SCHOOL OF EARTH AND ATMOSPHERIC SCIENCES

GEORGIA INSTITUTE OF TECHNOLOGY

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EARTH SYSTEM MODELING (EAS 4610/6310) — FALL 2017

Problem Sheet # 9 — Return date: Thursday, 02 November (before 09:30 am)

13. **Free fall in Earth's inhomogeneous gravitational field** **(5 points)**
The gravitational acceleration $a(z)$ of Earth changes as a function of altitude,

$$a(z) = -\frac{\gamma M}{(z + R_E)^2} \quad , \tag{1}$$

where γ is the gravitational constant. The parameters M and R_E denote the mass and the radius of the planet, respectively. The coordinate z represents the (vertical) distance to the surface of Earth. Thus, Newton's equation of motion for the position of a free-falling object reads

$$\frac{\mathrm{d}^2 z}{\mathrm{d}t^2} = -\frac{\gamma M}{(z + R_E)^2} \quad . \tag{2}$$

(a) Apply the discretization for the second derivative found in class to numerically solve equation (2). The initial conditions are $z(0) = 5R_E$ and $\dot{z}(0) = 0$. Plot the altitude $z(t)$ as a function of time. Continue the calculation until the object hits the surface $z = 0$ of Earth. How long does it take the object to reach the surface?

(b) Transform equation (2) into a system of *two* ODEs of *first order*. Solve that system by using the Euler-Forward method. Generate plots of $z(t)$ and $\dot{z}(t)$ and compare your results to the findings of part (a).

14. **Steady-state diffusion equation** **(5 points)**
We consider the steady-state diffusion equation introduced in class:

$$\frac{\mathrm{d}^2 T(x)}{\mathrm{d}x^2} = h(x) \tag{3}$$

for $h(x) = 0.1x$ and $0 \leq x \leq 10$. We introduce a set of "mixed" boundary conditions:

$$T(0) = 3 \qquad \text{and} \qquad \left.\frac{\mathrm{d}T}{\mathrm{d}x}\right|_{x=10} = -2 \quad .$$

(a) By using the material from the lecture, write this equation in the form $\underline{\underline{M}} \cdot \underline{T} = \underline{C}$, i.e., give the components of $\underline{\underline{M}}$ and $\underline{C}$.

(b) Solve for $\underline{T}$ by using the function for tridiagonal matrix inversion that we developed in problem 10(a) of sheet 7. Use a step size of $\Delta x = 0.05$. Generate a plot of $T(x)$.

(c) *(graduate students only, + 4 points)*
Find the *analytical* solution of equation (3) for the given boundary conditions. Generate a plot of the analytical solution to validate your results from part (b).