1D Muscle Model based on the Hill 2-Element Model

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Brief notes accompanying the MATLAB (The Mathworks, Inc.) script:
`One_dimensional_2_element_model.m`.

1 Model Overview

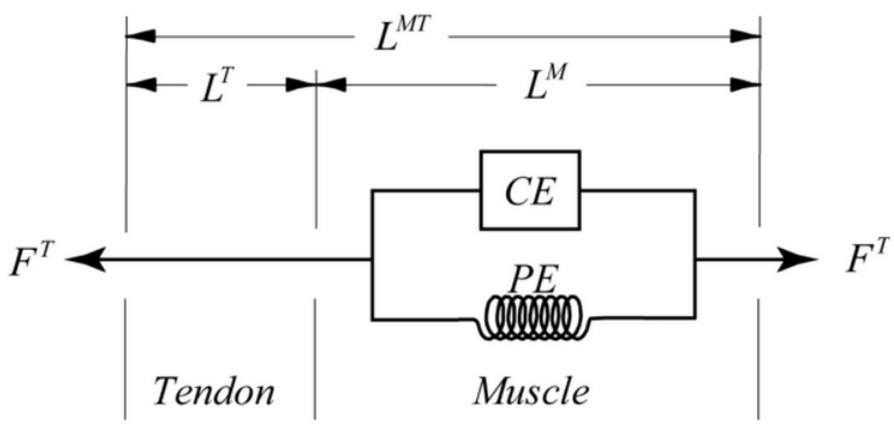


Figure 1: Schematic of 1D muscle-model

Schematic taken from Anderson (2007). The following code represents a model *without* tendons, i.e. $L^T = 0$. The total force is given by

$$F_{musc} = F_{PE} + F_{MAX} \times F_{CE}$$

where F_{MAX} is the maximum isometric force and F_{PE} and F_{CE} are the *normalised* passive and active forces, respectively.

2 Global muscle properties

These properties characterise the muscle itself and do not govern the relationships, for example the force-length relationship.

L_{REST}: In fact the resting length, here is set to the optimal length, i.e. the length at which peak force is produced

V_{MAX}: The maximum contractile velocity of the muscle

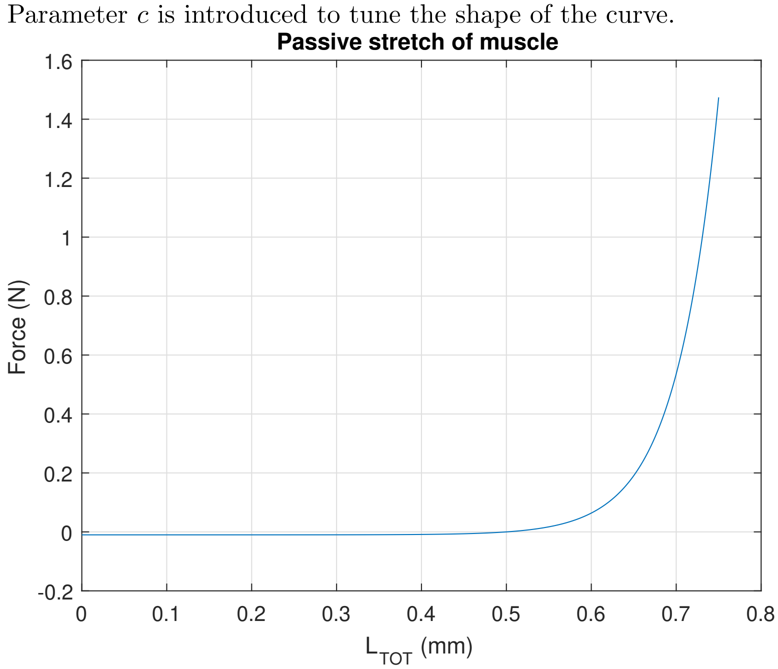
F_{MAX}: The peak isometric force of the muscle

3 Parallel elastic element

This element represents the passive response of the muscle upon stretching greater than resting length. Note that $L_{PE} = L_{TOT} = L_{CE}$ since the current model does not involve any tendons. The form of the passive relationship is not easily found in literature and is implemented in a straight forward way here to match the requirements of an exponential form which is non-zero above L_{REST};

$$F_{PE} = c \times [e^{L_{TOT}/L_{REST}-1} - 1]$$

Parameter c is introduced to tune the shape of the curve.



