

► Preamble

$$\begin{aligned}
 R1 &:= \text{simplify} \left(\text{DiagonalMatrix} \left(\sim_q \left(\text{Diagonal} \left(\frac{2}{3} \text{KP}(H1, \text{subs}(\lambda = \Lambda, H1)) + \frac{2}{3} \text{KP}(H2, \text{subs}(\mu = \mathbf{M}, H2)) + \frac{1}{3} \text{KP}(H2, \text{subs}(\lambda = \Lambda, H1)) + \frac{1}{3} \text{KP}(H1, \text{subs}(\mu = \mathbf{M}, H2)) \right) \right) \right) \right. \\
 &\quad \cdot (\text{IdentityMatrix}(64) + 2 \text{IKP}(E1, F1)) \cdot (\text{IdentityMatrix}(64) + 2 \text{IKP}(E3, F3)) \\
 &\quad \left. \cdot (\text{IdentityMatrix}(64) + 2 \text{IKP}(E2, F2)), \text{power} \right) : \\
 R &:= \sim \sim \text{simplify} \left(\sim \sim \text{expand} \left(\text{subs} \left(\mu = -\frac{2 \text{I} \ln(s)}{\pi}, \lambda = -\frac{2 \text{I} \ln(t)}{\pi}, \mathbf{M} = -\frac{2 \text{I} \ln(s)}{\pi}, \Lambda = -\frac{2 \text{I} \ln(t)}{\pi}, \frac{P \cdot R1}{R1_{8,8}} \right) \right) \right) : \\
 U &:= \text{ssimplify}(R^{-1}) : Ri := \text{KP}(R, id) : iR := \text{KP}(id, R) : Ui := \text{KP}(U, id) : iU := \text{KP}(id, U) : iiR := \text{KP}(id, iR) : iiU := \text{KP}(id, iU) : iRi := \text{KP}(id, Ri) : iUi := \text{KP}(id, Ui) : Rii := \text{KP}(Ri, id) : Uii := \text{KP}(Ui, id) : RR := \text{KP}(R, R) : UU := \text{KP}(U, U) : h := \text{simplify}(K1^{-2} \cdot K2^{-2}) : \\
 &\quad \text{simplify} \left(\frac{1}{8} \text{Trace}(\text{KP}(id, \kappa) \cdot R) \right);
 \end{aligned}$$

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(1)

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sl3 := proc(A, B) local n, m, k, l, v, i :
  n, m := op(1, A);
  k, l := op(1, B);
  if m ≠ k then error "incompatible dimensions"; end if;
  v := Vector(k) :
  for i from 1 to m do
    v[i] := DotProduct(Transpose(A)[.., i], B[.., i], conjugate=false);
  end do;
  return simplify(1/8 add(v));
end;
test := proc(poly) local a :
  return testeq(subs(s=a, t=s, a=t, poly) - poly=0), testeq(subs(s=1/s, t=1/t, poly) - poly=0),
    testeq(subs(s=1, poly) - subs(s=I/t, poly)=0), simplify(subs(s=1, poly));
end;
#test checks symmetry between t and s, t and its inverse, and whether evaluations at s=1 and s=It-1 are equal

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Invariants

$$\begin{aligned}
 A31 &:= KP(id, h).R : B31 := R^2 : K31 := sl3(A31, B31); test(K31); \\
 K31 &:= \frac{1}{s^4 t^4} ((t^8 - t^6 + t^4) s^8 + (-t^8 + 2t^6 - 2t^4 + t^2) s^6 + (t^8 - 2t^6 + t^4 - 2t^2 + 1) s^4 \\
 &\quad + (t^6 - 2t^4 + 2t^2 - 1) s^2 + t^4 - t^2 + 1) \\
 &\quad \text{true, true, true, } \frac{t^8 - t^4 + 1}{t^4}
 \end{aligned} \tag{2.1}$$

$$\begin{aligned}
 A41 &:= KP(id, KP(h, h)).spmm(iU, Ri) : B41 := spmm(iU, Ri) : K41 := sl3(A41, B41); \\
 &\quad test(K41); \\
 K41 &:= \frac{1}{s^4 t^4} ((t^8 - 3t^6 + t^4) s^8 + (-3t^8 + 12t^6 - 12t^4 + 3t^2) s^6 + (t^8 - 12t^6 + 25t^4 \\
 &\quad - 12t^2 + 1) s^4 + (3t^6 - 12t^4 + 12t^2 - 3) s^2 + t^4 - 3t^2 + 1) \\
 &\quad \text{true, true, true, } \frac{-t^8 + 3t^4 - 1}{t^4}
 \end{aligned} \tag{2.2}$$

$$\begin{aligned}
 A51 &:= KP(id, h).R^2 : B51 := R^3 : K51 := sort(sort(expand(sl3(A51, B51)), s), t); test(K51); \\
 K51 &:= s^8 t^8 - s^6 t^8 + s^4 t^8 - s^2 t^8 + t^8 - s^8 t^6 + 2s^6 t^6 - 2s^4 t^6 + 2s^2 t^6 + \frac{t^6}{s^2} - 2t^6 + s^8 t^4 \\
 &\quad - 2s^6 t^4 + s^4 t^4 - s^2 t^4 - \frac{2t^4}{s^2} + \frac{t^4}{s^4} + t^4 - s^8 t^2 + 2s^6 t^2 - s^4 t^2 + \frac{t^2}{s^2} - \frac{2t^2}{s^4} + \frac{t^2}{s^6} \\
 &\quad + \frac{s^6}{t^2} - \frac{2s^4}{t^2} + \frac{s^2}{t^2} - \frac{1}{s^4 t^2} + \frac{2}{s^6 t^2} - \frac{1}{s^8 t^2} + \frac{s^4}{t^4} - \frac{2s^2}{t^4} - \frac{1}{s^2 t^4} + \frac{1}{s^4 t^4} - \frac{2}{s^6 t^4} \\
 &\quad + \frac{1}{s^8 t^4} + \frac{1}{t^4} + \frac{s^2}{t^6} + \frac{2}{s^2 t^6} - \frac{2}{s^4 t^6} + \frac{2}{s^6 t^6} - \frac{1}{s^8 t^6} - \frac{2}{t^6} - \frac{1}{s^2 t^8} + \frac{1}{s^4 t^8} - \frac{1}{s^6 t^8} \\
 &\quad + \frac{1}{s^8 t^8} + \frac{1}{t^8} + s^8 - 2s^6 + s^4 + \frac{1}{s^4} - \frac{2}{s^6} + \frac{1}{s^8} + 1 \\
 &\quad \text{true, true, true, } \frac{t^{16} - t^{12} + t^8 - t^4 + 1}{t^8}
 \end{aligned} \tag{2.3}$$

$$\begin{aligned}
 A52 &:= KP(id, KP(h, h)).spmm(iR, Ui) : B52 := spmm(iR, KP(R^3, id)) : K52 := sl3(A52, B52); \\
 &\quad test(K52); \\
 K52 &:= \frac{1}{s^4 t^4} ((6t^8 - 10t^6 + 6t^4) s^8 + (-10t^8 + 26t^6 - 26t^4 + 10t^2) s^6 + (6t^8 - 26t^6 \\
 &\quad + 37t^4 - 26t^2 + 6) s^4 + (10t^6 - 26t^4 + 26t^2 - 10) s^2 + 6t^4 - 10t^2 + 6) \\
 &\quad \text{true, true, true, } \frac{2t^8 - 3t^4 + 2}{t^4}
 \end{aligned} \tag{2.4}$$

#Computed Externally

$$\begin{aligned}
 \#A61 &:= spmm(KP(id, KP(h, KP(h, h))), spmm(KP(id, spmm(iU, Ri)), UU)) : B61 := \\
 &\quad KP(spmm(iR, KP(R^2, id)), id) : K61 := sl3(A61, B61);
 \end{aligned}$$

$$\begin{aligned}
K61 &:= 6s^4t^4 - 14s^4t^2 - 14s^2t^4 + 6s^4 + 46s^2t^2 + 6t^4 - 46s^2 - 46t^2 + \frac{14s^2}{t^2} + \frac{14t^2}{s^2} - \frac{46}{s^2} \\
&\quad - \frac{46}{t^2} + \frac{6}{s^4} + \frac{6}{t^4} + \frac{46t^2}{s^2t^4} - \frac{14}{s^2t^4} - \frac{14}{s^4t^2} + \frac{6}{s^4t^4} + 85; \text{test}(K61); \\
K61 &:= 6s^4t^4 - 14s^4t^2 - 14s^2t^4 + 6s^4 + 46s^2t^2 + 6t^4 - 46s^2 - 46t^2 + \frac{14s^2}{t^2} + \frac{14t^2}{s^2} \\
&\quad - \frac{46}{s^2} - \frac{46}{t^2} + \frac{6}{s^4} + \frac{6}{t^4} + \frac{46}{t^2s^2} - \frac{14}{s^2t^4} - \frac{14}{s^4t^2} + \frac{6}{s^4t^4} + 85 \\
&\quad \text{true, true, true, } \frac{-2t^8 + 5t^4 - 2}{t^4}
\end{aligned} \tag{2.5}$$

$$A62 := \text{spmm}(KP(id, KP(h, h)), \text{spmm}(iU, Ri)) : B62 := \text{spmm}(iU, KP(R^3, id)) : K62 := \text{sort}(\text{sort}(\text{expand}(sl3(A62, B62)), s), t); \text{test}(K62);$$

$$\begin{aligned}
K62 &:= s^8t^8 - 3s^6t^8 + 3s^4t^8 - 3s^2t^8 + t^8 - 3s^8t^6 + 12s^6t^6 - 18s^4t^6 + 18s^2t^6 + \frac{3t^6}{s^2} \\
&\quad - 12t^6 + 3s^8t^4 - 18s^6t^4 + 35s^4t^4 - 37s^2t^4 - \frac{18t^4}{s^2} + \frac{3t^4}{s^4} + 35t^4 - 3s^8t^2 + 18s^6t^2 \\
&\quad - 37s^4t^2 + 38s^2t^2 + \frac{37t^2}{s^2} - \frac{18t^2}{s^4} + \frac{3t^2}{s^6} - 38t^2 + \frac{3s^6}{t^2} - \frac{18s^4}{t^2} + \frac{37s^2}{t^2} + \frac{38}{s^2t^2} \\
&\quad - \frac{37}{s^4t^2} + \frac{18}{s^6t^2} - \frac{3}{s^8t^2} - \frac{38}{t^2} + \frac{3s^4}{t^4} - \frac{18s^2}{t^4} - \frac{37}{s^2t^4} + \frac{35}{s^4t^4} - \frac{18}{s^6t^4} + \frac{3}{s^8t^4} \\
&\quad + \frac{35}{t^4} + \frac{3s^2}{t^6} + \frac{18}{s^2t^6} - \frac{18}{s^4t^6} + \frac{12}{s^6t^6} - \frac{3}{s^8t^6} - \frac{12}{t^6} - \frac{3}{s^2t^8} + \frac{3}{s^4t^8} - \frac{3}{s^6t^8} \\
&\quad + \frac{1}{s^8t^8} + \frac{1}{t^8} + s^8 - 12s^6 + 35s^4 - 38s^2 - \frac{38}{s^2} + \frac{35}{s^4} - \frac{12}{s^6} + \frac{1}{s^8} + 25 \\
&\quad \text{true, true, true, } \frac{-t^{16} + 3t^{12} - 3t^8 + 3t^4 - 1}{t^8}
\end{aligned} \tag{2.6}$$

$$A63 := \text{spmm}(KP(id, KP(h, h)), \text{spmm}(KP(id, U^2), Ri)) : B63 := \text{spmm}(iU, KP(R^2, id)) : K63 := \text{sort}(\text{sort}(\text{expand}(sl3(A63, B63)), s), t); \text{test}(K63);$$

$$\begin{aligned}
K63 &:= s^8t^8 - 3s^6t^8 + 5s^4t^8 - 3s^2t^8 + t^8 - 3s^8t^6 + 12s^6t^6 - 24s^4t^6 + 24s^2t^6 + \frac{3t^6}{s^2} \\
&\quad - 12t^6 + 5s^8t^4 - 24s^6t^4 + 53s^4t^4 - 71s^2t^4 - \frac{24t^4}{s^2} + \frac{5t^4}{s^4} + 53t^4 - 3s^8t^2 + 24s^6t^2 \\
&\quad - 71s^4t^2 + 124s^2t^2 + \frac{71t^2}{s^2} - \frac{24t^2}{s^4} + \frac{3t^2}{s^6} - 124t^2 + \frac{3s^6}{t^2} - \frac{24s^4}{t^2} + \frac{71s^2}{t^2} \\
&\quad + \frac{124}{s^2t^2} - \frac{71}{s^4t^2} + \frac{24}{s^6t^2} - \frac{3}{s^8t^2} - \frac{124}{t^2} + \frac{5s^4}{t^4} - \frac{24s^2}{t^4} - \frac{71}{s^2t^4} + \frac{53}{s^4t^4} - \frac{24}{s^6t^4} \\
&\quad + \frac{5}{s^8t^4} + \frac{53}{t^4} + \frac{3s^2}{t^6} + \frac{24}{s^2t^6} - \frac{24}{s^4t^6} + \frac{12}{s^6t^6} - \frac{3}{s^8t^6} - \frac{12}{t^6} - \frac{3}{s^2t^8} + \frac{5}{s^4t^8}
\end{aligned}$$

