Welford's Method for Computing Variance

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With all the steps missing from the derivation available here.

$$(N-1)s_N^2 - (N-2)S_{N-1}^2 = \sum_{i=1}^{N}(x_i - \overline{x}_N)^2 - \sum_{i=1}^{N-1}(x_i - \overline{x}_{N-1})^2 \tag{1}$$

$$(x_N - \overline{x}_N)^2 + \sum_{i=1}^{N-1}(x_i - \overline{x}_N)^2 - \sum_{i=1}^{N-1}(x_i - \overline{x}_{N-1})^2 \tag{2}$$

$$(x_N - \overline{x}_N)^2 + \sum_{i=1}^{N-1}(x_i - \overline{x}_N)^2 - (x_i - \overline{x}_{N-1})^2 \tag{3}$$

$$(x_N - \overline{x}_N)^2 + \sum_{i=1}^{N-1}(x_i - \overline{x}_N)^2 - (x_i - \overline{x}_{N-1})^2 \tag{4}$$

$$(x_N - \overline{x}_N)^2 + \sum_{i=1}^{N-1}(\overline{x}_{N-1} - \overline{x}_N)(2x_i - \overline{x}_N - \overline{x}_{N-1}) \tag{5}$$

(difference of squares)

$$(x_N - \overline{x}_N)^2 + \sum_{i=1}^{N-1}(\overline{x}_{N-1} - \overline{x}_N)(x_i - \overline{x}_N + x_i - \overline{x}_{N-1}) \tag{6}$$

$$(x_N - \overline{x}_N)^2 + (\overline{x}_{N-1} - \overline{x}_N)\sum_{i=1}^{N-1}(x_i - \overline{x}_N + x_i - \overline{x}_{N-1}) \tag{7}$$

$$(x_N - \overline{x}_N)^2 + (\overline{x}_{N-1} - \overline{x}_N)\sum_{i=1}^{N-1}((x_i - \overline{x}_N) + (x_i - \overline{x}_{N-1})) \tag{8}$$

$$(x_N - \overline{x}_N)^2 + (\overline{x}_{N-1} - \overline{x}_N)[\sum_{i=1}^{N-1}(x_i - \overline{x}_N) + \sum_{i=1}^{N-1}(x_i - \overline{x}_{N-1})] \tag{9}$$

$$(x_N - \overline{x}_N)^2 + (\overline{x}_{N-1} - \overline{x}_N)[\sum_{i=1}^{N-1}(x_i - \overline{x}_N) + 0] \tag{10}$$

$$(x_N - \overline{x}_N)^2 + (\overline{x}_{N-1} - \overline{x}_N)[\sum_{i=1}^{N-1}(x_i - \overline{x}_N) + (x_N - \overline{x}_N) - (x_N - \overline{x}_N)] \tag{11}$$

$$(x_N - \overline{x}_N)^2 + (\overline{x}_{N-1} - \overline{x}_N)[(\sum_{i=1}^{N-1}(x_i - \overline{x}_N) + (x_N - \overline{x}_N)) - (x_N - \overline{x}_N)] \tag{12}$$

$$(x_N - \overline{x}_N)^2 + (\overline{x}_{N-1} - \overline{x}_N)[\sum_{i=1}^{N}(x_i - \overline{x}_N) - (x_N - \overline{x}_N)] \tag{13}$$

$$(x_N - \overline{x}_N)^2 + (\overline{x}_{N-1} - \overline{x}_N)[0 - (x_N - \overline{x}_N)] \tag{14}$$

$$(x_N - \overline{x}_N)^2 + (\overline{x}_{N-1} - \overline{x}_N)(\overline{x}_N - x_N) \tag{15}$$

$$(x_N - \overline{x}_N)^2 + (\overline{x}_{N-1} - \overline{x}_N)(\overline{x}_N - x_N) \tag{16}$$

$$(x_N - \overline{x}_N)((x_N - \overline{x}_N) + (\overline{x}_{N-1} - \overline{x}_N)(-1)) \tag{17}$$

$$(x_N - \overline{x}_N)((x_N - \overline{x}_N) - (\overline{x}_{N-1} - \overline{x}_N)) \tag{18}$$

$$(x_N - \overline{x}_N)(x_N - \overline{x}_N) - \overline{x}_{N-1} + \overline{x}_N) \tag{19}$$

$$(x_N - \overline{x}_N)(x_N - \overline{x}_{N-1}) \tag{20}$$