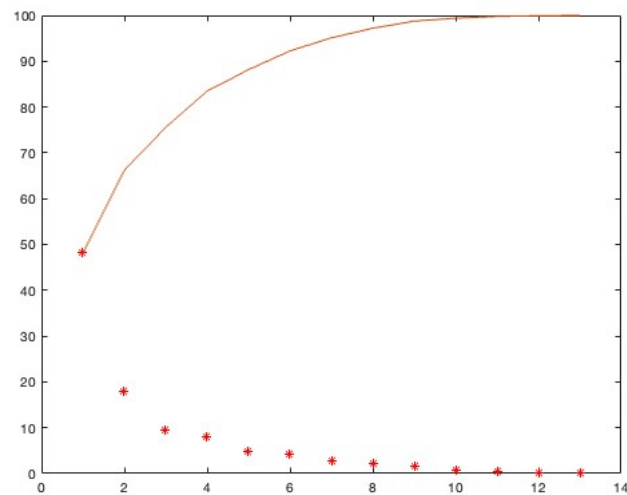


Figure 2 Scree Plot

We can then calculate the matrix of scores,  $T = U \cdot S$ , the results was displayed in table 4. We are using the first four principal components to compute a dimensionally reduced matrix of the data set, shown in table 5.

Next, to identify any outliers, residuals will be calculated for all 56 countries. A plot of the residuals was produced in Figure 3.

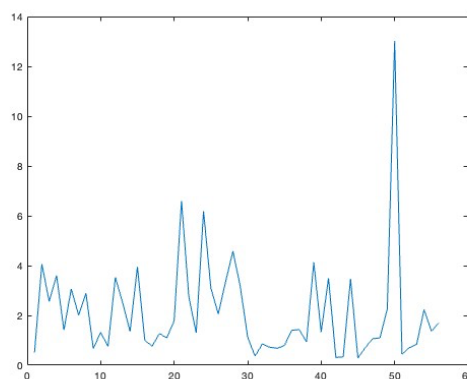


Figure 3 Residual Plot

The outlier countries were found using the matrix of scores. The countries that the relationships the PC1 identifies are not present are Malawi, Sri Lanka, and Haiti. The country outlier identified by PC2 is the Democratic Republic of the Congo.

Table 4. Matrix of scores (T)

