# 1D Muscle Lumped Parameter Muscle Model

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## 1 Model Overview

A one-dimensional muscle model is implemented in MATLAB and tested under various loading cases. The model consists of an active component incorporating force-length and force-velocity relationships, where dynamic force enhancement is considered under eccentric loading. Furthermore, a passive component provides resistance of the un-activated muscle. Lastly, for certain load-cases, residual force enhancement is implemented with a simple linear model. For example, when a fully activated muscle is loaded beyond it's internal force - the resulting behaviour is taken here as a linear spring with some residual stiffness.

The model is implemented in such a way that each component (force-length relation, force-velocity relation and residual force enhancement relation) can be modified relatively easily without large changes in the main code.

## 2 Global muscle properties

The properties required to characterise the muscle model are listed alongside a short description here.

$L_{REST}$: The length at which peak force is produced

$V_{MAX}$: The maximum contractile velocity of the muscle

$F_{MAX}$: The peak isometric force of the muscle

$c$: Characterisation parameter for passive response

$SK$: Characterisation parameter for the force-length relationship

$\gamma$: Characterisation parameter for the residual force-enhancement

The following parameters are computed internally based on assumptions and definitions.

$F_{MAX}^{ECC}$: Peak force parameter for residual force enhancement

$V_{MAX}^{LENGTHEN}$: The maximum lengthening velocity of the muscle

# 3 Muscle Model Equations

The overall muscle force is computed by:

$$F_{MUSC} = F_{ACT} + F_{PASS} + F_{ENH} \tag{1}$$

where $F_{ACT}$ is the active force, $F_{PASS}$ is the passive force and $F_{ENH}$ is the residual force enhancement component.

## 3.1 Active Force

The active force is given by:

$$F_{ACT} = \alpha \times F_{MAX} \times F_{VEL}(V) \times F_{LEN}(L_{TOT}) \tag{2}$$

where $\alpha$ is the activation level, $F_{VEL}$ is the force-velocity relationship (Section 3.1.1) and $F_{LEN}$ is the force-length relationship (Section 3.1.2)

### 3.1.1 Force-velocity

Taken from Challis (1994), based on Hill (1938).

$$F_{VEL} = \frac{a(V_{MAX} - V)}{(b - V)} \tag{3}$$

where $a$ and $b$ have been fit to $V_{MAX}$ and $F_{MAX}$. The shape parameters for the force-velocity relationship are computed with the $F_{MAX}$ such that;

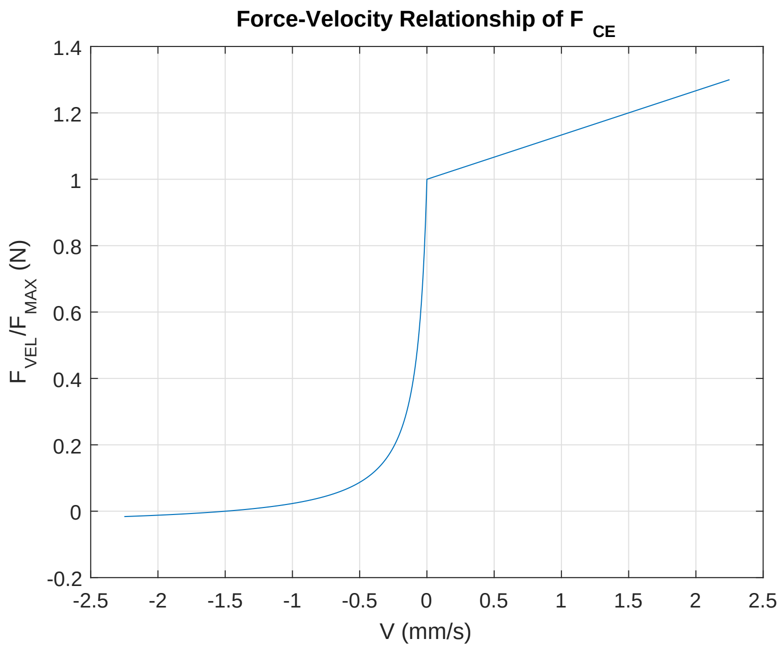
$$b = a \times V_{MAX}/F_{MAX} \tag{4}$$

where $a = 0.25$ is defined. This assures that the force-velocity relationship always gives $F_{MAX}$ when $V = 0$.

