SCHOOL OF EARTH AND ATMOSPHERIC SCIENCES

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Problem Sheet # 8 Return date: Thursday, 26 October (before 09:30 am)

11. **The daisy world system** **(6 points)**

The purpose of this problem is to numerically solve the system of equations for the daisy world discussed in class. We found the following set of equations:

$$
\begin{aligned}
\frac{\mathrm{d}C_W}{\mathrm{d}t} &= C_W(b_W C_G - D) \qquad ; \\
\frac{\mathrm{d}C_B}{\mathrm{d}t} &= C_B(b_B C_G - D) \qquad ; \\
C_G &= 1 - C_B - C_W \qquad ; \\
b_i &= 1 - b(T_0 - T_i)^2 \qquad (i = W, B) \qquad ; \\
T_i^4 &= (1-k)\frac{LS_0}{4\sigma}(A - A_i) + T_s^4 \qquad (i = W, B) \qquad ; \\
A &= A_G C_G + A_B C_B + A_W C_W \qquad ; \\
T_e^4 &= \frac{LS_0}{4\sigma}(1 - A) \qquad .
\end{aligned}
$$

In addition, the surface temperature T_s is defined as $T_s^4 = 2T_e^4$. The free parameters are provided in the following table:

Symbol	Value
S_0	1380
A_W	0.75
A_B	0.25
A_G	0.5
D	0.3
σ	$5.67 \cdot 10^{-8}$
T_0	295.7
b	0.003265
k	0.6

(a) Write a Matlab script that solves the daisy world equations by using the Euler-Forward method. The script should ask the user for the luminosity L as the only required input parameter. Use a time step of 1 year and simulate the evolution of the system for a total duration of 40 years. The initial conditions read $C_B(t = 0) = C_W(t = 0) = 0.01$.

(b) Run your script for $L = 1.2$. Generate plots that show the evolution of the total albedo A and the three areas (C_G, C_W, C_B) as a function of time.

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12. **The predator-prey problem** **(4 points)**

The relationship between a population of lions (predators) and a population of gazelles (prey) that reside in the same area can be modeled by a system of two ODEs. We consider a community that consists of N_L lions and N_G gazelles, with b_i and d_i (where $i = L, G$) representing the birth and death rates of the respective species. The rate of change (growth or decay) of the lion and gazelle populations can then be modeled by the equations

$$\frac{\mathrm{d}N_L}{\mathrm{d}t} = b_L N_L N_G - d_L N_L \quad ; \tag{1}$$

$$\frac{\mathrm{d}N_G}{\mathrm{d}t} = b_G N_G - d_G N_G N_L \quad . \tag{2}$$

(a) Explain the meaning of each term in equations (1) and (2). Especially, why do the birth and death terms in both equations have a different structure?

(b) By using the Euler-Forward method, determine the population of lions $N_L(t)$ and gazelles $N_G(t)$ as a function of time from $t = 0$ to $t = 25$ years. The initial conditions read $N_G(0) = 3000$ and $N_L(0) = 500$. The coefficients in the model are $b_G = 1.1/\mathrm{yr}$, $b_L = 0.00025/\mathrm{yr}$, $d_G = 0.0005/\mathrm{yr}$ and $d_L = 0.7/\mathrm{yr}$. Use a time step of $\Delta t = 0.001\mathrm{yr}$.

(c) Generate plots of $N_L(t)$ and $N_G(t)$ from $t = 0$ to $t = 25$ years. Discuss your results.