Homework Five

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STA4702
02.07.12

3.5a

X		eigval	eigvec		S		|S|
9	1	16.262087	0.9915228	0.1299328	16	-2	12
5	3	0.7379127	-0.129933	0.9915228	-2	1	
1	2						

3.5b

X		eigval	eigvec		S		|S|
3	4	11.408327	-0.471858	0.8816746	3	-4.5	6.75
6	-2	0.5916731	0.8816746	0.4718579	-4.5	9	
3	1						

3.6a

X			x-bar	one	one*x-bar		
-1	3	-2	2	1	2	3	1
2	4	2	3	1	2	3	1
5	2	3	1	1	2	3	1

residuals			|S|
-3	0	-3	0
0	1	1	
3	-1	2	

Since the determinant of X – 1(x-bar)’ = 0, the matrix is not of full rank.

3.6b

X			S			|S|
-1	3	-2	9	-1.5	7.5	0
2	4	2	-1.5	1	-0.5	
5	2	3	7.5	-0.5	7	

Since |S| = 0, the three-dimensional volume of the parallelogram is zero.

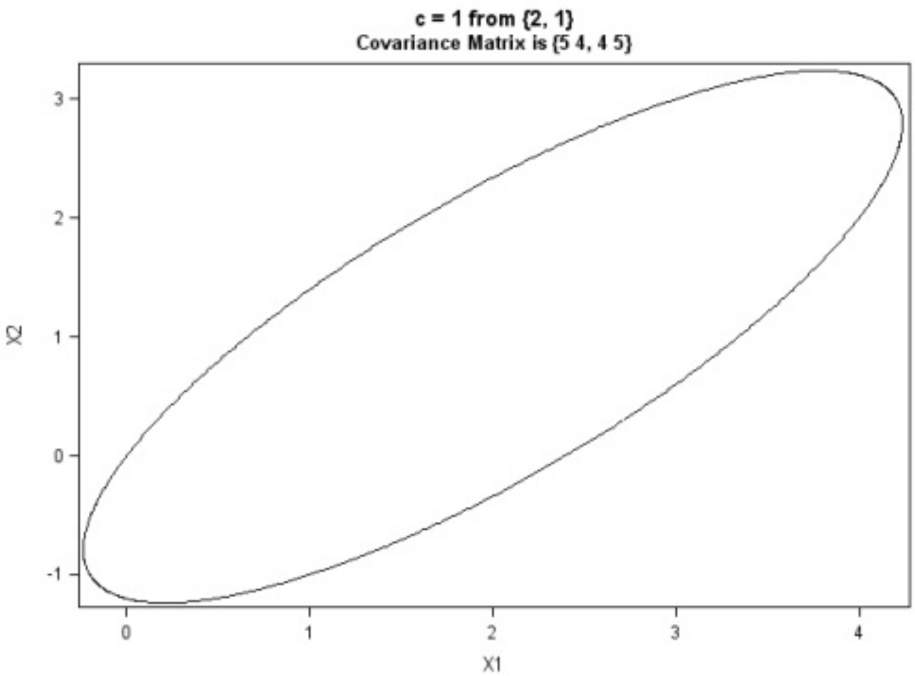
3.6c

Total variance = trace(S) = S11 + S22 + S33 = 9 + 1 +7 = 17

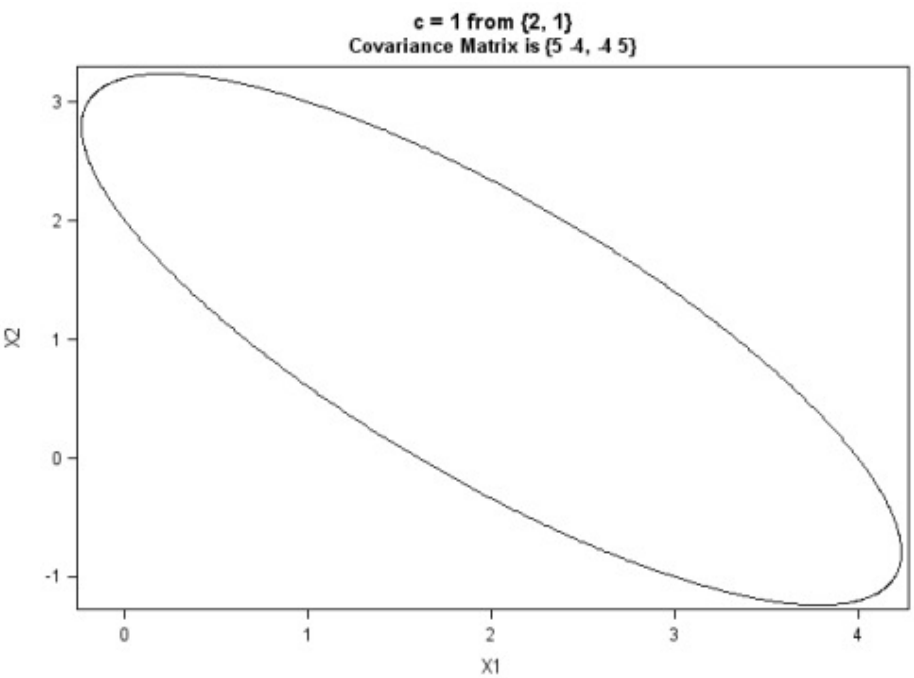