Note that velocity is taken as negative when the muscle is shortening, i.e. concentric contractions. To account for eccentric contractions a temporary adjustment is made. When velocity is positive:

$$F_{VEL} = \frac{F_{MAX}^{ECC} - F_{MAX}}{|V_{MAX}|} \times V + F_{MAX} \tag{5}$$

where $F_{MAX}^{ECC} > F_{MAX}$ is the force under eccentric contraction. And the steepness of the curve is defined such that $F_{MAX}^{ECC}$ is reached at $|V_{MAX}|$.

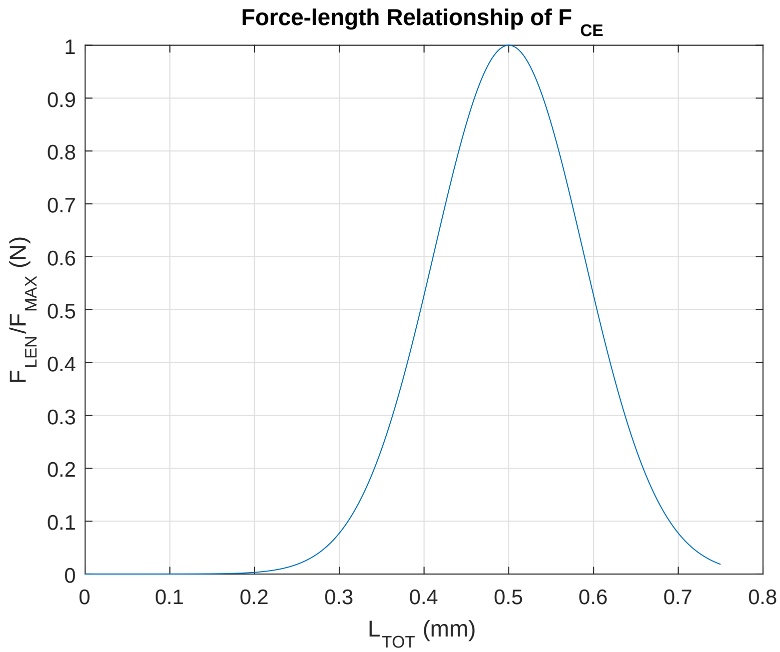



#### 3.1.2 Force-length

Taken from Challis (1994), based on Hatze (1981);

$$F_{LEN} = F_{MAX}[e^{-(\frac{Q-1}{SK})^2}] \tag{6}$$

where $Q = L_{CE}/L_{REST}$ and $SK$ is a material parameter.




Substitute into the active total force (Equation 2);

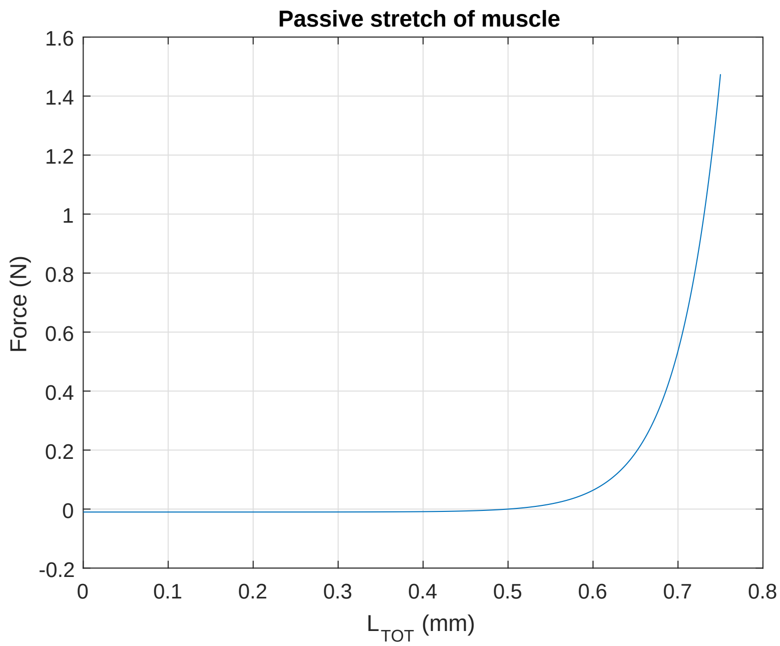
$$F_{ACT} = \alpha \times F_{MAX} \times \frac{a(V_{MAX} - V)}{(b - V)F_{MAX}} \times e^{-(\frac{L_{CE}/L_{REST}-1}{SK})^2} \tag{7}$$