$$\begin{aligned}
& -\frac{3}{s^6 t^8} + \frac{1}{s^8 t^8} + \frac{1}{t^8} + s^8 - 12 s^6 + 53 s^4 - 124 s^2 - \frac{124}{s^2} + \frac{53}{s^4} - \frac{12}{s^6} + \frac{1}{s^8} + 169 \\
& \text{true, true, true, } \frac{t^{16} - 3 t^{12} + 5 t^8 - 3 t^4 + 1}{t^8}
\end{aligned} \tag{2.7}$$

$$\begin{aligned}
& A71 := KP(id, h).R^2 : B71 := R^5 : K71 := \text{sort}(\text{sort}(\text{expand}(sl3(A71, B71)), s), t); \text{test}(K71); \\
& K71 := s^{12} t^{12} - s^{10} t^{12} + s^8 t^{12} - s^6 t^{12} + s^4 t^{12} - s^2 t^{12} + t^{12} - s^{12} t^{10} + 2 s^{10} t^{10} - 2 s^8 t^{10} \\
& + 2 s^6 t^{10} - 2 s^4 t^{10} + 2 s^2 t^{10} + \frac{t^{10}}{s^2} - 2 t^{10} + s^{12} t^8 - 2 s^{10} t^8 + s^8 t^8 - s^6 t^8 + s^4 t^8 - s^2 t^8 \\
& - \frac{2 t^8}{s^2} + \frac{t^8}{s^4} + t^8 - s^{12} t^6 + 2 s^{10} t^6 - s^8 t^6 + \frac{t^6}{s^2} - \frac{2 t^6}{s^4} + \frac{t^6}{s^6} + s^{12} t^4 - 2 s^{10} t^4 + s^8 t^4 \\
& + s^4 t^4 - s^2 t^4 + \frac{t^4}{s^4} - \frac{2 t^4}{s^6} + \frac{t^4}{s^8} + t^4 - s^{12} t^2 + 2 s^{10} t^2 - s^8 t^2 - s^4 t^2 + 2 s^2 t^2 + \frac{t^2}{s^2} \\
& + \frac{t^2}{s^6} - \frac{2 t^2}{s^8} + \frac{t^2}{s^{10}} - 2 t^2 + \frac{s^{10}}{t^2} - \frac{2 s^8}{t^2} + \frac{s^6}{t^2} + \frac{s^2}{t^2} + \frac{2}{s^2 t^2} - \frac{1}{s^4 t^2} - \frac{1}{s^8 t^2} \\
& + \frac{2}{s^{10} t^2} - \frac{1}{s^{12} t^2} - \frac{2}{t^2} + \frac{s^8}{t^4} - \frac{2 s^6}{t^4} + \frac{s^4}{t^4} - \frac{1}{s^2 t^4} + \frac{1}{s^4 t^4} + \frac{1}{s^8 t^4} - \frac{2}{s^{10} t^4} \\
& + \frac{1}{s^{12} t^4} + \frac{1}{t^4} + \frac{s^6}{t^6} - \frac{2 s^4}{t^6} + \frac{s^2}{t^6} - \frac{1}{s^8 t^6} + \frac{2}{s^{10} t^6} - \frac{1}{s^{12} t^6} + \frac{s^4}{t^8} - \frac{2 s^2}{t^8} - \frac{1}{s^2 t^8} \\
& + \frac{1}{s^4 t^8} - \frac{1}{s^6 t^8} + \frac{1}{s^8 t^8} - \frac{2}{s^{10} t^8} + \frac{1}{s^{12} t^8} + \frac{1}{t^8} + \frac{s^2}{t^{10}} + \frac{2}{s^2 t^{10}} - \frac{2}{s^4 t^{10}} + \frac{2}{s^6 t^{10}} \\
& - \frac{2}{s^8 t^{10}} + \frac{2}{s^{10} t^{10}} - \frac{1}{s^{12} t^{10}} - \frac{2}{t^{10}} - \frac{1}{s^2 t^{12}} + \frac{1}{s^4 t^{12}} - \frac{1}{s^6 t^{12}} + \frac{1}{s^8 t^{12}} - \frac{1}{s^{10} t^{12}} \\
& + \frac{1}{s^{12} t^{12}} + \frac{1}{t^{12}} + s^{12} - 2 s^{10} + s^8 + s^4 - 2 s^2 - \frac{2}{s^2} + \frac{1}{s^4} + \frac{1}{s^8} - \frac{2}{s^{10}} + \frac{1}{s^{12}} + 1 \\
& \text{true, true, true, } \frac{t^{24} - t^{20} + t^{16} - t^{12} + t^8 - t^4 + 1}{t^{12}}
\end{aligned} \tag{2.8}$$

#Computed Externally

$$\begin{aligned}
& \#iRUiiR := \text{ssimplify}(\text{spmm}(iR, \text{spmm}(Ui, iR))) : A72 := \text{spmm}(KP(id, KP(h, KP(h, h))), KP(id, \\
& iRUiiR)) : B72 := KP(\text{spmm}(iRUiiR, KP(R^3, id)), id) : K72 := sl3(A72, B72); \\
& K72 := \frac{1}{s^4 t^4} ((13 t^8 - 23 t^6 + 13 t^4) s^8 + (-23 t^8 + 64 t^6 - 64 t^4 + 23 t^2) s^6 + (13 t^8 - 64 t^6 \\
& + 97 t^4 - 64 t^2 + 13) s^4 + (23 t^6 - 64 t^4 + 64 t^2 - 23) s^2 + 13 t^4 - 23 t^2 + 13); \text{test}(K72); \\
& K72 := \frac{1}{s^4 t^4} ((13 t^8 - 23 t^6 + 13 t^4) s^8 + (-23 t^8 + 64 t^6 - 64 t^4 + 23 t^2) s^6 + (13 t^8 \\
& - 64 t^6 + 97 t^4 - 64 t^2 + 13) s^4 + (23 t^6 - 64 t^4 + 64 t^2 - 23) s^2 + 13 t^4 - 23 t^2 + 13) \\
& \text{true, true, true, } \frac{3 t^8 - 5 t^4 + 3}{t^4}
\end{aligned} \tag{2.9}$$

$$A73 := \text{spmm}(KP(id, KP(h, h)), \text{spmm}(iU, Ri)) : B73 := \text{spmm}(iU, KP(U^5, id)) : K73 :=$$

$sort(sort(expand(sl3(A73, B73)), s), t); test(K73);$

$$\begin{aligned}
K73 := & 6s^8t^8 - 10s^6t^8 + 10s^4t^8 - 10s^2t^8 + 6t^8 - 10s^8t^6 + 26s^6t^6 - 32s^4t^6 + 32s^2t^6 \\
& + \frac{10t^6}{s^2} - 26t^6 + 10s^8t^4 - 32s^6t^4 + 39s^4t^4 - 37s^2t^4 - \frac{32t^4}{s^2} + \frac{10t^4}{s^4} + 39t^4 \\
& - 10s^8t^2 + 32s^6t^2 - 37s^4t^2 + 24s^2t^2 + \frac{37t^2}{s^2} - \frac{32t^2}{s^4} + \frac{10t^2}{s^6} - 24t^2 + \frac{10s^6}{t^2} \\
& - \frac{32s^4}{t^2} + \frac{37s^2}{t^2} + \frac{24}{s^2t^2} - \frac{37}{s^4t^2} + \frac{32}{s^6t^2} - \frac{10}{s^8t^2} - \frac{24}{t^2} + \frac{10s^4}{t^4} - \frac{32s^2}{t^4} - \frac{37}{s^2t^4} \\
& + \frac{39}{s^4t^4} - \frac{32}{s^6t^4} + \frac{10}{s^8t^4} + \frac{39}{t^4} + \frac{10s^2}{t^6} + \frac{32}{s^2t^6} - \frac{32}{s^4t^6} + \frac{26}{s^6t^6} - \frac{10}{s^8t^6} - \frac{26}{t^6} \\
& - \frac{10}{s^2t^8} + \frac{10}{s^4t^8} - \frac{10}{s^6t^8} + \frac{6}{s^8t^8} + \frac{6}{t^8} + 6s^8 - 26s^6 + 39s^4 - 24s^2 - \frac{24}{s^2} + \frac{39}{s^4} \\
& - \frac{26}{s^6} + \frac{6}{s^8} + 13
\end{aligned}$$

$$true, true, true, \frac{2t^{16} - 3t^{12} + 3t^8 - 3t^4 + 2}{t^8} \quad (2.10)$$

#Computed Externally

$\#A74 := spmm(KP(id, KP(h, KP(h, h))), KP(id, spmm(iU, spmm(Ri, spmm(iU, Ui^2))))):$
 $B74 := KP(spmm(Ri, spmm(iU, KP(U^2, id))), id) : K74 := sort(sort(expand(sl3(A74,$
 $B74)), s), t);$

$$\begin{aligned}
K74 := & \frac{1}{s^4t^4} ((36t^8 - 68t^6 + 36t^4)s^8 + (-68t^8 + 196t^6 - 196t^4 + 68t^2)s^6 + (36t^8 - 196t^6 \\
& + 313t^4 - 196t^2 + 36)s^4 + (68t^6 - 196t^4 + 196t^2 - 68)s^2 + 36t^4 - 68t^2 + 36); \\
& test(K74);
\end{aligned}$$

$$\begin{aligned}
K74 := & \frac{1}{s^4t^4} ((36t^8 - 68t^6 + 36t^4)s^8 + (-68t^8 + 196t^6 - 196t^4 + 68t^2)s^6 + (36t^8 \\
& - 196t^6 + 313t^4 - 196t^2 + 36)s^4 + (68t^6 - 196t^4 + 196t^2 - 68)s^2 + 36t^4 - 68t^2 \\
& + 36)
\end{aligned}$$

$$true, true, true, \frac{4t^8 - 7t^4 + 4}{t^4} \quad (2.11)$$

$A75 := spmm(KP(id, KP(h, h)), spmm(KP(id, R^2), Ui)) : B75 := spmm(iR, KP(R^4, id)) :$
 $K75 := sort(sort(expand(sl3(A75, B75)), s), t); test(K75);$

$$\begin{aligned}
K75 := & 6s^8t^8 - 14s^6t^8 + 18s^4t^8 - 14s^2t^8 + 6t^8 - 14s^8t^6 + 44s^6t^6 - 68s^4t^6 + 68s^2t^6 \\
& + \frac{14t^6}{s^2} - 44t^6 + 18s^8t^4 - 68s^6t^4 + 114s^4t^4 - 132s^2t^4 - \frac{68t^4}{s^2} + \frac{18t^4}{s^4} + 114t^4 \\
& - 14s^8t^2 + 68s^6t^2 - 132s^4t^2 + 164s^2t^2 + \frac{132t^2}{s^2} - \frac{68t^2}{s^4} + \frac{14t^2}{s^6} - 164t^2 + \frac{14s^6}{t^2} \\
& - \frac{68s^4}{t^2} + \frac{132s^2}{t^2} + \frac{164}{s^2t^2} - \frac{132}{s^4t^2} + \frac{68}{s^6t^2} - \frac{14}{s^8t^2} - \frac{164}{t^2} + \frac{18s^4}{t^4} - \frac{68s^2}{t^4}
\end{aligned}$$

$$\begin{aligned}
& -\frac{132}{s^2 t^4} + \frac{114}{s^4 t^4} - \frac{68}{s^6 t^4} + \frac{18}{s^8 t^4} + \frac{114}{t^4} + \frac{14 s^2}{t^6} + \frac{68}{s^2 t^6} - \frac{68}{s^4 t^6} + \frac{44}{s^6 t^6} - \frac{14}{s^8 t^6} \\
& -\frac{44}{t^6} - \frac{14}{s^2 t^8} + \frac{18}{s^4 t^8} - \frac{14}{s^6 t^8} + \frac{6}{s^8 t^8} + \frac{6}{t^8} + 6 s^8 - 44 s^6 + 114 s^4 - 164 s^2 - \frac{164}{s^2} \\
& + \frac{114}{s^4} - \frac{44}{s^6} + \frac{6}{s^8} + 181 \\
& \text{true, true, true, } \frac{2 t^{16} - 4 t^{12} + 5 t^8 - 4 t^4 + 2}{t^8} \tag{2.12}
\end{aligned}$$

