Time reversal non-invariance in stochastic theories: Can GRW explain the irreversibility of macroscopic physics?

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Abstract

After explaining the notions of time reversal invariance and reversibility, I argue that the world is reversible in the microscopic view and irreversible in the macroscopic view, and that this poses a problem. Next, I describe Boltzmann's solution to this problem in the form of statistical mechanics, and the Past Hypothesis. In his book "Time and Chance", David Albert has suggested a new proposal to solve this problem: if one assumes that the theory of Ghirardi, Rimini, and Weber (GRW) provides the true dynamics of the world, one would obtain a mechanism by which abnormal microstates would be perturbed into normal microstates. If GRW is time reversal non-invariant, this can be done without the need for the Past Hypothesis. I compare three principles for stochastic time reversal invariance, viz microreversibility, retrodictability, and the principle by Holster and Bacciagaluppi, and argue that the last one is the correct one. It is proven that GRW is time reversal non-invariant according to that principle, meaning that, for the purpose of explaining the irreversibility of the macroscopic view, GRW can replace statistical mechanics without a need for the Past Hypothesis.

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Chapter 0

Introduction

One of the greatest problems in the literature of the foundations of physics is that in microscopic views the world seems to be reversible, whereas in macroscopic views the world seems to be irreversible. In this introduction a description is given of the concept of reversibility. In chapter 1 the problem is described in detail. Chapter 2 describes the historical suggestion by Boltzmann to solve the problem, objections to this suggestion, and the Past Hypothesis. More recently, David Albert ([1]) made another suggestion to solve the above problem by taking recourse to the stochastic mechanism developed by Ghirardi, Rimini, and Weber (GRW, [9]), claiming that it perturbs a system from abnormal microstates into normal microstates in such a manner that the tension between the microscopic and macroscopic view is removed. Albert's suggestion is described in chapter 3. For GRW to be a solution to the problem without a need for the Past Hypothesis, it has to be time reversal non-invariant. In chapter 4 an account of time reversal invariance for stochastic laws of nature is given. In chapter 5 it is proven that GRW is time reversal non-invariant, arriving at the conclusion that the perturbation by GRW of abnormal microstates into normal microstates is a solution to the problem and does not need the Past Hypothesis.

0.1 The order of time

In this section we rehearse some temporal notions that are central to a proper discussion of the problem. Through the order of states in a single process we will arrive at a time coordinate that can be assigned to any state in a process. Next, the reverse process is described and, finally, adjacent problems are brought up that however will be left aside.

In most physical theories, the *state* of a system gives a description of that system at an instant by specifying the values of certain properties the system possesses. A state has to be complete, i.e. all of the physical facts about the system at the time in question can be read off the state at that time. In this section I will occasionally use classical particle mechanics as an illustration. A state in classical mechanics of a system consisting of many particles is a list of the positions $\vec{r}$ and velocities $\vec{v}$ of all particles in the system at a certain instant. I will denote a state with a capital letter. *Phase space* is a $6n$-dimensional space, with n the number of particles, where every degree of freedom, i.e. the position or velocity in the x, y, or z direction for every particle, is represented as an axis. A state may then be represented by a point in phase space. A *process* is a sequence of states, which can be depicted as a trajectory in phase space. To simplify notation, I will, although this is not completely accurate because it suggests that a process consists of a finite number of states, write down a process as a sequence of capital letters, e.g. ABC.

To order the states in a process unambiguously a total order relation is needed. For non-relativistic time, this is the relation 'earlier than or simultaneous with' (see Kroes [14]). A relation $\precsim$ between two states in a given process is called a total order relation if for all states A, B, and C the following relations hold:

(1) reflexivity:	A$\precsim$A
(2) anti-symmetry:	A$\precsim$B and B$\precsim$A implies A=B
(3) transitivity:	A$\precsim$B and B$\precsim$C implies A$\precsim$C
(4) totality:	if A$\neq$B: A$\precsim$B or B$\precsim$A but not both

Strictly speaking, 'earlier than or simultaneous with' is not a total order relation because the condition of anti-symmetry is not met. A$\precsim$B and B$\precsim$A does not imply that A equals B, but that A is simultaneous with B. However, at a certain instant in a process a system can be in only one state and therefore A$\precsim$B and B$\precsim$A does imply that A equals B.

A fifth condition can be imposed. Since the number of states 'between' two states in a process is uncountably infinite, where a state B is 'between' A and C if A$\precsim$B and B$\precsim$C, or, C$\precsim$B and B$\precsim$A, the relation 'earlier than or simultaneous with' gives a continuous total order.

It is then possible to define a function t(A) that uses the relation 'earlier than or simultaneous with' in order to assign a time coordinate to the state A, so that

$$t(\mathrm{A}) \leq t(\mathrm{B}) \qquad \Leftrightarrow \qquad \mathrm{A}\precsim\mathrm{B}. \tag{0.1}$$

But if t is such a function, so is every monotonically increasing function $f(t)$. An aid in the search of one such function is the choice of a cyclical process that can serve as clock. A cyclical process is a process of a system in which the system repeatedly goes from an initial state A, through some intermediate states B,C,D,...,Z and then back to the initial state (see Schlegel [16]). A clock is a cyclical process chosen by intuition or convention as having a constant period. The motion of celestial bodies, the rotation of the earth, a swinging pendulum and, more recently, the cesium atom are all examples of clocks. We would like the function $f(t)$ to assign equal time intervals to the cycles of a clock. Therefore, $f(t)$ has to be a linear function.

Two things have to be specified further in order to single out the function $f(t)$: a unit (in order to assign a value to an interval) and an origin (to know which state corresponds to $t = 0$). The unit is usually based on the cycles of a clock. The origin can also be taken at will and usually corresponds with the first state of the process under investigation.

When a state in a process is 'simultaneous with' a state in a clock, it is possible to assign a universal time coordinate t(A), t(B), and t(C) to states A, B, and C, in processes (even when it's not the same process) and place them somewhere on a single axis.

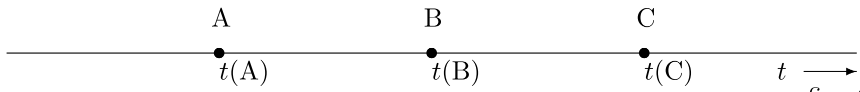

fig. (0.1)

For this thesis it is important to specify what the reverse process of a given process is. For instance, (in simplified notation) if the system was in the states A, B, and C, where $t_A < t_B < t_C$, the process is depicted as ABC, or (more accurately) for classical mechanics as the trajectory $\{(\vec{r}(t), \vec{v}(t))|t \in \mathbb{R}\}$ in phase space.

Now, one might expect the reverse process to just be the states of the process in reverse, i.e. the sequence CBA. However in classical mechanics this would mean that a process of a particle moving, at times $t_A < t_B < t_C$, with positions and velocities ($\vec{x}_A$, $\vec{v}_A$), ($\vec{x}_B$, $\vec{v}_B$), and ($\vec{x}_C$, $\vec{v}_C$) respectively, would be reversed into a process of a particle moving, at times $t_C < t_B < t_A$, with positions and velocities ($\vec{x}_C$, $\vec{v}_C$), ($\vec{x}_B$, $\vec{v}_B$), and ($\vec{x}_A$, $\vec{v}_A$) respectively. This means that if in the original process the position increases (positive velocities), the position decreases in the reverse process although the velocities are still positive.

Therefore, the operation of reversing a process should also take into account that a state can have information of how the state evolves. This operation and specifically the reversal of this information of the evolution of the state can be different for every theory. So, the reverse process is the reversed sequence of the reversed states, i.e. the sequence ^{R}C ^{R}B ^{R}A, where ^{R}A is the reverse of the state A. In the example of classical mechanics the reverse process would be $\{(\vec{x}(-t), -\vec{v}(-t))|t \in \mathbb{R}\}$.

It is often thought that since there is a directedness in time, i.e. there are two opposite directions, it is implied that there is movement; that time flows in a certain manner. This is not necessary. For instance, when I point towards the east, I specify a direction but in no way am I obliged to move in that direction.

An important question is whether the direction of a sequence, i.e. whether we are looking at the process as it occurred in reality or the reverse process, can be determined without reference to anything but the states in the sequence. If this is so, the ordering relation 'earlier than or simultaneous with' is called an intrinsic ordering relation. The search for 'the Arrow of Time' is the search for a physical correlate for the direction of time. Although it is closely related to the subject of irreversibility, it will not be discussed here.

In this thesis the notion of the order of time and that of the reverse process are used in the notions of reversibility and time reversal invariance. These are described in the next section.

0.2 Processes and laws of nature: reversibility and time reversal invariance

A subtle distinction has to be made between processes and laws of nature. Processes are reversible or irreversible and laws of nature are time reversal invariant or time reversal non-invariant.

According to Horwich [13], there are two ways in which a process can be irreversible: nomologically and de facto. A process is nomologically irreversible if, by at least one of the laws of nature that apply to it, the process, consisting of the sequence ABC of states, is allowed, but the reverse process ^{R}C ^{R}B ^{R}A is not. For nomological reversibility the reverse process must be allowed by all laws of nature that apply to the original process.

A process is de facto irreversible if, when that process is allowed by all laws of nature, the reverse process is also, but just never or hardly ever occurs because of initial or boundary conditions. This means that a nomologically reversible process can be de facto irreversible. However, after it is determined that a process is nomologically irreversible, there is, by definition, no need to examine the process for de facto irreversibility. One might similarly define a de facto reversible process by the stipulation that the reverse process is allowed by all laws of nature and does actually occur. However, this definition may be unpractical to work with and is hardly ever used.

According to Earman ([6]), a law of nature is time reversal invariant if, for every process, represented by the sequence ABC of states, that is allowed by the law of nature, the reverse process ^{R}C ^{R}B ^{R}A is allowed as well. A law of nature is time reversal non-invariant if, for one or more of the processes that are allowed by the law of nature, the reverse process is not allowed.

This means that a law of nature is time reversal invariant if all processes to which the law of nature applies are nomologically reversible, and conversely, a process is nomologically reversible if all laws of nature that apply to the process are time reversal invariant.

In this thesis the notions of reversibility and time reversal invariance will be applied to respectively processes and laws of nature in both the microscopic and the macroscopic view. In chapter 1 it will be shown that there is a contradiction between the microscopic and the macroscopic view in this respect.

Chapter 1

Problem: Why is the world irreversible in the macroscopic view but reversible in the microscopic view?

In the literature of the foundations of physics it is taken as one of the greatest problems that in thermodynamics (i.e. a macroscopic view) the world is irreversible, whereas in microscopic views the world is reversible. The introduction explored the notion of reversibility. This chapter explores how the world is reversible in the microscopic view, how the world is irreversible in the macroscopic view and why this apparent contradiction is a problem.

1.1 How is the world reversible in the microscopic view?

When we say the world is reversible in the microscopic view, we can mean one of two things: the laws of nature governing the world are time reversal invariant in the microscopic view, or, the processes in the world are reversible in the microscopic view.

But first, what is a microscopic view of the world? The line between microscopic and macroscopic is rather vague. It can be said that a microscopic view uses elementary constituents as the primary object of investigation. E.g. quantum mechanics is a microscopic view of the world. In a macroscopic view many particles together (usually taken at least to be in the order of Avogadro's number) are the primary object of investigation. E.g. thermodynamics is a macroscopic view of the world. Classical mechanics, electromagnetics and relativity theory view the world both microscopically and macroscopically.

Let us look at the time reversal invariance of the laws of classical (particle) mechanics. One way of describing classical mechanics is through Hamilton's equations (for simplicity a one-particle system is used):

$$\frac{\partial H}{\partial q(t)} = -\frac{dp(t)}{dt} \tag{1.1}$$

$$\frac{\partial H}{\partial p(t)} = \frac{dq(t)}{dt}, \tag{1.2}$$

where H is the Hamiltonian of the system. It is usually assumed that all Hamiltonians in nature are independent of time. Time dependent Hamiltonians only arise in experiments where, say, an experimenter pushes a button to change the Hamiltonian of the system of investigation. However, if the entire system (i.e. system of investigation and whatever the experimenter changes) is taken into consideration, the Hamiltonian turns out to be independent of time after all.

$q(t)$ and $p(t)$ are respectively the position and momentum of the particle at time t. The laws of classical mechanics are time reversal invariant if, for any process (with the trajectory $(q(t), p(t))$ in phase space) that is a solution of the equations, the reverse process (with the trajectory $(q(-t), -p(-t))$) is a solution too. As explained in section 0.1, the momentum is reversed, because in the reverse process we want the momentum to correspond with the change in position as well.

When we make the substitution $t \to -t$ in Hamilton's equations (1.1) and (1.2) we arrive at

$$\frac{\partial H}{\partial q(-t)} = -\frac{d(-p(-t))}{dt} \tag{1.3}$$

$$\frac{\partial H}{\partial(-p(-t))} = \frac{dq(-t)}{dt}. \tag{1.4}$$

This means that the reverse process with the trajectory $(q(-t), -p(-t))$ is indeed a solution of Hamilton's equations as well and the laws of classical mechanics are time reversal invariant.

