▶ Preamble

$x0 := ssimplify(E1 \cdot E2 \cdot E1)[..,-1] : X := simplify(\langle x0 | F1 \cdot x0 | F2 \cdot F1 \cdot x0 | F1 \cdot F2 \cdot F1 \cdot x0\rangle) :$
$Xplus := subs(t2 = 1, X) : Xminus := subs(t2 = -1, X) :$

$$f1 := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} : f2 := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} : e1 := \begin{bmatrix} 0 & \frac{t1^2+1}{2\,t1} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{-\frac{\mathrm{I}}{2}\left(t1^2-1\right)}{t1} \\ 0 & 0 & 0 & 0 \end{bmatrix} : e2 :=$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -t2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} :$$

$$k1 := \begin{bmatrix} \mathrm{I}\,t1 & 0 & 0 & 0 \\ 0 & -\mathrm{I}\,t1 & 0 & 0 \\ 0 & 0 & t1 & 0 \\ 0 & 0 & 0 & -t1 \end{bmatrix} : k2 := \begin{bmatrix} -t2 & 0 & 0 & 0 \\ 0 & -I\cdot t2 & 0 & 0 \\ 0 & 0 & I\cdot t2 & 0 \\ 0 & 0 & 0 & -t2 \end{bmatrix} : h1 :=$$

$$\begin{bmatrix} \lambda+1 & 0 & 0 & 0 \\ 0 & \lambda-1 & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda-2 \end{bmatrix} : h2 := \begin{bmatrix} (\mu-2) & 0 & 0 & 0 \\ 0 & (\mu-1) & 0 & 0 \\ 0 & 0 & (\mu-3) & 0 \\ 0 & 0 & 0 & (\mu-2) \end{bmatrix} :$$

$e3 := simplify(-(e1 \cdot e2 + I\cdot e2 \cdot e1)) : f3 := simplify(-(f2.f1 - I\cdot f1.f2)) :$
$check(br(e1, f1) - fl(k1)), check(br(e2, f2) - fl(k2)), check(br(e1, f2)), check(br(e2, f1)),$
$check(k1 \cdot f1 + f1 \cdot k1), check(k2 \cdot f2 + f2 \cdot k2), check(k1 \cdot e1 + e1 \cdot k1), check(k2 \cdot e2 + e2 \cdot k2);$
$check(e3^2), check(f3^2), check(k1 \cdot f2 - \mathrm{I} \cdot f2 \cdot k1), check(k2 \cdot f1 - \mathrm{I} \cdot f1 \cdot k2), check(k1 \cdot e2 + \mathrm{I} \cdot e2 \cdot k1), check(k2 \cdot e1 + \mathrm{I} \cdot e1 \cdot k2), check(e1^2), check(f1^2), check(e2^2), check(f2^2), check(br(h1, e1) - 2\,e1), check(br(h2, e1) + e1), check(br(h1, e2) + e2), check(br(h2, e2) - 2\,e2),$
$check(br(h1, f1) + 2\,f1), check(br(h2, f1) - f1), check(br(h1, f2) - f2), check(br(h2, f2) + 2\,f2)$

$$Good, Bad, \begin{bmatrix} \frac{-\frac{I}{2}(t2^2-1)}{t2} & 0 & 0 & 0 \\ 0 & \frac{-t2^2+1}{2\,t2} & 0 & 0 \\ 0 & 0 & \frac{t2^2-1}{2\,t2} & 0 \\ 0 & 0 & 0 & \frac{-\frac{I}{2}(t2^2-1)}{t2} \end{bmatrix}, Good, Good, Good,$$

Good, Good, Good

Good, Good, Good, Good, Good, Good, Good, Good, Good, Good, Good, Good, Good, Good, Good, Good, Good, Good **(1)**

$R1 := simplify\left(DiagonalMatrix\left(`\sim`_q\left(Diagonal\left(\frac{2}{3}KP(H1,H1)+\frac{2}{3}KP(H2,H2)+\frac{1}{3}KP(H2,H1)+\frac{1}{3}KP(H1,H2)\right)\right)\right).\ (IdentityMatrix(64)+2\,I\,KP(E1,F1))\cdot(IdentityMatrix(64)+2\,I\,KP(E3,F3))\cdot(IdentityMatrix(64)+2\,I\,KP(E2,F2)), power\right):$

$R := `\sim`_{simplify}\left(`\sim`_{expand}\left(subs\left(\mu=-\frac{2\,I\ln(t2)}{\pi}, \lambda=-\frac{2\,I\ln(t1)}{\pi}, \frac{P\cdot R1}{R1_{8,8}}\right)\right)\right):$

$Rplus := LinearSolve(KP(Xplus,Xplus), subs\sim(t2=1,R)\cdot KP(Xplus,Xplus)) : Rplus := simplify(subs(t1=-I\cdot t1, t2=1, Rplus)) : Rminus := LinearSolve(KP(Xminus,Xminus), subs\sim(t2=-1,R)\cdot KP(Xminus,Xminus)) : Rminus := simplify(subs(t1=-I\cdot t1, t2=1, Rminus)) :$ #weight relabel since Xplus is weight I·t1,-t2

$de1 := KroneckerProduct(e1,k1) + KroneckerProduct(id,e1):$

$de2 := KroneckerProduct(e2,k2) + KroneckerProduct(id,e2) : df1 := KroneckerProduct(f1,id) + KroneckerProduct\left(\frac{1}{k1}, f1\right) : df2 := KroneckerProduct(f2,id) + KroneckerProduct\left(\frac{1}{k2}, f2\right) : dk1 := KroneckerProduct(k1,k1) : dk2 := KroneckerProduct(k2,k2) : de3 := -de1\cdot de2 - (I\,de2).de1 : df3 := -df2\cdot df1 + (I\,df1).df2 :$