#Computed Externally

$$\begin{aligned}
& \#A76 := \text{spmm}(KP(id, KP(h, KP(h, h))), KP(id, \text{spmm}(iR, Ui))) : B76 := \text{spmm}(RR, \\
& \quad KP(\text{spmm}(iU, KP(R^2, id)), id)) : K76 := \text{sort}(\text{sort}(\text{expand}(sl3(A76, B76)), s), t); \\
& K76 := \frac{1}{s^8 t^8} (t^8 (t^8 - 5 t^6 + 7 t^4 - 5 t^2 + 1) s^{16} + (-5 t^{16} + 30 t^{14} - 60 t^{12} + 60 t^{10} - 30 t^8 \\
& \quad + 5 t^6) s^{14} + (7 t^{16} - 60 t^{14} + 163 t^{12} - 215 t^{10} + 163 t^8 - 60 t^6 + 7 t^4) s^{12} + (-5 t^{16} + 60 t^{14} \\
& \quad - 215 t^{12} + 366 t^{10} - 366 t^8 + 215 t^6 - 60 t^4 + 5 t^2) s^{10} + (t^{16} - 30 t^{14} + 163 t^{12} - 366 t^{10} \\
& \quad + 457 t^8 - 366 t^6 + 163 t^4 - 30 t^2 + 1) s^8 + (5 t^{14} - 60 t^{12} + 215 t^{10} - 366 t^8 + 366 t^6 - 215 t^4 \\
& \quad + 60 t^2 - 5) s^6 + (7 t^{12} - 60 t^{10} + 163 t^8 - 215 t^6 + 163 t^4 - 60 t^2 + 7) s^4 + (5 t^{10} - 30 t^8 \\
& \quad + 60 t^6 - 60 t^4 + 30 t^2 - 5) s^2 + t^8 - 5 t^6 + 7 t^4 - 5 t^2 + 1); \text{test}(K76); \\
& K76 := \frac{1}{s^8 t^8} (t^8 (t^8 - 5 t^6 + 7 t^4 - 5 t^2 + 1) s^{16} + (-5 t^{16} + 30 t^{14} - 60 t^{12} + 60 t^{10} - 30 t^8 \\
& \quad + 5 t^6) s^{14} + (7 t^{16} - 60 t^{14} + 163 t^{12} - 215 t^{10} + 163 t^8 - 60 t^6 + 7 t^4) s^{12} + (-5 t^{16} \\
& \quad + 60 t^{14} - 215 t^{12} + 366 t^{10} - 366 t^8 + 215 t^6 - 60 t^4 + 5 t^2) s^{10} + (t^{16} - 30 t^{14} \\
& \quad + 163 t^{12} - 366 t^{10} + 457 t^8 - 366 t^6 + 163 t^4 - 30 t^2 + 1) s^8 + (5 t^{14} - 60 t^{12} + 215 t^{10} \\
& \quad - 366 t^8 + 366 t^6 - 215 t^4 + 60 t^2 - 5) s^6 + (7 t^{12} - 60 t^{10} + 163 t^8 - 215 t^6 + 163 t^4 \\
& \quad - 60 t^2 + 7) s^4 + (5 t^{10} - 30 t^8 + 60 t^6 - 60 t^4 + 30 t^2 - 5) s^2 + t^8 - 5 t^6 + 7 t^4 - 5 t^2 \\
& \quad + 1) \\
& \text{true, true, true, } \frac{-t^{16} + 5 t^{12} - 7 t^8 + 5 t^4 - 1}{t^8} \tag{2.13}
\end{aligned}$$

#Computed Externally

$$\begin{aligned}
& \#A77 := \text{spmm}(KP(id, KP(h, KP(h, h))), \text{spmm}(iiU, \text{spmm}(iRi, iiU))) : B77 := \text{spmm}(iRi, \\
& \quad \text{spmm}(Uii, \text{spmm}(iRi, Uii))) : K77 := \text{sort}(\text{sort}(\text{expand}(sl3(A77, B77)), s), t); \\
& K77 := s^8 t^8 - 5 s^6 t^8 + 9 s^4 t^8 - 5 s^2 t^8 + t^8 - 5 s^8 t^6 + 30 s^6 t^6 - 70 s^4 t^6 + 70 s^2 t^6 + \frac{5 t^6}{s^2} - 30 t^6 \\
& \quad + 9 s^8 t^4 - 70 s^6 t^4 + 209 s^4 t^4 - 301 s^2 t^4 - \frac{70 t^4}{s^2} + \frac{9 t^4}{s^4} + 209 t^4 - 5 s^8 t^2 + 70 s^6 t^2 \\
& \quad - 301 s^4 t^2 + 608 s^2 t^2 + \frac{301 t^2}{s^2} - \frac{70 t^2}{s^4} + \frac{5 t^2}{s^6} - 608 t^2 + \frac{5 s^6}{t^2} - \frac{70 s^4}{t^2} + \frac{301 s^2}{t^2} + \frac{608}{s^2 t^2} \\
& \quad - \frac{301}{s^4 t^2} + \frac{70}{s^6 t^2} - \frac{5}{s^8 t^2} - \frac{608}{t^2} + \frac{9 s^4}{t^4} - \frac{70 s^2}{t^4} - \frac{301}{s^2 t^4} + \frac{209}{s^4 t^4} - \frac{70}{s^6 t^4} + \frac{9}{s^8 t^4} + \frac{209}{t^4}
\end{aligned}$$

$$+ \frac{5s^2}{t^6} + \frac{70}{s^2 t^6} - \frac{70}{s^4 t^6} + \frac{30}{s^6 t^6} - \frac{5}{s^8 t^6} - \frac{30}{t^6} - \frac{5}{s^2 t^8} + \frac{9}{s^4 t^8} - \frac{5}{s^6 t^8} + \frac{1}{s^8 t^8} + \frac{1}{t^8} + s^8 \\ - 30s^6 + 209s^4 - 608s^2 - \frac{608}{s^2} + \frac{209}{s^4} - \frac{30}{s^6} + \frac{1}{s^8} + 865; \text{test}(K77);$$

$$K77 := 865 - 608s^2 - 608t^2 + \frac{1}{t^8} - \frac{30}{t^6} + \frac{1}{s^8 t^8} + \frac{30}{s^6 t^6} - \frac{5}{s^8 t^6} + \frac{5s^6}{t^2} + \frac{5t^6}{s^2} + \frac{9s^4}{t^4} \\ + \frac{9t^4}{s^4} - \frac{70s^2}{t^4} - \frac{70t^2}{s^4} + \frac{5s^2}{t^6} + \frac{5t^2}{s^6} + \frac{9}{s^4 t^8} + \frac{9}{s^8 t^4} - \frac{5}{s^6 t^8} - \frac{70}{s^6 t^4} - \frac{301}{s^2 t^4} \\ + t^8 - 30t^6 + s^8 - 30s^6 + \frac{70}{s^6 t^2} - 301s^4 t^2 - 301s^2 t^4 + 608s^2 t^2 + \frac{70}{s^2 t^6} - \frac{608}{t^2} \\ - \frac{608}{s^2} - \frac{70}{s^4 t^6} + \frac{209}{s^4} + \frac{209}{t^4} + \frac{608}{s^2 t^2} - \frac{70s^4}{t^2} + s^8 t^8 - \frac{70t^4}{s^2} - \frac{301}{s^4 t^2} + \frac{209}{s^4 t^4} \\ - 70s^4 t^6 - 70s^6 t^4 - \frac{5}{s^2 t^8} - \frac{5}{s^8 t^2} + 209s^4 + \frac{301s^2}{t^2} + 209t^4 + \frac{301t^2}{s^2} + \frac{1}{s^8} - \frac{30}{s^6} \\ + 209s^4 t^4 + 30s^6 t^6 - 5s^8 t^2 - 5s^2 t^8 + 9s^4 t^8 + 9s^8 t^4 - 5s^6 t^8 - 5s^8 t^6 + 70s^2 t^6 \\ + 70s^6 t^2 \\ \text{true, true, true, } \frac{t^{16} - 5t^{12} + 9t^8 - 5t^4 + 1}{t^8} \quad (2.14)$$

$$A89 := \text{spmm}(KP(id, KP(h, h)), \text{spmm}(\text{spmm}(KP(id, U^3), Ri), iU)) : B89 := KP(R^3, id) :$$

$$K89 := \text{sort}(\text{sort}(\text{expand}(\text{sl3}(A89, B89)), s), t); \text{test}(K89);$$

$$K89 := s^{12}t^{12} - 3s^{10}t^{12} + 5s^8t^{12} - 7s^6t^{12} + 5s^4t^{12} - 3s^2t^{12} + t^{12} - 3s^{12}t^{10} + 12s^{10}t^{10} \\ - 24s^8t^{10} + 36s^6t^{10} - 36s^4t^{10} + 24s^2t^{10} + \frac{3t^{10}}{s^2} - 12t^{10} + 5s^{12}t^8 - 24s^{10}t^8 \\ + 53s^8t^8 - 83s^6t^8 + 101s^4t^8 - 83s^2t^8 - \frac{24t^8}{s^2} + \frac{5t^8}{s^4} + 53t^8 - 7s^{12}t^6 + 36s^{10}t^6 \\ - 83s^8t^6 + 128s^6t^6 - 164s^4t^6 + 164s^2t^6 + \frac{83t^6}{s^2} - \frac{36t^6}{s^4} + \frac{7t^6}{s^6} - 128t^6 + 5s^{12}t^4 \\ - 36s^{10}t^4 + 101s^8t^4 - 164s^6t^4 + 217s^4t^4 - 251s^2t^4 - \frac{164t^4}{s^2} + \frac{101t^4}{s^4} - \frac{36t^4}{s^6} \\ + \frac{5t^4}{s^8} + 217t^4 - 3s^{12}t^2 + 24s^{10}t^2 - 83s^8t^2 + 164s^6t^2 - 251s^4t^2 + 344s^2t^2 \\ + \frac{251t^2}{s^2} - \frac{164t^2}{s^4} + \frac{83t^2}{s^6} - \frac{24t^2}{s^8} + \frac{3t^2}{s^{10}} - 344t^2 + \frac{3s^{10}}{t^2} - \frac{24s^8}{t^2} + \frac{83s^6}{t^2} \\ - \frac{164s^4}{t^2} + \frac{251s^2}{t^2} + \frac{344}{s^2 t^2} - \frac{251}{s^4 t^2} + \frac{164}{s^6 t^2} - \frac{83}{s^8 t^2} + \frac{24}{s^{10} t^2} - \frac{3}{s^{12} t^2} - \frac{344}{t^2} \\ + \frac{5s^8}{t^4} - \frac{36s^6}{t^4} + \frac{101s^4}{t^4} - \frac{164s^2}{t^4} - \frac{251}{s^2 t^4} + \frac{217}{s^4 t^4} - \frac{164}{s^6 t^4} + \frac{101}{s^8 t^4} - \frac{36}{s^{10} t^4}$$

$$\begin{aligned}
& + \frac{5}{s^{12} t^4} + \frac{217}{t^4} + \frac{7 s^6}{t^6} - \frac{36 s^4}{t^6} + \frac{83 s^2}{t^6} + \frac{164}{s^2 t^6} - \frac{164}{s^4 t^6} + \frac{128}{s^6 t^6} - \frac{83}{s^8 t^6} + \frac{36}{s^{10} t^6} \\
& - \frac{7}{s^{12} t^6} - \frac{128}{t^6} + \frac{5 s^4}{t^8} - \frac{24 s^2}{t^8} - \frac{83}{s^2 t^8} + \frac{101}{s^4 t^8} - \frac{83}{s^6 t^8} + \frac{53}{s^8 t^8} - \frac{24}{s^{10} t^8} + \frac{5}{s^{12} t^8} \\
& + \frac{53}{t^8} + \frac{3 s^2}{t^{10}} + \frac{24}{s^2 t^{10}} - \frac{36}{s^4 t^{10}} + \frac{36}{s^6 t^{10}} - \frac{24}{s^8 t^{10}} + \frac{12}{s^{10} t^{10}} - \frac{3}{s^{12} t^{10}} - \frac{12}{t^{10}} \\
& - \frac{3}{s^2 t^{12}} + \frac{5}{s^4 t^{12}} - \frac{7}{s^6 t^{12}} + \frac{5}{s^8 t^{12}} - \frac{3}{s^{10} t^{12}} + \frac{1}{s^{12} t^{12}} + \frac{1}{t^{12}} + s^{12} - 12 s^{10} + 53 s^8 \\
& - 128 s^6 + 217 s^4 - 344 s^2 - \frac{344}{s^2} + \frac{217}{s^4} - \frac{128}{s^6} + \frac{53}{s^8} - \frac{12}{s^{10}} + \frac{1}{s^{12}} + 433 \\
& \text{true, true, true, } \frac{-t^{24} + 3 t^{20} - 5 t^{16} + 7 t^{12} - 5 t^8 + 3 t^4 - 1}{t^{12}}
\end{aligned} \tag{2.15}$$

