SCHOOL OF EARTH AND ATMOSPHERIC SCIENCES

GEORGIA INSTITUTE OF TECHNOLOGY

Sven Simon
Jacob J. Buffo

EARTH SYSTEM MODELING (EAS 4610/6310) FALL 2017

Problem Sheet # 11 Return date: Thursday, 16 November (before 09:30 am)

---

16. **Advection equation** **(4 points)**
We consider the one-dimensional advection equation discussed in the lecture:

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} = 0 \qquad , \quad \text{where} \qquad u = 1.5$$

and $0 \leq x \leq 20$. The initial condition is given by

$$T(x,0) = \exp\left\{-(x-4)^2\right\} \qquad .$$

(a) Write a Matlab script that solves the advection equation by applying the Euler-Forward upwind scheme introduced in class.

(b) By using your script and $\Delta x = 0.1$, calculate $T(x,t)$ for $0 \leq t \leq 10$ and different values of the Courant number: $\mathcal{C} = 0.1, 0.5, 1, 2$. Use the boundary condition $T(0,t) = T(0,0)$. Generate plots that show the time evolution of $T(x,t)$ for each value of $\mathcal{C}$. Discuss your results.

17. **Diffusion equation: Evolution of a hill slope** **(6 points)**
In geomorphology the diffusion equation

$$\frac{\partial h(x,t)}{\partial t} = \kappa \frac{\partial^2 h(x,t)}{\partial x^2} \tag{1}$$

is applied to study how hill slopes are smoothed due to erosion and sedimentation. As a simple example, we consider a hill whose initial altitude profile $h(x,0)$ is given by

$$h(x,0) = \begin{cases} \frac{H_{max}x}{L} & \text{for} \quad 0 \leq x \leq L \\ H_{max} - \frac{H_{max}(x-L)}{L} & \text{for} \quad L \leq x \leq 2L \end{cases} \qquad ,$$

where H_{max} and L are positive constants.

(a) Generate a plot of the altitude profile $h(x,0)$ for $L = 10$ and $H_{max} = 20$.

(b) Write a Matlab script that solves equation (1) by using the *implicit* scheme discussed in class. The boundary conditions read $h(0,t) = h(2L,t) = 0$. The required inversion of the tridiagonal matrix is again done with the Thomas algorithm (cf. sheet 7).

(c) Use your script to compute the time evolution of the hill for $L = 10$ and $H_{max} = 20$. Use a step size of $\Delta x = 0.1$ and make runs for three different values of the diffusion coefficient $\kappa > 0$. Continue your simulations until the summit of the hill is below $H_{max}/2$ for the smallest κ you chose. For each value of κ generate a plot that illustrates the evolution of the altitude profile $h(x,t)$ in time.