3.7

x-bar’ = [2, 1] for all S. c^2 = 1 for all S.

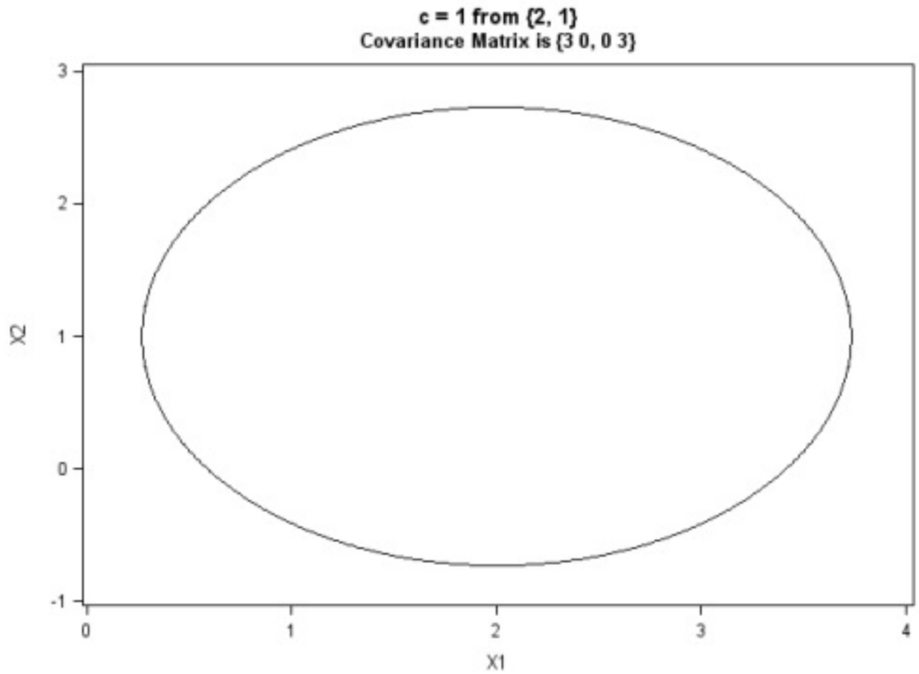
S = [5 4, 4 5], $\lambda^1 = 9$, $\lambda^2 = 1$, $e^{1\prime} = [1/\sqrt{2}, 1/\sqrt{2}]$, $e^{2\prime} = [1/\sqrt{2}, -1/\sqrt{2}]$, vectors $\sqrt{9}*\sqrt{1}*e^1$ and $\sqrt{1}*\sqrt{1}*e^2$



S = [5 −4, −4 5], $\lambda^1 = 9$, $\lambda^2 = 1$, $e^{1'} = [1/\sqrt{2}, -1/\sqrt{2}]$, $e^{2'} = [1/\sqrt{2}, 1/\sqrt{2}]$, vectors $\sqrt{9}*\sqrt{1}*e^1$ and $\sqrt{1}*\sqrt{1}*e^2$



S = [3 0, 0 3], $\lambda^1 = 3$, $\lambda^2 = 3$, $e^{1'} = [1, 0]$, $e^{2'} = [0, 1]$, vectors $\sqrt{3}*\sqrt{1}*e^1$ and $\sqrt{3}*\sqrt{1}*e^2$



3.8a

Sone			Sonetot	Stwo			Stwotot
1	0	0	3	1	-0.5	-0.5	3
0	1	0		-0.5	1	-0.5	
0	0	1		-0.5	-0.5	1	

As the diagonals of both matrices are identical, the total sample variances are equal.

3.8b.