# 4 Parallel elastic element

This element represents the passive response of the muscle upon stretching greater than resting length. The form of the passive relationship is not easily found in literature and is implemented in a straight forward way here to match the requirements of an exponential form, which is non-zero above $L_{REST}$;

$$F_{PE} = F_{MAX} c \times [e^{L_{TOT}/LREST-1} - 1] \tag{8}$$

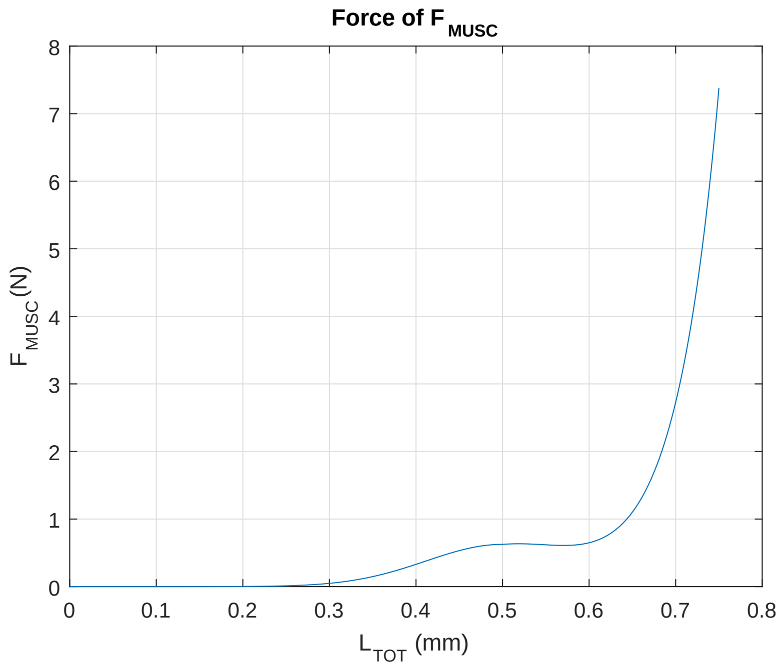
where the parameter $c$ is introduced to tune the shape of the curve.




# 5 Total muscle force

Making the appropriate substitutions into 1

$$F_{MUSC} = F_{MAX} \times \left\{ \frac{a\left(V_{MAX} - V\right)}{\left(b - V\right) F_{MAX}} \times e^{-\left(\frac{\frac{L_{CE}}{L_{REST}} - 1}{SK}\right)^2} + c\left[e^{\frac{L_{TOT}}{L_{REST} - 1}} - 1\right] \right\} \tag{9}$$




# 6 Load Cases of the Lumped Parameter Muscle Model

Consider an external loading as follows:

1. Stretch the muscle passively to some intiial length: $L_{INIT}$
2. Activate the muscle to some activity level: $\alpha_{FIXED}$ (for now, consider full activation) to give an initial (isometric) force: $F_{INIT}$
3. Free one end of the muscle and instantly apply an external load: $F_{EXT}$
4. Find the resulting final length of the muscle

Consider that the muscle has to undergo a dynamic contraction from $L_{INIT}$ to $L_{FIN}$ (while contracting against $F_{EXT}$). The velocity of this contraction is given by the force-velocity law; then the first step is to see if the external load would cause an eccentric or concentric contraction.

Find the velocity for the given external loading (assume for now that it is below the maximal eccentric force. Given that any number of force-velocity laws of the form $F_{VEL} = f(V, V_{MAX}, ...)$ can be used, and there may not exists a analytical solution in the form $V = f(F, V_{MAX}, ...)$. Therefore solve iteratively for a given external force to find the corresponding velocity.

