SCHOOL OF EARTH AND ATMOSPHERIC SCIENCES
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EARTH SYSTEM MODELING (EAS 4610/6310) FALL 2017

Problem Sheet # 3 Return date: Thursday, 14 September (before 09:30 am)

4. **Nuclear decay equation** **(10 points)**

In class we discussed the numerical treatment of the nuclear decay equation

$$\frac{\mathrm{d}N}{\mathrm{d}t} = -\lambda N \quad , \tag{1}$$

where $\lambda > 0$ is a constant. We also showed that the analytical solution of this equation reads

$$N(t) = N(0)\exp(-\lambda t) \quad .$$

(a) Implement the Euler-Forward method to numerically solve equation (1). Your script should ask the user to provide $N(t = 0)$, λ and the time step Δt as input parameters.

(b) By using the Euler-Forward method, compute the numerical solution of equation (1) for the following six combinations of parameters:

- $\Delta t = 1$ and $\lambda = 0.1$, $\lambda = 0.8$, $\lambda = 1.2$,
- $\Delta t = 2$ and $\lambda = 0.1$, $\lambda = 0.8$, $\lambda = 1.2$.

For simplicity you can assume $N(t = 0) = 1$. Generate plots of the numerical and the analytical solution for $0 \leq t \leq 20$ (in the same diagram) for each of the six cases.

(c) We will show in class that the Euler-Forward method becomes *numerically unstable*, if

$$\lambda \Delta t \geq 2 \quad .$$

Which of the six scenarios in problem (b) are numerically unstable? Describe how the numerical instability affects the shape of the computed solution.

(d) Write a script that solves equation (1) by using the Euler-Backward method. Again, the required input parameters are $N(t = 0)$, λ and Δt. Use this script to compute the numerical solution for the same six cases as discussed in problem (b). For each case, generate a plot of the numerical results and the corresponding analytical solution (see part (b)). Do your results show any signs of numerical instability?

(e) Finally, implement the Centered Euler method to solve equation (1). Compute the numerical solution for the set of six parameter combinations given in part (b). Generate plots of the analytical and the numerical solutions. Compare your results to the plots generated with the Euler-Backward method. What are your conclusions regarding the numerical stability of the Centered Euler scheme?

(f) Compare the three numerical methods discussed in this problem. Which scheme would you prefer to solve the nuclear decay equation?