Sone			|Sone|	Stwo			|Stwo|
1	0	0	1	1	-0.5	-0.5	0
0	1	0		-0.5	1	-0.5	
0	0	1		-0.5	-0.5	1	

The values of |S| are not equal for both matrices. This is in contrast to the total sample variances, as that only relies on the diagonals of the matrix (which are equivalent in both Ss), while the generalized sample variance is the determinant of the matrix. Since the off-diagonals of both matrices are not equivalent, the |S|'s will not be the same.

3.9a

X			x-bar			Xmeancorrected		
12	17	29	16	18	34	-4	-1	-5
18	20	38				2	2	4
14	16	30				-2	-2	-4
20	18	38				4	0	4
16	19	35				0	1	1

S			|S|	Seigval	Seigvec		
10	3	13	0	29.365425	0.5668448	-0.587668	0.5773503
3	2.5	5.5		1.6345754	0.225513	0.784736	0.5773503
13	5.5	18.5		1.122E-15	0.7923578	0.197068	-0.57735

Since |S| = 0, there is linear dependance of columns. In addition, the sum of the first two variables minus the third is always a constant, *c*, equal to 0, so the columns of the original data matrix satisfy a linear constraint with *c* = 0. This establishes the fact that the columns of the data matrix are linear dependent. As the scaled eigenvector associated with a zero (or near-zero) eigenvalue is *a*' = [1, 1, −1], and *Sa* = 0, this vector establishes the linear dependence.

```
 S                          a        S*a

10        3        13         1         0
 3      2.5       5.5         1         0
13      5.5      18.5        -1         0
```

3.9b

```
 S                          |S|

10        3        13         0
 3      2.5       5.5
13      5.5      18.5
```

Since |S| = 0, there is linear dependance of columns. As the scaled eigenvector associated with a zero (or near-zero) eigenvalue is *a*' = [1, 1, −1], and *Sa* = 0, this vector establishes the linear dependence.

```
 S                          a        S*a

10        3        13         1         0
 3      2.5       5.5         1         0
13      5.5      18.5        -1         0

Seigval   Seigvec

29.365425 0.5668448 -0.587668  0.5773503
1.6345754 0.225513   0.784736  0.5773503
1.122E-15 0.7923578  0.197068 -0.57735
```

3.9c

Since X*a* = 0 (where *a*' = [1, 1, −1], it is verified that the sum of columns X1 and X2 is equal to the third column (e.g., 12 +17 = 29, or 12 + 17 – 29 = 0).

```
 X                          a        X*a

12       17        29         1         0
18       20        38         1         0
14       16        30        -1         0
20       18        38                   0
16       19        35                   0
```