Let us now look at orthodox quantum mechanics. In quantum mechanics the time evolution of the state vector $|\phi\rangle$ is given by the Schrödingier equation:

$$H|\phi(t)\rangle = i\hbar\frac{\partial}{\partial t}|\phi(t)\rangle, \tag{1.5}$$

where H is the time independent Hamiltonian of the system. Quantum mechanics is time reversal invariant if, when $|\phi(t)\rangle$ is a solution of equation (1.5), $T|\phi(t)\rangle$ is too, with T the time reversal operator. We specify the action of T by:

$$T|\phi(t)\rangle = |\phi^*(-t)\rangle, \tag{1.6}$$

and not by $|\phi(-t)\rangle$ as might have been expected. This is so because in the state, $|\phi(-t)\rangle$, there is both information about position and momentum. Now, we want the position unchanged but the momentum reversed, just as in classical mechanics. We thus require:

$$\begin{aligned} T\hat{x}T^{-1} &= \hat{x} && (1.7) \\ T\hat{p}T^{-1} &= -\hat{p}, && (1.8) \end{aligned}$$

where $\hat{x}$ and $\hat{p}$ are the position and momentum operators respectively, denoted with the hat symbol to accentuate the fact that we are dealing with operators here. When we compare the commutation relations for the ordinary and the reverse operators:

$$\begin{aligned} [\hat{p},\hat{x}] &= i\hbar && (1.9) \\ [T\hat{p}T^{-1}, T\hat{x}T^{-1}] &= \begin{cases} [-\hat{p},\hat{x}] = -i\hbar \\ T[\hat{p},\hat{x}]T^{-1}, \end{cases} && (1.10) \end{aligned}$$

we arrive at

$$TiT^{-1} = -i. \tag{1.11}$$

Therefore, it is needed for T to perform the substitution $t \to -t$ and a complex conjugation on everything written after it (see Sachs [15]). It is noteworthy to mention that T is not a unitary operator but an anti-unitary operator, i.e. for two arbitrary state vectors $|\phi\rangle$ and $|\psi\rangle$:

$$\langle T\phi|T\psi\rangle = \langle\phi|\psi\rangle^*. \tag{1.12}$$

Applying T to the Schrödingier equation (1.5) gives

$$\begin{aligned} TH|\phi(t)\rangle &= Ti\hbar\frac{\partial}{\partial t}|\phi(t)\rangle \\ &= -i\hbar T\frac{\partial}{\partial t}|\phi(t)\rangle \\ &= i\hbar\frac{\partial}{\partial t}(T|\phi(t)\rangle). && (1.13) \end{aligned}$$

For $T|\phi(t)\rangle$ to be a solution of the Schrödingier equation (1.5), T and the Hamiltonian must commute. This means H has to be self-adjoint and time reversal invariant. All known physical processes have time reversal invariant Hamiltonians (except, allegedly, those involving K mesons). A Hamiltonian has to be self-adjoint in order to be observable. So, $T|\phi(t)\rangle$ is also a solution of equation (1.5) and the laws of quantum mechanics are time reversal invariant.

The time reversal invariance of electromagnetics and relativity theory can also be proven but will not be discussed here.

All processes can be viewed microscopically by means of the laws of classical mechanics, quantum mechanics, electromagnetics, or relativity theory. Since these are all time reversal invariant, it follows that if a process, described microscopically, is allowed by the laws of nature, then its time reversed process is also allowed; they are all nomologically reversible.

It might be argued that for nomologically reversible processes the reverse processes, viewed microscopically, do actually occur making them also de facto reversible. However intuitive this may seem, verifying it would be a tedious job.

So, the reversibility of the world in the microscopic view expresses itself in the time reversal invariant laws and in the nomologically and perhaps de facto reversible processes that take place in it.

1.2 How is the world irreversible in the macroscopic view?

Let us now look at the time reversal non-invariance of the laws and the irreversibility of the processes in the macroscopic view. The laws of classical mechanics, electromagnetics and relativity theory are still time reversal invariant on a macroscopic view, just as they were on a microscopic view, whereas the laws of quantum mechanics are not a macroscopic view. So, the laws of these theories cannot explain the irreversibility of the world on a macroscopic view. Four (other) macroscopic theories will be described and examined for time reversal non-invariance. After describing the laws, the processes will be described on a macroscopic view and examined for irreversibility.

The first theory to be described is thermodynamics. One formulation of the Second Law of thermodynamics is as follows: if a system in equilibrium at time t_1 is disturbed and evolves adiabatically until, at time t_2, it is in equilibrium again, then the entropy S_2 of the system at time t_2 is larger than or equal to the entropy S_1 of the system at time t_1:

$$S_2 \geq S_1 \qquad \text{if} \qquad t_2 > t_1. \tag{1.14}$$

It is clear that if the evolution of a system in a state with S_1 into a state with S_2 complies with equation (1.14), then the reverse process, the evolution of a system in a state with S_2 into a state with S_1 does not, except for the very special case where $S_1 = S_2$. So, since all reverse processes have to be allowed, it can be said that the laws of thermodynamics are time reversal non-invariant.

Of course, thermodynamics has been superseded by statistical mechanics. According to this theory, it is not impossible that a system in a state with entropy S_2 evolves into a state with a lower entropy S_1, it is merely very improbable. According to the definition for time reversal invariance we are maintaining at the moment, this would make statistical mechanics a time reversal invariant macroscopic theory. Chapter 2 gives a description of the attempts made, before Albert's, to reconcile the reversible microscopic descriptions of classical and orthodox quantum mechanics with the irreversible macroscopic description. Statistical mechanics is exactly that attempt at reconciliation between the microscopic and the macroscopic view.

The second macroscopic view described is Ohm's law for an electrical circuit, written in its continuum form as

$$\vec{J}(\vec{r}, t) = \sigma \vec{E}(\vec{r}, t), \tag{1.15}$$

where $\vec{J}(\vec{r},t)$ is the current density (current flowing through the (semi-)conductor per unit area) at position $\vec{r}$ and time t, σ the conductivity of the (semi-)conductor, and $\vec{E}(\vec{r},t)$ the electric field at position $\vec{r}$ and time t brought about by a changing electric potential ($\sigma\vec{E}(\vec{r},t) \neq \vec{0}$). In this theory a process is depicted as $\vec{J}(\vec{r},t)$. When time is reversed, the current will flow in the opposite direction, so we would expect the current density to be reversed, making the reverse process $-\vec{J}(\vec{r},-t)$.

The electric field will not be reversed, i.e. when $t \to -t$ then $\vec{E}(\vec{r},t) \to \vec{E}(\vec{r},-t)$. The reason is as follows. An electric field is usually brought about by a source, e.g. an electron (it may be argued that there are electric fields without sources, but this is outside the scope of this thesis). Imagine the process of this electron to bring about an increase in the electric field, then the reverse process would bring about a decreasing electric field, but this electric field would still be pointing in the same direction.

If Ohm's law is to be time reversal non-invariant, then, if the process $\vec{J}(\vec{r},t)$ is a solution of equation (1.15), the reverse process $-\vec{J}(\vec{r},-t)$ is not. Substituting $-t$ for t in equation (1.15), it is easy to see that $-\vec{J}(\vec{r},-t)$ is not a solution to the equation, making Ohm's law time reversal non-invariant.

The third macroscopic law we will look at is the equation for the diffusion of particles:

$$\frac{\partial\phi(\vec{r},t)}{\partial t} = D\nabla^2\phi(\vec{r},t), \tag{1.16}$$

where $\phi(\vec{r},t)$ is the density of particles at position $\vec{r}$ and time t and D is the diffusion coefficient (here taken as a constant and nonzero). For the time reversal non-invariance of equation (1.16), if $\phi(\vec{r},t)$ is a solution, then $\phi(\vec{r},-t)$ should not be. Making the substitution $t \to -t$ in equation (1.16) reveals that $\phi(\vec{r},-t)$ is not a solution and that the diffusion equation is time reversal non-invariant.

Fourthly and finally, the Cauchy equation of motion for any continuous medium is:

$$\rho\Big(\frac{\partial\vec{v}(\vec{r},t)}{\partial t} + \nabla\cdot(\vec{v}(\vec{r},t)\otimes\vec{v}(\vec{r},t))\Big) = \nabla\cdot\mathbb{P} + \rho\vec{f}(\vec{r},t), \tag{1.17}$$

where ρ is the mass density (here taken to be constant), $\vec{v}(\vec{r},t)$ is the velocity of the continuous medium at position $\vec{r}$ and time t, $\mathbb{P}$ the stress tensor, $\vec{f}(\vec{r},t)$ the force per unit mass at position $\vec{r}$ and time t, and $\nabla\cdot\mathbb{P} + \rho\vec{f}(\vec{r},t) \neq \vec{0}$.

For equation (1.17) to be time reversal non-invariant, we need, when the process $\vec{v}(\vec{r},t)$ is a solution, the reverse process $-\vec{v}(\vec{r},-t)$ not be a solution. Again performing the substitution $t \to -t$ shows that $-\vec{v}(\vec{r},-t)$ is not a solution and that the Cauchy equation is time reversal non-invariant.

After looking at the laws on a macroscopic view, we will now pay attention to the processes. First, the processes that are governed by the Second Law of thermodynamics will be reviewed. For processes that have an S_2 that is larger than S_1, it is not allowed by the Second Law that the reverse process occurs; these processes are therefore nomologically irreversible. For processes that have

$S_2 = S_1$ the reverse process is allowed by the Second Law, which makes these processes nomologically reversible.

It can be debated whether processes with $S_2 = S_1$ are de facto reversible as well, i.e. wether they actually occur in nature or not. Take for example a system with a gas in a cylindrical container with on one side a moveable piston, all of this placed in a heat bath. The system is isolated and has entropy S_1. If the piston is pushed very slowly, making sure no energy is added to the system through friction, the temperature of the gas and of the heat bath increases. Now the entropy of the entire system is S_2 but this is exactly the same as S_1, since the system is isolated. If the piston is released, the temperature of the gas and the heat bath will decrease, because warmth is converted in work and the piston is pushed back out again. The reverse process occurs which would make the process de facto reversible. However, it might be said that this process is an idealization, since the frictionless pushing of the piston does not occur in nature, countering the claim of de facto reversibility.

For the processes allowed by Ohm's law (1.15), the diffusion equation (1.16), or Cauchy's equation (1.17), the reverse process is not allowed by that law. Therefore, these processes are nomologically irreversible.

So, the world is irreversible on the macroscopic view because at least four important macroscopic laws are time reversal non-invariant and almost all processes governed by these laws are nomologically irreversible.

1.3 Why is this a problem?

In the previous two sections I have explained that the world is reversible in the microscopic view whereas it is irreversible in the macroscopic view. Now the question is, why is this a problem?

The problem is that the macroscopic view describes systems as a whole that consist of exactly the same constituents that are described individually in the microscopic view. Now, the process of the system is nothing more than the processes of all the objects taken together. So, if in the microscopic view processes and their reverse processes are allowed for the constituents of the system, then we would expect that for the entire system the corresponding process and its reverse process is allowed as well. But, according to laws in the macroscopic view, its reverse process is not allowed.

One way of solving this contradiction is by showing that it is possible to arrive at irreversibility in the macroscopic view from reversibility in the microscopic view. In chapter 2 a description is given of how statistical mechanics as developed by Boltzmann does exactly this. Objections to and improvements of the theory are also described.

Another way of solving this problem is a microscopic view that has time reversal non-invariant laws, so in the microscopic view the reverses of allowed processes are not allowed. Then there is no contradiction between the two different views. Chapter 3 explores to solve the problem through an expansion of a suggestion by Albert ([1]). According to this, the time reversal non-invariant laws of the microscopic stochastic GRW theory cause an irreversible perturbation of abnormal microstates into normal microstates, thereby eliminating the contradiction between the two views.

Chapter 2

Suggestion: Boltzmann and the Past Hypothesis

In this chapter it is described how Boltzmann saw statistical mechanics as the way to make reversible processes in the microscopic view compatible with irreversible processes in the macroscopic view. Of course there were objections to this theory which are described in the second section of this chapter. Improvement were made which led to the Past Hypothesis, described in the third section.

2.1 Boltzmann's suggestion to solve the problem

According to Boltzmann (and I will use Goldstein's account ([11])), a system can be described with microstates or macrostates, which Albert (p. 17 and 39) calls microconditions and macroconditions. We, however, will stick to the names microstates and macrostates, as in standard literature (e.g. Garrod [8]). A *microstate* X is a specification of the position and velocity of all particles and is depicted as one point in phase space. This is exactly the same as what in classical particle mechanics is referred to as a state. A *macrostate* is a region in phase space consisting of microstates with equal macroscopic properties (e.g. temperature or pressure). The macrostate to which a microstate X belongs is depicted as $\Gamma(X)$.

It is also possible to look at a microstate in μ-space. μ-space is a 6-dimensional space, in which the position and velocity (both 3 dimensions) of a single particle are represented by a point. If we divide the μ-space into partitions, we can specify the number of particles in each partition. This specification then determines a macrostate.

At first, Boltzmann used the language of microstates and macrostates to try and prove the second Law of Thermodynamics. But then two objections arose.

The first objection to Boltzmann's suggestion that I will discuss is by Loschmidt and goes by the name "Umkehreinwand" or reversibility objection. It states that since the Hamilton equations are time reversal invariant, the reverse process, i.e. where the microstate belonging to the equilibrium macrostate $\Gamma(X_2)$ evolves into the non-equilibrium microstate X_1, is also possible. Whereas the original process obeys the second Law of Thermodynamics, the reverse process does not, but is possible nonetheless and so Boltzmann is wrong.

The second objection is by Zermelo and goes by the name "Wiederkehreinwand" or recurrence objection. It uses Poincaré's recurrence theorem which says that a system with a finite energy and confined in a finite space will after a sufficiently long time return within an arbitrarily small neighbourhood of its initial state. This means that a system, which is initially in state X_1, may spend most of the subsequent time in a microstate of $\Gamma(X_2)$, but will eventually return to microstate X_1. Or at least, that the last statement holds for almost every state X_1 and that the class of states for which it does not hold has measure zero. This process would not obey the second Law of Thermodynamics and prove Boltzmann wrong.

What is proven here is that reverse processes are possible. This contradicts the Second Law of Thermodynamics, since it is a strict law: the entropy must increase or remain equal, but in any case is not allowed to decrease. Boltzmann then turned to a statistical view on reversible processes, statistical mechanics, saying that the reverse process is not impossible but merely extremely improbable. It goes like this.

