▶ Preamble

$$R1 := simplify\left(DiagonalMatrix\left(\grave{\sim}\grave{}_{q}\left(Diagonal\left(\frac{2}{3}\, KP(H1, subs(\lambda = \Lambda, H1)) + \frac{2}{3}\, KP(H2, subs(\mu = \mathrm{M}, H2)) + \frac{1}{3}\, KP(H2, subs(\lambda = \Lambda, H1)) + \frac{1}{3}\, KP(H1, subs(\mu = \mathrm{M}, H2)) \right) \right) \right) \cdot (IdentityMatrix(64) + 2\,\mathrm{I}\, KP(E1, F1)) \cdot (IdentityMatrix(64) + 2\,\mathrm{I}\, KP(E3, F3)) \cdot (IdentityMatrix(64) + 2\,\mathrm{I}\, KP(E2, F2)), power \right) :$$

$$R := \grave{\sim}\grave{}_{simplify}\left(\grave{\sim}\grave{}_{expand}\left(subs\left(\mu = -\frac{2\,\mathrm{I}\ln(s)}{\pi}, \lambda = -\frac{2\,\mathrm{I}\ln(t)}{\pi}, \mathrm{M} = -\frac{2\,\mathrm{I}\ln(s)}{\pi}, \Lambda = -\frac{2\,\mathrm{I}\ln(t)}{\pi}, \frac{P \cdot R1}{R1_{8,8}} \right) \right) \right) :$$

$$U := ssimplify\left(R^{-1}\right) : Ri := KP(R, id) : iR := KP(id, R) : Ui := KP(U, id) : iU := KP(id, U) : iiR := KP(id, iR) : iiU := KP(id, iU) : iRi := KP(id, Ri) : iUi := KP(id, Ui) : Rii := KP(Ri, id) : Uii := KP(Ui, id) : RR := KP(R, R) : UU := KP(U, U) : h := simplify\left(K1^{-2}.K2^{-2}\right) : simplify\left(\frac{1}{8}\, Trace\left(KP(id, h).R\right) \right);$$

1 **(1)**

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sl3 := proc(A, B) local n, m, k, l, v , i :
 n, m := op(1, A);
  k, l := op(1, B);
 if m ≠ k then error "incompatible dimensions"; end if;
 v := Vector(k) :
 for i from 1 to m do
 v[i] := DotProduct(Transpose(A)[ .., i], B[ .., i], conjugate = false);
end do:
 return simplify( 1/8 add(v) );
  end:
test :=proc(poly) local a :
 return testeq(subs(s = a, t = s, a = t, poly) − poly = 0), testeq( subs( s = 1/s, t = 1/t, poly ) − poly = 0 ),
      testeq( subs(s = 1, poly) - subs( s = I/t, poly ) = 0 ), simplify(subs(s = 1, poly)) ;
 end:
#test checks symmetry between t and s, t and its inverse, and whether evaluations at s=1 and s=It^-1 are
      equal
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Invariants

$A31 := KP(id, h).R : B31 := R^2 : K31 := sl3(A31, B31); test(K31);$

$$K31 := \frac{1}{s^4 t^4}\left(\left(t^8 - t^6 + t^4\right) s^8 + \left(-t^8 + 2\,t^6 - 2\,t^4 + t^2\right) s^6 + \left(t^8 - 2\,t^6 + t^4 - 2\,t^2 + 1\right) s^4 + \left(t^6 - 2\,t^4 + 2\,t^2 - 1\right) s^2 + t^4 - t^2 + 1\right)$$

$$true, true, true, \frac{t^8 - t^4 + 1}{t^4} \tag{2.1}$$

$A41 := KP(id, KP(h, h)).spmm(iU, Ri) : B41 := spmm(iU, Ri) : K41 := sl3(A41, B41);$
$test(K41);$

$$K41 := \frac{1}{s^4 t^4}\left(\left(t^8 - 3\,t^6 + t^4\right) s^8 + \left(-3\,t^8 + 12\,t^6 - 12\,t^4 + 3\,t^2\right) s^6 + \left(t^8 - 12\,t^6 + 25\,t^4 - 12\,t^2 + 1\right) s^4 + \left(3\,t^6 - 12\,t^4 + 12\,t^2 - 3\right) s^2 + t^4 - 3\,t^2 + 1\right)$$

$$true, true, true, \frac{-t^8 + 3\,t^4 - 1}{t^4} \tag{2.2}$$

$A51 := KP(id, h).R^2 : B51 := R^3 : K51 := sort(sort(expand(sl3(A51, B51))), s), t); test(K51);$

$$K51 := s^8 t^8 - s^6 t^8 + s^4 t^8 - s^2 t^8 + t^8 - s^8 t^6 + 2\,s^6 t^6 - 2\,s^4 t^6 + 2\,s^2 t^6 + \frac{t^6}{s^2} - 2\,t^6 + s^8 t^4 - 2\,s^6 t^4 + s^4 t^4 - s^2 t^4 - \frac{2\,t^4}{s^2} + \frac{t^4}{s^4} + t^4 - s^8 t^2 + 2\,s^6 t^2 - s^4 t^2 + \frac{t^2}{s^2} - \frac{2\,t^2}{s^4} + \frac{t^2}{s^6} + \frac{s^6}{t^2} - \frac{2\,s^4}{t^2} + \frac{s^2}{t^2} - \frac{1}{s^4 t^2} + \frac{2}{s^6 t^2} - \frac{1}{s^8 t^2} + \frac{s^4}{t^4} - \frac{2\,s^2}{t^4} - \frac{1}{s^2 t^4} + \frac{1}{s^4 t^4} - \frac{2}{s^6 t^4} + \frac{1}{s^8 t^4} + \frac{1}{t^4} + \frac{s^2}{t^6} + \frac{2}{s^2 t^6} - \frac{2}{s^4 t^6} + \frac{2}{s^6 t^6} - \frac{1}{s^8 t^6} - \frac{2}{t^6} - \frac{1}{s^2 t^8} + \frac{1}{s^4 t^8} - \frac{1}{s^6 t^8} + \frac{1}{s^8 t^8} + \frac{1}{t^8} + s^8 - 2\,s^6 + s^4 + \frac{1}{s^4} - \frac{2}{s^6} + \frac{1}{s^8} + 1$$

$$true, true, true, \frac{t^{16} - t^{12} + t^8 - t^4 + 1}{t^8} \tag{2.3}$$

$A52 := KP(id, KP(h, h)).spmm(iR, Ui) : B52 := spmm\left(iR, KP\left(R^3, id\right)\right) : K52 := sl3(A52, B52);$
$test(K52);$

$$K52 := \frac{1}{s^4 t^4}\left(\left(6\,t^8 - 10\,t^6 + 6\,t^4\right) s^8 + \left(-10\,t^8 + 26\,t^6 - 26\,t^4 + 10\,t^2\right) s^6 + \left(6\,t^8 - 26\,t^6 + 37\,t^4 - 26\,t^2 + 6\right) s^4 + \left(10\,t^6 - 26\,t^4 + 26\,t^2 - 10\right) s^2 + 6\,t^4 - 10\,t^2 + 6\right)$$

$$true, true, true, \frac{2\,t^8 - 3\,t^4 + 2}{t^4} \tag{2.4}$$

#Computed Externally

$\#A61 := spmm(KP(id, KP(h, KP(h, h))), spmm(KP(id, spmm(iU, Ri)), UU)) : B61 :=$
$KP\left(spmm\left(iR, KP\left(R^2, id\right)\right), id\right) : K61 := sl3(A61, B61);$

$$K61 := 6\,s^4\,t^4 - 14\,s^4\,t^2 - 14\,s^2\,t^4 + 6\,s^4 + 46\,s^2\,t^2 + 6\,t^4 - 46\,s^2 - 46\,t^2 + \frac{14\,s^2}{t^2} + \frac{14\,t^2}{s^2} - \frac{46}{s^2} - \frac{46}{t^2} + \frac{6}{s^4} + \frac{6}{t^4} + \frac{46\,t^2}{s^2\,t^4} - \frac{14}{s^2\,t^4} - \frac{14}{s^4\,t^2} + \frac{6}{s^4\,t^4} + 85;\ test(K61);$$

