$$I^j_{pred}(x) = \sum_n \alpha^j_n g(x - x_n)$$

$$I^j_{data}(x)$$

$$D = \sum_x \sum_j \left(I^j_{pred}(x) - I^j_{data}(x) \right)^2$$

$$= \underbrace{\sum_{x,j} I^{j\,2}_{data}(x)}_{①} \; \underbrace{- 2 \sum_j \sum_x I^j_{pred}(x) I^j_{data}(x)}_{②} + \underbrace{\sum_j \sum_x I^{j\,2}_{pred}(x)}_{③}$$

① ⇒ const

② $\Rightarrow -2 \sum_n \alpha^j_n \sum_x g(x - x_n) I^j_{data}(x)$

$= -2 \sum_n \sum_j \alpha^j_n F^j(x_n) = \left(\begin{matrix} \text{eval.} \\ \text{at} \\ n-1 \end{matrix} \right) - 2 \sum_j \alpha^j_n F^j(x_n)$

easy dot product. ↑ ↑ precompute

③ $\sum_{j,n=0} (\alpha^j_n)^2 g(0) + 2 \sum_{j,n>1} (\alpha^j_n \alpha^j_{n-1}) g(\underbrace{\delta_n}_{[x_n - x_{n-1}]})$

Again can be precomputed

$= \left(\begin{matrix} \text{eval.} \\ \text{at} \\ n-1 \end{matrix} \right) + (\alpha^j_n)^2 g(0) + 2 \left[\sum_j \alpha^j_n \alpha^j_{n-1} \right] g(\delta_n)$