The dynamics of the particles (e.g. in classical particles mechanics) do not favour one microstate over another; the probability of every microstate is equal. In contrast, not every macrostate is equally probable. Let us take for example a rectangular box filled with a gas. Then, whereas a microstate X_1 with all particles in one corner (a non-equilibrium microstate) is just as probable as a microstate X_2 with the particles dispersed uniformly throughout the box (an equilibrium microstate), the number of microstates that belong to the macrostate $\Gamma(X_1)$ is vastly smaller than the number of microstates that belong to the macrostate $\Gamma(X_2)$. For a system of 10^{20} particles it is of the order of $10^{10^{20}}$ times smaller. So, since every single microstate has an equal probability, the macrostate $\Gamma(X_2)$ is far more probable than the macrostate $\Gamma(X_1)$. The number of microstates that belong to a macrostate can be equated with the (hyper)volume of the macrostate in phase space. This volume can be determined by the Liouville measure.

The idea by Boltzmann how to reconcile the reversible processes in the microscopic view with the irreversible processes in the macroscopic view then becomes the following. If we were to start with the non-equilibrium microstate X_1, it would be amazingly improbable for the dynamics governed by the Hamilton equations not to lead this microstate into a microstate which is in the equilibrium macrostate $\Gamma(X_2)$. If we assume that the microstate X_1 was in equilibrium before it was in non-equilibrium (in our example, assume that it was bounded by a smaller box located in the corner which was subsequently released), then the entropy can be determined. Now we see that the second Law of Thermodynamics holds and reversible processes in the microscopic view have led to irreversible processes in the macroscopic view.

2.2 The Past Hypothesis

So, Boltzmann proved how incredibly probable it is for the volume of the macrostate in phase space to increase towards the future. But thought should be given to the fact that the underlying dynamics are time reversal invariant. If a microstate goes through a certain process, then the reverse microstate goes through the reverse process. This means that if a macrostate is probable to have an increase of volume in phase space towards the future, it is just as probable to have an increase towards the past. This was however never denied in statistical mechanics. According to Albert ([1], p.77), statistical mechanics is perfectly compatible with saying that the volume of the macrostate in phase space is as probable to increase towards the future as it is towards the past. This can be seen as that in the present the volume of the macrostate in phase space is in a local minimum and will, with high probability, increase both towards the future and towards the past.

This, however, sounds a bit strange. Let us again look at the gas-in-the-box system. We will use $\Gamma(X_1)$ to denote the initial non-equilibrium macrostate and the system will go through $\Gamma(X_2), \cdots, \Gamma(X_{m-1})$ until it reaches equilibrium in the macrostate $\Gamma(X_m)$. We know that the volume of the macrostate is small for $\Gamma(X_1)$ but steadily increases until it reaches the volume of $\Gamma(X_m)$. However, this is not how statistical mechanics tells it. If we look at the macrostate $\Gamma(X_{m-1})$, statistical mechanics says that the volume of $\Gamma(X_m)$ will be larger, in accord with what we have seen, but also says that the volume of $\Gamma(X_{m-2})$ will be larger. The latter contradicts with what we have seen.

This can be solved by positing that the macrostate was $\Gamma(X_{m-2})$, as we have seen it with a smaller volume in phase space, and the microstate is one of the microstates that belong to $\Gamma(X_{m-2})$. This does not contradict statistical mechanics, which merely says that the volume of $\Gamma(X_{m-2})$ being smaller than that of $\Gamma(X_{m-1})$ is improbable, not that it cannot occur. And we know that it occurred.

From the viewpoint of a $\Gamma(X_{m-2})$ with a smaller volume in phase space then, it is highly probable that $\Gamma(X_{m-1})$ will have a larger volume and $\Gamma(X_m)$ an even larger volume, which agrees with what we have seen. But it is also highly probable that $\Gamma(X_{m-3})$ has a larger volume. To solve this, we have to posit that the macrostate was $\Gamma(X_{m-3})$, as we have seen it with a smaller volume in phase space, and the microstate is one of the microstates that belong to $\Gamma(X_{m-3})$. From this viewpoint it is highly probable that the volumes of $\Gamma(X_{m-2})$, $\Gamma(X_{m-1})$, and $\Gamma(X_m)$ were larger and larger, which agrees with what we have seen.

Now, to have the entire process of going from $\Gamma(X_1)$ to $\Gamma(X_m)$ agree with what we have seen, we have to posit that the initial macrostate was $\Gamma(X_1)$, as we have seen it with a small volume in phase space, and the initial microstate is one of the microstates that belong to $\Gamma(X_1)$. And if we look outside of the gas-in-the-box system, if we want the process of the entire universe to agree with what we have seen, we have to posit that the universe first came into being in that kind of macrostate with a small volume in phase space for which statistical mechanics makes it highly probable that the process of the entire universe agrees with what we have seen. This is called the 'Past Hypothesis'.

Statistical mechanics proves that reversible processes in the microscopic view along with an improbable initial macrostate following the Past Hypothesis lead to irreversible processes as we see them in the macroscopic view. Of course objections can be raised against the Past Hypothesis. Also, it could be argued that, however amazingly probable it would be to see an entropy increasing process, there could be conservation laws which prevent a trajectory of the microstate in phase space to enter the enormous region of the macrostate with higher entropy. These objections are important, otherwise the problem would already be solved and the suggestion by Albert would be unnecessary. However, they will not be treated in this thesis, since they have nothing to do with the suggestion itself. Albert in fact doesn't even appear to disagree with anything said in this chapter. The next chapter describes the suggestion by Albert, a fundamentally different new way of reconciling the reversibility of the microscopic view with the irreversibility of the macroscopic view without the use of statistical mechanics.

Chapter 3

Suggestion: GRW perturbs abnormal microstates into normal microstates

In chapter 1 the problem was described. This chapter will deal with a suggestion by David Albert to solve the problem. For this purpose the first two sections describe the GRW theory. In the third section the suggestion by Albert is described, i.e. GRW perturbs abnormal microstates into normal microstates, and how this suggestion solves the problem. In the fourth section, objections are made against Albert's suggestion and it is explained why the time reversal non-invariance of GRW is needed, which is described in the following chapters.

3.1 What is GRW?

The term GRW refers to the theory set forth by G. C. Ghirardi, A. Rimini, and T. Weber in 1986 ([9], see also [3, 4, 10]). This theory is devised mainly to solve the so-called measurement problem. In orthodox quantum mechanics the measurement of a microscopic system in a superposition of states leads to a superposition of macroscopically different states of the measuring device. This is not what we would expect. Ghirardi, Rimini, and Weber propose to solve this problem by modifying the dynamics of orthodox quantum mechanics in such a way that 'collapses' to quasi-localized states occur spontaneously.

Constraining GRW is the fact that both orthodox quantum mechanics and classical mechanics have been validated extensively by experiments. Therefore, for microscopic systems the theory should, as far as we can tell at the moment, yield approximately the same experimental results as orthodox quantum mechanics, whereas for macroscopic systems it should, as far as we can tell at the moment, yield the same experimental results as classical physics. In the future it may be possible to obtain experimental results which will speak in favour of

GRW or in favour of orthodox quantum mechanics and classical mechanics.

GRW is a modification of orthodox quantum mechanics; the postulates of von Neumann that describe states and observables are retained. There is no special role for measurement in GRW, and so the postulates that describe measurement are dropped. Instead, the dynamics described by the Schrödingier equation are altered.

The theory deals with a system of n distinguishable particles that are described by a quantum mechanical wavefunction. The system evolves according to the Schrödingier equation, but, with a very small frequency (i.e. probability per time unit) λ_i, particle i experiences a spontaneous localization process, also called a jump, giving the particle a fairly well determined position.

It is proven ([3], p. 49) for a rigid system, e.g. an insulating solid, that if a subsystem experiences a jump, it brings about a jump in the composite system. In the proof, the system is described with center of mass and relative motion position coordinates and it is shown that the equation describing the dynamics of the center of mass has the same form as the equation describing the dynamics of a constituent. Now, it is also proven ([3], p. 50) that the fact that the system is rigid gives rise to a sharp localization of the relative motion positions. Therefore, a localization of a constituent of the system is equivalent to a localization of the center of mass. Because of this, the probability per unit time for the composite system to undergo a spontaneous localization increases proportionally to the number of particles.

In GRW there is no distinction between a measurement and an ordinary physical interaction; a measurement of a microscopic system in a superposition of states just means bringing it into contact with a macroscopic system, thereby inducing a jump in the microscopic system and giving its particles a quasi-definite position. The macroscopic measurement device will approximately give only one result by suppressing the other states of the superposition it would, in orthodox quantum mechanics, evolve into.

The microscopic world effectively still gives the same experimental results as orthodox quantum mechanics does (since here the localization frequency is small), whereas the macroscopic world has definite positions as we are used to (since here the localization frequency is large), giving approximately the same experimental results as classical physics.

When a GRW jump occurs in the composite system, three of the involved variables are random: the time at which the jump occurs, the particle that experiences the jump, and the centre of the jump.

A jump can be described by

$$\phi(\vec{r}_1, \ldots, \vec{r}_n) \quad \overset{jump}{\rightarrow} \quad \frac{j(\vec{x}-\vec{r}_i)\phi(\vec{r}_1, \ldots, \vec{r}_n)}{R_i(\vec{x})}, \tag{3.1}$$

with $\vec{r}_i$ the position coordinate of the randomly chosen particle i. The jump factor $j(\vec{x})$ has $\vec{r}_i$ from a randomly chosen particle i as its origin and is normalized:

$$\int d^3\vec{x}\, |j(\vec{x})|^2 = 1. \tag{3.2}$$

Ghirardi, Rimini, and Weber suggest a Gaussian shape for the jump factor (with Bell's ([4]) notation):

$$j(\vec{x}) = (\frac{\alpha}{\pi})^{3/4}\, e^{-\frac{\alpha}{2}\|\vec{x}\|^2}, \tag{3.3}$$

where $1/\sqrt{\alpha}$ is the localization amplitude. $R_i(\vec{x})$ is a renormalization factor:

$$|R_i(\vec{x})|^2 = \int d^3\vec{r}_1 \ldots d^3\vec{r}_n\, |j(\vec{x}-\vec{r}_i)\phi(\vec{r}_1,\ldots,\vec{r}_n)|^2. \tag{3.4}$$

The centre of the jump $\vec{x}$ is randomly chosen with normalized probability density

$$d^3\vec{x}\, |R_i(\vec{x})|^2. \tag{3.5}$$

For simplicity the localization frequency is taken to be equal for all particles:

$$\lambda_i = \lambda. \tag{3.6}$$

In order for GRW to solve the measurement problem and comply with microscopic and macroscopic experimental results, Ghirardi, Rimini, and Weber suggest orders of magnitude for the localization frequency and the localization amplitude, which are to be seen as new constants of nature,

$$\begin{aligned} \lambda &\approx 10^{-16}\ \mathrm{sec}^{-1} \\ 1/\sqrt{\alpha} &\approx 10^{-5}\ \mathrm{cm}. \end{aligned} \tag{3.7}$$

It can be remarked that the repeated localization of the particles in GRW adds energy to the system; energy conservation does not hold. According to ([3]) the expected amount of energy added per time unit is approximately

$$\frac{\delta E}{t} \approx 10^{-25}\ \mathrm{eV\ sec}^{-1}, \tag{3.8}$$

which is, anyway, too small to be measured at the moment, but may in the future be used to verify or falsify GRW.

3.2 The equation for the statistical operator

Let us consider one particle. If the particle is in a pure state, the localization process transforms it into a statistical mixture (switching to bra-ket notation):

$$\begin{aligned} |\phi\rangle\langle\phi| \quad &\overset{jump}{\rightarrow} \quad \int d^3\vec{x}\, |R_i(\vec{x})|^2 \frac{j(\vec{x}-\vec{r}_i)|\phi\rangle}{R_i(\vec{x})} \frac{\langle\phi|j(\vec{x}-\vec{r}_i)}{R_i(\vec{x})} \\ &= \quad \int d^3\vec{x}\, j(\vec{x}-\vec{r}_i)|\phi\rangle\langle\phi|j(\vec{x}-\vec{r}_i) \\ &\equiv \quad \mathcal{T}[|\phi\rangle\langle\phi|], \end{aligned} \tag{3.9}$$

where $\mathcal{T}$ is the jump superoperator. If the initial state of the particle is a statistical mixture, then the effect of a jump is the same as the one above:

$$\rho \quad \overset{jump}{\rightarrow} \quad \mathcal{T}[\rho]. \tag{3.10}$$

In between jumps the particle evolves according to von Neumann's equation:

$$\frac{d\rho(t)}{dt} = -\frac{i}{\hbar}[H, \rho(t)], \tag{3.11}$$

where H is the Hamiltonian for a free particle. Now we will derive the evolution equation for $\rho(t)$, by looking at how it will look like a time interval dt later. In the time interval dt, there is a probability of $\lambda\ dt$ that a jump occurs as described by equation (3.10), and a probability of $1 - \lambda\ dt$ that no jump occurs and the particle evolves according to von Neumann's equation (3.11). This gives:

$$\rho(t + dt) = (1 - \lambda\ dt)\Big(\rho(t) - \frac{i}{\hbar}[H, \rho(t)]\ dt\Big) + \lambda\ dt\ \mathcal{T}[\rho(t)]. \tag{3.12}$$

Rewriting $\rho(t + dt)$ as $\rho(t) + d\rho(t)$ leads to:

$$\frac{d}{dt}\rho(t) = -\frac{i}{\hbar}[H, \rho(t)] - \lambda\Big(\rho(t) - \mathcal{T}[\rho(t)]\Big). \tag{3.13}$$

This is the master equation of GRW; it describes the evolution of a single particle which undergoes jumps. In the coordinate representation (for simplicity in one dimension) it is:

$$\begin{aligned} \frac{\partial}{\partial t}\langle q'|\rho(t)|q''\rangle &= -\frac{\imath\hbar}{2m}\Big(\frac{\partial^2}{\partial q'^2} - \frac{\partial^2}{\partial q''^2}\Big)\langle q'|\rho(t)|q''\rangle \\ &\quad -\lambda\Big(1 - e^{-\frac{\alpha}{4}(q'-q'')^2}\Big)\langle .q'|\rho(t)|q''\rangle \end{aligned} \tag{3.14}$$

The solution of this equation can be expressed in terms of the solution $\langle q'|\rho_{Sch}(t)|q''\rangle$ of the pure Schrödingier equation ($\lambda = 0$) satisfying the same initial conditions:

$$\langle q'|\rho(t)|q''\rangle = \frac{1}{2\pi\hbar}\int_{-\infty}^{\infty} dk \int_{-\infty}^{\infty} dy\ e^{-\frac{i}{\hbar}ky} F(k, q' - q'', t)\langle q' + y|\rho_{Sch}(t)|q'' + y\rangle, \tag{3.15}$$

where

$$F(k, q, t) = e^{-\lambda t + \lambda \int_0^t d\tau\ e^{-\frac{\alpha}{4}(q - \frac{k\tau}{m})^2}}. \tag{3.16}$$

In order to get a better understanding of the evolution guided by the GRW master equation, one could calculate the mean values, spreads and correlations for the position and momentum operator for all times, compared to the evolution guided by the Schrödingier equation (see [3]).