$A817 := \text{spmm}(KP(id, KP(h, h)), \text{spmm}(KP(id, U^2), \text{spmm}(KP(R, id), KP(id, U)))) :$
 $B817 := \text{spmm}(\text{spmm}(KP(R, id), KP(id, U)), KP(R^2, id)) : K817 := \text{sl3}(A817, B817);$
 $\text{test}(K817);$

$$\begin{aligned}
K817 := & \frac{1}{s^{12} t^{12}} ((t^{24} - 4 t^{22} + 8 t^{20} - 11 t^{18} + 8 t^{16} - 4 t^{14} + t^{12}) s^{24} + (-4 t^{24} + 20 t^{22} \\
& - 48 t^{20} + 76 t^{18} - 76 t^{16} + 48 t^{14} - 20 t^{12} + 4 t^{10}) s^{22} + (8 t^{24} - 48 t^{22} + 132 t^{20} \\
& - 236 t^{18} + 292 t^{16} - 236 t^{14} + 132 t^{12} - 48 t^{10} + 8 t^8) s^{20} + (-11 t^{24} + 76 t^{22} - 236 t^{20} \\
& + 468 t^{18} - 672 t^{16} + 672 t^{14} - 468 t^{12} + 236 t^{10} - 76 t^8 + 11 t^6) s^{18} + (8 t^{24} - 76 t^{22} \\
& + 292 t^{20} - 672 t^{18} + 1108 t^{16} - 1328 t^{14} + 1108 t^{12} - 672 t^{10} + 292 t^8 - 76 t^6 + 8 t^4) \\
& s^{16} + (-4 t^{24} + 48 t^{22} - 236 t^{20} + 672 t^{18} - 1328 t^{16} + 1924 t^{14} - 1924 t^{12} + 1328 t^{10} \\
& - 672 t^8 + 236 t^6 - 48 t^4 + 4 t^2) s^{14} + (t^{24} - 20 t^{22} + 132 t^{20} - 468 t^{18} + 1108 t^{16} \\
& - 1924 t^{14} + 2353 t^{12} - 1924 t^{10} + 1108 t^8 - 468 t^6 + 132 t^4 - 20 t^2 + 1) s^{12} + (4 t^{22} \\
& - 48 t^{20} + 236 t^{18} - 672 t^{16} + 1328 t^{14} - 1924 t^{12} + 1924 t^{10} - 1328 t^8 + 672 t^6 - 236 t^4 \\
& + 48 t^2 - 4) s^{10} + (8 t^{20} - 76 t^{18} + 292 t^{16} - 672 t^{14} + 1108 t^{12} - 1328 t^{10} + 1108 t^8 \\
& - 672 t^6 + 292 t^4 - 76 t^2 + 8) s^8 + (11 t^{18} - 76 t^{16} + 236 t^{14} - 468 t^{12} + 672 t^{10} \\
& - 672 t^8 + 468 t^6 - 236 t^4 + 76 t^2 - 11) s^6 + (8 t^{16} - 48 t^{14} + 132 t^{12} - 236 t^{10} + 292 t^8 \\
& - 236 t^6 + 132 t^4 - 48 t^2 + 8) s^4 + (4 t^{14} - 20 t^{12} + 48 t^{10} - 76 t^8 + 76 t^6 - 48 t^4 \\
& + 20 t^2 - 4) s^2 + t^{12} - 4 t^{10} + 8 t^8 - 11 t^6 + 8 t^4 - 4 t^2 + 1) \\
& \text{true, true, true, } \frac{-t^{24} + 4 t^{20} - 8 t^{16} + 11 t^{12} - 8 t^8 + 4 t^4 - 1}{t^{12}}
\end{aligned} \tag{2.16}$$

#Computed Externally

$\#A := \text{spmm}(KP(id, \text{spmm}(KP(h, KP(h, h)), \text{spmm}(KP(id, U), KP(R, id)))) , KP(U, U)) : B :=$
 $KP(\text{spmm}(KP(id, R), \text{spmm}(KP(U, id), \text{spmm}(KP(id, R), KP(U^2, id)))) , id) :$
 $\#K932 := \text{sl3}(A, B);$

$$K932 := \frac{1}{s^{12} t^{12}} (t^{12} (t^{12} - 6 t^{10} + 14 t^8 - 17 t^6 + 14 t^4 - 6 t^2 + 1) s^{24} + (-6 t^{24} + 42 t^{22}$$

$$\begin{aligned}
& -120 t^{20} + 186 t^{18} - 186 t^{16} + 120 t^{14} - 42 t^{12} + 6 t^{10}) s^{22} + (14 t^{24} - 120 t^{22} + 420 t^{20} \\
& - 802 t^{18} + 970 t^{16} - 802 t^{14} + 420 t^{12} - 120 t^{10} + 14 t^8) s^{20} + (-17 t^{24} + 186 t^{22} - 802 t^{20} \\
& + 1870 t^{18} - 2718 t^{16} + 2718 t^{14} - 1870 t^{12} + 802 t^{10} - 186 t^8 + 17 t^6) s^{18} + (14 t^{24} - 186 t^{22} \\
& + 970 t^{20} - 2718 t^{18} + 4650 t^{16} - 5446 t^{14} + 4650 t^{12} - 2718 t^{10} + 970 t^8 - 186 t^6 + 14 t^4) s^{16} \\
& + (-6 t^{24} + 120 t^{22} - 802 t^{20} + 2718 t^{18} - 5446 t^{16} + 7296 t^{14} - 7296 t^{12} + 5446 t^{10} - 2718 t^8 \\
& + 802 t^6 - 120 t^4 + 6 t^2) s^{14} + (t^{24} - 42 t^{22} + 420 t^{20} - 1870 t^{18} + 4650 t^{16} - 7296 t^{14} \\
& + 8257 t^{12} - 7296 t^{10} + 4650 t^8 - 1870 t^6 + 420 t^4 - 42 t^2 + 1) s^{12} + (6 t^{22} - 120 t^{20} \\
& + 802 t^{18} - 2718 t^{16} + 5446 t^{14} - 7296 t^{12} + 7296 t^{10} - 5446 t^8 + 2718 t^6 - 802 t^4 + 120 t^2 \\
& - 6) s^{10} + (14 t^{20} - 186 t^{18} + 970 t^{16} - 2718 t^{14} + 4650 t^{12} - 5446 t^{10} + 4650 t^8 - 2718 t^6 \\
& + 970 t^4 - 186 t^2 + 14) s^8 + (17 t^{18} - 186 t^{16} + 802 t^{14} - 1870 t^{12} + 2718 t^{10} - 2718 t^8 \\
& + 1870 t^6 - 802 t^4 + 186 t^2 - 17) s^6 + (14 t^{16} - 120 t^{14} + 420 t^{12} - 802 t^{10} + 970 t^8 - 802 t^6 \\
& + 420 t^4 - 120 t^2 + 14) s^4 + (6 t^{14} - 42 t^{12} + 120 t^{10} - 186 t^8 + 186 t^6 - 120 t^4 + 42 t^2 \\
& - 6) s^2 + t^{12} - 6 t^{10} + 14 t^8 - 17 t^6 + 14 t^4 - 6 t^2 + 1); \text{test}(K932);
\end{aligned}$$

$$\begin{aligned}
K932 := & \frac{1}{s^{12} t^{12}} (t^{12} (t^{12} - 6 t^{10} + 14 t^8 - 17 t^6 + 14 t^4 - 6 t^2 + 1) s^{24} + (-6 t^{24} + 42 t^{22} \\
& - 120 t^{20} + 186 t^{18} - 186 t^{16} + 120 t^{14} - 42 t^{12} + 6 t^{10}) s^{22} + (14 t^{24} - 120 t^{22} \\
& + 420 t^{20} - 802 t^{18} + 970 t^{16} - 802 t^{14} + 420 t^{12} - 120 t^{10} + 14 t^8) s^{20} + (-17 t^{24} \\
& + 186 t^{22} - 802 t^{20} + 1870 t^{18} - 2718 t^{16} + 2718 t^{14} - 1870 t^{12} + 802 t^{10} - 186 t^8 \\
& + 17 t^6) s^{18} + (14 t^{24} - 186 t^{22} + 970 t^{20} - 2718 t^{18} + 4650 t^{16} - 5446 t^{14} + 4650 t^{12} \\
& - 2718 t^{10} + 970 t^8 - 186 t^6 + 14 t^4) s^{16} + (-6 t^{24} + 120 t^{22} - 802 t^{20} + 2718 t^{18} \\
& - 5446 t^{16} + 7296 t^{14} - 7296 t^{12} + 5446 t^{10} - 2718 t^8 + 802 t^6 - 120 t^4 + 6 t^2) s^{14} \\
& + (t^{24} - 42 t^{22} + 420 t^{20} - 1870 t^{18} + 4650 t^{16} - 7296 t^{14} + 8257 t^{12} - 7296 t^{10} \\
& + 4650 t^8 - 1870 t^6 + 420 t^4 - 42 t^2 + 1) s^{12} + (6 t^{22} - 120 t^{20} + 802 t^{18} - 2718 t^{16} \\
& + 5446 t^{14} - 7296 t^{12} + 7296 t^{10} - 5446 t^8 + 2718 t^6 - 802 t^4 + 120 t^2 - 6) s^{10} \\
& + (14 t^{20} - 186 t^{18} + 970 t^{16} - 2718 t^{14} + 4650 t^{12} - 5446 t^{10} + 4650 t^8 - 2718 t^6 \\
& + 970 t^4 - 186 t^2 + 14) s^8 + (17 t^{18} - 186 t^{16} + 802 t^{14} - 1870 t^{12} + 2718 t^{10} - 2718 t^8 \\
& + 1870 t^6 - 802 t^4 + 186 t^2 - 17) s^6 + (14 t^{16} - 120 t^{14} + 420 t^{12} - 802 t^{10} + 970 t^8 \\
& - 802 t^6 + 420 t^4 - 120 t^2 + 14) s^4 + (6 t^{14} - 42 t^{12} + 120 t^{10} - 186 t^8 + 186 t^6 \\
& - 120 t^4 + 42 t^2 - 6) s^2 + t^{12} - 6 t^{10} + 14 t^8 - 17 t^6 + 14 t^4 - 6 t^2 + 1) \\
& \text{true, true, true, } \frac{t^{24} - 6 t^{20} + 14 t^{16} - 17 t^{12} + 14 t^8 - 6 t^4 + 1}{t^{12}} \quad (2.17)
\end{aligned}$$

#Computed Externally

#A := spmm(KP(id, spmm(KP(h, KP(h, h)), spmm(KP(id, R), KP(U, id)))), KP(R, R)) : B :=
 KP(spmm(KP(id, U^2), spmm(KP(R, id), spmm(KP(id, U), KP(R, id)))), id) :
 #K933 := sl3(A, B);

$$\begin{aligned}
K933 := & \frac{1}{s^{12} t^{12}} (t^{12} (t^{12} - 6 t^{10} + 14 t^8 - 19 t^6 + 14 t^4 - 6 t^2 + 1) s^{24} + (-6 t^{24} + 42 t^{22} \\
& - 120 t^{20} + 198 t^{18} - 198 t^{16} + 120 t^{14} - 42 t^{12} + 6 t^{10}) s^{22} + (14 t^{24} - 120 t^{22} + 420 t^{20} \\
& - 834 t^{18} + 1046 t^{16} - 834 t^{14} + 420 t^{12} - 120 t^{10} + 14 t^8) s^{20} + (-19 t^{24} + 198 t^{22} - 834 t^{20} \\
& + 1972 t^{18} - 2992 t^{16} + 2992 t^{14} - 1972 t^{12} + 834 t^{10} - 198 t^8 + 19 t^6) s^{18} + (14 t^{24} - 198 t^{22}
\end{aligned}$$

$$\begin{aligned}
& + 1046 t^{20} - 2992 t^{18} + 5414 t^{16} - 6582 t^{14} + 5414 t^{12} - 2992 t^{10} + 1046 t^8 - 198 t^6 + 14 t^4) \\
& s^{16} + (-6 t^{24} + 120 t^{22} - 834 t^{20} + 2992 t^{18} - 6582 t^{16} + 9680 t^{14} - 9680 t^{12} + 6582 t^{10} \\
& - 2992 t^8 + 834 t^6 - 120 t^4 + 6 t^2) s^{14} + (t^{24} - 42 t^{22} + 420 t^{20} - 1972 t^{18} + 5414 t^{16} \\
& - 9680 t^{14} + 11737 t^{12} - 9680 t^{10} + 5414 t^8 - 1972 t^6 + 420 t^4 - 42 t^2 + 1) s^{12} + (6 t^{22} \\
& - 120 t^{20} + 834 t^{18} - 2992 t^{16} + 6582 t^{14} - 9680 t^{12} + 9680 t^{10} - 6582 t^8 + 2992 t^6 - 834 t^4 \\
& + 120 t^2 - 6) s^{10} + (14 t^{20} - 198 t^{18} + 1046 t^{16} - 2992 t^{14} + 5414 t^{12} - 6582 t^{10} + 5414 t^8 \\
& - 2992 t^6 + 1046 t^4 - 198 t^2 + 14) s^8 + (19 t^{18} - 198 t^{16} + 834 t^{14} - 1972 t^{12} + 2992 t^{10} \\
& - 2992 t^8 + 1972 t^6 - 834 t^4 + 198 t^2 - 19) s^6 + (14 t^{16} - 120 t^{14} + 420 t^{12} - 834 t^{10} \\
& + 1046 t^8 - 834 t^6 + 420 t^4 - 120 t^2 + 14) s^4 + (6 t^{14} - 42 t^{12} + 120 t^{10} - 198 t^8 + 198 t^6 \\
& - 120 t^4 + 42 t^2 - 6) s^2 + t^{12} - 6 t^{10} + 14 t^8 - 19 t^6 + 14 t^4 - 6 t^2 + 1) ; test(K933);
\end{aligned}$$