CODE

```
/* 3.5a */
proc iml;
a = {9 1, 5 3, 1 2};
Scov  = cov(a);
eigva = eigval(Scov);
eigve = eigvec(Scov);
Sdet  = det(Scov);
print a eigva eigve Scov Sdet;
run;

/* 3.5b */
proc iml;
a = {3 4, 6 -2, 3 1};
Scov  = cov(a);
eigva = eigval(Scov);
eigve = eigvec(Scov);
Sdet  = det(Scov);
print a eigva eigve Scov Sdet;
run;

/* 3.6a */
proc iml;
X = {-1 3 -2, 2 4 2, 5 2 3};
m = mean(X);
m = t(m);
```

```
one = t({1 1 1});
onexm = one*(t(m));
res = X - (one*(t(m)));
Sdet = det(res);
print X m one onem res Sdet;
run;

/* 3.6b */

proc iml;
X = {-1 3 -2, 2 4 2, 5 2 3};
Scov  = cov(X);
Sdet  = det(Scov);
print X  Scov Sdet;
run;

/* 3.6c */

proc iml;
X = {-1 3 -2, 2 4 2, 5 2 3};
Scov  = cov(X);
Sdet  = det(Scov);
Stot  = trace(Scov);
print X  Scov Sdet Stot;
run;

/* 3.7 */

%let Covariance = {5 4, 4 5};  /* Covariance matrix */
%let center     = {2, 1};               /* Center Point */
%let distance   = 1;                      /* Ordinary distance, not squared distance
*/
proc iml;
  A = &Covariance;
  Evec = Eigvec(A);
  Eval = diag(Eigval(A));
  try1 = Evec*Eval*Evec`;   /*print A, try1; * This demonstrates the spectral
decomposition! */
  center = &center;
  distance = &distance;
npoints = 1000;
free xbig;
do r = 1 to npoints;
    angle = 2*3.14159265 * (r/npoints);
    w1 = sin(angle);
    w2 = cos(angle);
    w = w1//w2;
    x = Evec*sqrt(Eval)*distance*w + center;
    xbig = xbig//x`;
end;
create plotdata from xbig;
append from xbig;
quit;
   title "c = &distance from &center";
   title2 "Covariance Matrix is &Covariance";
proc sgplot data=plotdata(rename=(Col1=X1 Col2=X2));
   series y=x2 x=x1;
  /* refline 10 /axis=y; refline 1/ axis=x;  */
run;
title; title2;

%let Covariance = {5 -4, -4 5};  /* Covariance matrix */
%let center     = {2, 1};               /* Center Point */
%let distance   = 1;                      /* Ordinary distance, not squared distance
*/
proc iml;
  A = &Covariance;
  Evec = Eigvec(A);
  Eval = diag(Eigval(A));
  try1 = Evec*Eval*Evec`;   /*print A, try1; * This demonstrates the spectral
decomposition! */
  center = &center;
  distance = &distance;
npoints = 1000;
free xbig;
do r = 1 to npoints;
```

```
    angle = 2*3.14159265 * (r/npoints);
    w1 = sin(angle);
    w2 = cos(angle);
    w = w1//w2;
    x = Evec*sqrt(Eval)*distance*w + center;
    xbig = xbig//x`;
end;
create plotdata from xbig;
append from xbig;
quit;
   title "c = &distance from &center";
   title2 "Covariance Matrix is &Covariance";
proc sgplot data=plotdata(rename=(Col1=X1 Col2=X2));
   series y=x2 x=x1;
  /* refline 10 /axis=y; refline 1/ axis=x;  */
run;
title; title2;

%let Covariance = {3 0, 0 3};  /* Covariance matrix */
%let center     = {2, 1};              /* Center Point */
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  A = &Covariance;
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decomposition! */
  center = &center;
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npoints = 1000;
free xbig;
do r = 1 to npoints;
    angle = 2*3.14159265 * (r/npoints);
    w1 = sin(angle);
    w2 = cos(angle);
    w = w1//w2;
    x = Evec*sqrt(Eval)*distance*w + center;
    xbig = xbig//x`;
end;
create plotdata from xbig;
append from xbig;
quit;
   title "c = &distance from &center";
   title2 "Covariance Matrix is &Covariance";
proc sgplot data=plotdata(rename=(Col1=X1 Col2=X2));
   series y=x2 x=x1;
  /* refline 10 /axis=y; refline 1/ axis=x;  */
run;
title; title2;

proc iml;
X = {5 4, 4 5};
Y = {5 -4, -4 5};
Z = {3 0, 0 3};
Xeigval = eigval(X);
Xeigvec = eigvec(X);
Yeigval = eigval(Y);
Yeigvec = eigvec(Y);
Zeigval = eigval(Z);
Zeigvec = eigvec(Z);
print Xeigval Xeigvec Yeigval Yeigvec Zeigval Zeigvec;
run;

/* 3.8a */

proc iml;
Sone = {1 0 0, 0 1 0, 0 0 1};
Sonetot  = trace(Sone);
Stwo = {1 -.5 -.5, -.5 1 -.5, -.5 -.5 1};
Stwotot  = trace(Stwo);
print Sone Sonetot Stwo Stwotot;
run;

/* 3.8b */
```

```
proc iml;
Sone = {1 0 0, 0 1 0, 0 0 1};
Sonedet  = det(Sone);
Stwo = {1 -.5 -.5, -.5 1 -.5, -.5 -.5 1};
Stwodet  = det(Stwo);
print Sone Sonedet Sonerank Stwo Stwodet Stworank;
run;

/* 3.9a */
proc iml;
X = {12 17 29, 18 20 38, 14 16 30, 20 18 38, 16 19 35};
m = mean(X);
Xmcor = X - m;
Scov  = cov(Xmcor);
Sdet  = det(Scov);
Seigval = eigval(Scov);
Seigvec = eigvec(Scov);
a = {1, 1, -1};
Sxa = Scov*a;
print X m Xmcor Scov Sdet Seigval Seigvec a Sxa;
run;

/* 3.9b */
proc iml;
X = {12 17 29, 18 20 38, 14 16 30, 20 18 38, 16 19 35};
m = mean(X);
Xmcor = X - m;
Scov  = cov(Xmcor);
Sdet  = det(Scov);
print X m Xmcor Scov Sdet;
run; quit;

/* 3.9c */
proc iml;
X = {12 17 29, 18 20 38, 14 16 30, 20 18 38, 16 19 35};
m = mean(X);
Xmcor = X - m;
a = {1, 1, -1};
Xmcorxa = Xmcor*a;
Xxa = X*a;
print X a Xxa;
run; quit;
```