However promising GRW's solving of the measurement problem may be, in this thesis GRW is presented for another reason, viz Albert suggested that GRW could remove the contradiction between the microscopic and the macroscopic view, as discussed in chapter 1.

3.3 What is the perturbation of abnormal microstates into normal microstates?

In "Time and Chance" ([1], p. 150) Albert suggests a way for reconciling the reversibility of the microscopic view with the irreversibility of the macroscopic view. In the previous chapter it was described how Boltzmann achieved this by explaining how the reversible processes in the microscopic view along with statistical mechanics and the Past Hypothesis lead to irreversible processes in the macroscopic view. Albert wants to replace the statistical mechanics of reversible processes with the GRW theory. He says that, when dynamics guide a system from one microstate to another, statistical mechanics is needed for calculating how probable a certain microstate is. However, GRW can do this all on its own; GRW can determine how a system evolves and determine the probability of a microstate. Then it is needed for GRW to perturb the system out of an abnormal microstate into a normal microstate.

What does this mean? Albert gives the following example. At time t_1 there are two macroscopic bodies whose temperatures differ. They are brought into thermal contact with each other and are not disturbed subsequently. Eventually, at which we will call time t_2, there is no temperature difference.

Now, Albert distinguishes two kinds of microstates of the two-body system; normal microstates are those who happen to be sitting on trajectories which pass, at time t_2, through a macrostate of the two-body system in which the temperature difference between the two bodies is lower. Abnormal states do not sit on these trajectories.

To me this seems too loose a definition; a microstate where the temperature difference is +10000K is not normal, even when it is sitting on a trajectory which eventually, at time t_2, passes through a macrostate where the temperature difference is 0K. It is better to say that normal microstates are those that sit on trajectories with macrostates that have a monotonically declining temperature difference, preferably according to Newton's law of cooling or a similar law. However, requiring a monotonically decreasing temperature difference makes the definition too strict, since in reality it often occurs that the temperature difference rises for a short time. A certain amount of deviation must be allowed.

According to Albert, the collection of normal microstates is vastly larger than the collection of abnormal microstates, not only in the collection of all possible microstates, but in every not-unimaginably-small neighbourhood, and even in every not-unimaginably-small neighbourhood of every abnormal microstate. Although this appears to be a reasonable assumption, I think a thorough mathematical investigation is required. However, I will leave it at this remark.

Albert claims that, because of the difference in size between a collection of normal microstates and a collection of abnormal microstates, being in a normal microstate is very stable under small perturbations and that being in an abnormal microstate is very unstable under small perturbations. This does not have to be true; the fact that there are many more normal states does not automatically mean that the system is more likely to evolve into one. Dynamics (e.g.

through conservation laws) could exclude them all from being entered, which makes being in a abnormal microstate not unstable at all. Although this should also be looked into, I will not do that here.

Let us get back to GRW. GRW should guide the microstate from being abnormal into being normal by perturbation. Albert thinks that this occurs because the volume in phase space of the macrostate to which the abnormal microstate belongs, is far smaller than the volume of the microstates towards which GRW is possible to perturb. One can seriously ask whether this perturbation instills the same probability distribution of microstates as statistical mechanics does. This is a very important question, whose answering unfortunately lies outside the scope of this thesis.

Since Albert ([1], p. 154) calls his suggestion with GRW "a (possible) new universal statistical mechanics", it must be assumed that it applies to gases, liquids, and solids. However, for gases and liquids GRW causes jumps far less than it does for solids. The rigid structure of a solid makes the jump of a single particle cause a jump for the entire solid. For gases I estimate, using the parameters from equation (3.7), the frequency with which every particle in the entire system is even affected by a jump of a nearby particle to be in the order of 10^{-12}sec^{-1}, and for liquids this isn't significantly more.[1] This means that for gases and liquids GRW jumps occur far too seldom to account for the irreversible processes in the macroscopic view.

Given that GRW jumps perturb solids from abnormal microstates to normal microstates, we will look further at Albert's proposal. Albert believes that GRW is capable of making predictions in the macroscopic view just as statistical mechanics is. However, in his proposal he includes the Past Hypothesis; he says (p. 162) that "the GRW contraption, minus the past-hypothesis, makes no claims whatsoever (statistical or otherwise) about the past." I do not agree with that. In the following chapters of this thesis it is shown that, although the GRW theory does not provide probabilities for inferring towards the past, they can be determined from probabilities for inferring towards the future, but only when certain information is known about the past. This is however not the same as the Past Hypothesis. We do not have to posit that the universe first came into being in that kind of macrostate with a small volume in phase space for which statistical mechanics makes it highly probable that the process of the entire universe agrees with what we have seen. What we need, which will be shown later, is the probability density of microstates at the time in the past of which we want to say something. If we want to say something about the complete history of the universe, then we need the probability density of microstates of when the universe first came into being. But there is no constraint to the probability density of the microstates. When GRW is used to make inferences

[1] At a temperature of 273 K and a pressure of 1 atm, the density of hydrogen is of the order of 10^{-1} kg/m^3. This means that the average volume per particle is 10^{-25}m^3. With $1/\sqrt{\alpha} \approx 10^{-7}$ m, this means that per jump a number in the order of 10^4 particles or 10^{-22} part of the system is affected by the jump. Multiplying this number with the number of jumps that occur per second in the system, $10^{-16}\text{sec}^{-1} \cdot 10^{26}$particles, gives for the average location frequency of the system: 10^{-12}sec^{-1}.

towards the past, it is not automatically needed that the initial macrostate had a small volume in phase space.

But perhaps there are other reasons for a need for the Past Hypothesis, similar to those that led to the Past Hypothesis in statistical mechanics. An important question then is of course the following: is GRW itself time reversal invariant or not? If it is time reversal invariant, reverse processes of allowed processes are also allowed, which means we will still need the Past Hypothesis for GRW to make correct inferences towards the past. Indeed, you imagine that this must be what Albert thinks for he includes the Past Hypothesis in his suggestion and not only the need for the probability density of the initial microstates.

But then the side-question arises of why orthodox quantum mechanics isn't sufficient. There is no difference in calculating probabilities between GRW and orthodox quantum mechanics. This makes them both able to replace statistical mechanics and need the probability density of the initial microstates. And if both are time reversal invariant then there is no reason for preferring one above the other, since both require the Past Hypothesis. Although Albert argues why other theories aren't sufficient to replace statistical mechanics, there is no mentioning of orthodox quantum mechanics.

In contrast with his using the Past Hypothesis, Albert (p. 162) thinks that GRW is time reversal non-invariant. If GRW were to be time reversal non-invariant and if GRW were true, we would not have to ask ourselves why the world is irreversible in the macroscopic view but reversible in the microscopic view, because the world is not reversible in the microscopic view anymore. If GRW is time reversal non-invariant, then GRW and the probability density of the initial microstates can do it all on their own, and the Past Hypothesis is not needed. This can be seen as an expansion of Albert's proposal with an even greater role for GRW. For this reason, chapter 5 investigates whether GRW is time reversal invariant or not. But first, since in GRW the system evolves stochastically, an account must be given of what it means for a stochastic law of nature to be time reversal non-invariant. This is done in chapter 4.

Chapter 4

Explication: Stochastic time reversal invariance

Since the time reversal invariance of the GRW theory is such an important requirement for the expansion of the suggestion by Albert, stochastic time reversal invariance requires a thorough investigation. This chapter will begin with an introduction of what it means for a theory to be stochastic. Next, a description of three principles for stochastic time reversal invariance, viz microreversibility, retrodictability, and the HB principle, is given and they are applied to orthodox quantum mechanics. Finally, these three principles are compared and one is chosen which is best at determining if GRW can replace statistical mechanics without a need for the Past Hypothesis.

4.1 Introduction to stochastic theories and stochastic time reversal invariance

According to van Kampen ([18]), a *stochastic variable* is an object X defined by:

(1) a set of possible values (called "phase space" in classical mechanics and "spectrum" in quantum mechanics);

(2) a probability distribution over this set.

The set of possible values may be discrete or continuous. Because our ultimate object of investigation will be GRW, I will use a continuous set of possible values. In mathematics, the probability density of the value x for X at time t is given by a function $p(x, t)$, which is nonnegative for all x and t:

$$p(x, t) \geq 0. \tag{4.1}$$

The probability that, at time t, X has a value between x and $x + dx$ is:

$$p(x, t)\ dx. \tag{4.2}$$

$p(x, t)$ is normalized for all t:

$$\int p(x, t)\ dx = 1, \tag{4.3}$$

where the integral extends over the entire set of possible states.

The probability density of X obtaining the values $x_1, \ldots, x_N$ at times $t_1 < \ldots < t_N$ is depicted as:

$$p(x_N, t_N; \ldots; x_1, t_1), \tag{4.4}$$

and is called the *joint probability density.*

The infinite hierarchy of joint probability densities of obtaining the value x_1 at time t_1, and x_2 at time t_2,$\cdots$,x_N at time t_N for all $N = 0, 1, 2, \cdots$ is called a *stochastic process.*

The probability density of X obtaining the values $x_n, \ldots, x_N$ at times $t_n < \ldots < t_N$, given the fact that X obtained the values $x_1, \ldots, x_{n-1}$ at times $x_1 < \ldots < x_{n-1}$, is depicted as:

$$p(x_N, t_N; \ldots; x_n, t_n | x_{n-1}, t_{n-1}; \ldots; x_1, t_1), \tag{4.5}$$

and is called the *conditional probability density.*

Equation (4.5) is called a probability[1] through prediction by Watanabe ([20]) and a future-directed probability by Holster ([12]), because it gives the probability of obtaining the values $x_N, \cdots, x_n$ at times $t_N, \cdots, t_n$ given the fact of obtaining the values $x_{n-1}, \cdots, x_1$ at earlier times $t_{n-1}, \cdots, t_1$. The probability of obtaining values given facts at later times is called, respectively, a probability through retrodiction or a past-directed probability, e.g.:

$$p(x_1, t_1; \ldots; x_{m-1}, t_{m-1} | x_m, t_m; \ldots; x_N, t_N), \tag{4.6}$$

where still $t_1 < \cdots < t_m < \cdots < t_N$. I will use Holster's terms because they don't have, as he remarks, an epistemological connotation.

The joint and the conditional probability distribution are connected according to Bayes' rule:

$$p(x_N, t_N; \ldots; x_n, t_n | x_{n-1}, t_{n-1}; \ldots; x_1, t_1) = \frac{p(x_N, t_N; \ldots; x_n, t_n)}{p(x_N, t_N; \ldots; x_1, t_1)}. \tag{4.7}$$

Particularly simple stochastic processes are singled out by the *Markov* property:

$$p(x_n, t_n | x_{n-1}, t_{n-1}; \ldots; x_1, t_1) = p(x_n, t_n | x_{n-1}, t_{n-1}), \tag{4.8}$$

[1]Although it is not correct I will, for sake of brevity, refer to probability densities as probabilities.

meaning that the conditional probability of obtaining x_n at t_n is determined only by x_{n-1} at t_{n-1} and not by values at earlier times. The right hand side is called a *transition probability.* Note that with the Markov property, a stochastic process can also be described with only the transition probabilities and the initial probability $p(x_1, t_1)$. In this thesis I will, from now on, limit myself to Markov processes.

A stochastic theory can be compared to a deterministic theory as follows. In a deterministic theory, a state gives a description of a system by specifying the value x of a property X of the system at time t (cf. chapter 0.1). In a stochastic theory, this property X is a stochastic variable and only the probability $p(x, t)\, dx$ of being in a state, i.e. that X possesses the value x, at time t can be given.

A process in a stochastic theory is not depicted as a sequence of states as in a deterministic theory, but as the hierarchy of joint probabilities for a variable amount of values, or for Markov processes as the sequence of transition probabilities.

In a deterministic theory there is usually a law of nature that expresses the evolution of the state. In a stochastic theory this would be for a Markov process the sequence of transition probabilities $p(x_n, t_n|x_{n-1}, t_{n-1})$ of obtaining the value x_n at time t_n given the fact that the value x_{n-1} is obtained at time t_{n-1}. If one knows one probability then one can determine the next probability through the law of nature with:

$$p(x_n, t_n) = \int p(x_n, t_n|x_{n-1}, t_{n-1})\, p(x_{n-1}, t_{n-1})\, dx_{n-1} \tag{4.9}$$

There is no distinction in the description of stochastic processes and stochastic laws of nature, and we will therefore only speak of the time reversal (non-)invariance of laws of nature.