4 Contractile element

This element is reponsible for the active force production in the muscle. The total force is given in the form;

$$F_{CE} = \alpha(t) \times F_{VEL}(V) \times F_{LEN}(L_{TOT})$$

, where $\alpha(t)$ is the activation, F_{VEL} is the force-velocity relationship and F_{LEN} is the force length relationship. As a first step, we assume that the muscle is fully activated, i.e. $\alpha(t) = 1$.

5 Force-velocity

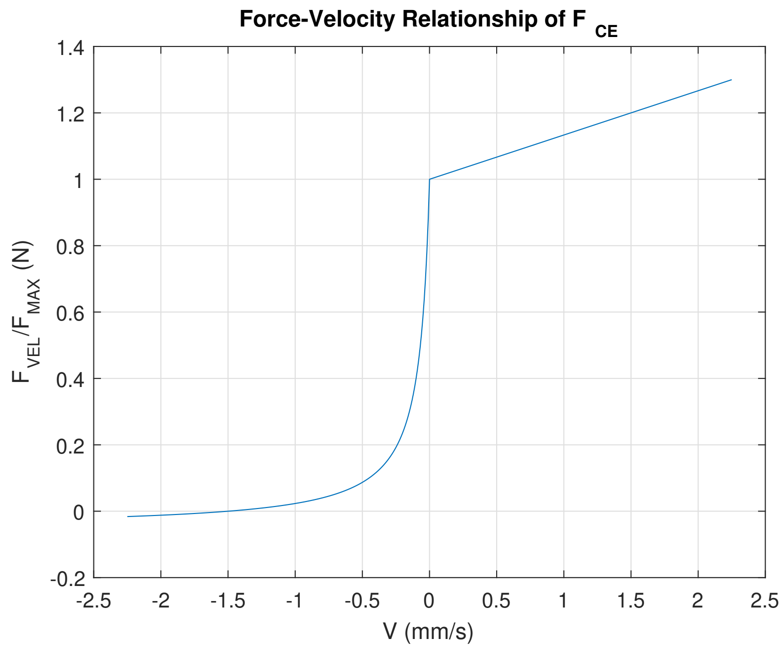
Taken from Challis (1994), based on Hill (1938).

$$F_{VEL} = \frac{1}{F_{MAX}} \times \frac{a(V_{MAX} - V)}{(b - V)}$$

where a and b have been fit to V_{MAX} and F_{MAX} and are the shape constants for the muscle force-velocity relationship. Note that velocity is taken as negative when the muscle is shortening, i.e. concentric contractions. To account for eccentric contractions a temporary adjustment is made. When velocity is positive, i.e. $V > 0$:

$$F_{VEL} = \frac{1}{F_{MAX}} \times \frac{F_{MAX}^{ECC} - F_{MAX}}{|V_{MAX}|} \times V + F_{MAX}$$

Where $F_{MAX}^{ECC} > F_{MAX}$ is the force under eccentric contraction. And the steepness of the curve is defined such that F_{MAX}^{ECC} is reached at $|V_{MAX}|$.