$$K61 := 6\,s^4\,t^4 - 14\,s^4\,t^2 - 14\,s^2\,t^4 + 6\,s^4 + 46\,s^2\,t^2 + 6\,t^4 - 46\,s^2 - 46\,t^2 + \frac{14\,s^2}{t^2} + \frac{14\,t^2}{s^2} - \frac{46}{s^2} - \frac{46}{t^2} + \frac{6}{s^4} + \frac{6}{t^4} + \frac{46}{t^2\,s^2} - \frac{14}{s^2\,t^4} - \frac{14}{s^4\,t^2} + \frac{6}{s^4\,t^4} + 85$$

$$true,\ true,\ true,\ \frac{-2\,t^8 + 5\,t^4 - 2}{t^4} \qquad \textbf{(2.5)}$$

$$A62 := spmm(KP(id, KP(h, h)), spmm(iU, Ri)) : B62 := spmm\left(iU, KP\left(R^3, id\right)\right) : K62 := sort(sort(expand(sl3(A62, B62)), s), t);\ test(K62);$$

$$K62 := s^8\,t^8 - 3\,s^6\,t^8 + 3\,s^4\,t^8 - 3\,s^2\,t^8 + t^8 - 3\,s^8\,t^6 + 12\,s^6\,t^6 - 18\,s^4\,t^6 + 18\,s^2\,t^6 + \frac{3\,t^6}{s^2} - 12\,t^6 + 3\,s^8\,t^4 - 18\,s^6\,t^4 + 35\,s^4\,t^4 - 37\,s^2\,t^4 - \frac{18\,t^4}{s^2} + \frac{3\,t^4}{s^4} + 35\,t^4 - 3\,s^8\,t^2 + 18\,s^6\,t^2 - 37\,s^4\,t^2 + 38\,s^2\,t^2 + \frac{37\,t^2}{s^2} - \frac{18\,t^2}{s^4} + \frac{3\,t^2}{s^6} - 38\,t^2 + \frac{3\,s^6}{t^2} - \frac{18\,s^4}{t^2} + \frac{37\,s^2}{t^2} + \frac{38}{s^2\,t^2} - \frac{37}{s^4\,t^2} + \frac{18}{s^6\,t^2} - \frac{3}{s^8\,t^2} - \frac{38}{t^2} + \frac{3\,s^4}{t^4} - \frac{18\,s^2}{t^4} - \frac{37}{s^2\,t^4} + \frac{35}{s^4\,t^4} - \frac{18}{s^6\,t^4} + \frac{3}{s^8\,t^4} + \frac{35}{t^4} + \frac{3\,s^2}{t^6} + \frac{18}{s^2\,t^6} - \frac{18}{s^4\,t^6} + \frac{12}{s^6\,t^6} - \frac{3}{s^8\,t^6} - \frac{12}{t^6} - \frac{3}{s^2\,t^8} + \frac{3}{s^4\,t^8} - \frac{3}{s^6\,t^8} + \frac{1}{s^8\,t^8} + \frac{1}{t^8} + s^8 - 12\,s^6 + 35\,s^4 - 38\,s^2 - \frac{38}{s^2} + \frac{35}{s^4} - \frac{12}{s^6} + \frac{1}{s^8} + 25$$

$$true,\ true,\ true,\ \frac{-t^{16} + 3\,t^{12} - 3\,t^8 + 3\,t^4 - 1}{t^8} \qquad \textbf{(2.6)}$$

$$A63 := spmm\left(KP(id, KP(h, h)), spmm\left(KP\left(id, U^2\right), Ri\right)\right) : B63 := spmm\left(iU, KP\left(R^2, id\right)\right) : K63 := sort(sort(expand(sl3(A63, B63)), s), t);\ test(K63);$$

$$K63 := s^8\,t^8 - 3\,s^6\,t^8 + 5\,s^4\,t^8 - 3\,s^2\,t^8 + t^8 - 3\,s^8\,t^6 + 12\,s^6\,t^6 - 24\,s^4\,t^6 + 24\,s^2\,t^6 + \frac{3\,t^6}{s^2} - 12\,t^6 + 5\,s^8\,t^4 - 24\,s^6\,t^4 + 53\,s^4\,t^4 - 71\,s^2\,t^4 - \frac{24\,t^4}{s^2} + \frac{5\,t^4}{s^4} + 53\,t^4 - 3\,s^8\,t^2 + 24\,s^6\,t^2 - 71\,s^4\,t^2 + 124\,s^2\,t^2 + \frac{71\,t^2}{s^2} - \frac{24\,t^2}{s^4} + \frac{3\,t^2}{s^6} - 124\,t^2 + \frac{3\,s^6}{t^2} - \frac{24\,s^4}{t^2} + \frac{71\,s^2}{t^2} + \frac{124}{s^2\,t^2} - \frac{71}{s^4\,t^2} + \frac{24}{s^6\,t^2} - \frac{3}{s^8\,t^2} - \frac{124}{t^2} + \frac{5\,s^4}{t^4} - \frac{24\,s^2}{t^4} - \frac{71}{s^2\,t^4} + \frac{53}{s^4\,t^4} - \frac{24}{s^6\,t^4} + \frac{5}{s^8\,t^4} + \frac{53}{t^4} + \frac{3\,s^2}{t^6} + \frac{24}{s^2\,t^6} - \frac{24}{s^4\,t^6} + \frac{12}{s^6\,t^6} - \frac{3}{s^8\,t^6} - \frac{12}{t^6} - \frac{3}{s^2\,t^8} + \frac{5}{s^4\,t^8}$$

$$-\frac{3}{s^6 t^8}+\frac{1}{s^8 t^8}+\frac{1}{t^8}+s^8-12 s^6+53 s^4-124 s^2-\frac{124}{s^2}+\frac{53}{s^4}-\frac{12}{s^6}+\frac{1}{s^8}+169$$

$$true,\ true,\ true,\ \frac{t^{16}-3 t^{12}+5 t^8-3 t^4+1}{t^8} \tag{2.7}$$

$A71 := KP(id, h).R^2 : B71 := R^5 : K71 := sort(sort(expand(sl3(A71, B71))), s), t); test(K71);$

