Start with intuitive definition from landscape:

$$\frac{P_{inside}}{P_{outside}} = \frac{A_{inside}}{A_{outside}} e^{d/RT} \tag{1}$$

Take the natural log of both sides:

$$ln\left(\frac{P_{inside}}{P_{outside}}\right) = ln\left(\frac{A_{inside}}{A_{outside}} e^{d/RT}\right) \tag{2}$$

Log rule $log(A \cdot B) = log(A) + log(B)$:

$$ln\left(\frac{P_{inside}}{P_{outside}}\right) = ln\left(\frac{A_{inside}}{A_{outside}}\right) + ln\left(e^{d/RT}\right) \tag{3}$$

Simplify:

$$ln\left(\frac{P_{inside}}{P_{outside}}\right) = ln\left(\frac{A_{inside}}{A_{outside}}\right) + \frac{d}{RT} \tag{4}$$

Multiply both sides by $-RT$:

$$-RT \cdot ln\left(\frac{P_{inside}}{P_{outside}}\right) = -RT \cdot ln\left(\frac{A_{inside}}{A_{outside}}\right) + \frac{-RTd}{RT} \tag{5}$$

Write some definitions:

$$\triangle G \equiv -RT \cdot ln\left(\frac{P_{inside}}{P_{outside}}\right) \tag{6}$$

$$\triangle S \equiv Rln\left(\frac{A_{inside}}{A_{outside}}\right) \tag{7}$$

$$\triangle H \equiv -d \tag{8}$$

Substitute:

$$\triangle G = \triangle H - T\triangle S \tag{9}$$