$$\begin{aligned}
K933 := & \frac{1}{s^{12} t^{12}} (t^{12} (t^{12} - 6 t^{10} + 14 t^8 - 19 t^6 + 14 t^4 - 6 t^2 + 1) s^{24} + (-6 t^{24} + 42 t^{22} \\
& - 120 t^{20} + 198 t^{18} - 198 t^{16} + 120 t^{14} - 42 t^{12} + 6 t^{10}) s^{22} + (14 t^{24} - 120 t^{22} \\
& + 420 t^{20} - 834 t^{18} + 1046 t^{16} - 834 t^{14} + 420 t^{12} - 120 t^{10} + 14 t^8) s^{20} + (-19 t^{24} \\
& + 198 t^{22} - 834 t^{20} + 1972 t^{18} - 2992 t^{16} + 2992 t^{14} - 1972 t^{12} + 834 t^{10} - 198 t^8 \\
& + 19 t^6) s^{18} + (14 t^{24} - 198 t^{22} + 1046 t^{20} - 2992 t^{18} + 5414 t^{16} - 6582 t^{14} + 5414 t^{12} \\
& - 2992 t^{10} + 1046 t^8 - 198 t^6 + 14 t^4) s^{16} + (-6 t^{24} + 120 t^{22} - 834 t^{20} + 2992 t^{18} \\
& - 6582 t^{16} + 9680 t^{14} - 9680 t^{12} + 6582 t^{10} - 2992 t^8 + 834 t^6 - 120 t^4 + 6 t^2) s^{14} \\
& + (t^{24} - 42 t^{22} + 420 t^{20} - 1972 t^{18} + 5414 t^{16} - 9680 t^{14} + 11737 t^{12} - 9680 t^{10} \\
& + 5414 t^8 - 1972 t^6 + 420 t^4 - 42 t^2 + 1) s^{12} + (6 t^{22} - 120 t^{20} + 834 t^{18} - 2992 t^{16} \\
& + 6582 t^{14} - 9680 t^{12} + 9680 t^{10} - 6582 t^8 + 2992 t^6 - 834 t^4 + 120 t^2 - 6) s^{10} \\
& + (14 t^{20} - 198 t^{18} + 1046 t^{16} - 2992 t^{14} + 5414 t^{12} - 6582 t^{10} + 5414 t^8 - 2992 t^6 \\
& + 1046 t^4 - 198 t^2 + 14) s^8 + (19 t^{18} - 198 t^{16} + 834 t^{14} - 1972 t^{12} + 2992 t^{10} \\
& - 2992 t^8 + 1972 t^6 - 834 t^4 + 198 t^2 - 19) s^6 + (14 t^{16} - 120 t^{14} + 420 t^{12} - 834 t^{10} \\
& + 1046 t^8 - 834 t^6 + 420 t^4 - 120 t^2 + 14) s^4 + (6 t^{14} - 42 t^{12} + 120 t^{10} - 198 t^8 \\
& + 198 t^6 - 120 t^4 + 42 t^2 - 6) s^2 + t^{12} - 6 t^{10} + 14 t^8 - 19 t^6 + 14 t^4 - 6 t^2 + 1) \\
& true, true, true, \frac{-t^{24} + 6 t^{20} - 14 t^{16} + 19 t^{12} - 14 t^8 + 6 t^4 - 1}{t^{12}}
\end{aligned} \tag{2.18}$$

#Computed Externally

#iRRi:=ssimplify(spmmm(iR, Ri)) : RiiRUi := ssimplify(spmmm(spmmm(Ri, iR), Ui)) : RiiURi :=
ssimplify(spmmm(Ri, spmmm(iU, Ri))) :

#A946:=spmmm(KP(id, KP(h, KP(h, h))), spmmm(KP(id, iRRi), KP(Ui, id))) : B946 :=
spmmm(KP(id, RiiRUi), KP(RiiURi, id)) : K946 := sl3(A946, B946);

$$\begin{aligned}
K946 := & 6 s^4 t^4 - 14 s^4 t^2 - 14 s^2 t^4 + 6 s^4 + 46 s^2 t^2 + 6 t^4 - 46 s^2 - 46 t^2 + \frac{14 s^2}{t^2} + \frac{14 t^2}{s^2} - \frac{46}{s^2} \\
& - \frac{46}{t^2} + \frac{6}{s^4} + \frac{6}{t^4} + \frac{46}{s^2 t^2} - \frac{14}{s^2 t^4} - \frac{14}{s^4 t^2} + \frac{6}{s^4 t^4} + 85; test(K946);
\end{aligned}$$

$$K946 := 6 s^4 t^4 - 14 s^4 t^2 - 14 s^2 t^4 + 6 s^4 + 46 s^2 t^2 + 6 t^4 - 46 s^2 - 46 t^2 + \frac{14 s^2}{t^2} + \frac{14 t^2}{s^2}$$

$$\begin{aligned}
& -\frac{46}{s^2} - \frac{46}{t^2} + \frac{6}{s^4} + \frac{6}{t^4} + \frac{46}{s^2 t^2} - \frac{14}{s^2 t^4} - \frac{14}{s^4 t^2} + \frac{6}{s^4 t^4} + 85 \\
& \text{true, true, true, } \frac{-2t^8 + 5t^4 - 2}{t^4}
\end{aligned} \tag{2.19}$$

#Computed Externally

$$\begin{aligned}
& \#UiiRRi := \text{ssimplify}(\text{spmm}(\text{spmm}(Ui, iR), Ri)) : R2iRU3i := \text{ssimplify}(\text{spmm}(\text{spmm}(KP(R^2, \\
& \quad id), iR), KP(U^3, id))) : \\
& \#A10132 := \text{spmm}(KP(id, KP(h, KP(h, h))), KP(id, \text{spmm}(KP(id, R^2), UiiRRi))) : B10132 := \\
& \quad KP(R2iRU3i, id) : K10132 := \text{sort}(\text{sort}(\text{expand}(\text{sl3}(A10132, B10132)), s), t); \\
& K10132 := \frac{1}{s^8 t^8} (t^8 (t^8 - t^6 + t^4 - t^2 + 1) s^{16} + (-t^{16} + 2t^{14} - 4t^{12} + 4t^{10} - 2t^8 + t^6) s^{14} \\
& \quad + t^4 (t^{12} - 4t^{10} + 9t^8 - 13t^6 + 9t^4 - 4t^2 + 1) s^{12} + (-t^{16} + 4t^{14} - 13t^{12} + 32t^{10} - 32t^8 \\
& \quad + 13t^6 - 4t^4 + t^2) s^{10} + (t^{16} - 2t^{14} + 9t^{12} - 32t^{10} + 49t^8 - 32t^6 + 9t^4 - 2t^2 + 1) s^8 \\
& \quad + (t^{14} - 4t^{12} + 13t^{10} - 32t^8 + 32t^6 - 13t^4 + 4t^2 - 1) s^6 + (t^{12} - 4t^{10} + 9t^8 - 13t^6 + 9t^4 \\
& \quad - 4t^2 + 1) s^4 + (t^{10} - 2t^8 + 4t^6 - 4t^4 + 2t^2 - 1) s^2 + t^8 - t^6 + t^4 - t^2 + 1); \text{test}(K10132); \\
& K10132 := \frac{1}{s^8 t^8} (t^8 (t^8 - t^6 + t^4 - t^2 + 1) s^{16} + (-t^{16} + 2t^{14} - 4t^{12} + 4t^{10} - 2t^8 + t^6) s^{14} \\
& \quad + t^4 (t^{12} - 4t^{10} + 9t^8 - 13t^6 + 9t^4 - 4t^2 + 1) s^{12} + (-t^{16} + 4t^{14} - 13t^{12} + 32t^{10} \\
& \quad - 32t^8 + 13t^6 - 4t^4 + t^2) s^{10} + (t^{16} - 2t^{14} + 9t^{12} - 32t^{10} + 49t^8 - 32t^6 + 9t^4 - 2t^2 \\
& \quad + 1) s^8 + (t^{14} - 4t^{12} + 13t^{10} - 32t^8 + 32t^6 - 13t^4 + 4t^2 - 1) s^6 + (t^{12} - 4t^{10} + 9t^8 \\
& \quad - 13t^6 + 9t^4 - 4t^2 + 1) s^4 + (t^{10} - 2t^8 + 4t^6 - 4t^4 + 2t^2 - 1) s^2 + t^8 - t^6 + t^4 - t^2 \\
& \quad + 1) \\
& \text{true, true, true, } \frac{t^{16} - t^{12} + t^8 - t^4 + 1}{t^8}
\end{aligned} \tag{2.20}$$

$$\begin{aligned}
& A10155 := \text{spmm}(KP(id, KP(h, h)), \text{spmm}(KP(U^3, id), \text{spmm}(KP(id, U), KP(R^2, id)))) : \\
& B10155 := \text{spmm}(KP(id, U), \text{spmm}(KP(R^2, id), KP(id, U))) : K10155 := \\
& \quad \text{sort}(\text{sort}(\text{expand}(\text{sl3}(A10155, B10155)), s), t); \text{test}(K10155);
\end{aligned}$$