-2.9141	0.6164	1.4235	-2.2146	-0.1363	-0.4331	0.0575	-0.5273	0.0846	-0.0421	-0.0563	-0.076	0.0357
0.6759	0.7166	-2.0956	0.6757	0.2691	-0.9924	-1.148	0.3857	0.4648	-0.9726	-0.6079	0.0416	0.003
-4.9135	0.7403	-0.2045	2.477	0.9927	0.485	0.6469	0.8086	0.306	0.3768	0.0926	0.0887	0.131
-0.5783	0.4307	0.1306	-0.7961	0.2907	-0.7735	-1.3219	1.0342	-0.1437	0.1378	0.22	-0.1326	0.02
-4.0746	0.7898	-0.0492	-0.4797	0.4501	-0.8411	-0.5104	0.1408	-0.3827	0.2774	0.0289	0.0314	0.0102
-4.9781	-0.7145	-0.3862	0.8557	1.0409	-0.8379	-0.0202	0.1752	-0.3018	1.066	-0.0874	0.0174	-0.0995
-2.1105	0.3948	0.1234	-1.4591	0.383	-0.5129	-0.8494	0.8012	0.2969	-0.1661	-0.3374	-0.0575	0.0334
2.1833	0.8897	-1.606	-0.7862	-0.1898	-1.3957	-0.3282	0.7251	-0.4962	0.0792	0.139	0.0643	-0.0371
-1.9972	0.8198	0.335	-1.9177	-0.2636	-0.5067	0.3065	0.459	-0.0422	-0.1671	0.0184	0.0089	-0.0213
-0.0372	0.3201	0.6147	-1.0349	0.1174	-0.6292	-0.8072	-0.4305	-0.0542	-0.2104	0.165	-0.0064	0.0683
-0.8795	-0.2913	-0.3779	1.6324	0.1044	0.0626	-0.7493	-0.2655	-0.2859	-0.0066	0.0295	-0.1577	0.0016
-3.0295	0.5947	0.505	0.2753	0.8936	-0.652	1.0799	-0.8488	-0.4944	-0.3451	0.0867	0.2187	-0.0557
-2.7977	0.4584	0.1848	1.0626	-0.111	-0.0935	-0.7058	-0.3682	-1.2993	-0.1282	0.3212	0.1555	0.0926
-4.1873	-0.4299	-0.3855	-0.2065	0.6509	0.5046	0.0936	0.4965	-0.4453	-0.3364	-0.1388	0.0639	-0.2992
-0.0896	0.608	-4.0739	1.397	-1.8698	0.3811	0.3612	-0.3611	-0.003	-0.0961	-0.1732	-0.0574	0.0005
-2.6078	0.4802	1.3098	0.7204	0.283	0.463	0.3527	-0.3964	-0.6047	-0.1023	-0.1858	0.0843	0.0955
-2.5783	-0.3087	-0.7182	1.1169	0.178	0.242	0.2036	-0.7245	0.0504	-0.0363	-0.2632	-0.0499	0.1834
-1.5883	0.742	0.3022	-0.5641	-0.187	-0.4194	-0.7476	0.1699	-0.0685	-0.3912	0.5584	0.0437	0.0184
-1.8491	0.0223	0.4929	1.1081	0.0744	0.4846	-0.6621	0.5566	-0.0658	-0.225	0.1407	-0.0603	0.1775
-3.1282	0.402	-1.5821	1.3974	0.5162	-0.7924	0.0611	-0.7909	0.3922	0.2382	-0.2135	0.0257	-0.0555
-1.5352	0.6131	-2.9811	-1.556	-2.4577	0.2524	0.0247	-0.5047	-0.1133	0.3785	0.2737	-0.0118	-0.0474
-4.5333	0.1321	0.8115	-2.7161	-0.7242	0.909	-0.5204	-0.5848	0.4157	0.6386	-0.4189	-0.1232	0.0372
-1.7725	0.5883	0.1173	-1.2223	-0.6825	-0.0476	0.3739	-0.2247	0.7955	-0.0981	0.0082	-0.0291	0.0328
-1.2355	-0.9874	-1.3022	-0.1901	0.9765	2.1282	-0.2151	0.7179	0.2253	-0.0859	0.2151	-0.1412	-0.128
-1.293	0.3353	-0.7719	-0.0964	-0.7929	0.9991	0.4582	1.0064	0.1153	-0.2985	0.3276	-0.065	0.1608
-1.2805	-0.1915	1.1965	0.0865	-0.7261	0.2266	0.6431	-0.8217	0.0582	-0.4653	0.4018	-0.0632	-0.0535
-1.2111	-9.7036	0.6847	0.1855	-1.2236	-1.1802	0.1081	0.4409	0.4844	0.0052	0.0378	0.1506	0.0193
2.4203	0.7129	-0.0998	0.1297	-1.4463	0.956	0.5516	0.5535	-0.9269	0.1564	-0.2092	0.1834	0.0061