$$K71 := s^{12} t^{12}-s^{10} t^{12}+s^8 t^{12}-s^6 t^{12}+s^4 t^{12}-s^2 t^{12}+t^{12}-s^{12} t^{10}+2 s^{10} t^{10}-2 s^8 t^{10}$$
$$+2 s^6 t^{10}-2 s^4 t^{10}+2 s^2 t^{10}+\frac{t^{10}}{s^2}-2 t^{10}+s^{12} t^8-2 s^{10} t^8+s^8 t^8-s^6 t^8+s^4 t^8-s^2 t^8$$
$$-\frac{2 t^8}{s^2}+\frac{t^8}{s^4}+t^8-s^{12} t^6+2 s^{10} t^6-s^8 t^6+\frac{t^6}{s^2}-\frac{2 t^6}{s^4}+\frac{t^6}{s^6}+s^{12} t^4-2 s^{10} t^4+s^8 t^4$$
$$+s^4 t^4-s^2 t^4+\frac{t^4}{s^4}-\frac{2 t^4}{s^6}+\frac{t^4}{s^8}+t^4-s^{12} t^2+2 s^{10} t^2-s^8 t^2-s^4 t^2+2 s^2 t^2+\frac{t^2}{s^2}$$
$$+\frac{t^2}{s^6}-\frac{2 t^2}{s^8}+\frac{t^2}{s^{10}}-2 t^2+\frac{s^{10}}{t^2}-\frac{2 s^8}{t^2}+\frac{s^6}{t^2}+\frac{s^2}{t^2}+\frac{2}{s^2 t^2}-\frac{1}{s^4 t^2}-\frac{1}{s^8 t^2}$$
$$+\frac{2}{s^{10} t^2}-\frac{1}{s^{12} t^2}-\frac{2}{t^2}+\frac{s^8}{t^4}-\frac{2 s^6}{t^4}+\frac{s^4}{t^4}-\frac{1}{s^2 t^4}+\frac{1}{s^4 t^4}+\frac{1}{s^8 t^4}-\frac{2}{s^{10} t^4}$$
$$+\frac{1}{s^{12} t^4}+\frac{1}{t^4}+\frac{s^6}{t^6}-\frac{2 s^4}{t^6}+\frac{s^2}{t^6}-\frac{1}{s^8 t^6}+\frac{2}{s^{10} t^6}-\frac{1}{s^{12} t^6}+\frac{s^4}{t^8}-\frac{2 s^2}{t^8}-\frac{1}{s^2 t^8}$$
$$+\frac{1}{s^4 t^8}-\frac{1}{s^6 t^8}+\frac{1}{s^8 t^8}-\frac{2}{s^{10} t^8}+\frac{1}{s^{12} t^8}+\frac{1}{t^8}+\frac{s^2}{t^{10}}+\frac{2}{s^2 t^{10}}-\frac{2}{s^4 t^{10}}+\frac{2}{s^6 t^{10}}$$
$$-\frac{2}{s^8 t^{10}}+\frac{2}{s^{10} t^{10}}-\frac{1}{s^{12} t^{10}}-\frac{2}{t^{10}}-\frac{1}{s^2 t^{12}}+\frac{1}{s^4 t^{12}}-\frac{1}{s^6 t^{12}}+\frac{1}{s^8 t^{12}}-\frac{1}{s^{10} t^{12}}$$
$$+\frac{1}{s^{12} t^{12}}+\frac{1}{t^{12}}+s^{12}-2 s^{10}+s^8+s^4-2 s^2-\frac{2}{s^2}+\frac{1}{s^4}+\frac{1}{s^8}-\frac{2}{s^{10}}+\frac{1}{s^{12}}+1$$

$$true,\ true,\ true,\ \frac{t^{24}-t^{20}+t^{16}-t^{12}+t^8-t^4+1}{t^{12}} \tag{2.8}$$

#Computed Externally

$\#iRUiiR := ssimplify(spmm(iR, spmm(Ui, iR))) : A72 := spmm(KP(id, KP(h, KP(h, h))), KP(id, iRUiiR)) : B72 := KP(spmm(iRUiiR, KP(R^3, id)), id) : K72 := sl3(A72, B72);$

$$K72 := \frac{1}{s^4 t^4}\left(\left(13 t^8-23 t^6+13 t^4\right) s^8+\left(-23 t^8+64 t^6-64 t^4+23 t^2\right) s^6+\left(13 t^8-64 t^6\right.\right.$$
$$\left.\left.+97 t^4-64 t^2+13\right) s^4+\left(23 t^6-64 t^4+64 t^2-23\right) s^2+13 t^4-23 t^2+13\right); test(K72);$$

$$K72 := \frac{1}{s^4 t^4}\left(\left(13 t^8-23 t^6+13 t^4\right) s^8+\left(-23 t^8+64 t^6-64 t^4+23 t^2\right) s^6+\left(13 t^8\right.\right.$$
$$\left.\left.-64 t^6+97 t^4-64 t^2+13\right) s^4+\left(23 t^6-64 t^4+64 t^2-23\right) s^2+13 t^4-23 t^2+13\right)$$

$$true,\ true,\ true,\ \frac{3 t^8-5 t^4+3}{t^4} \tag{2.9}$$

$A73 := spmm(KP(id, KP(h, h)), spmm(iU, Ri)) : B73 := spmm(iU, KP(U^5, id)) : K73 :=$

$sort(sort(expand(sl3(A73, B73))), s), t); test(K73);$

$$K73 := 6 s^8 t^8 - 10 s^6 t^8 + 10 s^4 t^8 - 10 s^2 t^8 + 6 t^8 - 10 s^8 t^6 + 26 s^6 t^6 - 32 s^4 t^6 + 32 s^2 t^6 + \frac{10 t^6}{s^2} - 26 t^6 + 10 s^8 t^4 - 32 s^6 t^4 + 39 s^4 t^4 - 37 s^2 t^4 - \frac{32 t^4}{s^2} + \frac{10 t^4}{s^4} + 39 t^4 - 10 s^8 t^2 + 32 s^6 t^2 - 37 s^4 t^2 + 24 s^2 t^2 + \frac{37 t^2}{s^2} - \frac{32 t^2}{s^4} + \frac{10 t^2}{s^6} - 24 t^2 + \frac{10 s^6}{t^2} - \frac{32 s^4}{t^2} + \frac{37 s^2}{t^2} + \frac{24}{s^2 t^2} - \frac{37}{s^4 t^2} + \frac{32}{s^6 t^2} - \frac{10}{s^8 t^2} - \frac{24}{t^2} + \frac{10 s^4}{t^4} - \frac{32 s^2}{t^4} - \frac{37}{s^2 t^4} + \frac{39}{s^4 t^4} - \frac{32}{s^6 t^4} + \frac{10}{s^8 t^4} + \frac{39}{t^4} + \frac{10 s^2}{t^6} + \frac{32}{s^2 t^6} - \frac{32}{s^4 t^6} + \frac{26}{s^6 t^6} - \frac{10}{s^8 t^6} - \frac{26}{t^6} - \frac{10}{s^2 t^8} + \frac{10}{s^4 t^8} - \frac{10}{s^6 t^8} + \frac{6}{s^8 t^8} + \frac{6}{t^8} + 6 s^8 - 26 s^6 + 39 s^4 - 24 s^2 - \frac{24}{s^2} + \frac{39}{s^4} - \frac{26}{s^6} + \frac{6}{s^8} + 13$$

$$true, true, true, \frac{2 t^{16} - 3 t^{12} + 3 t^8 - 3 t^4 + 2}{t^8} \quad \textbf{(2.10)}$$

#Computed Externally

$A74 := spmm(KP(id, KP(h, KP(h, h))), KP(id, spmm(iU, spmm(Ri, spmm(iU, Ui^2))))) :$
$B74 := KP(spmm(Ri, spmm(iU, KP(U^2, id))), id) : K74 := sort(sort(expand(sl3(A74, B74))), s), t);$

$$K74 := \frac{1}{s^4 t^4}\left(\left(36 t^8 - 68 t^6 + 36 t^4\right) s^8 + \left(-68 t^8 + 196 t^6 - 196 t^4 + 68 t^2\right) s^6 + \left(36 t^8 - 196 t^6 + 313 t^4 - 196 t^2 + 36\right) s^4 + \left(68 t^6 - 196 t^4 + 196 t^2 - 68\right) s^2 + 36 t^4 - 68 t^2 + 36\right);$$