$$\begin{aligned}
& K10155 := s^{12} t^{12} - 3s^{10} t^{12} + 5s^8 t^{12} - 7s^6 t^{12} + 5s^4 t^{12} - 3s^2 t^{12} + t^{12} - 3s^{12} t^{10} \\
& \quad + 12s^{10} t^{10} - 24s^8 t^{10} + 36s^6 t^{10} - 36s^4 t^{10} + 24s^2 t^{10} + \frac{3t^{10}}{s^2} - 12t^{10} + 5s^{12} t^8 \\
& \quad - 24s^{10} t^8 + 53s^8 t^8 - 79s^6 t^8 + 93s^4 t^8 - 79s^2 t^8 - \frac{24t^8}{s^2} + \frac{5t^8}{s^4} + 53t^8 - 7s^{12} t^6 \\
& \quad + 36s^{10} t^6 - 79s^8 t^6 + 108s^6 t^6 - 124s^4 t^6 + 124s^2 t^6 + \frac{79t^6}{s^2} - \frac{36t^6}{s^4} + \frac{7t^6}{s^6} - 108t^6 \\
& \quad + 5s^{12} t^4 - 36s^{10} t^4 + 93s^8 t^4 - 124s^6 t^4 + 105s^4 t^4 - 91s^2 t^4 - \frac{124t^4}{s^2} + \frac{93t^4}{s^4} \\
& \quad - \frac{36t^4}{s^6} + \frac{5t^4}{s^8} + 105t^4 - 3s^{12} t^2 + 24s^{10} t^2 - 79s^8 t^2 + 124s^6 t^2 - 91s^4 t^2 + 24s^2 t^2 \\
& \quad - 12t^2 + \frac{5t^2}{s^2} - \frac{36t^2}{s^4} + \frac{5t^2}{s^6} - 12t^2 + 5s^{12} t^0 - 24s^{10} t^0 + 36s^8 t^0 - 36s^6 t^0 + 24s^4 t^0 - 12s^2 t^0 + t^0 - 3s^{12} t^{-2} \\
& \quad + 12s^{10} t^{-2} - 24s^8 t^{-2} + 36s^6 t^{-2} - 36s^4 t^{-2} + 24s^2 t^{-2} + \frac{3t^{-2}}{s^2} - 12t^{-2} + 5s^{12} t^{-4} \\
& \quad - 24s^{10} t^{-4} + 53s^8 t^{-4} - 79s^6 t^{-4} + 93s^4 t^{-4} - 79s^2 t^{-4} - \frac{24t^{-4}}{s^2} + \frac{5t^{-4}}{s^4} + 53t^{-4} - 7s^{12} t^{-6} \\
& \quad + 36s^{10} t^{-6} - 79s^8 t^{-6} + 108s^6 t^{-6} - 124s^4 t^{-6} + 124s^2 t^{-6} + \frac{79t^{-6}}{s^2} - \frac{36t^{-6}}{s^4} + \frac{7t^{-6}}{s^6} - 108t^{-6} \\
& \quad + 5s^{12} t^{-8} - 36s^{10} t^{-8} + 93s^8 t^{-8} - 124s^6 t^{-8} + 105s^4 t^{-8} - 91s^2 t^{-8} - \frac{124t^{-8}}{s^2} + \frac{93t^{-8}}{s^4} \\
& \quad - \frac{36t^{-8}}{s^6} + \frac{5t^{-8}}{s^8} + 105t^{-8} - 3s^{12} t^{-10} + 24s^{10} t^{-10} - 79s^8 t^{-10} + 124s^6 t^{-10} - 91s^4 t^{-10} + 24s^2 t^{-10} \\
& \quad - 12t^{-10} + \frac{5t^{-10}}{s^2} - \frac{36t^{-10}}{s^4} + \frac{5t^{-10}}{s^6} - 12t^{-10} + 5s^{12} t^{-12} - 24s^{10} t^{-12} + 36s^8 t^{-12} - 36s^6 t^{-12} + 24s^4 t^{-12} - 12s^2 t^{-12} \\
& \quad + t^{-12} - 3s^{12} t^{-14} + 12s^{10} t^{-14} - 24s^8 t^{-14} + 36s^6 t^{-14} - 36s^4 t^{-14} + 24s^2 t^{-14} + \frac{3t^{-14}}{s^2} - 12t^{-14} + 5s^{12} t^{-16} \\
& \quad - 24s^{10} t^{-16} + 53s^8 t^{-16} - 79s^6 t^{-16} + 93s^4 t^{-16} - 79s^2 t^{-16} - \frac{24t^{-16}}{s^2} + \frac{5t^{-16}}{s^4} + 53t^{-16} - 7s^{12} t^{-18} \\
& \quad + 36s^{10} t^{-18} - 79s^8 t^{-18} + 108s^6 t^{-18} - 124s^4 t^{-18} + 124s^2 t^{-18} + \frac{79t^{-18}}{s^2} - \frac{36t^{-18}}{s^4} + \frac{7t^{-18}}{s^6} - 108t^{-18} \\
& \quad + 5s^{12} t^{-20} - 36s^{10} t^{-20} + 93s^8 t^{-20} - 124s^6 t^{-20} + 105s^4 t^{-20} - 91s^2 t^{-20} - \frac{124t^{-20}}{s^2} + \frac{93t^{-20}}{s^4} \\
& \quad - \frac{36t^{-20}}{s^6} + \frac{5t^{-20}}{s^8} + 105t^{-20} - 3s^{12} t^{-22} + 24s^{10} t^{-22} - 79s^8 t^{-22} + 124s^6 t^{-22} - 91s^4 t^{-22} + 24s^2 t^{-22} \\
& \quad - 12t^{-22} + \frac{5t^{-22}}{s^2} - \frac{36t^{-22}}{s^4} + \frac{5t^{-22}}{s^6} - 12t^{-22} + 5s^{12} t^{-24} - 24s^{10} t^{-24} + 36s^8 t^{-24} - 36s^6 t^{-24} + 24s^4 t^{-24} - 12s^2 t^{-24} \\
& \quad + t^{-24} - 3s^{12} t^{-26} + 12s^{10} t^{-26} - 24s^8 t^{-26} + 36s^6 t^{-26} - 36s^4 t^{-26} + 24s^2 t^{-26} + \frac{3t^{-26}}{s^2} - 12t^{-26} + 5s^{12} t^{-28} \\
& \quad - 24s^{10} t^{-28} + 53s^8 t^{-28} - 79s^6 t^{-28} + 93s^4 t^{-28} - 79s^2 t^{-28} - \frac{24t^{-28}}{s^2} + \frac{5t^{-28}}{s^4} + 53t^{-28} - 7s^{12} t^{-30} \\
& \quad + 36s^{10} t^{-30} - 79s^8 t^{-30} + 108s^6 t^{-30} - 124s^4 t^{-30} + 124s^2 t^{-30} + \frac{79t^{-30}}{s^2} - \frac{36t^{-30}}{s^4} + \frac{7t^{-30}}{s^6} - 108t^{-30} \\
& \quad + 5s^{12} t^{-32} - 36s^{10} t^{-32} + 93s^8 t^{-32} - 124s^6 t^{-32} + 105s^4 t^{-32} - 91s^2 t^{-32} - \frac{124t^{-32}}{s^2} + \frac{93t^{-32}}{s^4} \\
& \quad - \frac{36t^{-32}}{s^6} + \frac{5t^{-32}}{s^8} + 105t^{-32} - 3s^{12} t^{-34} + 24s^{10} t^{-34} - 79s^8 t^{-34} + 124s^6 t^{-34} - 91s^4 t^{-34} + 24s^2 t^{-34} \\
& \quad - 12t^{-34} + \frac{5t^{-34}}{s^2} - \frac{36t^{-34}}{s^4} + \frac{5t^{-34}}{s^6} - 12t^{-34} + 5s^{12} t^{-36} - 24s^{10} t^{-36} + 36s^8 t^{-36} - 36s^6 t^{-36} + 24s^4 t^{-36} - 12s^2 t^{-36} \\
& \quad + t^{-36} - 3s^{12} t^{-38} + 12s^{10} t^{-38} - 24s^8 t^{-38} + 36s^6 t^{-38} - 36s^4 t^{-38} + 24s^2 t^{-38} + \frac{3t^{-38}}{s^2} - 12t^{-38} + 5s^{12} t^{-40} \\
& \quad - 24s^{10} t^{-40} + 53s^8 t^{-40} - 79s^6 t^{-40} + 93s^4 t^{-40} - 79s^2 t^{-40} - \frac{24t^{-40}}{s^2} + \frac{5t^{-40}}{s^4} + 53t^{-40} - 7s^{12} t^{-42} \\
& \quad + 36s^{10} t^{-42} - 79s^8 t^{-42} + 108s^6 t^{-42} - 124s^4 t^{-42} + 124s^2 t^{-42} + \frac{79t^{-42}}{s^2} - \frac{36t^{-42}}{s^4} + \frac{7t^{-42}}{s^6} - 108t^{-42} \\
& \quad + 5s^{12} t^{-44} - 36s^{10} t^{-44} + 93s^8 t^{-44} - 124s^6 t^{-44} + 105s^4 t^{-44} - 91s^2 t^{-44} - \frac{124t^{-44}}{s^2} + \frac{93t^{-44}}{s^4} \\
& \quad - \frac{36t^{-44}}{s^6} + \frac{5t^{-44}}{s^8} + 105t^{-44} - 3s^{12} t^{-46} + 24s^{10} t^{-46} - 79s^8 t^{-46} + 124s^6 t^{-46} - 91s^4 t^{-46} + 24s^2 t^{-46} \\
& \quad - 12t^{-46} + \frac{5t^{-46}}{s^2} - \frac{36t^{-46}}{s^4} + \frac{5t^{-46}}{s^6} - 12t^{-46} + 5s^{12} t^{-48} - 24s^{10} t^{-48} + 36s^8 t^{-48} - 36s^6 t^{-48} + 24s^4 t^{-48} - 12s^2 t^{-48} \\
& \quad + t^{-48} - 3s^{12} t^{-50} + 12s^{10} t^{-50} - 24s^8 t^{-50} + 36s^6 t^{-50} - 36s^4 t^{-50} + 24s^2 t^{-50} + \frac{3t^{-50}}{s^2} - 12t^{-50} + 5s^{12} t^{-52} \\
& \quad - 24s^{10} t^{-52} + 53s^8 t^{-52} - 79s^6 t^{-52} + 93s^4 t^{-52} - 79s^2 t^{-52} - \frac{24t^{-52}}{s^2} + \frac{5t^{-52}}{s^4} + 53t^{-52} - 7s^{12} t^{-54} \\
& \quad + 36s^{10} t^{-54} - 79s^8 t^{-54} + 108s^6 t^{-54} - 124s^4 t^{-54} + 124s^2 t^{-54} + \frac{79t^{-54}}{s^2} - \frac{36t^{-54}}{s^4} + \frac{7t^{-54}}{s^6} - 108t^{-54} \\
& \quad + 5s^{12} t^{-56} - 36s^{10} t^{-56} + 93s^8 t^{-56} - 124s^6 t^{-56} + 105s^4 t^{-56} - 91s^2 t^{-56} - \frac{124t^{-56}}{s^2} + \frac{93t^{-56}}{s^4} \\
& \quad - \frac{36t^{-56}}{s^6} + \frac{5t^{-56}}{s^8} + 105t^{-56} - 3s^{12} t^{-58} + 24s^{10} t^{-58} - 79s^8 t^{-58} + 124s^6 t^{-58} - 91s^4 t^{-58} + 24s^2 t^{-58} \\
& \quad - 12t^{-58} + \frac{5t^{-58}}{s^2} - \frac{36t^{-58}}{s^4} + \frac{5t^{-58}}{s^6} - 12t^{-58} + 5s^{12} t^{-60} - 24s^{10} t^{-60} + 36s^8 t^{-60} - 36s^6 t^{-60} + 24s^4 t^{-60} - 12s^2 t^{-60} \\
& \quad + t^{-60} - 3s^{12} t^{-62} + 12s^{10} t^{-62} - 24s^8 t^{-62} + 36s^6 t^{-62} - 36s^4 t^{-62} + 24s^2 t^{-62} + \frac{3t^{-62}}{s^2} - 12t^{-62} + 5s^{12} t^{-64} \\
& \quad - 24s^{10} t^{-64} + 53s^8 t^{-64} - 79s^6 t^{-64} + 93s^4 t^{-64} - 79s^2 t^{-64} - \frac{24t^{-64}}{s^2} + \frac{5t^{-64}}{s^4} + 53t^{-64} - 7s^{12} t^{-66} \\
& \quad + 36s^{10} t^{-66} - 79s^8 t^{-66} + 108s^6 t^{-66} - 124s^4 t^{-66} + 124s^2 t^{-66} + \frac{79t^{-66}}{s^2} - \frac{36t^{-66}}{s^4} + \frac{7t^{-66}}{s^6} - 108t^{-66} \\
& \quad + 5s^{12} t^{-68} - 36s^{10} t^{-68} + 93s^8 t^{-68} - 124s^6 t^{-68} + 105s^4 t^{-68} - 91s^2 t^{-68} - \frac{124t^{-68}}{s^2} + \frac{93t^{-68}}{s^4} \\
& \quad - \frac{36t^{-68}}{s^6} + \frac{5t^{-68}}{s^8} + 105t^{-68} - 3s^{12} t^{-70} + 24s^{10} t^{-70} - 79s^8 t^{-70} + 124s^6 t^{-70} - 91s^4 t^{-70} + 24s^2 t^{-70} \\
& \quad - 12t^{-70} + \frac{5t^{-70}}{s^2} - \frac{36t^{-70}}{s^4} + \frac{5t^{-70}}{s^6} - 12t^{-70} + 5s^{12} t^{-72} - 24s^{10} t^{-72} + 36s^8 t^{-72} - 36s^6 t^{-72} + 24s^4 t^{-72} - 12s^2 t^{-72} \\
& \quad + t^{-72} - 3s^{12} t^{-74} + 12s^{10} t^{-74} - 24s^8 t^{-74} + 36s^6 t^{-74} - 36s^4 t^{-74} + 24s^2 t^{-74} + \frac{3t^{-74}}{s^2} - 12t^{-74} + 5s^{12} t^{-76} \\
& \quad - 24s^{10} t^{-76} + 53s^8 t^{-76} - 79s^6 t^{-76} + 93s^4 t^{-76} - 79s^2 t^{-76} - \frac{24t^{-76}}{s^2} + \frac{5t^{-76}}{s^4} + 53t^{-76} - 7s^{12} t^{-78} \\
& \quad + 36s^{10} t^{-78} - 79s^8 t^{-78} + 108s^6 t^{-78} - 124s^4 t^{-78} + 124s^2 t^{-78} + \frac{79t^{-78}}{s^2} - \frac{36t^{-78}}{s^4} + \frac{7t^{-78}}{s^6} - 108t^{-78} \\
& \quad + 5s^{12} t^{-80} - 36s^{10} t^{-80} + 93s^8 t^{-80} - 124s^6 t^{-80} + 105s^4 t^{-80} - 91s^2 t^{-80} - \frac{124t^{-80}}{s^2} + \frac{93t^{-80}}{s^4} \\
& \quad - \frac{36t^{-80}}{s^6} + \frac{5t^{-80}}{s^8} + 105t^{-80} - 3s^{12} t^{-82} + 24s^{10} t^{-82} - 79s^8 t^{-82} + 124s^6 t^{-82} - 91s^4 t^{-82} + 24s^2 t^{-82} \\
& \quad - 12t^{-82} + \frac{5t^{-82}}{s^2} - \frac{36t^{-82}}{s^4} + \frac{5t^{-82}}{s^6} - 12t^{-82} + 5s^{12} t^{-84} - 24s^{10} t^{-84} + 36s^8 t^{-84} - 36s^6 t^{-84} + 24s^4 t^{-84} - 12s^2 t^{-84} \\
& \quad + t^{-84} - 3s^{12} t^{-86} + 12s^{10} t^{-86} - 24s^8 t^{-86} + 36s^6 t^{-86} - 36s^4 t^{-86} + 24s^2 t^{-86} + \frac{3t^{-86}}{s^2} - 12t^{-86} + 5s^{12} t^{-88} \\
& \quad - 24s^{10} t^{-88} + 53s^8 t^{-88} - 79s^6 t^{-88} + 93s^4 t^{-88} - 79s^2 t^{-88} - \frac{24t^{-88}}{s^2} + \frac{5t^{-88}}{s^4} + 53t^{-88} - 7s^{12} t^{-90} \\
& \quad + 36s^{10} t^{-90} - 79s^8 t^{-90} + 108s^6 t^{-90} - 124s^4 t^{-90} + 124s^2 t^{-90} + \frac{79t^{-90}}{s^2} - \frac{36t^{-90}}{s^4} + \frac{7t^{-90}}{s^6} - 108t^{-90} \\
& \quad + 5s^{12} t^{-92} - 36s^{10} t^{-92} + 93s^8 t^{-92} - 124s^6 t^{-92} + 105s^4 t^{-92} - 91s^2 t^{-92} - \frac{124t^{-92}}{s^2} + \frac{93t^{-92}}{s^4} \\
& \quad - \frac{36t^{-92}}{s^6} + \frac{5t^{-92}}{s^8} + 105t^{-92} - 3s^{12} t^{-94} + 24s^{10} t^{-94} - 79s^8 t^{-94} + 124s^6 t^{-94} - 91s^4 t^{-94} + 24s^2 t^{-94} \\
& \quad - 12t^{-94} + \frac{5t^{-94}}{s^2} - \frac{36t^{-94}}{s^4} + \frac{5t^{-94}}{s^6} - 12t^{-94} + 5s^{12} t^{-96} - 24s^{10} t^{-96} + 36s^8 t^{-96} - 36s^6 t^{-96} + 24s^4 t^{-96} - 12s^2 t^{-96} \\
& \quad + t^{-96} - 3s^{12} t^{-98} + 12s^{10} t^{-98} - 24s^8 t^{-98} + 36s^6 t^{-98} - 36s^4 t^{-98} + 24s^2 t^{-98} + \frac{3t^{-98}}{s^2} - 12t^{-98} + 5s^{12} t^{-100} \\
& \quad - 24s^{10} t^{-100} + 53s^8 t^{-100} - 79s^6 t^{-100} + 93s^4 t^{-100} - 79s^2 t^{-100} - \frac{24t^{-100}}{s^2} + \frac{5t^{-100}}{s^4} + 53t^{-100} - 7s^{12} t^{-102} \\
& \quad + 36s^{10} t^{-102} - 79s^8 t^{-102} + 108s^6 t^{-102} - 124s^4 t^{-102} + 124s^2 t^{-102} + \frac{79t^{-102}}{s^2} - \frac{36t^{-102}}{s^4} + \frac{7t^{-102}}{s^6} - 108t^{-102} \\
& \quad + 5s^{12} t^{-104} - 36s^{10} t^{-104} + 93s^8 t^{-104} - 124s^6 t^{-104} + 105s^4 t^{-104} - 91s^2 t^{-104} - \frac{124t^{-104}}{s^2} + \frac{93t^{-104}}{s^4} \\
& \quad - \frac{36t^{-104}}{s^6} + \frac{5t^{-104}}{s^8} + 105t^{-104} - 3s^{12} t^{-106} + 24s^{10} t^{-106} - 79s^8 t^{-106} + 124s^6 t^{-106} - 91s^4 t^{-106} + 24s^2 t^{-106} \\
& \quad - 12t^{-106} + \frac{5t^{-106}}{s^2} - \frac{36t^{-106}}{s^4} + \frac{5t^{-106}}{s^6} - 12t^{-106} + 5s^{12} t^{-108} - 24s^{10} t^{-108} + 36s^8 t^{-108} - 36s^6 t^{-108} + 24s^4 t^{-108} - 12s^2 t^{-108} \\
& \quad + t^{-108} - 3s^{12} t^{-110} + 12s^{10} t^{-110} - 24s^8 t^{-110} + 36s^6 t^{-110} - 36s^4 t^{-110} + 24s^2 t^{-110} + \frac{3t^{-110}}{s^2} - 12t^{-110} + 5s^{12} t^{-112} \\
& \quad - 24s^{10} t^{-112} + 53s^8 t^{-112} - 79s^6 t^{-112} + 93s^4 t^{-112} - 79s^2 t^{-112} - \frac{24t^{-112}}{s^2} + \frac{5t^{-112}}{s^4} + 53t^{-112} - 7s^{12} t^{-114} \\
& \quad + 36s^{10} t^{-114} - 79s^8 t^{-114} + 108s^6 t^{-114} - 124s^4 t^{-114} + 124s^2 t^{-114} + \frac{79t^{-114}}{s^2} - \frac{36t^{-114}}{s^4} + \frac{7t^{-114}}{s^6} - 108t^{-114} \\
& \quad + 5s^{12} t^{-116} - 36s^{10} t^{-116} + 93s^8 t^{-116} - 124s^6 t^{-116} + 105s^4 t^{-116} - 91s^2 t^{-116} - \frac{124t^{-116}}{s^2} + \frac{93t^{-116}}{s^4} \\
& \quad - \frac{36t^{-116}}{s^6} + \frac{5t^{-116}}{s^8} + 105t^{-116} - 3s^{12} t^{-118} + 24s^{10} t^{-118} - 79s^8 t^{-118} + 124s^6 t^{-118} - 91s^4 t^{-118} + 24s^2 t^{-118} \\
& \quad - 12t^{-118} + \frac{5t^{-118}}{s^2} - \frac{36t^{-118}}{s^4} + \frac{5t^{-118}}{s^6} - 12t^{-118} + 5s^{12} t^{-120} - 24s^{10} t^{-120} + 36s^8 t^{-120} - 36s^6 t^{-120} + 24s^4 t^{-120} - 12s^2 t^{-120} \\
& \quad + t^{-120} - 3s^{12} t^{-122} + 12s^{10} t^{-122} - 24s^8 t^{-122} + 36s^6 t^{-122} - 36s^4 t^{-122} + 24s^2 t^{-122} + \frac{3t^{-122}}{s^2} - 12t^{-122} + 5s^{12} t^{-124} \\
& \quad - 24s^{10} t^{-124} + 53s^8 t^{-124} - 79s^6 t^{-124} + 93s^4 t^{-124} - 79s^2 t^{-124} - \frac{24t^{-124}}{s^2} + \frac{5t^{-124}}{s^4} + 53t^{-124} - 7s^{12} t^{-126} \\
& \quad + 36s^{10} t^{-126} - 79s^8 t^{-126} + 108s^6 t^{-126} - 124s^4 t^{-126} + 124s^2 t^{-126} + \frac{79t^{-126}}{s^2} - \frac{36t^{-126}}{s^4} + \frac{7t^{-126}}{s^6} - 108t^{-126} \\
& \quad + 5s^{12} t^{-128} - 36s^{10} t^{-128} + 93s^8 t^{-128} - 124s^6 t^{-128} + 105s^4 t^{-128} - 91s^2 t^{-128} - \frac{124t^{-128}}{s^2} + \frac{93t^{-128}}{s^4} \\
& \quad - \frac{36t^{-128}}{s^6} + \frac{5t^{-128}}{s^8} + 105t^{-128} - 3s^{12} t^{-130} + 24s^{10} t^{-130} - 79s^8 t^{-130} + 124s^6 t^{-130} - 91s^4 t^{-130} + 24s^2 t^{-130} \\
& \quad - 12t^{-130} + \frac{5t^{-130}}{s^2} - \frac{36t^{-130}}{s^4} + \frac{5t^{-130}}{s^6} - 12t^{-130} + 5s^{12} t^{-132} - 24s^{10} t^{-132} + 36s^8 t^{-132} - 36s^6 t^{-132} + 24s^4 t^{-132} - 12s^2 t^{-132} \\
& \quad + t^{-132} - 3s^{12} t^{-134} + 12s^{10} t^{-134} - 24s^8 t^{-134} + 36s^6 t^{-134} - 36s^4 t^{-134} + 24s^2 t^{-134} + \frac{3t^{-134}}{s^2} - 12t^{-134} + 5s^{12} t^{-136} \\
& \quad - 24s^{10} t^{-136} + 53s^8 t^{-136} - 79s^6 t^{-136} + 93s^4 t^{-136} - 79s^2 t^{-136} - \frac{24t^{-136}}{s^2} + \frac{5t^{-136}}{s^4} + 53t^{-136} - 7s^{12} t^{-138} \\
& \quad + 36s^{10} t^{-138} - 79s^8 t^{-138} + 108s^6 t^{-138} - 124s^4 t^{-138} + 124s^2 t^{-138} + \frac{79t^{-138}}{s^2} - \frac{36t^{-138}}{s^4} + \frac{7t^{-138}}{s^6} - 108t^{-138} \\
& \quad + 5s^{12} t^{-140} - 36s^{10} t^{-140} + 93s^8 t^{-140} - 124s^6 t^{-140} + 105s^4 t^{-140} - 91s^2 t^{-140} - \frac{124t^{-140}}{s^2} + \frac{93t^{-140}}{s^4} \\
& \quad - \frac{36t^{-140}}{s^6} + \frac{5t^{-140}}{s^8} + 105t^{-140} - 3s^{12} t^{-142} + 24s^{10} t^{-142} - 79s^8 t^{-142} + 124s^6 t^{-142} - 91s^4 t^{-142} + 24s^2 t^{-142} \\
& \quad - 12t^{-142} + \frac{5t^{-142}}{s^2} - \frac{36t^{-142}}{s^4} + \frac{5t^{-142}}{s^6} - 12t^{-142} + 5s^{12} t^{-144} - 24s^{10} t^{-144} + 36s^8 t^{-144} - 36s^6 t^{-144} + 24s^4 t^{-144} - 12s^2 t^{-144} \\
& \quad + t^{-144} - 3s^{12} t^{-146} + 12s^{10} t^{-146} - 24s^8 t^{-146} + 36s^6 t^{-146} - 36s^4 t^{-146} + 24s^2 t^{-146} + \frac{3t^{-146}}{s^2} - 12t^{-146} + 5s^{12} t^{-148} \\
& \quad - 24s^{10} t^{-148} + 53s^8 t^{-148} - 79s^6 t^{-148} + 93s^4 t^{-148} - 79s^2 t^{-148} - \frac{24t^{-148}}{s^2} + \frac{5t^{-148}}{s^4} + 53t^{-148} - 7s^{12} t^{-150} \\
& \quad + 36s^{10} t^{-150} - 79s^8 t^{-150} + 108s^6 t^{-150} - 124s^4 t^{-150} + 124s^2 t^{-150} + \frac{79t^{-150}}{s^2} - \frac{36t^{-150}}{s^4} + \frac{7t^{-150}}{s^6} - 108t^{-150} \\
& \quad + 5s^{12} t^{-152} - 36s^{10} t^{-152} + 93s^8 t^{-152} - 124s^6 t^{-152} + 105s^4 t^{-152} - 91s^2 t^{-152} - \frac{124t^{-152}}{s^2} + \frac{93t^{-152}}{s^4} \\
& \quad - \frac{36t^{-152}}{s^6} + \frac{5t^{-152}}{s^8} + 105t^{-152} - 3s^{12} t^{-154} + 24s^{10} t^{-154} - 79s^8 t^{-15$$