As stated in section 0.1, a deterministic law of nature is time reversal invariant if, for a process that is allowed by the law of nature, the reverse process is allowed as well. Analogously, a stochastic law of nature is time reversal invariant if, for a process (in the stochastic meaning of a sequence of probabilities) that is allowed by the law of nature, the reverse process is allowed as well, and with the same probability. This means that, for all states with values x_n, in the law of nature the conditional probabilities of reaching a state with value x_n given an earlier state with value x_{n-1}, thus governing the process, must equal the conditional probabilities governing the reverse process.

In the next sections, I will review three different views on what a reverse process entails in stochastic theories leading to three principles for time reversal invariance of stochastic theories, viz microreversibility, retrodictability and the HB principle.

4.2 The principle of microreversibility

The first principle of the three for the time reversal invariance of stochastic laws of nature that I will discuss, is the principle of *microreversibility*. It is based on the definition of time reversal invariance as mentioned in section 0.2, i.e. a law of nature is time reversal invariant if, for every process it allows, it also allows the reverse process. Here, the reverse process goes through the reverse states in the reverse order, but, since it is not possible to physically reverse time, the reverse process is still future-directed.

Applied to a stochastic theory the principle of microreversibility states that a law of nature is time reversal invariant if:

> *for every state represented by the vector $\vec{x}_n$, the conditional probability of finding by measurement a state $\vec{x}_n$ given an earlier prepared state $\vec{x}_{n-1}$ equals the conditional probability of finding by measurement the reverse state ${}^R\vec{x}_{n-1}$ given an earlier prepared reverse state ${}^R\vec{x}_n$,*

or, symbolically:

$$p(\vec{x}_n, t_n | \vec{x}_{n-1}, t_{n-1}) = p({}^R\vec{x}_{n-1}, t_n | {}^R\vec{x}_n, t_{n-1}). \tag{4.10}$$

How the state $\vec{x}_n$ should be reversed into ${}^R\vec{x}_n$ is determined by the theory in question. For instance in classical mechanics, the position is not reversed but the momentum is. In quantum mechanics on the other hand, the state $|i\rangle$ is reversed into $K|i\rangle$. Again, it is important here that the reverse process is governed by the reverse sequence of the conditional probabilities but that the time t is still future-directed, i.e. $t_n > t_{n-1}$. Two future-directed conditional probabilities are compared here.

The principle of microreversibility is closely related to the notion of *detailed balance*. Tolman ([17], pp. 161-165) in fact calls equation (4.10) detailed balance. For 'microscopic reversibility' it is needed that the transition probability of a number of particles from being in certain partitions of μ-space to being in certain other partitions is equal to the transition probability of the same number of particles from being in the reverses of the latter partitions to being in the reverses of the former partitions.

Gardiner ([7]) equates joint probabilities for both notions:

$$p(\vec{x}_n, t_n; \vec{x}_{n-1}, t_{n-1}) = p({}^R\vec{x}_{n-1}, t_n; {}^R\vec{x}_n, t_{n-1}), \tag{4.11}$$

with the difference between the two notions being that for detailed balance both probabilities have to be obtained from the same stationary solution whereas for 'time reversal invariance' it is allowed to obtain the probabilities from different stationary solutions. Whereas in our notion (equation (4.10)) it is determined if the transition is equally probable given the fact that the initial state obtained, according to the equation by Gardiner it is necessary that the transitions actually occur in nature with equal probability.

Watanabe ([19]) uses the same expression for microreversibility (calling it 'reversibility') as we do, but uses for detailed balance the following, often used, expression:

$$p(\vec{x}_n, t_n|\vec{x}_{n-1}, t_{n-1}) = p(\vec{x}_{n-1}, t_n|\vec{x}_n, t_{n-1}). \tag{4.12}$$

It is important here to note that the reverse states ${}^R\vec{x}_{n-1}$ and ${}^R\vec{x}_n$ are taken to equal the original states $\vec{x}_{n-1}$ and $\vec{x}_n$ respectively.

4.3 Applying microreversibility to orthodox quantum mechanics

An often quoted author on the principle of microreversibility is Davies ([5]). In this section, I will use his work as a guideline and will note where I think that he has gone astray.

Davies calls the probability on the left hand side of equation (4.10) for the principle of microreversibility w, and the reverse probability on the right hand side w_{rev}, with which the principle of microreversibility can be expressed as:

$$w = w_{rev}. \tag{4.13}$$

Suppose at time t_1 a system is prepared in the pure state $|i\rangle$. Then, w is the probability of finding by measurement the pure state $|f\rangle$ at time t_2:

$$w = \text{prob}(|i\rangle_{t_1} \to |f\rangle_{t_2}). \tag{4.14}$$

This is the notation as used by Davies and I will stick to it. Although it is rather unclear, it should be read as a conditional probability. The only thing I altered is the explicit mentioning of the time at which the state obtains. While not so important to Davies, it will be to us for the principles in the other sections of this chapter.

It is important to notice that at time t_2 the state $|f\rangle$ is found by measurement. Davies does not mention explicitly the act of measurement although this is of fundamental importance in orthodox quantum mechanics.

Now we will consider the reverse probability w_{rev} of finding by measurement the state $K|i\rangle$ at time t_2 out of the state $K|f\rangle$ at time t_1:

$$w_{rev} = \text{prob}(K|f\rangle_{t_1} \to K|i\rangle_{t_2}). \tag{4.15}$$

K was already used implicitly in equation (1.6) for the time reversal operator T in section (1.2) and is a nonlinear operator that reverses the states by carrying out a complex conjugation on all numbers to the right of it. For two arbitrary states ϕ and ψ, and $a, b \in \mathbb{C}$:

$$K(a|\phi\rangle + b|\psi\rangle) = a^*|\phi^*\rangle + b^*|\psi^*\rangle. \tag{4.16}$$

K, just as T in section (1.2), is a anti-unitary operator, meaning that:

$$\langle K\psi|K\phi\rangle = \langle\psi^*|\phi^*\rangle = \langle\psi|\phi\rangle^*. \tag{4.17}$$

Of course, $KK = \mathbb{I}$, with $\mathbb{I}$ being the identity operator, and although $K^\dagger K \neq \mathbb{I}$, it is true that $K^\dagger K$ and $\mathbb{I}$ give rise to the same probability:

$$|\langle\psi|K^\dagger K|\phi\rangle|^2 = |\langle\psi|\mathbb{I}|\phi\rangle|^2. \tag{4.18}$$

Both sides of the equation yield the same probability and thus the physical content of quantum mechanics is left unchanged.

To test whether orthodox quantum mechanics is microreversible or not w and w_{rev} have to be determined. In the time between t_1 and t_2 the state $|i\rangle$ will have evolved in a particular way into:

$$U(t_2, t_1)|i\rangle, \tag{4.19}$$

where $U(t_2, t_1)$ is the evolution operator:

$$U(t_2, t_1) = e^{-\frac{i}{\hbar}H(t_2 - t_1)}, \tag{4.20}$$

with H the Hamiltonian. Then, according to Davies, the probability w is

$$w = \langle f|U(t_2, t_1)|i\rangle \qquad \text{(false)}, \tag{4.21}$$

which is of course not the way a probability is determined in quantum mechanics. The correct probability w is:

$$w = |\langle f|U(t_2, t_1)|i\rangle|^2. \tag{4.22}$$

Davies expresses the reverse probability w_{rev} similarly to equation (4.21). Correct is:

$$\begin{aligned} w_{rev} &= |\langle Ki|U(t_2, t_1)|Kf\rangle|^2 \\ &= |\langle i|K^\dagger U(t_2, t_1)K|f\rangle|^2 \\ &= |\langle f|K^\dagger U^\dagger(t_2, t_1)K|i\rangle^*|^2 \end{aligned} \tag{4.23}$$

Davies now gives the following sufficient but not necessary condition: orthodox quantum mechanics is microreversible if:

$$U(t_2, t_1) \stackrel{?}{=} K^\dagger U^\dagger(t_2, t_1)K. \tag{4.24}$$

This equation cannot be solved on its own, i.e. without vectors to which the operators apply. Let us look at the expression of w_{rev} again:

$$\begin{aligned} w_{rev} &= |\langle f|K^\dagger U^\dagger(t_2, t_1)K|i\rangle^*|^2 \\ &= |\langle f^*|U^\dagger(t_2, t_1)|i^*\rangle^*|^2 \\ &= |\langle f|U^{\mathrm{T}}(t_2, t_1)|i\rangle|^2, \end{aligned} \tag{4.25}$$

where $^{\mathrm{T}}$ denotes the transpose. This means that w and w_{rev} are equal when $U = U^{\mathrm{T}}$, or in other words, when U is symmetrical. This, expressed as,

$$e^{-\frac{i}{\hbar}H(t_2 - t_1)} = e^{-\frac{i}{\hbar}H^{\mathrm{T}}(t_2 - t_1)} \tag{4.26}$$

together with the self-adjointness of H, leads to the statement that orthodox quantum mechanics is microreversible if H is real.

Until now, we have only spoken about pure states. For the next chapter it is important to develop the principle of microreversibility for mixed states as well. But first, it is important to look at equation (4.24), the sufficient condition for microreversibility, again. We have seen that this equation, if H is real, holds for all vectors $|i\rangle$ and $|f\rangle$ that are applied to both sides. Therefore:

$$K^\dagger U^\dagger(t_2,t_1)K = U(t_2,t_1), \tag{4.27}$$

and similarly

$$K^\dagger U(t_2,t_1)K = U^\dagger(t_2,t_1). \tag{4.28}$$

For mixed states the principle of microreversibility can be expressed as:

$$\text{prob}((\rho_i)_{t_1} \to (\rho_f)_{t_2}) = \text{prob}((K\rho_f K^\dagger)_{t_1} \to (K\rho_i K^\dagger)_{t_2}). \tag{4.29}$$

w_{rev}, i.e. the right hand side of this equation can be written and manipulated as (with Tr meaning trace):

$$\begin{aligned}
\text{Tr}\Big(K\rho_i K^\dagger\, U(t_2,t_1)\, K\rho_f K^\dagger\, U^\dagger(t_2,t_1)\Big) &= \text{Tr}\Big(\rho_i K^\dagger\, U(t_2,t_1)\, K\rho_f K^\dagger\, U^\dagger(t_2,t_1)K\Big) \\
&= \text{Tr}\Big(\rho_i\, U^\dagger(t_2,t_1)\, \rho_f\, U(t_2,t_1)\Big) \\
&= \text{Tr}\Big(U^\dagger(t_2,t_1)\, \rho_f\, U(t_2,t_1)\, \rho_f\Big) \\
&= \text{Tr}\Big(\rho_f\, U(t_2,t_1)\, \rho_i\, U^\dagger(t_2,t_1)\Big),
\end{aligned} \tag{4.30}$$

which is exactly the expression for w. This means that for mixed states orthodox quantum mechanics is microreversible as well.

4.4 The principle of retrodictability

The following principle for the time reversal invariance of stochastic laws of nature that I will discuss, is the principle of *retrodictability* as introduced by Watanabe ([20]). Important here is not that both a process and the reverse process are equally probable according to the law of nature, but that for one process inference of the future observational data from the present ones (prediction) equals inference of the past observational data from the present ones (this is what Watanabe calls retrodiction). The principle of retrodictability states that a stochastic law of nature is time reversal invariant if:

> *for every state represented by the vector $\vec{x}_n$, the conditional probability of finding by measurement a state $\vec{x}_n$ given an earlier prepared state $\vec{x}_{n-1}$ equals the conditional probability of having prepared the state $\vec{x}_{n-1}$ given finding by measurement the later state $\vec{x}_n$,*

or, symbolically:

$$p(\vec{x}_n, t_n|\vec{x}_{n-1}, t_{n-1}) = p(\vec{x}_{n-1}, t_{n-1}|\vec{x}_n, t_n). \tag{4.31}$$

Here, future-directed probabilities of a process are compared with past-directed probabilities.

Let us first discuss the term 'prepare'. This term was already used in the formulation of the principle of microreversibility but needs no special attention there, because two future-directed probabilities are compared and it doesn't matter if there is a difference between the way the initial and the final state is acquired. In Watanabe's account of retrodictability both the initial system at time t_{n-1} and the final system at time t_n are described as being observed and yielding a result. However, there is a difference between these two mentionings of the term 'observe'. The initial state $\vec{x}_{n-1}$ is prepared with a certain weight of occurrence $p(\vec{x}_{n-1}, t_{n-1})$, whereas the final state $\vec{x}_n$ is found by measurement. To me, even classically, these two things seem to be different. A measurement gives the state in classical mechanics and one of the eigenstates in orthodox quantum mechanics, from the system as it exists in nature. This state is supplied by nature and we have no influence over it. When preparing the system in a certain state, it is irrelevant how the system existed in nature because we have so much influence over it that we can, with a certain weight of occurrence, force the state onto the system.

Now the question is how to find the past-directed probabilities. Usually a theory supplies a way to determine the future-directed probabilities, but determining the past-directed probability is a bit more difficult. Consider the following case: there is a system with two states A and B and future-directed conditional probabilities $p(A, t_2|A, t_1) = p(A, t_2|B, t_1) = p(B, t_2|A, t_1) = p(B, t_2|B, t_1) = \frac{1}{2}$. If the system is found by measurement at time t_2 to be in state A half the time and in state B half the time (as we would expect), still nothing can be said about the probability of having been found by measurement in state A or B at time t_1. For, although we know the probability of moving from state A at time t_1 to state B at time t_2, it is nowhere said that state A is prepared at all at time t_1. The difficulty comes from the fact that we do not know the weight $p(A, t_1)$ or $p(B, t_1)$, with which each of the states A or B is prepared.