Once the velocity of contraction is found *at the current muscle length*, the next steps are:

1. Update the muscle length

2. Update the force-velocity relationship
3. Find the new contraction velocity (for the fixed $F_{EXT}$)

What is expected is that the velocity of contraction tends towards zero - and the length is found where $F_{EXT}$ is the $F_{MAX}^{LOCAL}$ - and the muscle is at rest.

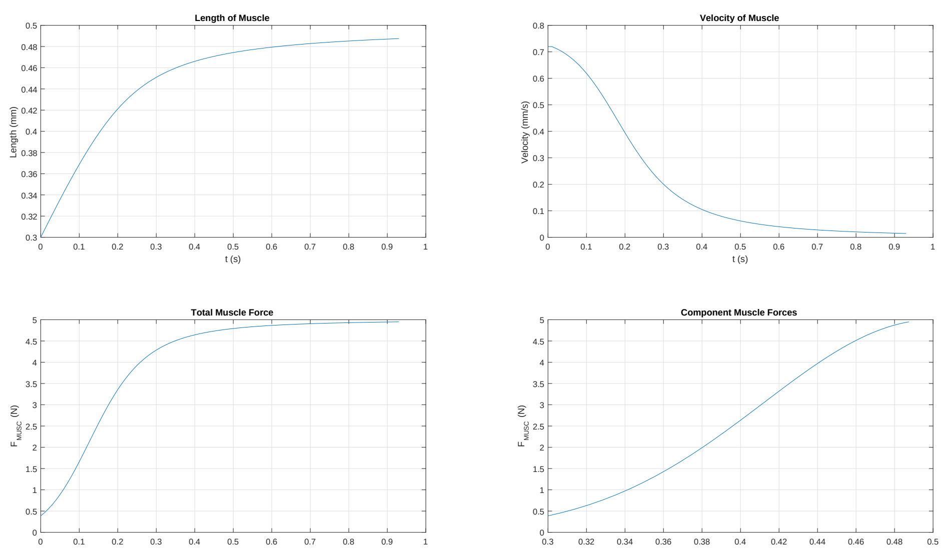
**Remark**: Here the acceleration and deceleration phases of the muscle are not explicitly modelled. In that the muscle has no mass. Any acceleration or deceleration that arises is due to the dynamic change in the force-velocity relationship; i.e. $F_{VEL} = f(V, L, V_{MAX}, F_{MAX}^{LOCAL})$

**Force enhancement** occurs for the case when a fully activated muscle is undergoing an eccentric contraction; in fact only on the descending limb of the force-length response. Therefore, the computation steps are:

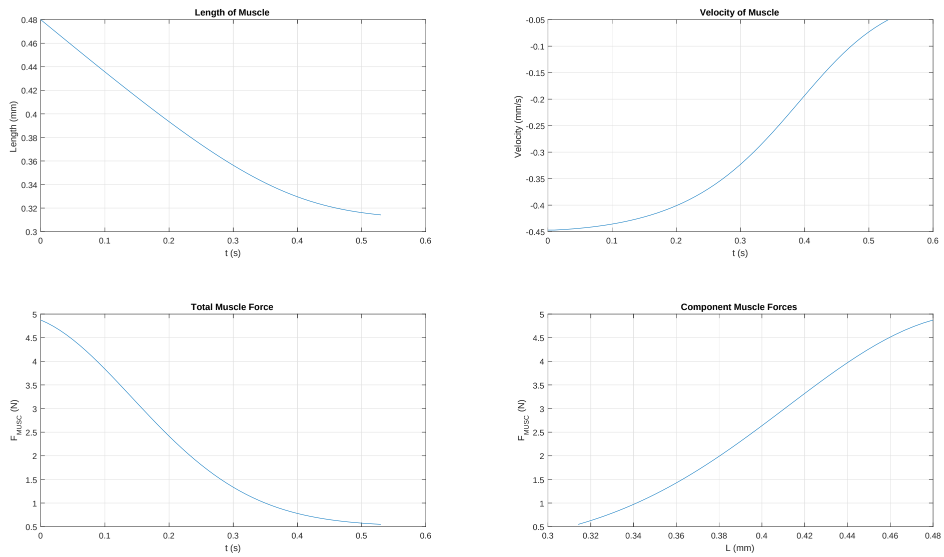
1. Check if current muscle length $L_{TOT}$ is on the decending limb by taking a small perturbation in the length. If on descending limb, continue, if not escape.
2. Check if the velocity will cause an elongation of the muscle
3. Determine the change in length; based on the velocity and time-step
4. Use the force-enhancement law to compute the resulting force enhancement $F_{ENH}$

This is then added to the total muscle force as in equation 1.

