

1. Solve the following problem.

Water is leaking out of an inverted conical tank at a rate of $11,500 \text{ cm}^3/\text{min}$ at the same time that water is being pumped into the tank at a constant rate. The tank has height 6 m and the diameter at the top is 4 m. If the water level is rising at a rate of 20 cm/min when the height of the water is 2 m, find the rate at which water is being pumped into the tank. (Round your answer to the nearest integer.)

Hint:

We know that $V_{\text{cone}} = \frac{1}{3}\pi r^2 h$. Thus $\frac{dV}{dt} = \frac{\pi}{3} \frac{d(r^2 h)}{dt} = \frac{\pi}{3} (2rh \frac{dh}{dt} + r^2 \frac{dr}{dt})$ with product rule.

In a cone, r is proportional to h , which means $r = \alpha h$. From $h = 6\text{m}$ and $r = 2\text{m}$, we can calculate $\alpha = \frac{1}{3}$. Thus $r = \frac{1}{3}h$, and $\frac{dr}{dt} = \frac{1}{3} \frac{dh}{dt}$.

Thus $\frac{dV}{dt}$ can be written as $\frac{\pi}{3} (\frac{2h^2}{3} \frac{dh}{dt} + \frac{h^2}{9} \frac{dh}{dt})$.

From the question, h is known here to be 2m and $\frac{dh}{dt}$ is known to be 20cm/min and we can calculate $\frac{dV}{dt}$.

We also notice that $\frac{dV}{dt} = \text{VolumeChange}_{\text{in}} - \text{VolumeChange}_{\text{out}}$, where $\text{VolumeChange}_{\text{out}}$ is a known number of $11,500 \text{ cm}^3/\text{min}$. It will be easy to calculate the volume out speed.