Contents

1D Muscle Model based on the Hill 2-Element Model

Model Overview

Schematic taken from: Anderson, C. (2007). Physics-based Simulation of Biological Structures Equations for Modeling the Forces Generated by Muscles and Tendons. In *BIOE215*. Stanford.

Global plotting options

Global muscle properties

L_{rest} is the muscle resting length, P_{max} is the maximum force the muscle can produce and v_{max} is the maximum velocity of shortening.

a and b are shape constants for the muscle force-velocity relationship

Parallel elastic element

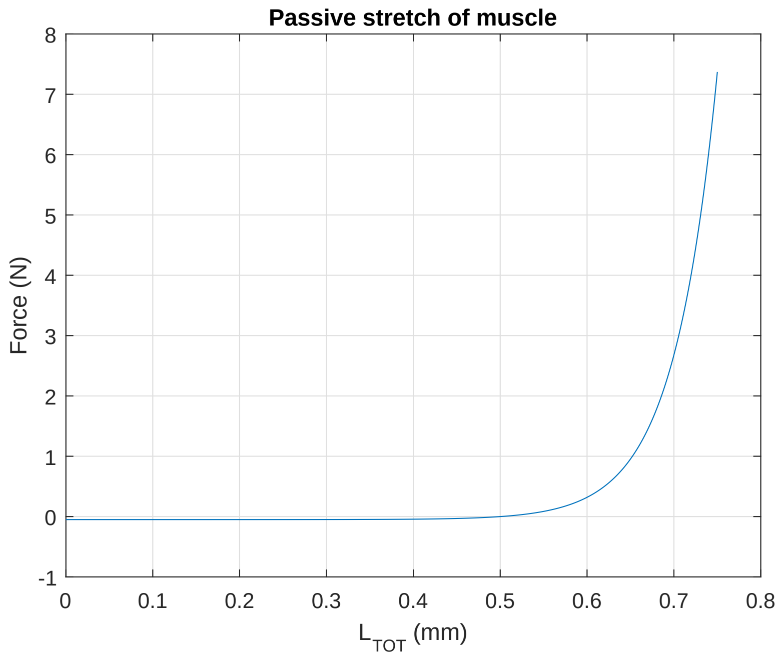
This element represents the passive response of the muscle upon streching greater than resting length.

Note that $L_{PE} = L_{TOT} = L_{CE}$

The form of the passive relationship is not easily found in literature and is implemented in a straight forward way here to match the requirements of an exponential form which is non-zero above L_{rest};

$$F_{PE} = F_{MAX} c \times [e^{L_{TOT}/LREST-1} - 1]$$

Parameter c is introduced to tune the shape of the curve.



Contractile element

This element is reponsible for the active force production in the muscle. The total force is given in the form;

$$F_{CE} = \alpha(t) \times F_{vel}(V) \times F_{len}(L_{TOT})$$

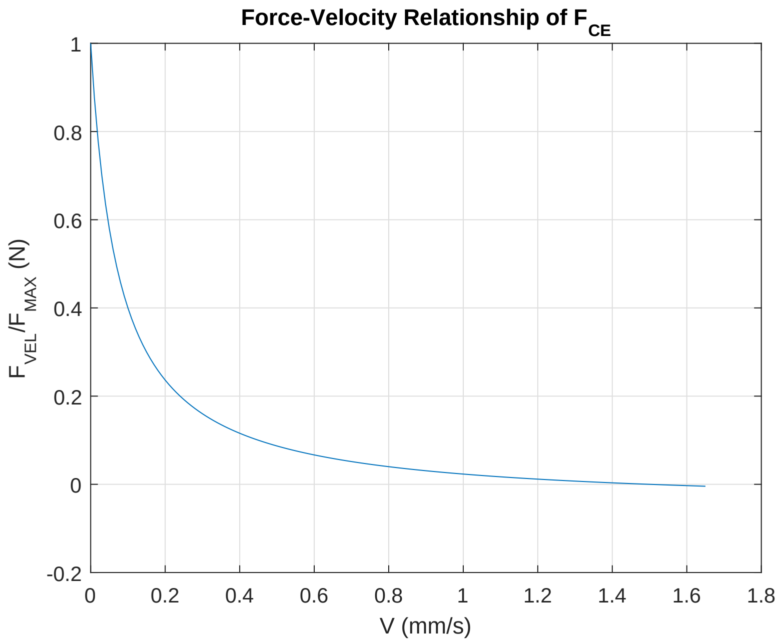
As a first step, we assume that the muscle is fully tetanized, i.e. $\alpha(t) = 1$.

Force-velocity* *

Taken from *"Muscle modelling basics"* by Challis, J. (1994), who based it on on Hill (1938);

$$F_{VEL} = \frac{a(V_{MAX} - V)}{(b - V)}$$

where a and b have been fit to V_{MAX} and F_{MAX}.