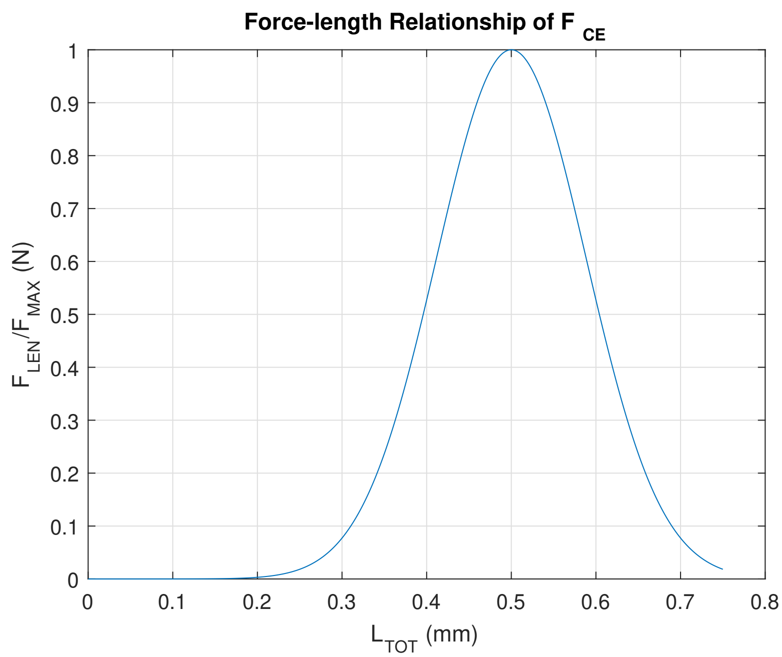


6 Force-Length

Taken from Challis (1994), based on Hatze (1981);

$$F_{LEN} = e^{-(\frac{Q-1}{SK})^2}$$

where $Q = L_{CE}/L_{REST}$ and SK is a material parameter.



Now, returning to the contractile element total force (normalising the individual relationships to maximum force);

$$F_{CE} = \alpha(t) \times \frac{1}{F_{MAX}} \times \frac{a(V_{MAX} - V)}{(b - V)} \times e^{-(\frac{Q-1}{SK})^2}$$

recall that the muscle is currently fully tetanised.

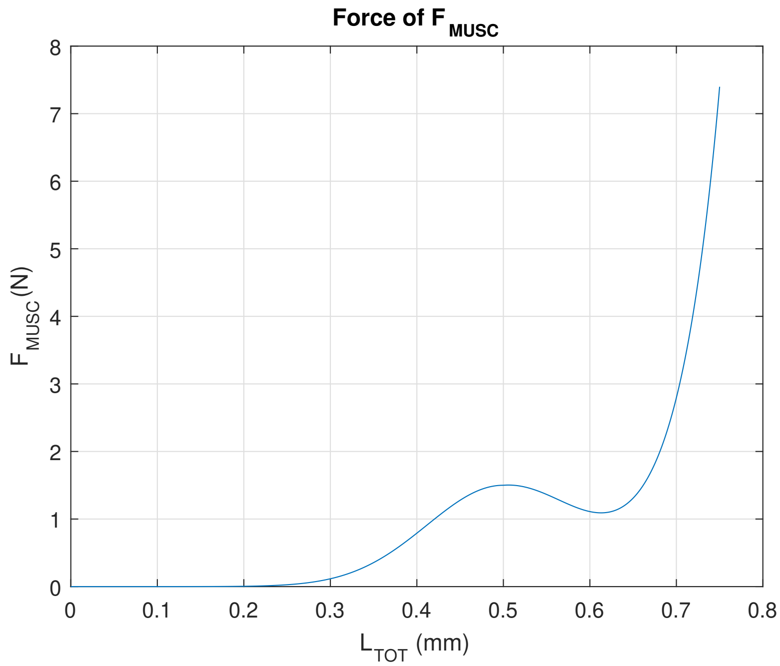
7 Total muscle force

Total force of the muscle is given by summing the passive and active parts;

$$F_{musc} = F_{PE} + F_{MAX} \times F_{CE}$$

$$F_{MUSC} = c \times \{e^{L_{TOT}/L_{REST}-1} - 1\} + F_{MAX} \times \{\alpha(t) \times \frac{1}{F_{MAX}} \times \frac{a(V_{MAX} - V)}{(b - V)} \times e^{-(\frac{Q-1}{SK})^2}\}$$

Now, consider the muscle shortening at a constant velocity.



References

Anderson, C. (2007). Physics-based Simulation of Biological Structures Equations for Modeling the Forces Generated by Muscles and Tendons. In *BIOE215*. Stanford.

Challis, J. (1994). Modeling Muscle : Basics. In *Modeling in Biomechanics*, (pp. 1–22).

Hatze, H. (1981). Estimation of Myodynamic Parameter Values from Observations on Isometrically Contracting Muscle Groups. *Eur J Appl Physiol*, *46*, 325–338.

Hill, A. (1938). The heat of shortening and the dynamic constants of muscle. *Proceedings of the Royal Society of London*, *126*, 136–195.