We will now determine the general past-directed probabilities $p(\vec{x}_{n-1}, t_{n-1}|\vec{x}_n, t_n)$. Let us call $p(\vec{x}_{n-1}, t_{n-1})$ the weight with which the state $\vec{x}_{n-1}$ was found by measurement at time t_{n-1}, with:

$$\int p(\vec{x}_{n-1}, t_{n-1})\, d\vec{x}_{n-1} = 1. \tag{4.32}$$

For the joint probability of finding by measurement the state $\vec{x}_{n-1}$ at time t_{n-1} and finding by measurement the state $\vec{x}_n$ at time t_n, it holds that (cf. Bayes' rule, equation (4.7)):

$$p(\vec{x}_{n-1}, t_{n-1}; \vec{x}_n, t_n) = p(\vec{x}_{n-1}, t_{n-1})\, p(\vec{x}_n, t_n|\vec{x}_{n-1}, t_{n-1}). \tag{4.33}$$

The probability of finding by measurement the state $\vec{x}_n$ at time t_n can be found by integrating the joint probability over all $\vec{x}_{n-1}$ (cf. equation (4.9)):

$$p(\vec{x}_n, t_n) = \int p(\vec{x}_{n-1}, t_{n-1})\, p(\vec{x}_n, t_n|\vec{x}_{n-1}, t_{n-1})\, d\vec{x}_{n-1}. \tag{4.34}$$

Again using Bayes' rule, equation (4.7), and using the last two expressions, we arrive at the following expression for determining the past-directed probabilities out of the forward-directed probabilities:

$$p(\vec{x}_{n-1}, t_{n-1}|\vec{x}_n, t_n) = \frac{p(\vec{x}_{n-1}, t_{n-1})\, p(\vec{x}_n, t_n|\vec{x}_{n-1}, t_{n-1})}{\int p(\vec{x}_{n-1}, t_{n-1})\, p(\vec{x}_n, t_n|\vec{x}_{n-1}, t_{n-1})\, d\vec{x}_{n-1}}. \tag{4.35}$$

There is a difference in what Watanabe calls 'the rules of the game' for prediction and retrodiction. Whereas in prediction it is not allowed to withhold occurrences of certain states found by measurement at time t_n, in retrodiction the occurrence of a state at time t_{n-1} is explicitly determined by $p(\vec{x}_{n-1}, t_{n-1})$. The rules of the game can be changed by assuming that every state $\vec{x}_{n-1}$ is prepared with a uniform $p(\vec{x}_{n-1}, t_{n-1}$ by picking it at random from the microcanonical ensemble. Watanabe calls this 'blind retrodiction'. In that case equation (4.35) turns into:

$$p(\vec{x}_{n-1}, t_{n-1}|\vec{x}_n, t_n) = \frac{p(\vec{x}_n, t_n|\vec{x}_{n-1}, t_{n-1})}{\int p(\vec{x}_n, t_n|\vec{x}_{n-1}, t_{n-1})\, d\vec{x}_{n-1}}. \tag{4.36}$$

Because the states $\vec{x}_{n-1}$ and $\vec{x}_n$ are assumed to be complete, we have the normalization:

$$\int p(\vec{x}_n, t_n|\vec{x}_{n-1}, t_{n-1})\, d\vec{x}_n = 1 \tag{4.37}$$

However, the inverse normalization,

$$\int p(\vec{x}_n, t_n|\vec{x}_{n-1}, t_{n-1})\, d\vec{x}_{n-1} = 1, \tag{4.38}$$

which can be used to simplify equation (4.36), is not self-evident. According to Watanabe ([19]), there are three conditions for the inverse normalization to hold. Firstly, in orthodox quantum mechanics it can be derived from the unitarity of the transition matrix, which will be discussed in the following section. Secondly, the condition of detailed balance:

$$p(\vec{x}_n, t_n|\vec{x}_{n-1}, t_{n-1}) = p(\vec{x}_{n-1}, t_n|\vec{x}_n, t_{n-1}) \tag{4.39}$$

is obviously sufficient. Thirdly, the condition of microreversibility, as described in equation (4.10), is sufficient, because $\vec{x}_n$ and ${}^R\vec{x}_n$ on the one hand, and $\vec{x}_{n-1}$ and ${}^R\vec{x}_{n-1}$ on the other hand, belong to the same complete set of states.

These conditions however are not independent. The unitarity of the transition matrix immediately leads to microreversibility in orthodox quantum mechanics. The condition of detailed balance is the same as the principle of microreversibility, only for detailed balance it is assumed by Watanabe (see also the last paragraph of section 4.2) that the reverse states ${}^R\vec{x}_{n-1}$ and ${}^R\vec{x}_n$ are taken to equal the original states $\vec{x}_{n-1}$ and $\vec{x}_n$ respectively. We therefore cannot use the condition of detailed balance later for orthodox quantum mechanics and GRW.

If any one of the conditions hold, then equation (4.36) reduces to:

$$p(\vec{x}_{n-1}, t_{n-1}|\vec{x}_n, t_n) = p(\vec{x}_n, t_n|\vec{x}_{n-1}, t_{n-1}), \tag{4.40}$$

which is exactly the principle of retrodictability. Summarizing gives the following expression with sufficient conditions:

$$\text{uniform } p(\vec{x}_{n-1}, t_{n-1}) + \left\{ \begin{array}{c} \underline{\text{inverse normalization through:}} \\ \\ \text{unitary transition matrix (OQM)} \\ \text{or detailed balance} \\ \text{or microreversibility} \end{array} \right\} \longrightarrow \text{retrodictability.} \tag*{\textit{fig. (4.1)}}$$

Another way of looking at the principle of retrodictability is through the following equation based on Bayes' rule (equation (4.7)) (see also Watanabe ([21])):

$$\begin{aligned} p(\vec{x}_{n-1}, t_{n-1}|\vec{x}_n, t_n)\, p(\vec{x}_n, t_n) &= p(\vec{x}_{n-1}, t_{n-1}; \vec{x}_n, t_n) \qquad (4.41) \\ &= p(\vec{x}_n, t_n|\vec{x}_{n-1}, t_{n-1})\, p(\vec{x}_{n-1}, t_{n-1}), \end{aligned}$$

which leads to a necessary condition for retrodictability:

$$p(\vec{x}_n, t_n) = p(\vec{x}_{n-1}, t_{n-1}) \neq 0, \tag{4.42}$$

meaning that for every process the initial state should be as probable as the final state. This is false for many stochastic theories. A theory for which it would hold, is a theory that for example describes an equilibrium process. Basic use of probability theory has led us to the conditions of uniform $p(\vec{x}_{n-1}, t_{n-1})$ and equal probability of initial and final states for retrodictability which are not always met by a theory. There could exist another way of determining the past-directed probabilities and then comparing them to the future-directed probabilities, but they are then likely to contradict, since equation (4.35) shows that if a theory provides for the $p(\vec{x}_{n-1}, t_{n-1})$ and the future-directed probabilities $p(\vec{x}_n, t_n|\vec{x}_{n-1}, t_{n-1})$, the past-directed probabilities $p(\vec{x}_{n-1}, t_{n-1}|\vec{x}_n, t_n)$ are completely determined. As Watanabe ([21]) puts it, "this irretrodictability [originates] from a simple property of the very notion of conditional probability, independent of the structure of dynamical theories."

Important to note here is that, in order to determine the past-directed probabilities, we need to know the $p(\vec{x}_{n-1}, t_{n-1})$. In order to infer towards the past we need to know the probability densities of that time in the past of which we want to know something.

Let us diverge a bit and look at how the principle of retrodictability should be applied to the time reversal invariant theory of classical particle mechanics, underlying statistical mechanics. In statistical mechanics it is assumed that the

$p(\vec{x}_{n-1}, t_{n-1})$ are uniform; there is not one state that is more probable than the other. Furthermore, the future-directed probability $p(\vec{x}_n, t_n|\vec{x}_{n-1}, t_{n-1})$ is unity for one value of x_n and zero for all others, by definition of the deterministic theory of classical particle mechanics. The equation for deriving past-directed probabilities out of future-directed probabilities (equation (4.35)) then gives one or zero for the past-directed probability, depending on whether the chosen process is allowed by the theory, i.e. whether the future-directed probability is respectively one or zero. This means that statistical mechanics is retrodictable.

4.5 Applying retrodictability to orthodox quantum mechanics

Let us call the left hand side of equation (4.31) the future-directed probability w_{fd}, which is the same as w as mentioned in section (4.3), and let us call the right hand side the past-directed probability w_{pd}. Then, retrodictability can be expressed as:

$$w_{fd} = w_{pd} \tag{4.43}$$

Suppose again that at time t_1 a system is prepared in the pure state $|i\rangle$. Then, w_{fd} is the future-directed probability of finding by measurement the pure state $|f\rangle$ at time t_2:

$$w_{fd} = w = \mathrm{prob}(|i\rangle_{t_1} \to |f\rangle_{t_2}). \tag{4.44}$$

Now we will consider the past-directed probability w_{pd} of, when finding by measurement the state $|f\rangle$ at time t_2, that the system was prepared in the state $i\rangle$ at time t_1:

$$w_{pd} = \mathrm{prob}(|f\rangle_{t_2} \to |i\rangle_{t_1}). \tag{4.45}$$

In quantum mechanics a measurement determines the state of the system immediately after the measurement. Little can be said about what the state of the system was, right before the measurement. However, Watanabe ([20], p. 184) does not recognize this fact and determines w_{pd} as follows. The retrodictive state, i.e. the state derived from the final state at time t_2, can, according to him, be determined at time t_1:

$$U(t_1, t_2)|f\rangle, \tag{4.46}$$

which is a expression similar to equation (4.19). The important difference is however that, although it is possible to specify how a system evolves towards the future, based on information from right after the measurement, it is not possible to specify how a system evolves towards the past, based on information from right after the measurement. Watanabe however continues by writing and manipulating w_{pd} as:

$$\begin{aligned} |\langle i|U(t_1, t_2)|f\rangle|^2 &= |\langle i|U^\dagger(t_2, t_1)|f\rangle|^2 \\ &= |\langle f|U(t_2, t_1)|i\rangle|^2, \end{aligned} \tag{4.47}$$

thus claiming to have proven the retrodictability of orthodox quantum mechanics. Here we see that the unitarity of the transition matrix $U(t_2, t_1)$ is required as mentioned in the previous section. It is amazing to see that Watanabe does not explicitly acknowledge this result to be a confirmation of the retrodictability of orthodox quantum mechanics, especially given the fact that he denies it in later articles. But let us again look at the way retrodiction is treated by Watanabe. It is true that if at time t_2 the system is in the state $|f\rangle$ (not measured, but somehow mathematically derived) , then at time t_1 it was in the state $U(t_1, t_2)|f\rangle$. However, this is not the way retrodiction is used in the definition of the principle of retrodictability. There we want to know in what state the system was prepared at time t_1 given that the state $|f\rangle$ is found by measurement at time t_2, and nothing can be said about in what state the system was just before the measurement. This cannot be a valid proof for the retrodictability of orthodox quantum mechanics.

So, the past-directed probabilities cannot be determined directly in orthodox quantum mechanics. In the previous section it was shown how to determine the past-directed probabilities out of the future-directed probabilities. Let us begin with looking at the conditions for retrodictability according to 4.1. As said the unitarity of the transition matrix is sufficient for the inverse normalization:

$$\begin{aligned}\int p(\vec{x}_n, t_n|\vec{x}_{n-1}, t_{n-1})\, d\vec{x}_{n-1} &= \int |\langle f|U(t_2, t_1)|i\rangle|^2\, d|i\rangle \\ &= \int |\langle i|U^\dagger(t_2, t_1)|f\rangle|^2\, d|i\rangle \\ &= \int |\langle i|U(t_2, t_1)|f\rangle|^2\, d|i\rangle \\ &= \int p(\vec{x}_{n-1}, t_n|\vec{x}_n, t_{n-1})\, d\vec{x}_{n-1} \\ &= 1. \end{aligned} \tag{4.48}$$

Then, if the $p(\vec{x}_{n-1}, t_{n-1})$ were set to be uniform, orthodox quantum mechanics would be retrodictable. However, as Watanabe ([21]) states, "such a retrodictive quantum mechanics can find applicability only in very exceptional cases."

Looking at the necessary condition for retrodictability (equation (4.42)) gives as a requirement:

$$|\langle f|U(t_2, t_1)|i\rangle|^2 = |\langle i|i\rangle|^2 = 1, \tag{4.49}$$

which should hold for every $|i\rangle$ and $|f\rangle$. Of course this cannot be true and orthodox quantum mechanics is irretrodictable yet again.

4.6 The HB principle

The third and last principle for the time reversal invariance of stochastic laws of nature that I will discuss, I will call the *HB principle* as it was discussed by Holster ([12]) and Bacciagaluppi ([2]). Holster himself calls it the correct criterion for time reversal invariance, which seems to be a bit presumptuous.