$test(K74);$

$$K74 := \frac{1}{s^4 t^4}\left(\left(36 t^8 - 68 t^6 + 36 t^4\right) s^8 + \left(-68 t^8 + 196 t^6 - 196 t^4 + 68 t^2\right) s^6 + \left(36 t^8 - 196 t^6 + 313 t^4 - 196 t^2 + 36\right) s^4 + \left(68 t^6 - 196 t^4 + 196 t^2 - 68\right) s^2 + 36 t^4 - 68 t^2 + 36\right)$$

$$true, true, true, \frac{4 t^8 - 7 t^4 + 4}{t^4} \quad \textbf{(2.11)}$$

$A75 := spmm(KP(id, KP(h, h)), spmm(KP(id, R^2), Ui)) : B75 := spmm(iR, KP(R^4, id)) :$
$K75 := sort(sort(expand(sl3(A75, B75))), s), t); test(K75);$

$$K75 := 6 s^8 t^8 - 14 s^6 t^8 + 18 s^4 t^8 - 14 s^2 t^8 + 6 t^8 - 14 s^8 t^6 + 44 s^6 t^6 - 68 s^4 t^6 + 68 s^2 t^6 + \frac{14 t^6}{s^2} - 44 t^6 + 18 s^8 t^4 - 68 s^6 t^4 + 114 s^4 t^4 - 132 s^2 t^4 - \frac{68 t^4}{s^2} + \frac{18 t^4}{s^4} + 114 t^4 - 14 s^8 t^2 + 68 s^6 t^2 - 132 s^4 t^2 + 164 s^2 t^2 + \frac{132 t^2}{s^2} - \frac{68 t^2}{s^4} + \frac{14 t^2}{s^6} - 164 t^2 + \frac{14 s^6}{t^2} - \frac{68 s^4}{t^2} + \frac{132 s^2}{t^2} + \frac{164}{s^2 t^2} - \frac{132}{s^4 t^2} + \frac{68}{s^6 t^2} - \frac{14}{s^8 t^2} - \frac{164}{t^2} + \frac{18 s^4}{t^4} - \frac{68 s^2}{t^4}$$

$$-\frac{132}{s^2 t^4}+\frac{114}{s^4 t^4}-\frac{68}{s^6 t^4}+\frac{18}{s^8 t^4}+\frac{114}{t^4}+\frac{14 s^2}{t^6}+\frac{68}{s^2 t^6}-\frac{68}{s^4 t^6}+\frac{44}{s^6 t^6}-\frac{14}{s^8 t^6}$$
$$-\frac{44}{t^6}-\frac{14}{s^2 t^8}+\frac{18}{s^4 t^8}-\frac{14}{s^6 t^8}+\frac{6}{s^8 t^8}+\frac{6}{t^8}+6 s^8-44 s^6+114 s^4-164 s^2-\frac{164}{s^2}$$
$$+\frac{114}{s^4}-\frac{44}{s^6}+\frac{6}{s^8}+181$$

$$\mathit{true}, \mathit{true}, \mathit{true}, \frac{2 t^{16}-4 t^{12}+5 t^8-4 t^4+2}{t^8} \tag{2.12}$$

#Computed Externally

#A76:=spmm(KP(id, KP(h, KP(h, h)))), KP(id, spmm(iR, Ui)))) : B76 := spmm(RR, KP(spmm(iU, KP(R^2, id))), id)) : K76 := sort(sort(expand(sl3(A76, B76))), s), t);

$$K76 := \frac{1}{s^8 t^8}\left(t^8\left(t^8-5t^6+7t^4-5t^2+1\right)s^{16}+\left(-5t^{16}+30t^{14}-60t^{12}+60t^{10}-30t^8+5t^6\right)s^{14}+\left(7t^{16}-60t^{14}+163t^{12}-215t^{10}+163t^8-60t^6+7t^4\right)s^{12}+\left(-5t^{16}+60t^{14}-215t^{12}+366t^{10}-366t^8+215t^6-60t^4+5t^2\right)s^{10}+\left(t^{16}-30t^{14}+163t^{12}-366t^{10}+457t^8-366t^6+163t^4-30t^2+1\right)s^8+\left(5t^{14}-60t^{12}+215t^{10}-366t^8+366t^6-215t^4+60t^2-5\right)s^6+\left(7t^{12}-60t^{10}+163t^8-215t^6+163t^4-60t^2+7\right)s^4+\left(5t^{10}-30t^8+60t^6-60t^4+30t^2-5\right)s^2+t^8-5t^6+7t^4-5t^2+1\right); test(K76);$$

$$K76 := \frac{1}{s^8 t^8}\left(t^8\left(t^8-5t^6+7t^4-5t^2+1\right)s^{16}+\left(-5t^{16}+30t^{14}-60t^{12}+60t^{10}-30t^8+5t^6\right)s^{14}+\left(7t^{16}-60t^{14}+163t^{12}-215t^{10}+163t^8-60t^6+7t^4\right)s^{12}+\left(-5t^{16}+60t^{14}-215t^{12}+366t^{10}-366t^8+215t^6-60t^4+5t^2\right)s^{10}+\left(t^{16}-30t^{14}+163t^{12}-366t^{10}+457t^8-366t^6+163t^4-30t^2+1\right)s^8+\left(5t^{14}-60t^{12}+215t^{10}-366t^8+366t^6-215t^4+60t^2-5\right)s^6+\left(7t^{12}-60t^{10}+163t^8-215t^6+163t^4-60t^2+7\right)s^4+\left(5t^{10}-30t^8+60t^6-60t^4+30t^2-5\right)s^2+t^8-5t^6+7t^4-5t^2+1\right)$$

$$\mathit{true}, \mathit{true}, \mathit{true}, \frac{-t^{16}+5t^{12}-7t^8+5t^4-1}{t^8} \tag{2.13}$$

#Computed Externally

#A77:=spmm(KP(id, KP(h, KP(h, h)))), spmm(iiU, spmm(iRi, iiU))) : B77 := spmm(iRi, spmm(Uii, spmm(iRi, Uii))) : K77 := sort(sort(expand(sl3(A77, B77))), s), t);

$$K77 := s^8 t^8-5s^6 t^8+9s^4 t^8-5s^2 t^8+t^8-5s^8 t^6+30s^6 t^6-70s^4 t^6+70s^2 t^6+\frac{5t^6}{s^2}-30t^6$$
$$+9s^8 t^4-70s^6 t^4+209s^4 t^4-301s^2 t^4-\frac{70t^4}{s^2}+\frac{9t^4}{s^4}+209t^4-5s^8 t^2+70s^6 t^2$$
$$-301s^4 t^2+608s^2 t^2+\frac{301t^2}{s^2}-\frac{70t^2}{s^4}+\frac{5t^2}{s^6}-608t^2+\frac{5s^6}{t^2}-\frac{70s^4}{t^2}+\frac{301s^2}{t^2}+\frac{608}{s^2 t^2}$$
$$-\frac{301}{s^4 t^2}+\frac{70}{s^6 t^2}-\frac{5}{s^8 t^2}-\frac{608}{t^2}+\frac{9s^4}{t^4}-\frac{70s^2}{t^4}-\frac{301}{s^2 t^4}+\frac{209}{s^4 t^4}-\frac{70}{s^6 t^4}+\frac{9}{s^8 t^4}+\frac{209}{t^4}$$

$$+\frac{5\,s^2}{t^6}+\frac{70}{s^2\,t^6}-\frac{70}{s^4\,t^6}+\frac{30}{s^6\,t^6}-\frac{5}{s^8\,t^6}-\frac{30}{t^6}-\frac{5}{s^2\,t^8}+\frac{9}{s^4\,t^8}-\frac{5}{s^6\,t^8}+\frac{1}{s^8\,t^8}+\frac{1}{t^8}+s^8$$
$$-30\,s^6+209\,s^4-608\,s^2-\frac{608}{s^2}+\frac{209}{s^4}-\frac{30}{s^6}+\frac{1}{s^8}+865;\ test(K77);$$