2.1783	0.5542	0.9184	1.8767	-0.4126	0.6417	-0.8361	-0.566	1.2247	0.1073	0.1739	-0.0744	0.0482
0.0509	0.2903	1.4098	0.2983	-0.4671	0.7775	-0.3005	0.0208	0.1594	0.0371	0.3566	-0.2049	-0.1443
2.4684	-0.7071	0.5876	0.4186	0.3328	-0.0753	0.2776	-0.1836	-0.235	0.129	-0.1006	-0.14	0.1912
1.3585	-3.1258	0.0985	0.3713	-0.4079	-0.1787	0.1983	-0.0346	-0.7376	-0.1729	-0.0543	-0.1935	0.0043
-0.6287	0.5314	1.1212	0.4219	0.1864	0.5353	0.411	0.2384	0.0469	-0.1865	-0.1275	-0.3496	0.0389
3.5616	0.3589	0.5957	-0.2776	0.3885	-0.6215	-0.0528	-0.1982	0.2534	-0.022	-0.1212	0.1473	-0.0235
1.6575	-1.0122	0.2078	1.1639	-0.2033	-0.7229	-0.0977	-0.2915	0.0183	-0.053	-0.2011	-0.2645	-0.1229
2.4847	-0.2143	1.2372	0.2395	-0.2444	0.9477	-0.3627	0.3271	-0.1993	0.0168	-0.2523	0.292	-0.1386
1.531	-0.6545	1.3903	0.6326	-0.6006	0.7002	0.3052	-0.0679	-0.6419	-0.0219	-0.0708	-0.196	-0.1638
2.3801	0.7227	-0.7877	-0.0906	-0.4359	-0.3435	0.6238	0.4658	0.0344	0.1395	0.0385	0.0077	0.0169
-2.7707	0.6756	-0.1662	0.2033	0.5245	-0.6208	1.4265	-0.4196	1.0123	-0.3925	0.2058	0.1796	-0.0673
2.721	0.8471	-0.2628	0.1577	-0.7154	-0.3516	-0.3792	-0.2706	0.5996	0.2003	0.2032	0.1837	-0.0532
2.4412	0.7533	1.0319	1.2356	-0.7087	0.7341	-1.4796	-0.0351	0.1388	0.3112	-0.0812	0.3624	-0.0298
3.3584	0.6192	0.6595	-0.0306	0.0812	-0.1217	0.4047	-0.1197	0.017	0.3256	-0.0114	0.0701	-0.077
2.2136	0.6722	0.2569	-0.1398	0.0821	-0.3606	0.3395	0.116	0.154	-0.1606	0.061	0.1132	-0.0421
0.7366	0.5302	0.6878	-0.2216	0.1236	0.704	0.8024	1.3793	0.5924	0.0227	-0.1639	0.1761	0.0377
1.5286	0.5271	0.8901	-0.022	0.1024	0.0275	0.2925	-0.1311	-0.3664	-0.0626	0.1895	0.0515	-0.0274
3.3368	-0.6138	-0.0222	-0.8469	0.6361	0.0575	0.0319	-0.3422	-0.0111	0.1896	0.103	0.1326	0.3337
1.4449	0.5876	0.5524	-0.1989	0.0868	0.2595	0.7934	0.3671	0.2861	0.0783	-0.307	-0.2109	0.0116
3.1555	0.6741	0.1617	0.0361	0.3724	-0.7164	0.4163	-0.3242	0.2337	0.2916	0.097	-0.1174	-0.065
1.8396	0.2428	-0.4833	0.47	1.3133	-0.3859	-0.448	0.0767	0.3146	0.1262	0.1945	-0.0948	-0.1165
2.8297	-3.1061	-2.4864	-2.0733	2.6145	2.2134	-0.4869	-1.0046	-0.0975	-0.1001	0.0733	0.1101	-0.012
2.6051	-0.0903	-0.9874	-0.3046	0.3562	-0.022	0.5172	-0.0941	-0.15	0.0733	-0.024	-0.0243	0.0977
2.6104	0.5021	0.1439	0.1772	0.3616	-0.1389	0.5816	0.3605	0.2101	0.1977	0.0787	-0.0109	-0.0143
2.786	0.1956	-0.1288	-0.3185	0.5975	-0.5778	0.2349	-0.0526	0.201	0.1417	0.1554	-0.065	-0.0446
3.0622	0.2161	-0.4498	-1.0006	0.4856	-0.5852	0.9765	0.2542	-0.7682	-0.0213	-0.1939	-0.0518	0.0532
2.8293	0.6845	0.6626	0.3614	-0.0526	-0.4832	-0.7364	-0.62	-0.3462	-0.1299	-0.1951	-0.1658	0.0044
-1.8502	0.5587	1.5635	-0.4197	-0.8069	0.7212	-0.1906	-0.4729	0.0952	-0.2454	-0.3992	0.1903	-0.0254