$$\begin{aligned}
& + \frac{91 t^2}{s^2} - \frac{124 t^2}{s^4} + \frac{79 t^2}{s^6} - \frac{24 t^2}{s^8} + \frac{3 t^2}{s^{10}} - 24 t^2 + \frac{3 s^{10}}{t^2} - \frac{24 s^8}{t^2} + \frac{79 s^6}{t^2} \\
& - \frac{124 s^4}{t^2} + \frac{91 s^2}{t^2} + \frac{24}{s^2 t^2} - \frac{91}{s^4 t^2} + \frac{124}{s^6 t^2} - \frac{79}{s^8 t^2} + \frac{24}{s^{10} t^2} - \frac{3}{s^{12} t^2} - \frac{24}{t^2} + \frac{5 s^8}{t^4} \\
& - \frac{36 s^6}{t^4} + \frac{93 s^4}{t^4} - \frac{124 s^2}{t^4} - \frac{91}{s^2 t^4} + \frac{105}{s^4 t^4} - \frac{124}{s^6 t^4} + \frac{93}{s^8 t^4} - \frac{36}{s^{10} t^4} + \frac{5}{s^{12} t^4} \\
& + \frac{105}{t^4} + \frac{7 s^6}{t^6} - \frac{36 s^4}{t^6} + \frac{79 s^2}{t^6} + \frac{124}{s^2 t^6} - \frac{124}{s^4 t^6} + \frac{108}{s^6 t^6} - \frac{79}{s^8 t^6} + \frac{36}{s^{10} t^6} - \frac{7}{s^{12} t^6} \\
& - \frac{108}{t^6} + \frac{5 s^4}{t^8} - \frac{24 s^2}{t^8} - \frac{79}{s^2 t^8} + \frac{93}{s^4 t^8} - \frac{79}{s^6 t^8} + \frac{53}{s^8 t^8} - \frac{24}{s^{10} t^8} + \frac{5}{s^{12} t^8} + \frac{53}{t^8} \\
& + \frac{3 s^2}{t^{10}} + \frac{24}{s^2 t^{10}} - \frac{36}{s^4 t^{10}} + \frac{36}{s^6 t^{10}} - \frac{24}{s^8 t^{10}} + \frac{12}{s^{10} t^{10}} - \frac{3}{s^{12} t^{10}} - \frac{12}{t^{10}} - \frac{3}{s^2 t^{12}} \\
& + \frac{5}{s^4 t^{12}} - \frac{7}{s^6 t^{12}} + \frac{5}{s^8 t^{12}} - \frac{3}{s^{10} t^{12}} + \frac{1}{s^{12} t^{12}} + \frac{1}{t^{12}} + s^{12} - 12 s^{10} + 53 s^8 - 108 s^6 \\
& + 105 s^4 - 24 s^2 - \frac{24}{s^2} + \frac{105}{s^4} - \frac{108}{s^6} + \frac{53}{s^8} - \frac{12}{s^{10}} + \frac{1}{s^{12}} - 23 \\
& \text{true, true, true, } \frac{-t^{24} + 3 t^{20} - 5 t^{16} + 7 t^{12} - 5 t^8 + 3 t^4 - 1}{t^{12}}
\end{aligned} \tag{2.21}$$