$$K77 := 865-608\,s^2-608\,t^2+\frac{1}{t^8}-\frac{30}{t^6}+\frac{1}{s^8\,t^8}+\frac{30}{s^6\,t^6}-\frac{5}{s^8\,t^6}+\frac{5\,s^6}{t^2}+\frac{5\,t^6}{s^2}+\frac{9\,s^4}{t^4}$$
$$+\frac{9\,t^4}{s^4}-\frac{70\,s^2}{t^4}-\frac{70\,t^2}{s^4}+\frac{5\,s^2}{t^6}+\frac{5\,t^2}{s^6}+\frac{9}{s^4\,t^8}+\frac{9}{s^8\,t^4}-\frac{5}{s^6\,t^8}-\frac{70}{s^6\,t^4}-\frac{301}{s^2\,t^4}$$
$$+t^8-30\,t^6+s^8-30\,s^6+\frac{70}{s^6\,t^2}-301\,s^4\,t^2-301\,s^2\,t^4+608\,s^2\,t^2+\frac{70}{s^2\,t^6}-\frac{608}{t^2}$$
$$-\frac{608}{s^2}-\frac{70}{s^4\,t^6}+\frac{209}{s^4}+\frac{209}{t^4}+\frac{608}{s^2\,t^2}-\frac{70\,s^4}{t^2}+s^8\,t^8-\frac{70\,t^4}{s^2}-\frac{301}{s^4\,t^2}+\frac{209}{s^4\,t^4}$$
$$-70\,s^4\,t^6-70\,s^6\,t^4-\frac{5}{s^2\,t^8}-\frac{5}{s^8\,t^2}+209\,s^4+\frac{301\,s^2}{t^2}+209\,t^4+\frac{301\,t^2}{s^2}+\frac{1}{s^8}-\frac{30}{s^6}$$
$$+209\,s^4\,t^4+30\,s^6\,t^6-5\,s^8\,t^2-5\,s^2\,t^8+9\,s^4\,t^8+9\,s^8\,t^4-5\,s^6\,t^8-5\,s^8\,t^6+70\,s^2\,t^6$$
$$+70\,s^6\,t^2$$

$$true,\ true,\ true,\ \frac{t^{16}-5\,t^{12}+9\,t^8-5\,t^4+1}{t^8} \qquad \textbf{(2.14)}$$

$A89 := spmm(KP(id, KP(h, h)), spmm(spmm(KP(id, U^3), Ri), iU))$: $B89 := KP(R^3, id)$:
$K89 := sort(sort(expand(sl3(A89, B89)), s), t)$; $test(K89)$;

$$K89 := s^{12}\,t^{12}-3\,s^{10}\,t^{12}+5\,s^8\,t^{12}-7\,s^6\,t^{12}+5\,s^4\,t^{12}-3\,s^2\,t^{12}+t^{12}-3\,s^{12}\,t^{10}+12\,s^{10}\,t^{10}$$
$$-24\,s^8\,t^{10}+36\,s^6\,t^{10}-36\,s^4\,t^{10}+24\,s^2\,t^{10}+\frac{3\,t^{10}}{s^2}-12\,t^{10}+5\,s^{12}\,t^8-24\,s^{10}\,t^8$$
$$+53\,s^8\,t^8-83\,s^6\,t^8+101\,s^4\,t^8-83\,s^2\,t^8-\frac{24\,t^8}{s^2}+\frac{5\,t^8}{s^4}+53\,t^8-7\,s^{12}\,t^6+36\,s^{10}\,t^6$$
$$-83\,s^8\,t^6+128\,s^6\,t^6-164\,s^4\,t^6+164\,s^2\,t^6+\frac{83\,t^6}{s^2}-\frac{36\,t^6}{s^4}+\frac{7\,t^6}{s^6}-128\,t^6+5\,s^{12}\,t^4$$
$$-36\,s^{10}\,t^4+101\,s^8\,t^4-164\,s^6\,t^4+217\,s^4\,t^4-251\,s^2\,t^4-\frac{164\,t^4}{s^2}+\frac{101\,t^4}{s^4}-\frac{36\,t^4}{s^6}$$
$$+\frac{5\,t^4}{s^8}+217\,t^4-3\,s^{12}\,t^2+24\,s^{10}\,t^2-83\,s^8\,t^2+164\,s^6\,t^2-251\,s^4\,t^2+344\,s^2\,t^2$$
$$+\frac{251\,t^2}{s^2}-\frac{164\,t^2}{s^4}+\frac{83\,t^2}{s^6}-\frac{24\,t^2}{s^8}+\frac{3\,t^2}{s^{10}}-344\,t^2+\frac{3\,s^{10}}{t^2}-\frac{24\,s^8}{t^2}+\frac{83\,s^6}{t^2}$$
$$-\frac{164\,s^4}{t^2}+\frac{251\,s^2}{t^2}+\frac{344}{s^2\,t^2}-\frac{251}{s^4\,t^2}+\frac{164}{s^6\,t^2}-\frac{83}{s^8\,t^2}+\frac{24}{s^{10}\,t^2}-\frac{3}{s^{12}\,t^2}-\frac{344}{t^2}$$
$$+\frac{5\,s^8}{t^4}-\frac{36\,s^6}{t^4}+\frac{101\,s^4}{t^4}-\frac{164\,s^2}{t^4}-\frac{251}{s^2\,t^4}+\frac{217}{s^4\,t^4}-\frac{164}{s^6\,t^4}+\frac{101}{s^8\,t^4}-\frac{36}{s^{10}\,t^4}$$

$$+\frac{5}{s^{12}\,t^4}+\frac{217}{t^4}+\frac{7\,s^6}{t^6}-\frac{36\,s^4}{t^6}+\frac{83\,s^2}{t^6}+\frac{164}{s^2\,t^6}-\frac{164}{s^4\,t^6}+\frac{128}{s^6\,t^6}-\frac{83}{s^8\,t^6}+\frac{36}{s^{10}\,t^6}$$
$$-\frac{7}{s^{12}\,t^6}-\frac{128}{t^6}+\frac{5\,s^4}{t^8}-\frac{24\,s^2}{t^8}-\frac{83}{s^2\,t^8}+\frac{101}{s^4\,t^8}-\frac{83}{s^6\,t^8}+\frac{53}{s^8\,t^8}-\frac{24}{s^{10}\,t^8}+\frac{5}{s^{12}\,t^8}$$
$$+\frac{53}{t^8}+\frac{3\,s^2}{t^{10}}+\frac{24}{s^2\,t^{10}}-\frac{36}{s^4\,t^{10}}+\frac{36}{s^6\,t^{10}}-\frac{24}{s^8\,t^{10}}+\frac{12}{s^{10}\,t^{10}}-\frac{3}{s^{12}\,t^{10}}-\frac{12}{t^{10}}$$
$$-\frac{3}{s^2\,t^{12}}+\frac{5}{s^4\,t^{12}}-\frac{7}{s^6\,t^{12}}+\frac{5}{s^8\,t^{12}}-\frac{3}{s^{10}\,t^{12}}+\frac{1}{s^{12}\,t^{12}}+\frac{1}{t^{12}}+s^{12}-12\,s^{10}+53\,s^8$$
$$-128\,s^6+217\,s^4-344\,s^2-\frac{344}{s^2}+\frac{217}{s^4}-\frac{128}{s^6}+\frac{53}{s^8}-\frac{12}{s^{10}}+\frac{1}{s^{12}}+433$$

$$\mathit{true},\ \mathit{true},\ \mathit{true},\ \frac{-t^{24}+3\,t^{20}-5\,t^{16}+7\,t^{12}-5\,t^8+3\,t^4-1}{t^{12}} \qquad \textbf{(2.15)}$$