Table 5. The reduced matrix from only the first four components.

-0.6763	-0.5367	0.3814	-2.0382	-1.5636	-	1.2012	-2.0653	-	0.4081	1.1666	-0.7469	0.5947	-0.2143	-	0.3459
-0.2662	-0.0895	-0.5428	0.6776	0.5227		1.7638	0.6616	-0.5672	-0.0637	-0.2956	0.6114	0.4784	0.3845		
-1.8262	2.3056	-0.3438	-0.6592	-0.9013		0.3947	-0.6861	-0.0154	1.6163	2.1874	2.6975	-2.1775	-0.8997		
-0.1744	0.0245	-0.2942	-0.5424	-0.3833		-0.1133	-0.5577	-0.3247	0.2962	0.0594	-0.2587	0.1469	-0.0544		
-1.4359	1.3438	-0.4879	-1.5999	-1.4108		0.0934	-1.6277	-0.4011	1.584	1.3836	0.9215	-0.8025	-0.2584		
-1.7662	1.9814	0.5188	-1.3102	-1.4062		0.3194	-1.2849	0.6811	1.7522	2.0318	1.8708	-1.4869	0.1949		
-0.6816	0.4252	-0.2821	-1.3383	-1.0563		-0.1442	-1.3523	-0.3249	0.9229	0.5225	-0.1734	0.0378	0.1047		
0.3793	-0.9592	-0.6994	0.5493	0.6138		1.2947	0.5261	-0.8527	-0.5172	-1.0902	-0.6484	1.3166	0.4839		
-0.6104	0.2772	-0.5625	-1.5122	-1.1303		-0.3049	-1.5416	-0.625	0.9234	0.3848	-0.4676	0.1967	-0.0527		
0.1247	-0.2234	-0.2138	-0.4949	-0.297		-0.5241	-0.5083	-0.2616	0.0655	-0.1503	-0.6335	0.2443	-0.138		
-0.4049	0.6838	0.2384	0.4056	0.1472		0.3781	0.4165	0.3652	0.1893	0.6027	1.1462	-0.7829	-0.1267		
-0.9697	1.138	-0.317	-0.9765	-0.9066		-0.3223	-1	-0.184	1.0597	1.1616	0.8985	-1.0056	-0.5677		
-0.9698	1.2336	-0.2178	-0.5445	-0.6164		-0.0326	-0.5622	-0.0455	0.9247	1.2001	1.2978	-1.1813	-0.5663		
-1.4849	1.4583	0.2886	-1.4797	-1.4159		0.2754	-1.4637	0.3562	1.5786	1.539	1.1179	-0.8543	0.3384		
-0.9565	0.3547	-0.5355	0.8823	0.5469		3.374	0.8783	-0.5601	0.348	0.0179	1.5924	0.5425	1.0491		
-0.6607	1.0844	-0.1914	-0.7164	-0.6937		-0.9465	-0.7396	-0.0085	0.7722	1.1276	0.8413	-1.3069	-0.9465		
-1.044	1.1782	0.2295	-0.3586	-0.5254		0.6231	-0.3457	0.3478	0.8983	1.1345	1.4217	-0.9011	0.1215		
-0.5067	0.4341	-0.4669	-0.8124	-0.6437		-0.2054	-0.8395	-0.4393	0.6405	0.4625	0.116	-0.246	-0.3278		
-0.5575	0.9071	0.0694	-0.2208	-0.3299		-0.3154	-0.2246	0.2209	0.5201	0.8994	0.9799	-1.0511	-0.5012		
-1.4601	1.4343	-0.2524	-0.3683	-0.5596		1.3887	-0.3766	-0.1121	1.1948	1.2935	1.9191	-0.9093	0.0891		
-1.1481	0.2081	-0.5833	-0.9189	-0.747		2.3336	-0.9267	-0.7567	1.0546	0.1184	0.2585	1.0854	1.2993		
-1.32	1.0114	-0.1046	-2.7393	-2.2158		-0.7789	-2.747	-0.1625	1.8522	1.2699	-0.2952	-0.2449	0.2123		
-0.5838	0.3552	-0.4006	-1.1276	-0.8753		-0.1102	-1.1483	-0.4294	0.785	0.4223	-0.1377	0.0279	-0.0295		
-0.6378	0.4146	0.5735	-0.3755	-0.4378		0.9459	-0.3355	0.5182	0.5635	0.4317	0.4755	0.1274	0.9853		
-0.6209	0.4411	-0.2441	-0.4214	-0.3955		0.6413	-0.4298	-0.2378	0.5798	0.4041	0.4782	-0.0667	0.1743		
-0.1778	0.4747	0.1969	-0.5025	-0.4548		-0.9566	-0.5014	0.2757	0.3284	0.5667	0.1483	-0.6883	-0.4394		
0.2596	0.5661	6.2702	-0.2551	-0.8143		-1.3422	0.0793	6.0962	-0.0826	1.1913	-0.0534	-0.6128	3.863		
0.7645	-0.8438	-0.4799	0.8804	0.8565		0.1369	0.8563	-0.5183	-0.8589	-0.9365	-0.5586	0.5815	-0.2684		
