

**8. Linear system of ODEs**

**(5 points)**

Consider the linear system

$$\frac{d\mathbf{M}}{dt} = \mathbf{K} \cdot \mathbf{M} \quad ,$$

where  $\mathbf{M}(t) = (M_1(t), M_2(t), M_3(t))$  and

$$\mathbf{K} = \begin{pmatrix} 0 & \sqrt{2} & 0 \\ \sqrt{2} & 0 & \sqrt{2} \\ 0 & \sqrt{2} & 0 \end{pmatrix} \quad .$$

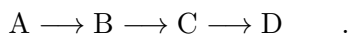
The initial condition reads  $M_1(0) = M_2(0) = M_3(0) = 1$ .

- (a) Write a Matlab script that calculates the eigenvectors and eigenvalues of matrix  $\mathbf{K}$ .
- (b) Use the results from part (a) to determine the analytical solution  $\mathbf{M}(t)$  of the system for the given initial condition. Apply the equations we derived in class.
- (c) By applying the Euler-Forward method, solve the system numerically for  $\Delta t = 0.001$ . Evaluate the evolution of  $\mathbf{M}(t)$  for 1000 time steps. Generate plots of both, the numerical and the analytical solution.
- (d) Is the Euler-Forward method stable for this problem? Explain your answer.

**9. Nuclear decay chain**

**(5 points)**

We consider a nuclear decay chain with four isotopes:



The element D is stable and the decay constants read  $\lambda_A$ ,  $\lambda_B$  and  $\lambda_C$ .

- (a) Give the linear system of equations that describes how the numbers of atoms  $N_A(t)$ ,  $N_B(t)$ ,  $N_C(t)$  and  $N_D(t)$  evolve in time.
- (b) Write a Matlab script that solves this system of equations by using the Euler-Backward method. Use  $\lambda_A = 0.1$ ,  $\lambda_B = 0.8$ ,  $\lambda_C = 1.2$  and  $N_A(0) = 1000$ ,  $N_B(0) = N_C(0) = N_D(0) = 0$ . Continue the calculation until  $N_A(t) < 10$ .
- (c) Use your results to generate a plot of the functions  $N_A(t)$ ,  $N_B(t)$ ,  $N_C(t)$  and  $N_D(t)$ . Explain the shape of these curves.