SCHOOL OF EARTH AND ATMOSPHERIC SCIENCES

GEORGIA INSTITUTE OF TECHNOLOGY

Sven Simon
Jacob J. Buffo

EARTH SYSTEM MODELING (EAS 4610/6310) FALL 2017

Problem Sheet # 5 Return date: Thursday, 28 September (before 09:30 am)

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7. **Motion of a ball in Earth's gravitational field** **(10 points)**
At the surface of the Earth ($z = 0$), a ball is thrown at an angle ϕ with respect to the horizontal. Thus, the initial velocity of the ball is given by $v_x(0) = v_0 \cos\phi$ and $v_z(0) = v_0 \sin\phi$, where v_0 is a constant. The evolution of the velocity components in time is then described by Newton's equations of motion, i.e.,

$$\frac{\mathrm{d}v_x}{\mathrm{d}t} = 0 \qquad \text{and} \qquad \frac{\mathrm{d}v_z}{\mathrm{d}t} = -g \qquad , \tag{1}$$

where $g = 9.81\,\mathrm{m/s^2}$ is Earth's gravitational acceleration.

(a) Write a Matlab script that solves the set of equations (1) by using the Euler-Forward scheme. Use this script to calculate the evolution of the ball's velocity in time for $\phi = 45°$ and $v_0 = 24\,\mathrm{m/s}$. Use a time step of $\Delta t = 0.002\,\mathrm{s}$.

(b) Modify your script to identify those points in time (t_1 and t_2) which fulfill $v_z(t_1) = 0$ and $v_z(t_1) = -v_0 \sin\phi$. Use an accuracy of 10^{-3}. What is the physical meaning of these two points?

(c) The position $(x(t), z(t))$ of the ball is related to its velocity $(v_x(t), v_z(t))$ through

$$\frac{\mathrm{d}x}{\mathrm{d}t} = v_x \qquad \text{and} \qquad \frac{\mathrm{d}z}{\mathrm{d}t} = v_z \qquad . \tag{2}$$

Use the Euler-Forward scheme to discretize these equations. Since the velocity $(v_x(t), v_z(t))$ has been computed in part (b), we can now use the discretized equations (2) to calculate the position $(x(t), z(t))$ of the ball as a function of time. Expand your Matlab script to perform this operation and plot the trajectory $(x(t), z(t))$ of the ball until it hits the surface $z = 0$. The initial condition reads $x(0) = z(0) = 0$. Mark the two points along the trajectory which correspond to t_1 and t_2 (see part (b)).
Note: In this problem we have –for the first time– solved a system of *coupled* differential equations (namely (1) and (2)).

(d) Finally, let us take into account the influence of *air resistance* on the trajectory of the ball. Air resistance typically increases with speed, and the modified equations of motion (1) then read

$$\frac{\mathrm{d}v_x}{\mathrm{d}t} = -Cv_x\sqrt{v_x^2 + v_z^2} \qquad \text{and} \qquad \frac{\mathrm{d}v_z}{\mathrm{d}t} = -g - Cv_z\sqrt{v_x^2 + v_z^2} \qquad , \tag{3}$$

where C is a constant. For $\phi = 45°$, $v_0 = 24\,\mathrm{m/s}$ and $C = 10^{-2}\,\mathrm{m^{-1}}$ use the Euler-Forward method to calculate the velocity $(v_x(t), v_z(t))$ of the ball. By using the same idea as in problem (c), you can then obtain its trajectory $(x(t), z(t))$. Plot the modified trajectory of the ball until it hits the surface $z = 0$. Use your script to calculate the position x_{end} where the ball hits the surface.

(e) *(graduate students only, + 3 points)*
For the case *without* air resistance calculate the analytical solution $(x(t), z(t))$ for the ball's trajectory and include it in your plot. Also calculate the analytical values of t_1 and t_2 and compare your results against the numerical solution.