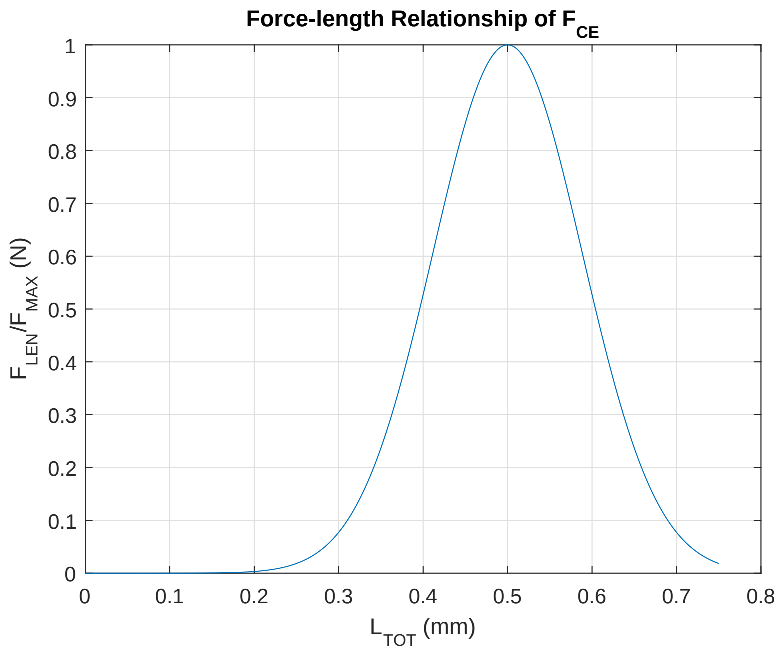


Force-Length

The force-length relationship is also taken from *"Muscle modelling basics"* by Challis, J. (1994), who based it on on Hatze (1981);

$$F_{LEN} = F_{MAX}[e^{-(\frac{Q-1}{SK})^2}]$$

where $Q = L_{CE}/L_{REST}$ and\$SK\$ is a material parameter.



Now, returning to the contractile elment total force (normalising the indiviual relatioships to maximum force);

$$F_{CE} = F_{MAX} \times \frac{a(V_{MAX} - V)}{(b - V)F_{MAX}} \times e^{-(\frac{L_{CE}/L_{REST}-1}{SK})^2}$$

recall that the muscle is currently fully tetanised.

Total muscle force

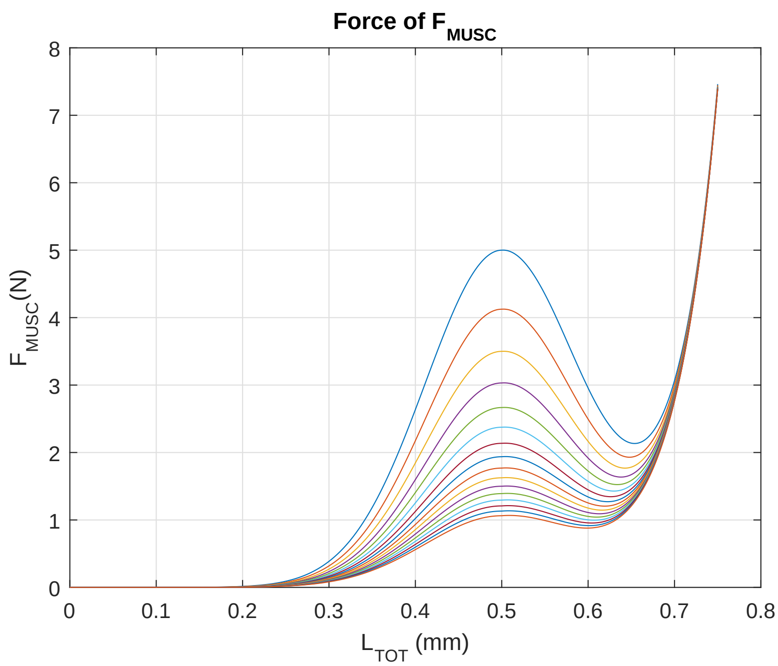
Total force of the muscle is given by summing the passive and active parts;

$$F_{MUSC} = F_{CE} + F_{PE}$$

$$F_{MUSC} = F_{MAX} \times \{\frac{a(V_{MAX} - V)}{(b - V)F_{MAX}} \times e^{-(\frac{L_{CE}/L_{REST}-1}{SK})^2} + c[e^{L_{TOT}/LREST-1} - 1]\}$$

Constant velocity

Now, consider the muscle shortening at a constant velocity.



Isometric Contraction

Now, consider an isometric contraction, that is, velocity $= 0$. How does the force evolve over time?