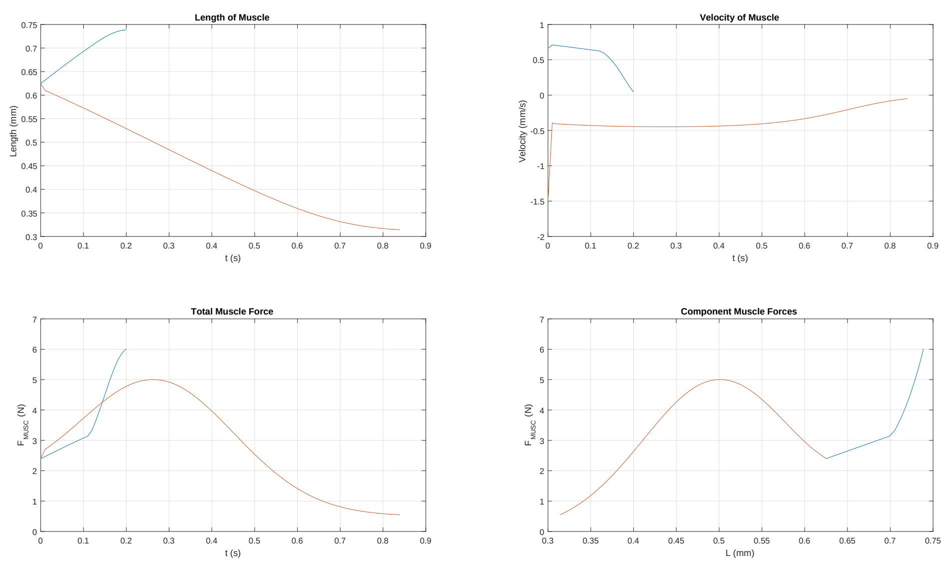
## 6.1 Ascending limb eccentric



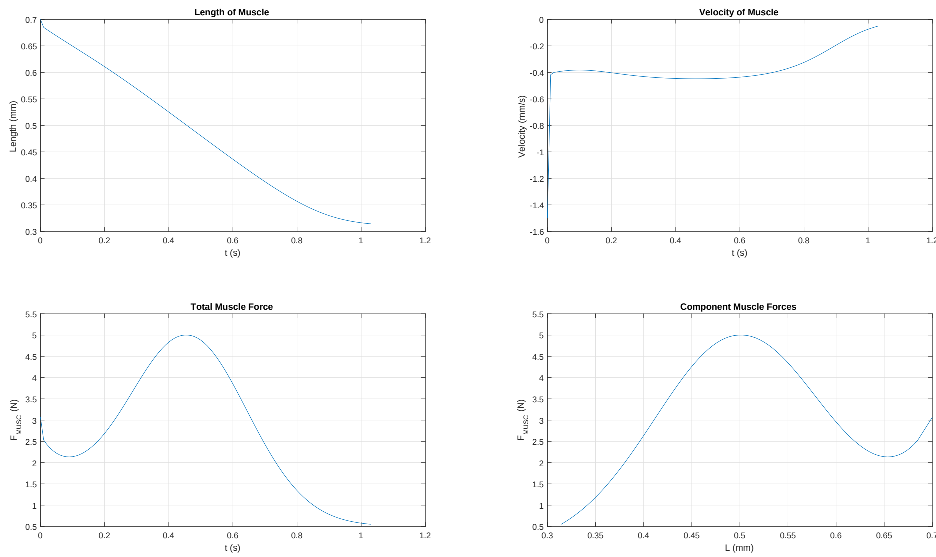
## 6.2 Ascending limb concentric




## 6.3 Descending limb eccentric



## 6.4 Descending limb concentric




# References

Challis, J. (1994). Modeling Muscle : Basics. In *Modeling in Biomechanics*, pages 1–22.

Hatze, H. (1981). Estimation of Myodynamic Parameter Values from Observations on Isometrically Contracting Muscle Groups. *Eur J Appl Physiol*, 46:325–338.

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