Bacciagaluppi just describes it by (FTB=BTP), meaning forward transition probability equals backward transition probability. The HB principle is a combination of microreversibility and retrodictability. Two processes are compared, the original one with the reverse one, as in microreversibility. The probability for the original process is future-directed, whereas the probability for the reverse process is past-directed as in retrodictability. The HB principle states that a stochastic law of nature is time reversal invariant if:

> *for every state represented by the vector $\vec{x}_n$, the conditional probability of finding by measurement a state $\vec{x}_n$ given an earlier prepared state $\vec{x}_{n-1}$ equals the conditional probability of having prepared the reverse state ${}^R\vec{x}_n$ given finding by measurement the later reverse state ${}^R\vec{x}_{n-1}$,*

or, symbolically:

$$p(\vec{x}_n, t_n|\vec{x}_{n-1}, t_{n-1}) = p({}^R\vec{x}_n, t_{n-1}|{}^R\vec{x}_{n-1}, t_n). \tag{4.50}$$

Here, the future-directed probability of a process is compared with the past-directed probability of the reverse process. Finding the past-directed probabilities faces the same problems as with retrodictability. Similarly to the expression for determining past-directed probabilities out of future-directed probabilities (equation (4.35)), it is possible to express the right hand side of equation (4.50) as:

$$p({}^R\vec{x}_n, t_{n-1}|{}^R\vec{x}_{n-1}, t_n) = \frac{p({}^R\vec{x}_n, t_{n-1})\, p({}^R\vec{x}_{n-1}, t_n|{}^R\vec{x}_n, t_{n-1})}{\int p({}^R\vec{x}_n, t_{n-1})\, p({}^R\vec{x}_{n-1}, t_n|{}^R\vec{x}_n, t_{n-1})\; d^R\vec{x}_n} \tag{4.51}$$

Again it can be seen that in order to determine the past-directed probabilities, one needs to know the $p({}^R\vec{x}_n, t_{n-1})$. This expression reduces to the left hand side of equation (4.50) if:

- the $p({}^R\vec{x}_n, t_{n-1})$ are uniform,
- and the inverse normalization $\int p({}^R\vec{x}_{n-1}, t_n|{}^R\vec{x}_n, t_{n-1})\; d^R\vec{x}_n = 1$ holds,
- and $p({}^R\vec{x}_{n-1}, t_n|{}^R\vec{x}_n, t_{n-1}) = p(\vec{x}_n, t_n|\vec{x}_{n-1}, t_{n-1})$.

But the third condition is exactly the principle of microreversibility and, if it is assumed that the sets ${}^R\vec{x}_n$ and ${}^R\vec{x}_{n-1}$ are complete, the first two condition are exactly the same as those for the principle of retrodictability. This means that if a theory is both microreversible and retrodictable it is also time reversal invariant according to the HB principle:

$$\text{microreversibility} + \text{retrodictability} \longrightarrow \text{HB principle}. \tag{4.2}$$

So, microreversibility and retrodictability are sufficient for time reversal invariance according to the HB principle, but it does not mean that they are necessary. Other relations between the three principles are discussed in the next two sections of this chapter.

4.7 Applying the HB principle to orthodox quantum mechanics

For orthodox quantum mechanics, microreversibility holds as was proven in section 4.3, but retrodictability does not hold as was proven in section 4.5. Let us look at this case, where microreversibility holds but retrodictability does not. The latter, according to equation (4.31), means that:

$$p(\vec{x}_n, t_n|\vec{x}_{n-1}, t_{n-1}) \neq p(\vec{x}_{n-1}, t_{n-1}|\vec{x}_n, t_n). \tag{4.52}$$

If we 'apply' the principle of microreversibility (equation (4.10)) to the right side, i.e. replace $\vec{x}_{n-1}$ by ${}^R\vec{x}_n$ and $\vec{x}_n$ by ${}^R\vec{x}_{n-1}$ but leave the times alone, we arrive at:

$$p(\vec{x}_n, t_n|\vec{x}_{n-1}, t_{n-1}) \neq p({}^R\vec{x}_n, t_{n-1}|{}^R\vec{x}_{n-1}, t_n), \tag{4.53}$$

which exactly means that the HB principle does not hold and that orthodox quantum mechanics is time reversal non-invariant according to the HB principle.

4.8 Comparing the principles for stochastic time reversal invariance

In this section I will compare the three principles for stochastic time reversal invariance. I will begin by showing that every two of them entail the other one. Suppose that microreversibility and retrodictability hold, then, similarly to the previous section, if we start with the equation for microreversibility (equation 4.10):

$$p(\vec{x}_n, t_n|\vec{x}_{n-1}, t_{n-1}) = p({}^R\vec{x}_{n-1}, t_n|{}^R\vec{x}_n, t_{n-1}), \tag{4.54}$$

and we 'apply' the principle of retrodictability (equation (4.31)) to the right side, i.e. exchange ${}^R\vec{x}_{n-1}, t_n$ and ${}^R\vec{x}_n, t_{n-1}$, we arrive at:

$$p(\vec{x}_n, t_n|\vec{x}_{n-1}, t_{n-1}) = p({}^R\vec{x}_n, t_{n-1}|{}^R\vec{x}_{n-1}, t_n), \tag{4.55}$$

which is exactly the HB principle. Similarly, all other entailments can be proven, meaning:

$$\text{principle A} + \text{principle B} \longrightarrow \text{principle C},$$

fig. (4.3)

where principles A, B, and C are any one of the three principles but none are the same one.

Let us now look at the case where one of the principles holds and a second doesn't. In the previous section it was shown that microreversibility and irretrodictability entail time reversal non-invariance according to the HB principle. This can easily be proven for all other entailments, with $\neg$ meaning that the principle does not hold:

$$\text{principle A} + \neg\text{principle B} \longrightarrow \neg\text{principle C.}$$

fig. (4.4)

We will now investigate if entailments can be found when two principles do not hold. Imagine the following case, of which I am not sure that the probabilities are consistent:

$$\begin{aligned}
p(\vec{x}_n, t_n|\vec{x}_{n-1}, t_{n-1}) &= p({}^R\vec{x}_{n-1}, t_n|{}^R\vec{x}_n, t_{n-1}) - \frac{1}{1000} \\
p(\vec{x}_n, t_n|\vec{x}_{n-1}, t_{n-1}) &= p(\vec{x}_{n-1}, t_{n-1}|\vec{x}_n, t_n) + \frac{1}{1000}.
\end{aligned} \tag{4.56}$$

It is immediately clear that microreversibility and retrodictability do not hold. However, it can be shown, similar to proofs above, that the HB principle does hold. This leads to:

$$\neg\text{principle A} + \neg\text{principle B} \not\longrightarrow \neg\text{principle C.}$$

fig. (4.5)

This proves that, for a principle to hold, either both of the other principles must hold or none of them. Because of countless examples we can also safely say that:

$$\neg\text{principle A} + \neg\text{principle B} \not\longrightarrow \text{principle C.}$$

fig. (4.6)

I will now continue by giving examples. They show what systems look like for which the three principles variably hold or do not hold, and are often used to disprove a certain principle. They also illustrate the aforementioned entailments. Two of these examples are by Holster ([12]), who uses them to try to disprove the principle of microreversibility.

Holster argues that microreversibility is neither necessary nor sufficient for time reversal invariance. To disprove the necessity Holster constructs a theory that is, according to him, obviously time reversal invariant, but is not microreversible. In this theory there are three states A, B, and C, all of which are equal to their reverse state: A = ^{R}A, B = ^{R}B, and C = ^{R}C and the following future-directed probabilities:

$$\begin{aligned}
p(B, t_n|A, t_{n-1}) = p(C, t_n|A, t_{n-1}) = \frac{1}{2} \\
p(A, t_n|B, t_{n-1}) = p(A, t_n|C, t_{n-1}) = 1 \\
p(C, t_n|B, t_{n-1}) = p(B, t_n|C, t_{n-1}) = 0.
\end{aligned} \tag{4.57}$$

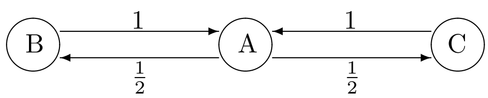

fig. (4.6)

Holster argues that this theory is time reversal invariant, because the probability of any given process is equal to the probability of the reverse process. With this he means the HB principle: the sequence of states in the future-direction is just as probable as the reverse sequence of reverse states in the past-direction. This can be proven by looking at the past-directed probabilities:

$$\begin{aligned} p(B,t_{n-1}|A,t_n) = p(C,t_{n-1}|A,t_n) &= \frac{1}{2} \\ p(A,t_{n-1}|B,t_n) = p(A,t_{n-1}|C,t_n) &= 1 \\ p(C,t_{n-1}|B,t_n) = p(B,t_{n-1}|C,t_n) &= 0. \end{aligned} \tag{4.58}$$

However, the theory is not microreversible and therefore, since microreversibility does not recognize this time reversal invariance, it is not necessary. To me this can not be seen as a proof since Holster equates the HB principle with time reversal invariance and then proves that microreversibility does not hold, arriving at the conclusion that microreversibility is not necessary. It can be seen as a means of persuasion; if this theory feels time reversal invariant and microreversibility does not hold, then microreversibility cannot be necessary for time reversal invariance.

It can also be proven that the theory is irretrodictable. This example illustrates the validity of figure (4.4), i.e. if one principle holds and another does not, the third principle does not hold either, and the validity of figure (4.5), i.e. if two principles do not hold, the third can still hold.

Holster tries to disprove the sufficiency of the principle of microreversibility for time reversal invariance with the following system. From a state S there are states in three directions, denoted by S_{down}, $\mathrm{S}_{up-right}$, and $\mathrm{S}_{up-left}$, with future-directed probabilities:

$$p(\mathrm{S}_{up-right},t_n|\mathrm{S},t_{n-1}) = p(\mathrm{S}_{up-left},t_n|\mathrm{S},t_{n-1}) = p(\mathrm{S}_{down},t_n|\mathrm{S},t_{n-1}) = \frac{1}{3}, \tag{4.59}$$

immediately implying microreversibility, and past-directed probabilities (which Holster shows to be consistent with the future-directed probabilities):

$$\begin{aligned} p(\mathrm{S}_{up-right},t_{n-1}|\mathrm{S},t_n) = p(\mathrm{S}_{up-left},t_{n-1}|\mathrm{S},t_n) = &\quad \frac{1}{6} \\ p(\mathrm{S}_{down},t_{n-1}|\mathrm{S},t_n) = &\quad \frac{2}{3}. \end{aligned} \tag{4.60}$$

Now, towards the future the system evolves in the up-direction with an average rate of climb of $\frac{1}{3}$ per transition. The time reversal non-invariance follows, according to Holster, from the fact that towards the past the system evolves in the up-direction with an average rate of climb of $\frac{1}{3}$ per transition as well. Therefore, although the theory is microreversible, it is time reversal non-invariant according to Holster and it is shown that microreversibility is not sufficient for time reversal invariance.

It can also be proven that retrodictability does not hold. This example is an illustration of the entailment shown in figure (4.4), i.e. if one of the principles holds and another doesn't, then the third is entailed not to hold as well, and of the entailment shown in figure (4.5), i.e. if two principles do not hold, it is not entailed that the third does not hold as well.

A final example, not by Holster, is one that is used often to show that microreversibility does not capture some intuitive notions of time reversal invariance. Again, in this theory there are three states A, B, and C, all of which are equal to their reverse state: A = ^{R}A, B = ^{R}B, and C = ^{R}C and the future-directed probabilities:

$$
\begin{aligned}
p(B,t_n|A,t_{n-1}) = p(C,t_n|B,t_{n-1}) = p(A,t_n|C,t_{n-1}) = \frac{3}{4} \\
p(A,t_n|B,t_{n-1}) = p(B,t_n|C,t_{n-1}) = p(C,t_n|A,t_{n-1}) = \frac{1}{4}
\end{aligned} \tag{4.61}
$$

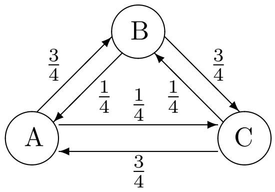

fig. (4.6)

The theory is not microreversible. In this system it is true that, if $p(A,t_n) = p(B,t_n) = p(C,t_n) = \frac{1}{3}$ for any time t_n, then it is true for every time t_n. Then the past-directed probabilities can be determined:

$$
\begin{aligned}
p(B,t_{n-1}|A,t_n) = p(C,t_{n-1}|B,t_n) = p(A,t_{n-1}|C,t_n) = \frac{1}{4} \\
p(A,t_{n-1}|B,t_n) = p(B,t_{n-1}|C,t_n) = p(C,t_{n-1}|A,t_n) = \frac{3}{4},
\end{aligned} \tag{4.62}
$$

which show that the theory is retrodictable but time reversal non-invariant according to the HB principle. This is another illustration of figures (4.4) and (4.5).

4.9 Deciding on a principle for stochastic time reversal invariance

It appears to me that choosing one of the three principles as a definition of stochastic time reversal invariance can be done in one of two ways. Above, I showed Holster trying to disprove microreversibility as the correct principle and prove the HB principle, through comparing them both mathematically with an intuitive notion of time reversal invariance, which, of course, looks like the HB principle. This cannot be the correct way.

Correct seems to be to me to think of time reversal and time reversal invariance as a philosophical notion. It should first be elucidated in words what is meant by time reversal invariance and when this is done only then can this view be described in a mathematical form. I will certainly not claim to know the answer, but I will now argue for one of the three principles.

Firstly, I think time reversal invariance should include time reversal, i.e. replacing t with $-t$ or moving through states in the opposite temporal direction or inversion of the temporal axis. Although it is physically not possible (yet?) to reverse time, this should not withhold us from using this concept theoretically. If in physics problems are being solved using processes reversed only in order but not in temporal direction, this should bear another name. To me microreversibility can therefore not be an adequate principle for stochastic time reversal.

Secondly, retrodictability does not suffice. It compares whether looking in one temporal direction equals looking in the other temporal direction, but looking at one and the same process. This says little about the system that is observed and a lot about the observation. It also arises from the fact that the origin of irretrodictability lies in the notion of conditional probability and not in the structure of dynamical theories.

Finally, I think the HB principle is the correct one for stochastic time reversal. It compares a process in the future direction with that process in the past direction. It both involves a literal time reversal and emphasizes the dynamical theory and not the act of observation.

The reason for deciding on a principle for stochastic time reversal invariance is in order to apply it to GRW. If GRW allows a process towards the future with the same probability as it does the reverse process towards the past, then it is shown in chapter 2 that the Past Hypothesis is needed to account for macroscopic behaviour. If GRW has different probabilities for processes towards the future and reverse processes towards the past, then the Past Hypothesis may not be needed and the microscopic view of GRW can, along with the probability densities of the past microstates, account for the irreversibility of the macroscopic view. For this purpose it is needed that the principle for stochastic time reversal invariance that is used, is able to compare future-directed and past-directed processes. The HB principle does exactly this.