$A817 := spmm(KP(id, KP(h, h)), spmm(KP(id, U^2), spmm(KP(R, id), KP(id, U))))$:
$B817 := spmm(spmm(KP(R, id), KP(id, U)), KP(R^2, id))$: $K817 := sl3(A817, B817)$;
$test(K817)$;

$$K817 := \frac{1}{s^{12}\,t^{12}}\left(\left(t^{24}-4\,t^{22}+8\,t^{20}-11\,t^{18}+8\,t^{16}-4\,t^{14}+t^{12}\right)s^{24}+\left(-4\,t^{24}+20\,t^{22}\right.\right.$$
$$-48\,t^{20}+76\,t^{18}-76\,t^{16}+48\,t^{14}-20\,t^{12}+4\,t^{10}\Big)\,s^{22}+\left(8\,t^{24}-48\,t^{22}+132\,t^{20}\right.$$
$$-236\,t^{18}+292\,t^{16}-236\,t^{14}+132\,t^{12}-48\,t^{10}+8\,t^8\Big)\,s^{20}+\left(-11\,t^{24}+76\,t^{22}-236\,t^{20}\right.$$
$$+468\,t^{18}-672\,t^{16}+672\,t^{14}-468\,t^{12}+236\,t^{10}-76\,t^8+11\,t^6\Big)\,s^{18}+\left(8\,t^{24}-76\,t^{22}\right.$$
$$+292\,t^{20}-672\,t^{18}+1108\,t^{16}-1328\,t^{14}+1108\,t^{12}-672\,t^{10}+292\,t^8-76\,t^6+8\,t^4\Big)$$
$$s^{16}+\left(-4\,t^{24}+48\,t^{22}-236\,t^{20}+672\,t^{18}-1328\,t^{16}+1924\,t^{14}-1924\,t^{12}+1328\,t^{10}\right.$$
$$-672\,t^8+236\,t^6-48\,t^4+4\,t^2\Big)\,s^{14}+\left(t^{24}-20\,t^{22}+132\,t^{20}-468\,t^{18}+1108\,t^{16}\right.$$
$$-1924\,t^{14}+2353\,t^{12}-1924\,t^{10}+1108\,t^8-468\,t^6+132\,t^4-20\,t^2+1\Big)\,s^{12}+\left(4\,t^{22}\right.$$
$$-48\,t^{20}+236\,t^{18}-672\,t^{16}+1328\,t^{14}-1924\,t^{12}+1924\,t^{10}-1328\,t^8+672\,t^6-236\,t^4$$
$$+48\,t^2-4\Big)\,s^{10}+\left(8\,t^{20}-76\,t^{18}+292\,t^{16}-672\,t^{14}+1108\,t^{12}-1328\,t^{10}+1108\,t^8\right.$$
$$-672\,t^6+292\,t^4-76\,t^2+8\Big)\,s^8+\left(11\,t^{18}-76\,t^{16}+236\,t^{14}-468\,t^{12}+672\,t^{10}\right.$$
$$-672\,t^8+468\,t^6-236\,t^4+76\,t^2-11\Big)\,s^6+\left(8\,t^{16}-48\,t^{14}+132\,t^{12}-236\,t^{10}+292\,t^8\right.$$
$$-236\,t^6+132\,t^4-48\,t^2+8\Big)\,s^4+\left(4\,t^{14}-20\,t^{12}+48\,t^{10}-76\,t^8+76\,t^6-48\,t^4\right.$$
$$\left.+20\,t^2-4\Big)\,s^2+t^{12}-4\,t^{10}+8\,t^8-11\,t^6+8\,t^4-4\,t^2+1\right)$$

$$\mathit{true},\ \mathit{true},\ \mathit{true},\ \frac{-t^{24}+4\,t^{20}-8\,t^{16}+11\,t^{12}-8\,t^8+4\,t^4-1}{t^{12}} \qquad \textbf{(2.16)}$$

#Computed Externally
#A :=spmm(KP(id, spmm(KP(h, KP(h, h)), spmm(KP(id, U), KP(R, id)))), KP(U, U)) : *B* :=
$KP(spmm(KP(id, R), spmm(KP(U, id), spmm(KP(id, R), KP(U^2, id)))), id)$:
#K932 :=sl3(A, B);

$$K932 := \frac{1}{s^{12}\,t^{12}}\left(t^{12}\left(t^{12}-6\,t^{10}+14\,t^8-17\,t^6+14\,t^4-6\,t^2+1\right)s^{24}+\left(-6\,t^{24}+42\,t^{22}\right.\right.$$

$$-120\,t^{20}+186\,t^{18}-186\,t^{16}+120\,t^{14}-42\,t^{12}+6\,t^{10})\,s^{22}+(14\,t^{24}-120\,t^{22}+420\,t^{20}-802\,t^{18}+970\,t^{16}-802\,t^{14}+420\,t^{12}-120\,t^{10}+14\,t^{8})\,s^{20}+(-17\,t^{24}+186\,t^{22}-802\,t^{20}+1870\,t^{18}-2718\,t^{16}+2718\,t^{14}-1870\,t^{12}+802\,t^{10}-186\,t^{8}+17\,t^{6})\,s^{18}+(14\,t^{24}-186\,t^{22}+970\,t^{20}-2718\,t^{18}+4650\,t^{16}-5446\,t^{14}+4650\,t^{12}-2718\,t^{10}+970\,t^{8}-186\,t^{6}+14\,t^{4})\,s^{16}+(-6\,t^{24}+120\,t^{22}-802\,t^{20}+2718\,t^{18}-5446\,t^{16}+7296\,t^{14}-7296\,t^{12}+5446\,t^{10}-2718\,t^{8}+802\,t^{6}-120\,t^{4}+6\,t^{2})\,s^{14}+(t^{24}-42\,t^{22}+420\,t^{20}-1870\,t^{18}+4650\,t^{16}-7296\,t^{14}+8257\,t^{12}-7296\,t^{10}+4650\,t^{8}-1870\,t^{6}+420\,t^{4}-42\,t^{2}+1)\,s^{12}+(6\,t^{22}-120\,t^{20}+802\,t^{18}-2718\,t^{16}+5446\,t^{14}-7296\,t^{12}+7296\,t^{10}-5446\,t^{8}+2718\,t^{6}-802\,t^{4}+120\,t^{2}-6)\,s^{10}+(14\,t^{20}-186\,t^{18}+970\,t^{16}-2718\,t^{14}+4650\,t^{12}-5446\,t^{10}+4650\,t^{8}-2718\,t^{6}+970\,t^{4}-186\,t^{2}+14)\,s^{8}+(17\,t^{18}-186\,t^{16}+802\,t^{14}-1870\,t^{12}+2718\,t^{10}-2718\,t^{8}+1870\,t^{6}-802\,t^{4}+186\,t^{2}-17)\,s^{6}+(14\,t^{16}-120\,t^{14}+420\,t^{12}-802\,t^{10}+970\,t^{8}-802\,t^{6}+420\,t^{4}-120\,t^{2}+14)\,s^{4}+(6\,t^{14}-42\,t^{12}+120\,t^{10}-186\,t^{8}+186\,t^{6}-120\,t^{4}+42\,t^{2}-6)\,s^{2}+t^{12}-6\,t^{10}+14\,t^{8}-17\,t^{6}+14\,t^{4}-6\,t^{2}+1);\ test(K932);$$

$$K932 := \frac{1}{s^{12}\,t^{12}}(t^{12}(t^{12}-6\,t^{10}+14\,t^{8}-17\,t^{6}+14\,t^{4}-6\,t^{2}+1)\,s^{24}+(-6\,t^{24}+42\,t^{22}-120\,t^{20}+186\,t^{18}-186\,t^{16}+120\,t^{14}-42\,t^{12}+6\,t^{10})\,s^{22}+(14\,t^{24}-120\,t^{22}+420\,t^{20}-802\,t^{18}+970\,t^{16}-802\,t^{14}+420\,t^{12}-120\,t^{10}+14\,t^{8})\,s^{20}+(-17\,t^{24}+186\,t^{22}-802\,t^{20}+1870\,t^{18}-2718\,t^{16}+2718\,t^{14}-1870\,t^{12}+802\,t^{10}-186\,t^{8}+17\,t^{6})\,s^{18}+(14\,t^{24}-186\,t^{22}+970\,t^{20}-2718\,t^{18}+4650\,t^{16}-5446\,t^{14}+4650\,t^{12}-2718\,t^{10}+970\,t^{8}-186\,t^{6}+14\,t^{4})\,s^{16}+(-6\,t^{24}+120\,t^{22}-802\,t^{20}+2718\,t^{18}-5446\,t^{16}+7296\,t^{14}-7296\,t^{12}+5446\,t^{10}-2718\,t^{8}+802\,t^{6}-120\,t^{4}+6\,t^{2})\,s^{14}+(t^{24}-42\,t^{22}+420\,t^{20}-1870\,t^{18}+4650\,t^{16}-7296\,t^{14}+8257\,t^{12}-7296\,t^{10}+4650\,t^{8}-1870\,t^{6}+420\,t^{4}-42\,t^{2}+1)\,s^{12}+(6\,t^{22}-120\,t^{20}+802\,t^{18}-2718\,t^{16}+5446\,t^{14}-7296\,t^{12}+7296\,t^{10}-5446\,t^{8}+2718\,t^{6}-802\,t^{4}+120\,t^{2}-6)\,s^{10}+(14\,t^{20}-186\,t^{18}+970\,t^{16}-2718\,t^{14}+4650\,t^{12}-5446\,t^{10}+4650\,t^{8}-2718\,t^{6}+970\,t^{4}-186\,t^{2}+14)\,s^{8}+(17\,t^{18}-186\,t^{16}+802\,t^{14}-1870\,t^{12}+2718\,t^{10}-2718\,t^{8}+1870\,t^{6}-802\,t^{4}+186\,t^{2}-17)\,s^{6}+(14\,t^{16}-120\,t^{14}+420\,t^{12}-802\,t^{10}+970\,t^{8}-802\,t^{6}+420\,t^{4}-120\,t^{2}+14)\,s^{4}+(6\,t^{14}-42\,t^{12}+120\,t^{10}-186\,t^{8}+186\,t^{6}-120\,t^{4}+42\,t^{2}-6)\,s^{2}+t^{12}-6\,t^{10}+14\,t^{8}-17\,t^{6}+14\,t^{4}-6\,t^{2}+1)$$

