16. Advection equation

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} = 0 \quad , u = 1.5$$

$$0 \leq x \leq 20 \quad , \quad T(x,0) = e^{-(x-4)^2}$$

a) EF - upwind scheme $\quad \dfrac{T_k^{n+1} - T_k^n}{\Delta t} + u \dfrac{T_k^n - T_{k-1}^n}{\Delta x} = 0$

$$\rightarrow T_k^{n+1} = T_k^n - u \frac{T_k^n - T_{k-1}^n}{\Delta x} \Delta t$$

$$C_1 = u \cdot \frac{\Delta t}{\Delta x}$$

$$\boxed{\rightarrow T_k^{n+1} = T_k^n - C_1 \left(T_k^n - T_{k-1}^n \right)}$$

Untitled

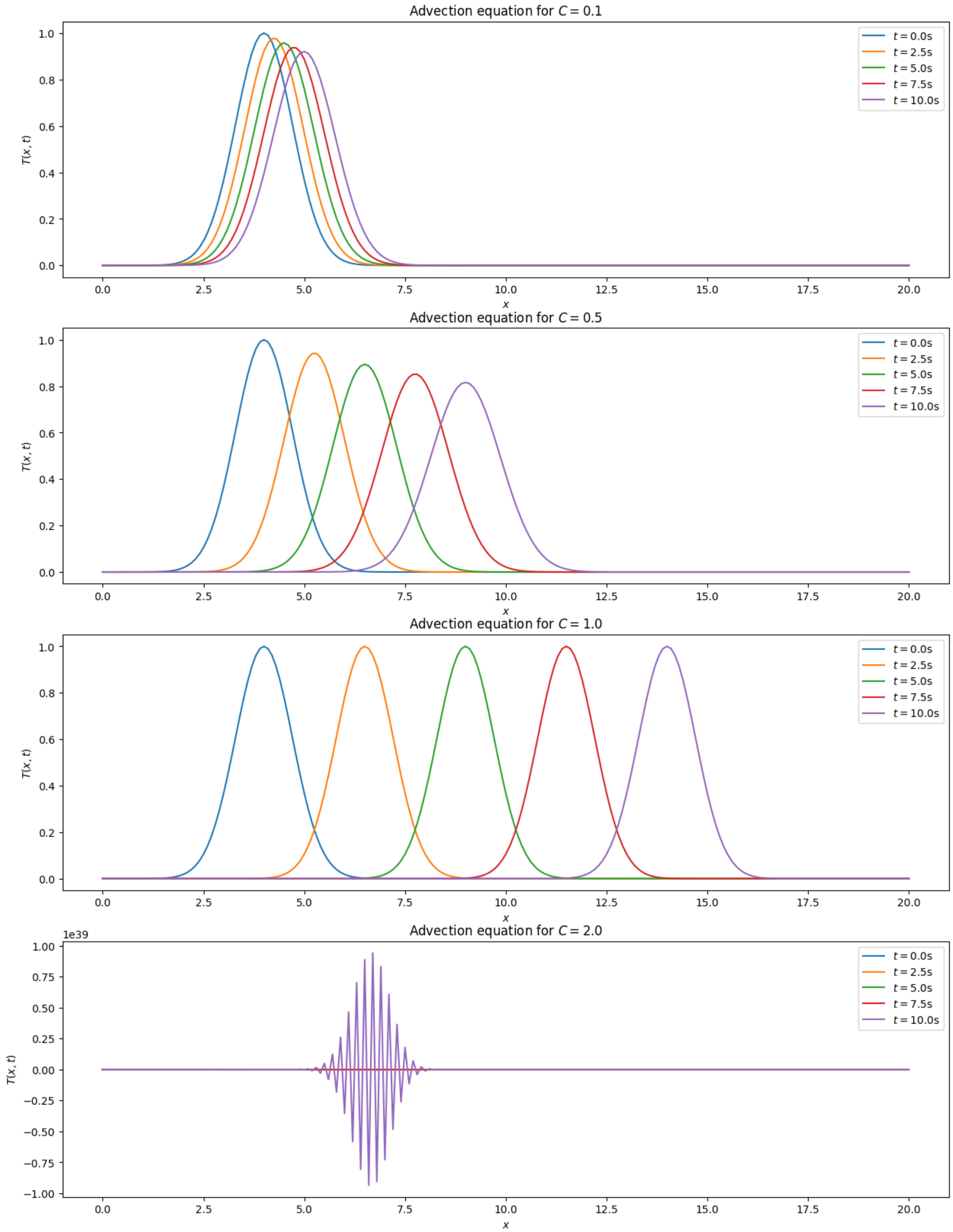
November 10, 2017

```
In [1]: using PyPlot

In [2]: function efupwind(dx,x,dt,t,n,C)
            #format XYZ, X-columns, Y-rows, Z-plotnumber
            z=length(C)*100+10
            figure(1,figsize=(15,20))

            #for all courant numbers
            for (zn,c) in enumerate(C)
                #from the sheet
                t0(x)=exp(-(x-4)^2)
                #steps in time and space
                nx=floor(Int,x/dx)
                nt=floor(Int,t/dt)
                T=zeros(nt+1,nx+1)
                #t(0,x)
                for (i, x) in enumerate(0:dx:x)
                    T[1,i]=t0(x)
                end

                for i in 1:nt
                    #T(t,0)=T(0,0)
                    T[i+1,1]=T[1,1]
                    for j in 2:nx
                        #T(t+1,x) formula derived
                        T[i+1,j]=T[i,j]-c*(T[i,j]-T[i,j-1])
                    end
                end
                #plot comands
                subplot(z+zn)
                for i in 0:n
                    d2t=floor(Int,((t/n)/dt))
                    nt=i*d2t+1
                    plot(0:dx:x,T[nt,1:nx+1],label=L"$t=$"*string((nt-1)*dt)*"s")
                    title(L"Advection equation for $C=$"*string(c))
                    xlabel(L"$x$")
                    ylabel(L"$T(x,t)$")
                    legend()
```

```
            end
        end
    end
```

Out[2]: efupwind (generic function with 1 method)

In [3]:
```
#number of curves per plot (n+1 curves in the plot, because t=0 is extra)
n=4
C=[0.1 0.5 1 2] #courant number
#constants
dx=0.1
x=20
dt=0.1
t=10
efupwind(dx,x,dt,t,n,C)
```


Advection equation for $C = 0.1$
Advection equation for $C = 0.5$
Advection equation for $C = 1.0$
Advection equation for $C = 2.0$
$T(x, t)$
x
1e39
$t = 0.0$s
$t = 2.5$s
$t = 5.0$s
$t = 7.5$s
$t = 10.0$s