$check(br(de1,df1)-fl(dk1)), check(subs(t2=1, br(de2,df2)-fl(dk2))), check(subs(t2=-1, br(de2,df2)-fl(dk2))), check(br(de1,df2)), check(br(de2,df1)), check(dk1\cdot df1+df1\cdot dk1), check(dk2\cdot df2+df2\cdot dk2), check(dk1\cdot de1+de1\cdot dk1),$
$check(dk2\cdot de2+de2\cdot dk2), check(dk1\cdot df2-I\cdot df2\cdot dk1), check(dk2\cdot df1-I\cdot df1\cdot dk2), check(dk1\cdot de2+I\cdot de2\cdot dk1), check(dk2\cdot de1+I\cdot de1\cdot dk2), check(de1^2), check(df1^2), check(de2^2), check(df2^2), check(de3^2), check(df3^2);$

Good, Good, Good, Good, Good, Good, Good, Good, Good, Good, Good, Good, Good, Good, Good, Good, Good, Good, Good **(2)**

$de21 := de2\cdot de1 : de1212 := de21^2 : dflist := [idd, df1, df2\cdot df1, df1\cdot df2\cdot df1] :$
$longdflist := [idd, df1, df2, df1\cdot df2, df2\cdot df1, df1\cdot df2\cdot df1, df2\cdot df1\cdot df2, df2\cdot df1\cdot df2\cdot df1] :$
$v1 := IdentityMatrix(16)_{..,1} : v2 := de1\cdot de2\cdot de1\cdot KP(f1\cdot f2\cdot f1, f1\cdot f2\cdot f1)_{..,1} :$
$v3 := de1\cdot KP(f1,f1)_{..,1} :$

$GammaX := Matrix(16)$: **for** j **to** 4 **do**
$GammaX_{..,j} := dflist_j \cdot v1;$
$GammaX_{..,j+4} := dflist_j \cdot v2$
end do: **for** j **to** 8 **do**
$GammaX_{..,j+8} := longdflist_j \cdot v3$
end do: $GammaX := simplify(GammaX)$: $GammaXinv := \frac{1}{GammaX}$:

$signsred := simplify\left(subs\left(t1 = -I \cdot t1, tr2\left(GammaXinv \cdot KP\left(id, \left(\frac{1}{k1^2}\right) \cdot \left(\frac{1}{k2^2}\right)\right) \cdot GammaX\right)\right)\right);$

$$signsred := \left[\left[\frac{t1^2}{(t1^4+1)\,t2^2}, 0, \frac{-\frac{I}{2}\,t1\,(t1^2-1)}{(t1^2+1)\,(t1^4+1)\,t2^2}, 0\right],\right. \tag{3}$$

$$\left[0, \frac{(-2\,t1^4+t1^2-1)\,t2^2+t1^2+1}{2\,(t1^4+1)\,(t1^2\,t2^2-1)\,t2^2}, 0, \frac{I\,(t1^2-1)}{t2^3\,(t1^4+1)\,(t1^2\,t2^2-1)\,t1}\right],$$

$$\left[\frac{-I\,(t1^6\,t2^2-2\,t1^4\,t2^2+t1^4-t1^2\,t2^2+2\,t1^2-1)}{(t1^2\,t2^2-1)\,t2^2\,t1\,(t1^8-1)}, 0, \frac{(-t1^2+1)\,t2^2+t1^2-1}{2\,(t1^4+1)\,(t1^2\,t2^2-1)\,t2^2}, 0\right],$$

$$\left.\left[0, \frac{\frac{I}{2}\,t1\,t2}{(t1^2\,t2^2-1)\,(t1^2-1)}, 0, 0\right]\right]$$

$Rplusredrel := simplify\left(subs\left(t2 = 1, tr2\left(subs(t1 = -I \cdot t1, GammaXinv).\left(Rplus - \frac{1}{Rplus} - \left(t1^2 - \frac{1}{t1^2}\right) IdentityMatrix(16)\right).subs(t1 = -I \cdot t1, GammaX)\right)\right)\right);$#adjust for using subrep of shift

$$Rplusredrel := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-t1^4+1}{t1^2} & 0 \\ 0 & 0 & 0 & \frac{-t1^4+1}{t1^2} \end{bmatrix} \tag{4}$$

$Rminusredrel := simplify\left(subs\left(t2 = -1, tr2\left(subs(t1 = -I \cdot t1, GammaXinv).\left(Rminus - \frac{1}{Rminus} - \left(t1^2 - \frac{1}{t1^2}\right) IdentityMatrix(16)\right).subs(t1 = -I \cdot t1, GammaX)\right)\right)\right);$#using '+' in skein relation

$$Rminusredrel := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-t1^4+1}{t1^2} & 0 \\ 0 & 0 & 0 & \frac{-t1^4+1}{t1^2} \end{bmatrix} \tag{5}$$

simplify(*subs*~(*t2* = 1, *signsred* • *Rplusredrel*)), *simplify*(*subs*~(*t2* =-1, *signsred* • *Rminusredrel*));

$$\begin{bmatrix} 0 & 0 & \frac{\frac{\mathrm{I}}{2}\,(t1^2-1)^2}{(t1^4+1)\,t1} & 0 \\ 0 & 0 & 0 & \frac{-\mathrm{I}\,(t1^4-1)}{(t1^4+1)\,t1^3} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & \frac{\frac{\mathrm{I}}{2}\,(t1^2-1)^2}{(t1^4+1)\,t1} & 0 \\ 0 & 0 & 0 & \frac{\mathrm{I}\,(t1^4-1)}{(t1^4+1)\,t1^3} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{6}$$

#any intertwiner multiplied with this gives zero, so it satisfies '+' Conway relation