$$true,\ true,\ true,\ \frac{t^{24}-6\,t^{20}+14\,t^{16}-17\,t^{12}+14\,t^{8}-6\,t^{4}+1}{t^{12}} \qquad \textbf{(2.17)}$$

#Computed Externally

#A:=spmm(KP(id, spmm(KP(h, KP(h, h))), spmm(KP(id, R), KP(U, id)))), KP(R, R)) : B :=
KP(spmm(KP(id, U^2), spmm(KP(R, id), spmm(KP(id, U), KP(R, id)))), id) :

#K933:=sl3(A, B);

$$K933 := \frac{1}{s^{12}\,t^{12}}(t^{12}(t^{12}-6\,t^{10}+14\,t^{8}-19\,t^{6}+14\,t^{4}-6\,t^{2}+1)\,s^{24}+(-6\,t^{24}+42\,t^{22}-120\,t^{20}+198\,t^{18}-198\,t^{16}+120\,t^{14}-42\,t^{12}+6\,t^{10})\,s^{22}+(14\,t^{24}-120\,t^{22}+420\,t^{20}-834\,t^{18}+1046\,t^{16}-834\,t^{14}+420\,t^{12}-120\,t^{10}+14\,t^{8})\,s^{20}+(-19\,t^{24}+198\,t^{22}-834\,t^{20}+1972\,t^{18}-2992\,t^{16}+2992\,t^{14}-1972\,t^{12}+834\,t^{10}-198\,t^{8}+19\,t^{6})\,s^{18}+(14\,t^{24}-198\,t^{22}$$

$$+1046\,t^{20}-2992\,t^{18}+5414\,t^{16}-6582\,t^{14}+5414\,t^{12}-2992\,t^{10}+1046\,t^{8}-198\,t^{6}+14\,t^{4})$$
$$s^{16}+(-6\,t^{24}+120\,t^{22}-834\,t^{20}+2992\,t^{18}-6582\,t^{16}+9680\,t^{14}-9680\,t^{12}+6582\,t^{10}$$
$$-2992\,t^{8}+834\,t^{6}-120\,t^{4}+6\,t^{2})\,s^{14}+(t^{24}-42\,t^{22}+420\,t^{20}-1972\,t^{18}+5414\,t^{16}$$
$$-9680\,t^{14}+11737\,t^{12}-9680\,t^{10}+5414\,t^{8}-1972\,t^{6}+420\,t^{4}-42\,t^{2}+1)\,s^{12}+(6\,t^{22}$$
$$-120\,t^{20}+834\,t^{18}-2992\,t^{16}+6582\,t^{14}-9680\,t^{12}+9680\,t^{10}-6582\,t^{8}+2992\,t^{6}-834\,t^{4}$$
$$+120\,t^{2}-6)\,s^{10}+(14\,t^{20}-198\,t^{18}+1046\,t^{16}-2992\,t^{14}+5414\,t^{12}-6582\,t^{10}+5414\,t^{8}$$
$$-2992\,t^{6}+1046\,t^{4}-198\,t^{2}+14)\,s^{8}+(19\,t^{18}-198\,t^{16}+834\,t^{14}-1972\,t^{12}+2992\,t^{10}$$
$$-2992\,t^{8}+1972\,t^{6}-834\,t^{4}+198\,t^{2}-19)\,s^{6}+(14\,t^{16}-120\,t^{14}+420\,t^{12}-834\,t^{10}$$
$$+1046\,t^{8}-834\,t^{6}+420\,t^{4}-120\,t^{2}+14)\,s^{4}+(6\,t^{14}-42\,t^{12}+120\,t^{10}-198\,t^{8}+198\,t^{6}$$
$$-120\,t^{4}+42\,t^{2}-6)\,s^{2}+t^{12}-6\,t^{10}+14\,t^{8}-19\,t^{6}+14\,t^{4}-6\,t^{2}+1\;;\;test(K933);$$

$$K933:=\frac{1}{s^{12}\,t^{12}}(t^{12}\,(t^{12}-6\,t^{10}+14\,t^{8}-19\,t^{6}+14\,t^{4}-6\,t^{2}+1)\,s^{24}+(-6\,t^{24}+42\,t^{22}$$
$$-120\,t^{20}+198\,t^{18}-198\,t^{16}+120\,t^{14}-42\,t^{12}+6\,t^{10})\,s^{22}+(14\,t^{24}-120\,t^{22}$$
$$+420\,t^{20}-834\,t^{18}+1046\,t^{16}-834\,t^{14}+420\,t^{12}-120\,t^{10}+14\,t^{8})\,s^{20}+(-19\,t^{24}$$
$$+198\,t^{22}-834\,t^{20}+1972\,t^{18}-2992\,t^{16}+2992\,t^{14}-1972\,t^{12}+834\,t^{10}-198\,t^{8}$$
$$+19\,t^{6})\,s^{18}+(14\,t^{24}-198\,t^{22}+1046\,t^{20}-2992\,t^{18}+5414\,t^{16}-6582\,t^{14}+5414\,t^{12}$$
$$-2992\,t^{10}+1046\,t^{8}-198\,t^{6}+14\,t^{4})\,s^{16}+(-6\,t^{24}+120\,t^{22}-834\,t^{20}+2992\,t^{18}$$
$$-6582\,t^{16}+9680\,t^{14}-9680\,t^{12}+6582\,t^{10}-2992\,t^{8}+834\,t^{6}-120\,t^{4}+6\,t^{2})\,s^{14}$$
$$+(t^{24}-42\,t^{22}+420\,t^{20}-1972\,t^{18}+5414\,t^{16}-9680\,t^{14}+11737\,t^{12}-9680\,t^{10}$$
$$+5414\,t^{8}-1972\,t^{6}+420\,t^{4}-42\,t^{2}+1)\,s^{12}+(6\,t^{22}-120\,t^{20}+834\,t^{18}-2992\,t^{16}$$
$$+6582\,t^{14}-9680\,t^{12}+9680\,t^{10}-6582\,t^{8}+2992\,t^{6}-834\,t^{4}+120\,t^{2}-6)\,s^{10}$$
$$+(14\,t^{20}-198\,t^{18}+1046\,t^{16}-2992\,t^{14}+5414\,t^{12}-6582\,t^{10}+5414\,t^{8}-2992\,t^{6}$$
$$+1046\,t^{4}-198\,t^{2}+14)\,s^{8}+(19\,t^{18}-198\,t^{16}+834\,t^{14}-1972\,t^{12}+2992\,t^{10}$$
$$-2992\,t^{8}+1972\,t^{6}-834\,t^{4}+198\,t^{2}-19)\,s^{6}+(14\,t^{16}-120\,t^{14}+420\,t^{12}-834\,t^{10}$$
$$+1046\,t^{8}-834\,t^{6}+420\,t^{4}-120\,t^{2}+14)\,s^{4}+(6\,t^{14}-42\,t^{12}+120\,t^{10}-198\,t^{8}$$
$$+198\,t^{6}-120\,t^{4}+42\,t^{2}-6)\,s^{2}+t^{12}-6\,t^{10}+14\,t^{8}-19\,t^{6}+14\,t^{4}-6\,t^{2}+1)$$
$$true,\,true,\,true,\,\frac{-t^{24}+6\,t^{20}-14\,t^{16}+19\,t^{12}-14\,t^{8}+6\,t^{4}-1}{t^{12}} \qquad \textbf{(2.18)}$$

