- You have N dice.
- Of these N dice, $N_{⚅}$ are currently showing ⚅.
- The probability of a dice coming up as ⚅ is $p = 1/6$.
- For each turn, you randomly pick up one dice and roll it.

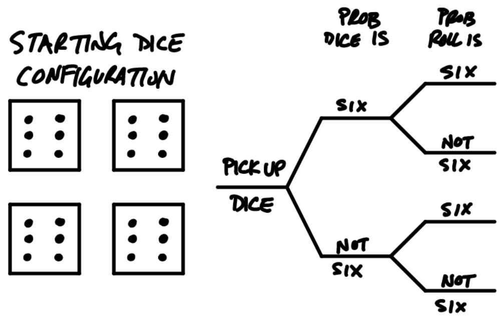
1. What is the probability—in terms of N, $N_{⚅}$, and p—of a non-⚅ dice becoming a ⚅ over one turn?

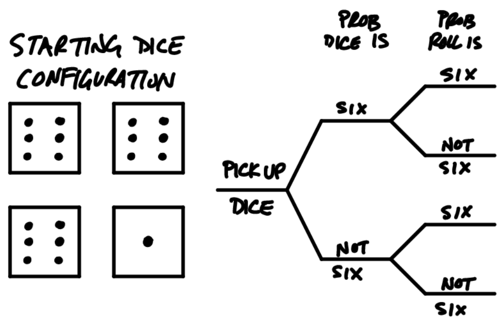
2. What is the probability—in terms of N, $N_{⚅}$, and p—of a ⚅ dice becoming a non-⚅ over one turn?

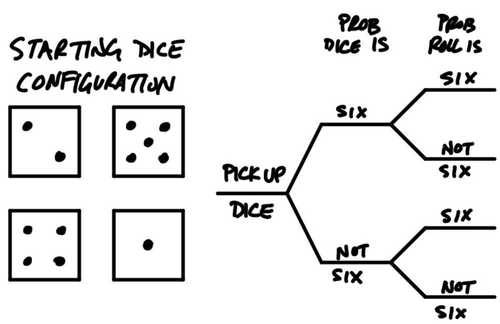
3. What is the predicted overall change in $N_{⚅}$ —in terms of N, $N_{⚅}$, and p—over a turn?

4. Under what conditions will the predicted change in $N_{⚅}$ will be zero?

You have a collection of four dice. You randomly pick up one dice and roll it. Assuming the dice are fair ($p = 1/6$ for all faces), fill out the contingency trees below and calculate the probability of each possible outcome.







Based on this calculation, why does a dice reaction equilibrate away from configurations like the top configuration?

1. You roll a dice 4 times and write down the observed sequence of faces you see (for example, ⚀, ⚃, ⚁, ⚅). How many different sequences of are possible?

2. What is the probability of seeing the following four rolls, in this order?

   ⚀ ⚅ ⚂ ⚅

3. What is the probability of seeing the following four rolls, in this order?

   {⚀, ⚁, ⚂, ⚃, ⚄} ⚅ {⚀, ⚁, ⚂, ⚃, ⚄} ⚅

   {⚀, ⚁, ⚂, ⚃, ⚄} indicates that any of these faces come up. Hint: Try to come up with an approach that does not equire you to make all possible combinations of {⚀, ⚁, ⚂, ⚃, ⚄}.

4. How many different ways are there to roll four dice and see the following come up in *any order*?

   ⚅, ⚅, {⚀, ⚁, ⚂, ⚃, ⚄}, {⚀, ⚁, ⚂, ⚃, ⚄}

5. What is the probability of seeing the following four rolls, in *any order*? (Put another way, what is the probability of observing exactly two ⚅ in four rolls?)

   ⚅, ⚅, {⚀, ⚁, ⚂, ⚃, ⚄}, {⚀, ⚁, ⚂, ⚃, ⚄}

$$P(n,k) = \binom{n}{k} p^k (1-p)^{n-k}$$

where n is the number of dice, k is the number of dice showing 6, p is the probability of obtaining 6 on a roll, and $\binom{n}{k}$ is the number of ways you can draw k dice out of n total.

1. You have four dice. Enumerate all possible combinations of ⚅ vs. not-⚅ you can obtain for $k \in \{0, 1, 2, 3, 4\}$.

2. You have $n = 600$ fair dice ($p = 1/6$). In a plotting program of your choice, graph each of the following as a function of k:

$$\binom{n}{k}$$

$$p^k (1-p)^{n-k}$$

$$P(n,k) = \binom{n}{k} p^k (1-p)^{n-k}$$

   You might want to put the y-axis on a log scale. (In Excel, the $\binom{n}{k}$ function is 'COMBIN'; in python it is 'scipy.special.comb'; in R it is 'choose').

3. Explain why each graph looks like it does. (Why does it have the overall shape it does? Why are it's minima/maxima/etc. where they are?)