Regular Langs

Problem 1

Prove that the class of regular languages is closed under the union ($\cup$) operation. (Hint: Use proof by construction.)

Problem 2

Provide the regular expression that corresponds to each of the following language descriptions. Assume that Σ is $\{0, 1\}$:

- $\{w|w$ has exactly a single $1\}$
- $\{w|w$ contains the string 001 as a substring$\}$
- $\{w|w$ is a string of even length$\}$
- $\{w|w$ starts and ends with the same symbol$\}$

Problem 3

Convert the regular expression $(a \cup b)^\star$ to a nondeterministic finite state automaton. Note that you do not have to draw the NFA if you do not wish to do so. You may provide the mathematical specification $M = (Q, \Sigma, \delta, q_0, F)$. For δ you may provide a table of the transitions.

Problem 4

Using the pumping lemma prove that $F = \{ww|w \in \{0, 1\}^\star\}$ is nonregular.

CFGs

Problem 1

Given the following grammar:

$E \rightarrow E + T | T$
$T \rightarrow T * F | F$
$F \rightarrow (E) | a$

Give the parse trees for each string:

- a
- $a + a$
- $a + a + a$
- $((a))$

Problem 2

Give CFG's that generate the following languages. In each case the alphabet Σ is $\{0, 1\}$.

- $\{w | w$ contains at least three 1s$\}$
- $\{w | w$ starts and ends with the same symbol$\}$
- $\{w | w$ contains more 1s than 0s$\}$

Problem 3

Design a PDA to recognize the language $\{ww^r | w \in \{0,1\}^*\}$, where w^r means w written backwards.

Problem 4

Prove that the class of context free languages is closed under the union operation.

Problem 5

The **pumping lemma for context-free languages** states that if A is a CFL, then there is a number p (the pumping length) where, if s is any string in A of length at least p, then s may be divided into five pieces $s = uvxyz$ satisfying the conditions:

- For each $i \geq 0, uv^ixy^iz \in A$

- $|vy| > 0$
- $|vxy| \leq p$

Use this pumping lemma for CFL's to prove that the language $B = \{a^nb^nc^n | n \geq 0\}$ is not context free.

NP Complete

Problem 1

In the vertex covering problem we are given a graph G consisting of a set of vertices V and edges E. We would like to determine if for a given integer k there is a subset V' of V with the size of V' less than k such that every edge has at least one endpoint in V'.

Prove that vertex cover is NP complete by showing 3-SAT is polynomial time reducible to it.