0.8616	-0.3696	-0.2598	1.4301	1.1847		-0.5794	1.4063	-0.1246	-1.0545	-0.4908	0.1896	-0.4771	-1.0379		
0.2909	0.0399	-0.1061	0.0152	0.0454		-1.0785	-0.0012	-0.0255	-0.1817	0.0825	-0.1386	-0.5001	-0.7686		
0.9968	-0.7856	0.4743	0.9775	0.8532		-0.5115	0.9997	0.4462	-1.0406	-0.7726	-0.6146	0.2776	-0.0015		
0.6485	-0.3734	2.009	0.6551	0.3926		-0.3239	0.7635	1.9271	-0.6865	-0.2231	-0.3174	0.1451	1.241		
-0.0187	0.3096	-0.2636	-0.1457	-0.119		-0.8186	-0.1694	-0.1605	0.0966	0.3256	0.1804	-0.6387	-0.7946		
1.331	-1.3427	-0.2452	1.0498	1.073		-0.475	1.0354	-0.3278	-1.3265	-1.3708	-1.2377	0.8374	-0.2839		
0.6377	-0.3243	0.6871	1.0427	0.7889		-0.1868	1.0767	0.7156	-0.79	-0.3432	0.0583	-0.125	0.0939		
1.1164	-0.8388	0.1834	0.8468	0.8048		-0.9974	0.8491	0.1774	-1.0745	-0.8098	-0.8258	0.1765	-0.4346		
0.8369	-0.406	0.4969	0.6749	0.5718		-1.1282	0.6916	0.5403	-0.7986	-0.3495	-0.4071	-0.2666	-0.4134		
0.6109	-0.876	-0.5278	0.8348	0.8166		0.6714	0.8135	-0.607	-0.7504	-0.994	-0.5196	0.8529	0.0589		
-1.0251	1.0313	-0.407	-0.8612	-0.808		0.2126	-0.8846	-0.3087	1.0453	1.0132	0.9297	-0.7246	-0.3099		
0.825	-0.9459	-0.5756	1.0068	0.9748		0.2781	0.9789	-0.6223	-0.949	-1.0616	-0.5896	0.691	-0.2633		
0.9803	-0.6093	-0.4086	1.2443	1.1055		-0.6927	1.2134	-0.3202	-1.0911	-0.7043	-0.2276	-0.1801	-1.0117		
1.2545	-1.2176	-0.3976	1.0723	1.0809		-0.4881	1.0485	-0.4471	-1.2711	-1.2684	-1.0587	0.6693	-0.4848		
0.7779	-0.8311	-0.4445	0.669	0.7009		-0.1655	0.6449	-0.4862	-0.7956	-0.8834	-0.7135	0.5407	-0.3329		
0.3731	-0.3211	-0.3215	0.0947	0.1743		-0.5187	0.0735	-0.319	-0.2965	-0.3139	-0.439	0.0854	-0.4453		
0.6819	-0.5607	-0.3096	0.4308	0.4747		-0.671	0.4088	-0.3049	-0.6282	-0.5657	-0.5976	0.145	-0.564		
1.1917	-1.3775	0.3283	0.806	0.8338		-0.0989	0.8285	0.1607	-1.1677	-1.3462	-1.3754	1.1816	0.5224		
0.5832	-0.5695	-0.3707	0.3574	0.4188		-0.4078	0.3349	-0.3883	-0.5421	-0.5854	-0.594	0.2906	-0.4116		
1.0756	-1.1286	-0.4527	1.0719	1.0572		-0.083	1.0483	-0.5087	-1.1487	-1.2106	-0.8649	0.7301	-0.3255		
0.5026	-0.5539	-0.179	0.8608	0.7532		0.4252	0.8546	-0.2022	-0.6602	-0.6496	-0.1619	0.4021	0.0008		
0.667	-1.4358	1.763	0.3759	0.3435		1.5789	0.4963	1.3613	-0.717	-1.3146	-1.4487	2.2305	2.8902		
0.6959	-0.9956	-0.0269	0.8529	0.8054		0.7481	0.861	-0.1561	-0.8279	-1.0676	-0.6796	1.0483	0.5465		
0.8913	-0.9002	-0.3328	0.9472	0.9103		-0.0724	0.9295	-0.3689	-0.9683	-0.9724	-0.6441	0.5407	-0.2849		
0.9231	-1.0697	-0.1703	0.8316	0.8353		0.0879	0.826	-0.2666	-0.9696	-1.1136	-0.9018	0.8616	0.0803		
0.9658	-1.3198	-0.2276	0.6732	0.7608		0.3025	0.6687	-0.3947	-0.9698	-1.3487	-1.2614	1.2987	0.3701		
1.0608	-0.9413	-0.4194	1.0508	1.015		-0.4593	1.0244	-0.4234	-1.1125	-1.0096	-0.7097	0.3765	-0.6159		
-0.3242	0.5568	-0.2796	-0.9479	-0.7552		-1.2135	-0.9743	-0.1925	0.5828	0.6613	-0.0012	-0.7221	-0.7979		