Chapter 5

Assumption: GRW is time reversal non-invariant

In this chapter it is determined whether GRW is time reversal invariant or not according to microreversibility, retrodictability, and the HB principle. Finally, a conclusion is drawn as to the question of GRW's ability to explain the irreversibility of the macroscopic world.

5.1 Applying microreversibility to GRW

In this section it is investigated if the principle of microreversibility holds for GRW. For simplicity I will look at a system consisting of a single particle. In order to determine w and w_{rev}, first the evolution specifically for GRW has to be established, as Bell ([4]) described. In GRW spontaneous collapses, also called jumps, occur at certain times t_n multiplying the wave function with the jump factor $j(\vec{x}_n - \vec{r})$ as in equation (3.3). In the preceding, intermediate and following time intervals the wave function evolves according to the Schrödingier equation through $U(t_2, t_1)$ as in equation (4.20). This gives $E(f, i)$ as the operator guiding the GRW evolution between the initial time i and the final time f when m jumps occur:

$$E(f, i) = U(f, t_m) j(\vec{x}_m - \vec{r}) \dots U(t_2, t_1) j(\vec{x}_1 - \vec{r}) U(t_1, i). \tag{5.1}$$

This means that at time f the state will have evolved from the initial pure state $|i\rangle$ as:

$$E(f, i)|i\rangle. \tag{5.2}$$

The probability of finding by measurement at time f the final pure state $|f\rangle$ then is:

$$\begin{aligned} w &= e^{\lambda(i-f)} |\langle f|E(f, i)|i\rangle|^2 \\ &= e^{\lambda(i-f)} |\langle i|E^\dagger(f, i)|f\rangle|^2 \end{aligned} \tag{5.3}$$

where $e^{\lambda(i-f)}$ comes from the product of the probabilities $e^{-\lambda(t_{n+1}-t_n)}$ of having no jump in the preceding, intermediate and following time intervals, and λ is the frequency with which jumps occur.[1]

When determining the reverse probability, i.e. the probability of reaching $K|i\rangle$ out of $K|f\rangle$, attention should be devoted to the question of what the evolution between these two states looks like. To avoid confusion with the prior evolution we will look at state $K|f\rangle$ at time f and $K|i\rangle$ at time i' with $i < f < i'$ and $f - i = i' - f$. I think the evolution should also be reversed. This means that the jumps will occur not at times $f + t_1, f + t_2, \ldots, f + t_m$ but at times $i' - t_m, \ldots, i' - t_2, i' - t_1$, giving:

$$E(i', f) = U(i', i' - t_1)j(\vec{x}_1 - \vec{r}) \ldots U(i' - t_{m-1}, i' - t_m)j(\vec{x}_m - \vec{r})U(i' - t_m, f), \tag{5.4}$$

which gives us for the reverse probability:

$$\begin{aligned} w_{rev} &= e^{\lambda(f-i')}|\langle Ki|E(i', f)|Kf\rangle|^2 \\ &= e^{\lambda(i-f)}|\langle i|K^\dagger E(i', f)K|f\rangle|^2. \end{aligned} \tag{5.5}$$

So the question of whether GRW is microreversible or not depends on the outcome of the following equation:

$$E^\dagger(f, i) \stackrel{?}{=} K^\dagger E(i', f)K. \tag{5.6}$$

[1]Imagine that between t_n and t_{n+1} one attempt at spontaneous collapse is done. The probability of no jump is $1 - \lambda(t_{n+1} - t_n)$. Now imagine the time interval is divided in two and two jumps can occur. The probability of no jump is $(1 - \frac{\lambda(t_{n+1}-t_n)}{2})^2$. With GRW infinitely many jumps may occur, so the probability of no jump is $\lim_{j\to\infty}(1 - \frac{\lambda(t_{n+1}-t_n)}{j})^j = e^{-\lambda(t_{n+1}-t_n)}$.

Doing the math,

$$\begin{aligned}
K^\dagger E(i',f)K &= K^\dagger U(i',i'-t_1)j(\vec{x}_1-\vec{r})\dots \\
&\quad U(i'-t_{m-1},i'-t_m)j(\vec{x}_m-\vec{r})U(i'-t_m,f)K \\
&= K^\dagger U(i',i'-t_1)Kj(\vec{x}_1-\vec{r})\dots \\
&\quad K^\dagger U(i'-t_{m-1},i'-t_m)Kj(\vec{x}_m-\vec{r})K^\dagger U(i'-t_m,f)K
\end{aligned}$$

(making the substitution $KK^\dagger \to \mathbb{I}$ (cf. equation (4.18)) and using $[K, j(\vec{x}_n-\vec{r})] = [K^\dagger, j(\vec{x}_n-\vec{r})] = 0$ since $j(\vec{x}_n-\vec{r})$ is a real valued function)

$$\begin{aligned}
&= U^\dagger(i',i'-t_1)j(\vec{x}_1-\vec{r})\dots \\
&\quad U^\dagger(i'-t_{m-1},i'-t_m)j(\vec{x}_m-\vec{r})U^\dagger(i'-t_m,f)
\end{aligned}$$

(using $K^\dagger U(t_2,t_1)K = U^\dagger(t_2,t_1)$ (equation (4.28)))

$$\begin{aligned}
&= \Big(U(f,t_m)j(\vec{x}_m-\vec{r})\dots \\
&\quad U(t_2,t_1)j(\vec{x}_1-\vec{r})U(t_1,i)\Big)^\dagger
\end{aligned}$$

(using $A^\dagger B^\dagger = (BA)^\dagger$ and seeing that $U(t_2,t_1)$ actually only depends on the difference t_2-t_1)

$$= E^\dagger(f,i), \tag{5.7}$$

proves that GRW is microreversible.

Until now, we have only spoken of pure states, so we will continue with mixed states. Repeating equation (3.9) gives the superoperator for jump m:

$$\mathcal{T}_m[\rho] = \int j(\vec{x}_m-\vec{r})\ \rho\ j(\vec{x}_m-\vec{r})\ d^3\vec{x}_m. \tag{5.8}$$

A similar expression can be given for the evolution through the Schrödingier equation:

$$\mathcal{U}(t_2,t_1)[\rho] = U(t_2,t_1)\ \rho\ U^\dagger(t_2,t_1). \tag{5.9}$$

I will use the circle symbol $\circ$ to denote the composition of two superoperators:

$$(\mathcal{A}\circ\mathcal{B})[\rho] = \mathcal{A}[\mathcal{B}[\rho]], \tag{5.10}$$

making it possible to denote the superoperator for the GRW evolution $\mathcal{E}(f,i)$ similar to equation (5.1):

$$\mathcal{E}(f,i) = \mathcal{U}(f,t_m)\circ\mathcal{T}_m\circ\dots\circ\mathcal{U}(t_2,t_1)\circ\mathcal{T}_1\circ\mathcal{U}(t_1,i), \tag{5.11}$$

and the superoperator for the evolution of the reverse process, $\mathcal{E}(i',f)$ as:

$$\mathcal{E}(i',f) = \mathcal{U}(i',i'-t_1)\circ\mathcal{T}_1\circ\dots\circ\mathcal{U}(i'-t_{m-1},i'-t_m)\circ\mathcal{T}_m\circ\mathcal{U}(i'-t_m,f). \tag{5.12}$$

The principle for microreversibility,

$$\text{prob}((\rho_i)_i \to (\rho_f)_f) = \text{prob}((K\rho_f K^\dagger)_f \to (K\rho_i K^\dagger)_{i'}), \tag{5.13}$$

then becomes the question:

$$\mathrm{Tr}(\rho_f\ \mathcal{E}(f,i)[\rho_i]) \stackrel{?}{=} \mathrm{Tr}(K\rho_i K^\dagger\ \mathcal{E}(i',f)[K\rho_f K^\dagger]) \tag{5.14}$$

Writing and manipulating the right hand side then gives:

$$\begin{aligned}
&\mathrm{Tr}(K\rho_i K^\dagger\ \mathcal{E}(i',f)[K\rho_f K^\dagger]) = \\
&\mathrm{Tr}(K\rho_i K^\dagger\ (\mathcal{U}(i',i'-t_1)\circ\mathcal{T}_1\circ\ldots\circ\mathcal{U}(i'-t_{m-1},i'-t_m)\circ\mathcal{T}_m\circ\mathcal{U}(i'-t_m,f))[K\rho_f K^\dagger]) = \\
&\mathrm{Tr}(K\rho_i K^\dagger\ U(i',i'-t_1)\int j(\vec{x}_1-\vec{r})\cdots U(i'-t_{m-1},i'-t_m)\int j(\vec{x}_m-\vec{r})U(i'-t_m,f)K^\dagger\rho_f K \\
&\qquad U^\dagger(i'-t_m,f)j(\vec{x}_m-\vec{r})\ d^3\vec{x}_m\ U^\dagger(i'-t_{m-1},i'-t_m)\cdots j(\vec{x}_1-\vec{r})\ d^3\vec{x}_1\ U^\dagger(i',i'-t_1))
\end{aligned}$$

(now making the substitution $KK^\dagger\rightarrow\mathbb{I}$ (cf. equation (4.18)) and using $[K,j_n(\mathbf{x_n})] = [K^\dagger,j_n(\mathbf{x_n})] = 0$ since $j_n(\mathbf{x_n})$ is a real valued function, and using the fact that the integral and the trace are interchangeable since the wavefunction is independent of the variables of integration.)

$$\begin{aligned}
= \quad & \int\cdots\int \mathrm{Tr}(K\rho_i\ K^\dagger U(i',i'-t_1)Kj(\vec{x}_1-\vec{r})\cdots \\
& K^\dagger U(i'-t_{m-1},i'-t_m)Kj(\vec{x}_m-\vec{r})K^\dagger U(i'-t_m,f)K\rho_f K^\dagger \\
& U^\dagger(i'-t_m,f)K^\dagger j(\vec{x}_m-\vec{r})KU^\dagger(i'-t_{m-1},i'-t_m)K\cdots \\
& j(\vec{x}_1-\vec{r})\ K^\dagger U^\dagger(i',i'-t_1)\ d^3\vec{x}_m\cdots d^3\vec{x}_1)
\end{aligned} \tag{5.15}$$

(now using $K^\dagger U(t_2,t_1)K = U^\dagger(t_2,t_1)$ and $K^\dagger U^\dagger(t_2,t_1)K = U(t_2,t_1)$ from equations (4.27) and (4.28)), seeing that $U(t_2,t_1)$ actually only depends on the difference t_2-t_1, moving ρ_f to the front using the fact that cyclic permutation is allowed inside a trace, and interchanging the integral and the trace back again.)

$$\begin{aligned}
= \quad & \mathrm{Tr}(\rho_f U(f,t_m)\int j(\vec{x}_m-\vec{r})U(i'-t_m,t_{m-1})\cdots\int j(\vec{x}_1-\vec{r})U(t_1,i) \\
& \rho_i U^\dagger(t_1,i)j(\vec{x}_1-\vec{r})\ d^3\vec{x}_1\cdots U^\dagger(t_m,t_{m-1})j(\vec{x}_m)d^3\vec{x}_m U^\dagger(f,t_m) \\
= \quad & \mathrm{Tr}(\rho_f\ \mathcal{E}(f,i)[\rho_i]),
\end{aligned} \tag{5.16}$$

which means that for mixed states GRW is microreversible as well.

5.2 Applying retrodictability to GRW

Let us now apply the principle of retrodictability to GRW. The theory of GRW provides for both the $p(\vec{x}_{n-1},t_{n-1})$ and the future-directed probabilities $p(\vec{x}_n,t_n|\vec{x}_{n-1},t_{n-1})$. This means that the past-directed probabilities are completely determined and we will have to examine whether GRW complies with the conditions brought forth by elementary probability theory. The condition of uniform $p(\vec{x}_{n-1},t_{n-1})$ makes for just as inapplicable a theory for GRW as it is for orthodox quantum mechanics. This means that, although the condition for the inverse normalization is met since GRW is microreversible, this condition is irrelevant. The necessary condition of equal probabilities for the initial and final states (equation (4.42)), expressed in GRW as:

$$|\langle f|E(t_2,t_1)|i\rangle|^2 = |\langle i|i\rangle|^2 = 1, \tag{5.17}$$

is also rarely met for all $|i\rangle$ and $|f\rangle$. Since orthodox quantum mechanics (cf. equation (4.49)) and GRW are both stochastic theories and they even determine probabilities in the same way, it is no surprise to find that GRW is irretrodictable.

5.3 Applying the HB principle to GRW

Determining if GRW is time reversal invariant according to the HB principle again becomes a matter of entailment. Microreversibility holds as was proven in section 5.1, but retrodictability does not hold as was proven in section 5.2. This entails, as illustrated in figure (4.4), that GRW is time reversal non-invariant according to the HB principle.

5.4 Summary

In the previous chapter it was argued that the HB principle captured best the features of stochastic time reversal invariance needed for establishing an account of statistical mechanics based solely on GRW without the Past Hypothesis. This leads to the following conclusion of this thesis.

If:

- the collection of normal microstates is vastly larger than the collection of abnormal microstates,
- the dynamics of GRW doesn't exclude the normal microstates from being entered into,
- the GRW perturbation instills the same probability distribution of microstates as statistical mechanics does,
- it is only applied to solids,
- the probability density of the initial state is known,

then, because:

- GRW accounts for the dynamics of processes in the microscopic view.
- GRW perturbs abnormal microstates into normal microstates.
- GRW determines the probability distribution of microstates on its own, just as statistical mechanics does, towards both the future and the past,

GRW can replace statistical mechanics and classical particle mechanics to describe processes in the macroscopic view, and because:

- it is proven in this thesis that GRW is time reversal non-invariant according to the HB principle,

it can explain the irreversibility of processes in the macroscopic view without the need of the Past Hypothesis.

5.5 Acknowledgements

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