SCHOOL OF EARTH AND ATMOSPHERIC SCIENCES

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EARTH SYSTEM MODELING (EAS 4610/6310) FALL 2017

Problem Sheet # 7 Return date: Thursday, 12 October (before 09:30 am)

10. **Inversion of a tridiagonal matrix: Thomas algorithm** **(10 points)**
We consider a linear system of equations

$$\underline{\underline{A}} \cdot \underline{x} = \underline{y} \quad , \tag{1}$$

where $\underline{x}, \underline{y} \in \mathbb{R}^N$ and $\underline{\underline{A}} \in \mathbb{R}^{N\times N}$ is a *tridiagonal* matrix. Using the nomenclature introduced in class we define a vector $\underline{d}$ that contains the diagonal elements of the matrix, i.e.,

$$d_i = A_{i,i} \quad ; \quad i = 1, \ldots, N \quad .$$

We also introduce a vector $\underline{a}$ that contains the elements of $\underline{\underline{A}}$ *above* the diagonal,

$$a_i = A_{i,i+1} \quad ; \quad i = 1, \ldots, N-1 \quad \text{and} \quad a_N = 0 \quad ,$$

as well as a vector $\underline{b}$ that contains the elements of $\underline{\underline{A}}$ *below* the diagonal:

$$b_1 = 0 \quad \text{and} \quad b_i = A_{i,i-1} \quad ; \quad i = 2, \ldots, N \quad .$$

(a) Write a Matlab function that solves the linear system (1) by using the Thomas algorithm. Follow the "recipe" outlined in class. The *input parameters* of your function should be the vectors $\underline{d}$, $\underline{a}$, $\underline{b}$, $\underline{y}$ as well as the rank N of the matrix $\underline{\underline{A}}$. The *output values* of the function should be the components of the solution vector $\underline{x}$.
Note: The input parameter N could be eliminated by using the *length* function in Matlab.

(b) Use the tool developed in part (a) to calculate the solution $\underline{x}$ for

$$\underline{\underline{A}} = \begin{pmatrix} 2.04 & -1 & 0 & 0 \\ -1 & 2.04 & -1 & 0 \\ 0 & -1 & 2.04 & -1 \\ 0 & 0 & -1 & 2.04 \end{pmatrix} \quad \text{and} \quad \underline{y} = \begin{pmatrix} 40.8 \\ 0.8 \\ 0.8 \\ 200.8 \end{pmatrix} \quad .$$

Verify your result by solving equation (1) with the *inv* function available in Matlab.

(c) We now consider a tridiagonal matrix $\underline{\underline{A}} \in \mathbb{R}^{N\times N}$ where all non-vanishing components of $\underline{d}$, $\underline{a}$ and $\underline{b}$ have the *same* values γ, δ, ϵ (see, e.g., the example from part (b)). Thus, the non-vanishing entries of $\underline{\underline{A}}$ read

$$d_i = \gamma \quad ; \quad i = 1, \ldots, N \quad ,$$

$$a_i = \delta \quad ; \quad i = 1, \ldots, N-1 \quad \text{and} \quad a_N = 0 \quad ,$$

and

$$b_1 = 0 \quad \text{and} \quad b_i = \epsilon \quad ; \quad i = 2, \ldots, N \quad .$$

We also assume that the components of $\underline{y}$ are given by

$$y_i = \gamma^i \quad ; \quad i = 1, \ldots, N \quad .$$

Give the recurrence relations of the Thomas algorithm for this case. Develop a Matlab function which solves the system (1) and requires γ, δ, ϵ and N as the only input parameters. Select a (non-trivial) example to test your function with the same method as in part (b).