## Appendix

MATLAB Code used in this PCA

%%PCA

```

% Import raw data from excel file
filename = 'SotTCombined2010';
Xshift = readmatrix(filename)
% Remove the country names column
Xshift = Xshift(:,2:14);
% Excluding countries with missing values
Xshift=Xshift(sum(isnan(Xshift),2)==0,:);
%%
% Compute means and variance of each column
colmeans = mean(Xshift,1);
colvars = var(Xshift,1);

```

```
% Compute the X matrix from the raw data
X = (Xshift - repmat(colmeans, size(Xshift,1), 1)) ./
repmat(sqrt(colvars), size(Xshift,1), 1)

% calculate the correlation
Cor = corrcoef(X)
f1 = figure;
heatmap(Cor,"Colormap",copper);

% Perform the SVD of the scaled matrix
[U,S,V] = svd(X)

%Find the loading scores between variables and first principal component
%vector
PC1 = V(:,1)

% Obtaining singular value
SV = diag(S)
weights = SV.^2

% Compute the ratio of each component to the total sum of the weights
prop_weights = weights / sum(weights)*100
% Cumulative sum demonstrates how much of the space is represented by sums
% of 1st, 1st and 2nd, all components
cumprop_weights = cumsum(prop_weights)

% Plot the Scree Plot
f2 = figure;
plot(1:13,prop_weights,'*r',1:13,cumprop_weights)

% Compute the score matrix
T = U*S

% Compute the reduced matrix from only first four components
reduced = U * S(:,1:4) * V(:,1:4)

% Compute the residuals
residuals = sum((X - reduced).^2,2)
f3 = figure;
plot(residuals)
%%

%Finding the outlier countries
[r,c] = find(abs(T(:,9:13))>1)
```