SCHOOL OF EARTH AND ATMOSPHERIC SCIENCES

GEORGIA INSTITUTE OF TECHNOLOGY

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Earth System Modeling (EAS 4610/6310) Fall 2017

Problem Sheet # 6 Return date: Thursday, 05 October (before 09:30 am)

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8. **Linear system of ODEs** **(5 points)**

Consider the linear system

$$\frac{\mathrm{d}\underline{M}}{\mathrm{d}t} = \underline{\underline{K}} \cdot \underline{M} \qquad ,$$

where $\underline{M}(t) = (M_1(t), M_2(t), M_3(t))$ and

$$\underline{\underline{K}} = \begin{pmatrix} 0 & \sqrt{2} & 0 \\ \sqrt{2} & 0 & \sqrt{2} \\ 0 & \sqrt{2} & 0 \end{pmatrix} \qquad .$$

The initial condition reads $M_1(0) = M_2(0) = M_3(0) = 1$.

(a) Write a Matlab script that calculates the eigenvectors and eigenvalues of matrix $\underline{\underline{K}}$.

(b) Use the results from part (a) to determine the analytical solution $\underline{M}(t)$ of the system for the given initial condition. Apply the equations we derived in class.

(c) By applying the Euler-Forward method, solve the system numerically for $\Delta t = 0.001$. Evaluate the evolution of $\underline{M}(t)$ for 1000 time steps. Generate plots of both, the numerical and the analytical solution.

(d) Is the Euler-Forward method stable for this problem? Explain your answer.

9. **Nuclear decay chain** **(5 points)**

We consider a nuclear decay chain with four isotopes:

$$\mathrm{A} \longrightarrow \mathrm{B} \longrightarrow \mathrm{C} \longrightarrow \mathrm{D} \qquad .$$

The element D is stable and the decay constants read λ_A, λ_B and λ_C.

(a) Give the linear system of equations that describes how the numbers of atoms $N_A(t)$, $N_B(t)$, $N_C(t)$ and $N_D(t)$ evolve in time.

(b) Write a Matlab script that solves this system of equations by using the Euler-Backward method. Use $\lambda_A = 0.1$, $\lambda_B = 0.8$, $\lambda_C = 1.2$ and $N_A(0) = 1000$, $N_B(0) = N_C(0) = N_D(0) = 0$. Continue the calculation until $N_A(t) < 10$.

(c) Use your results to generate a plot of the functions $N_A(t)$, $N_B(t)$, $N_C(t)$ and $N_D(t)$. Explain the shape of these curves.