#Computed Externally

#UiiRUi:=ssimplify(spmm(Ui, spmm(iR, Ui))) :

#A1134:=spmm(spmm(KP(id, KP(h, KP(h, h))), KP(U^2, R^2)), KP(id, KP(R^2, id))) :

B1134 := spmm(KP(U, R), KP(id, UiiRUi)) : K1134 := sort(sort(expand(sl3(A1134, B1134)), s), t);

$$\begin{aligned}
K1134 := & 2 s^8 t^{12} - 4 s^6 t^{12} + 2 s^4 t^{12} - 4 s^{10} t^{10} + 4 s^8 t^{10} + 8 s^6 t^{10} - 8 s^4 t^{10} - 4 s^2 t^{10} + 4 t^{10} \\
& + 2 s^{12} t^8 + 4 s^{10} t^8 - 20 s^8 t^8 + 8 s^6 t^8 + 12 s^4 t^8 + 8 s^2 t^8 + \frac{4 t^8}{s^2} + \frac{2 t^8}{s^4} - 20 t^8 - 4 s^{12} t^6 \\
& + 8 s^{10} t^6 + 8 s^8 t^6 - 4 s^6 t^6 - 46 s^4 t^6 + 46 s^2 t^6 - \frac{8 t^6}{s^2} - \frac{8 t^6}{s^4} + \frac{4 t^6}{s^6} + 4 t^6 + 2 s^{12} t^4 \\
& - 8 s^{10} t^4 + 12 s^8 t^4 - 46 s^6 t^4 + 164 s^4 t^4 - 248 s^2 t^4 - \frac{46 t^4}{s^2} + \frac{12 t^4}{s^4} - \frac{8 t^4}{s^6} + \frac{2 t^4}{s^8} + 164 t^4 \\
& - 4 s^{10} t^2 + 8 s^8 t^2 + 46 s^6 t^2 - 248 s^4 t^2 + 476 s^2 t^2 + \frac{248 t^2}{s^2} - \frac{46 t^2}{s^4} - \frac{8 t^2}{s^6} + \frac{4 t^2}{s^8} - 476 t^2 \\
& + \frac{4 s^8}{t^2} - \frac{8 s^6}{t^2} - \frac{46 s^4}{t^2} + \frac{248 s^2}{t^2} + \frac{476}{s^2 t^2} - \frac{248}{s^4 t^2} + \frac{46}{s^6 t^2} + \frac{8}{s^8 t^2} - \frac{4}{s^{10} t^2} - \frac{476}{t^2} \\
& + \frac{2 s^8}{t^4} - \frac{8 s^6}{t^4} + \frac{12 s^4}{t^4} - \frac{46 s^2}{t^4} - \frac{248}{s^2 t^4} + \frac{164}{s^4 t^4} - \frac{46}{s^6 t^4} + \frac{12}{s^8 t^4} - \frac{8}{s^{10} t^4} + \frac{2}{s^{12} t^4} \\
& + \frac{164}{t^4} + \frac{4 s^6}{t^6} - \frac{8 s^4}{t^6} - \frac{8 s^2}{t^6} + \frac{46}{s^2 t^6} - \frac{46}{s^4 t^6} - \frac{4}{s^6 t^6} + \frac{8}{s^8 t^6} + \frac{8}{s^{10} t^6} - \frac{4}{s^{12} t^6} + \frac{4}{t^6}
\end{aligned}$$

$$\begin{aligned}
& + \frac{2s^4}{t^8} + \frac{4s^2}{t^8} + \frac{8}{s^2t^8} + \frac{12}{s^4t^8} + \frac{8}{s^6t^8} - \frac{20}{s^8t^8} + \frac{4}{s^{10}t^8} + \frac{2}{s^{12}t^8} - \frac{20}{t^8} - \frac{4}{s^2t^{10}} - \frac{8}{s^4t^{10}} \\
& + \frac{8}{s^6t^{10}} + \frac{4}{s^8t^{10}} - \frac{4}{s^{10}t^{10}} + \frac{4}{t^{10}} + \frac{2}{s^4t^{12}} - \frac{4}{s^6t^{12}} + \frac{2}{s^8t^{12}} + 4s^{10} - 20s^8 + 4s^6 + 164s^4 \\
& - 476s^2 - \frac{476}{s^2} + \frac{164}{s^4} + \frac{4}{s^6} - \frac{20}{s^8} + \frac{4}{s^{10}} + 649; \text{test}(K1134);
\end{aligned}$$

$$\begin{aligned}
K1134 := & 649 - 476s^2 - 476t^2 - \frac{20}{t^8} + \frac{4}{t^6} + \frac{2}{s^{12}t^4} - \frac{20}{s^8t^8} - \frac{4}{s^6t^6} + \frac{8}{s^8t^6} - \frac{8s^6}{t^2} \\
& - \frac{8t^6}{s^2} + \frac{12s^4}{t^4} + \frac{12t^4}{s^4} - \frac{46s^2}{t^4} - \frac{46t^2}{s^4} - \frac{8s^2}{t^6} - \frac{8t^2}{s^6} + \frac{12}{s^4t^8} + \frac{12}{s^8t^4} + \frac{8}{s^6t^8} \\
& - \frac{46}{s^6t^4} - \frac{248}{s^2t^4} - 20t^8 + 4t^6 - 20s^8 + 4s^6 + \frac{46}{s^6t^2} - 248s^4t^2 - 248s^2t^4 + 476s^2t^2 \\
& + \frac{46}{s^2t^6} - \frac{476}{t^2} - \frac{476}{s^2} - \frac{46}{s^4t^6} + \frac{164}{s^4} + \frac{164}{t^4} + \frac{476}{s^2t^2} - \frac{46s^4}{t^2} - 20s^8t^8 - \frac{46t^4}{s^2} \\
& + 2s^4t^{12} - 4s^2t^{10} - \frac{248}{s^4t^2} + \frac{164}{s^4t^4} - 46s^4t^6 - 46s^6t^4 + \frac{8}{s^2t^8} + \frac{8}{s^8t^2} + 164s^4 \\
& + \frac{248s^2}{t^2} + \frac{4t^8}{s^2} + \frac{4s^8}{t^2} + 164t^4 + \frac{248t^2}{s^2} - \frac{20}{s^8} + \frac{4}{s^6} + 164s^4t^4 + \frac{4}{s^{10}} - 4s^6t^6 \\
& + 4s^{10} + \frac{2}{s^4t^{12}} + 4t^{10} + \frac{4}{t^{10}} + \frac{2s^8}{t^4} + \frac{2t^8}{s^4} - \frac{8s^6}{t^4} - \frac{8t^6}{s^4} + \frac{4s^6}{t^6} + \frac{4t^6}{s^6} - \frac{8s^4}{t^6} \\
& - \frac{8t^4}{s^6} + \frac{2s^4}{t^8} + \frac{2t^4}{s^8} + \frac{4s^2}{t^8} + \frac{4t^2}{s^8} - \frac{4}{s^2t^{10}} - \frac{4}{s^{10}t^2} - \frac{8}{s^4t^{10}} - \frac{8}{s^{10}t^4} + \frac{8}{s^6t^{10}} \\
& + \frac{8}{s^{10}t^6} - \frac{4}{s^6t^{12}} + \frac{4}{s^8t^{10}} + \frac{4}{s^{10}t^8} - \frac{4}{s^{12}t^6} + \frac{2}{s^8t^{12}} + \frac{2}{s^{12}t^8} - \frac{4}{s^{10}t^{10}} + 2s^{12}t^4 \\
& - 8s^{10}t^4 - 4s^{10}t^2 - 4s^{10}t^{10} - 8s^4t^{10} + 8s^8t^2 + 8s^2t^8 + 2s^8t^{12} - 4s^{12}t^6 + 4s^{10}t^8 \\
& + 4s^8t^{10} - 4s^6t^{12} + 8s^{10}t^6 + 8s^6t^{10} + 2s^{12}t^8 + 12s^4t^8 + 12s^8t^4 + 8s^6t^8 + 8s^8t^6 \\
& + 46s^2t^6 + 46s^6t^2
\end{aligned}$$

true, true, true, 1

(2.22)

#Computed Externally

#U2iiR2Ui:=spmm(KP(U², id), spmm(KP(id, R²), Ui)) : R2iiURiiURi := spmm(KP(R², id),
spmm(iU, spmm(Ri, spmm(iU, Ri)))) :
A1142:=spmm(KP(id, KP(h, KP(h, h))), spmm(KP(U, ID), KP(id, U2iiR2Ui))) : B1142 :=
spmm(KP(R, ID), KP(id, R2iiURiiURi)) : K1142 := sort(sort(expand(sl3(A1142, B1142)),
s), t);

$$\begin{aligned}
K1142 := & \frac{1}{s^6t^6} ((12t^{12} - 34t^{10} + 34t^8 - 12t^6)s^{12} + (-34t^{12} + 148t^{10} - 228t^8 + 148t^6 \\
& - 34t^4)s^{10} + (34t^{12} - 228t^{10} + 496t^8 - 496t^6 + 228t^4 - 34t^2)s^8 + (-12t^{12} + 148t^{10} \\
& - 496t^8 + 721t^6 - 496t^4 + 148t^2 - 12)s^6 + (-34t^{10} + 228t^8 - 496t^6 + 496t^4 - 228t^2 \\
& + 34)s^4 + (-34t^8 + 148t^6 - 228t^4 + 148t^2 - 34)s^2 - 12t^6 + 34t^4 - 34t^2 + 12); \\
& \text{test}(K1142);
\end{aligned}$$

$$\begin{aligned}
K1142 := & \frac{1}{s^6 t^6} \left((12 t^{12} - 34 t^{10} + 34 t^8 - 12 t^6) s^{12} + (-34 t^{12} + 148 t^{10} - 228 t^8 + 148 t^6 \right. \\
& - 34 t^4) s^{10} + (34 t^{12} - 228 t^{10} + 496 t^8 - 496 t^6 + 228 t^4 - 34 t^2) s^8 + (-12 t^{12} \\
& + 148 t^{10} - 496 t^8 + 721 t^6 - 496 t^4 + 148 t^2 - 12) s^6 + (-34 t^{10} + 228 t^8 - 496 t^6 \\
& + 496 t^4 - 228 t^2 + 34) s^4 + (-34 t^8 + 148 t^6 - 228 t^4 + 148 t^2 - 34) s^2 - 12 t^6 + 34 t^4 \\
& \left. - 34 t^2 + 12 \right) \\
& \text{true, true, true, 1} \tag{2.23}
\end{aligned}$$

$$\begin{aligned}
T22 := & \text{sl3}(KP(id, h).R, R); \text{test}(T22); \\
T22 := & \frac{(t-1)(t+1)(s^2 t^2 + 1)(s-1)(s+1)}{t^2 s^2} \\
& \text{true, true, true, 0} \tag{2.24}
\end{aligned}$$

$$\begin{aligned}
T24 := & \text{sl3}(KP(id, h).R^3, R); \text{test}(T24); \text{expand}\left(\text{simplify}\left(\frac{T24}{T22}\right)\right); \\
T24 := & \frac{(t+1)((t^8 + t^4) s^8 + (t^8 + 1) s^4 + t^4 + 1)(t-1)(s^2 t^2 + 1)(s+1)(s-1)}{s^6 t^6} \\
& \text{true, true, true, 0} \\
& s^4 t^4 + s^4 + t^4 + \frac{1}{t^4} + \frac{1}{s^4} + \frac{1}{s^4 t^4} \tag{2.25}
\end{aligned}$$