SCHOOL OF EARTH AND ATMOSPHERIC SCIENCES

GEORGIA INSTITUTE OF TECHNOLOGY

Sven Simon
Jacob J. Buffo

EARTH SYSTEM MODELING (EAS 4610/6310) FALL 2017

Problem Sheet # 10 Return date: Thursday, 09 November (before 09:30 am)

15. **Celestial mechanics** **(10 points)**
The motion of an object (e.g., planet, comet, asteroid) in the gravitational field of the Sun (mass M) is described by Newton's equations of motion,

$$\frac{\mathrm{d}\underline{x}}{\mathrm{d}t} = \underline{v} \qquad \text{and} \qquad \frac{\mathrm{d}\underline{v}}{\mathrm{d}t} = -\frac{\gamma M}{|\underline{x}|^3}\,\underline{x} \qquad , \tag{1}$$

where γ is the gravitational constant. The vectors $\underline{x} = (x_1, x_2, x_3)$ and $\underline{v} = (v_1, v_2, v_3)$ denote the object's position and velocity, respectively. The purpose of this problem is to solve these coupled equations for different initial conditions.

(a) Give the Euler-Forward discretization of (1). These discretized equations will help you solve the following problems.

(b) Write a Matlab script that solves the equations of motion (1) by using the *Predictor-Corrector Scheme*. The object is initially located at $\underline{x}(t = 0) = (d, 0, 0)$, where $d = 1\,\mathrm{AU} = 1.496{\cdot}10^{11}\,\mathrm{m}$ is (approximately) the distance between Earth and the Sun. The initial velocity vector $\underline{v} = (0, v_0, 0)$ is aligned with the y axis and the value of v_0 should be provided by the user.

(c) The shape of the orbit $\underline{x}(t)$ is determined by the sum of the object's kinetic and potential energy

$$E = \frac{1}{2}mv_0^2 - \frac{\gamma M m}{d} \qquad , \tag{2}$$

where m is the mass of the object. Here we set $m = 5.972 \cdot 10^{24}\,\mathrm{kg}$ (mass of Earth). Run your Matlab script for each of the following four cases and generate plots of the orbits in the $z = 0$ plane:

i. $E > 0$,
ii. $E = 0$,
iii. $-\frac{\gamma M m}{2d} < E < 0$,
iv. $E = -\frac{\gamma M m}{2d}$.

For each of these four cases, use equation (2) to obtain a suitable value of v_0. The total time for each simulation is 5 years. Discuss the different types of orbits $\underline{x}(t)$.

(d) Use the results of part (c) to demonstrate that the object's motion is confined to the $z = 0$ plane.

(e) Write a Matlab script that solves (1) by using the *Euler-Richardson Algorithm*. Calculate the orbit $\underline{x}(t)$ of the object for the four scenarios outlined in part (c) and generate plots of your results.

(f) *(graduate students only, + 10 points)*
Write a Matlab script that solves (1) by using the *Runge-Kutta Scheme of Fourth Order*. Calculate the orbit $\underline{x}(t)$ of the object for the four scenarios outlined in part (c) and generate plots of your results.