#Computed Externally
#iRRi:=ssimplify(spmm(iR, Ri)) : RiiRUi := ssimplify(spmm(spmm(Ri, iR), Ui)) : RiiURi := ssimplify(spmm(Ri, spmm(iU, Ri))) :
#A946:=spmm(KP(id, KP(h, KP(h, h))), spmm(KP(id, iRRi), KP(Ui, id))) : B946 := spmm(KP(id, RiiRUi), KP(RiiURi, id)) : K946 := sl3(A946, B946);

$$K946:=6\,s^{4}\,t^{4}-14\,s^{4}\,t^{2}-14\,s^{2}\,t^{4}+6\,s^{4}+46\,s^{2}\,t^{2}+6\,t^{4}-46\,s^{2}-46\,t^{2}+\frac{14\,s^{2}}{t^{2}}+\frac{14\,t^{2}}{s^{2}}-\frac{46}{s^{2}}$$
$$-\frac{46}{t^{2}}+\frac{6}{s^{4}}+\frac{6}{t^{4}}+\frac{46}{s^{2}\,t^{2}}-\frac{14}{s^{2}\,t^{4}}-\frac{14}{s^{4}\,t^{2}}+\frac{6}{s^{4}\,t^{4}}+85;\;test(K946);$$

$$K946:=6\,s^{4}\,t^{4}-14\,s^{4}\,t^{2}-14\,s^{2}\,t^{4}+6\,s^{4}+46\,s^{2}\,t^{2}+6\,t^{4}-46\,s^{2}-46\,t^{2}+\frac{14\,s^{2}}{t^{2}}+\frac{14\,t^{2}}{s^{2}}$$

$$-\frac{46}{s^2}-\frac{46}{t^2}+\frac{6}{s^4}+\frac{6}{t^4}+\frac{46}{s^2\,t^2}-\frac{14}{s^2\,t^4}-\frac{14}{s^4\,t^2}+\frac{6}{s^4\,t^4}+85$$

$$true,\ true,\ true,\ \frac{-2\,t^8+5\,t^4-2}{t^4} \tag{2.19}$$

#Computed Externally

#UiiRRi:=ssimplify(spmm(spmm(Ui, iR), Ri)) : R2iiRU3i := ssimplify(spmm(spmm(KP(R^2, id), iR), KP(U^3, id))) :

#A10132:=spmm(KP(id, KP(h, KP(h, h))), KP(id, spmm(KP(id, R^2), UiiRRi))) : B10132 := KP(R2iiRU3i, id) : K10132 := sort(sort(expand(sl3(A10132, B10132)), s), t);

$$K10132 := \frac{1}{s^8\,t^8}\left(t^8\left(t^8-t^6+t^4-t^2+1\right)s^{16}+\left(-t^{16}+2\,t^{14}-4\,t^{12}+4\,t^{10}-2\,t^8+t^6\right)s^{14}\right.$$
$$+t^4\left(t^{12}-4\,t^{10}+9\,t^8-13\,t^6+9\,t^4-4\,t^2+1\right)s^{12}+\left(-t^{16}+4\,t^{14}-13\,t^{12}+32\,t^{10}-32\,t^8\right.$$
$$\left.+13\,t^6-4\,t^4+t^2\right)s^{10}+\left(t^{16}-2\,t^{14}+9\,t^{12}-32\,t^{10}+49\,t^8-32\,t^6+9\,t^4-2\,t^2+1\right)s^8$$
$$+\left(t^{14}-4\,t^{12}+13\,t^{10}-32\,t^8+32\,t^6-13\,t^4+4\,t^2-1\right)s^6+\left(t^{12}-4\,t^{10}+9\,t^8-13\,t^6+9\,t^4\right.$$
$$\left.\left.-4\,t^2+1\right)s^4+\left(t^{10}-2\,t^8+4\,t^6-4\,t^4+2\,t^2-1\right)s^2+t^8-t^6+t^4-t^2+1\right);\ test(K10132);$$

$$K10132 := \frac{1}{s^8\,t^8}\left(t^8\left(t^8-t^6+t^4-t^2+1\right)s^{16}+\left(-t^{16}+2\,t^{14}-4\,t^{12}+4\,t^{10}-2\,t^8+t^6\right)s^{14}\right.$$
$$+t^4\left(t^{12}-4\,t^{10}+9\,t^8-13\,t^6+9\,t^4-4\,t^2+1\right)s^{12}+\left(-t^{16}+4\,t^{14}-13\,t^{12}+32\,t^{10}\right.$$
$$\left.-32\,t^8+13\,t^6-4\,t^4+t^2\right)s^{10}+\left(t^{16}-2\,t^{14}+9\,t^{12}-32\,t^{10}+49\,t^8-32\,t^6+9\,t^4-2\,t^2\right.$$
$$\left.+1\right)s^8+\left(t^{14}-4\,t^{12}+13\,t^{10}-32\,t^8+32\,t^6-13\,t^4+4\,t^2-1\right)s^6+\left(t^{12}-4\,t^{10}+9\,t^8\right.$$
$$\left.\left.-13\,t^6+9\,t^4-4\,t^2+1\right)s^4+\left(t^{10}-2\,t^8+4\,t^6-4\,t^4+2\,t^2-1\right)s^2+t^8-t^6+t^4-t^2\right.$$
$$\left.+1\right)$$

$$true,\ true,\ true,\ \frac{t^{16}-t^{12}+t^8-t^4+1}{t^8} \tag{2.20}$$

$$A10155 := spmm(KP(id, KP(h, h)), spmm(KP(U^3, id), spmm(KP(id, U), KP(R^2, id)))) :$$
$$B10155 := spmm(KP(id, U), spmm(KP(R^2, id), KP(id, U))) : K10155 := sort(sort(expand(sl3(A10155, B10155)), s), t); test(K10155);$$

$$K10155 := s^{12}\,t^{12}-3\,s^{10}\,t^{12}+5\,s^8\,t^{12}-7\,s^6\,t^{12}+5\,s^4\,t^{12}-3\,s^2\,t^{12}+t^{12}-3\,s^{12}\,t^{10}$$
$$+12\,s^{10}\,t^{10}-24\,s^8\,t^{10}+36\,s^6\,t^{10}-36\,s^4\,t^{10}+24\,s^2\,t^{10}+\frac{3\,t^{10}}{s^2}-12\,t^{10}+5\,s^{12}\,t^8$$
$$-24\,s^{10}\,t^8+53\,s^8\,t^8-79\,s^6\,t^8+93\,s^4\,t^8-79\,s^2\,t^8-\frac{24\,t^8}{s^2}+\frac{5\,t^8}{s^4}+53\,t^8-7\,s^{12}\,t^6$$
$$+36\,s^{10}\,t^6-79\,s^8\,t^6+108\,s^6\,t^6-124\,s^4\,t^6+124\,s^2\,t^6+\frac{79\,t^6}{s^2}-\frac{36\,t^6}{s^4}+\frac{7\,t^6}{s^6}-108\,t^6$$
$$+5\,s^{12}\,t^4-36\,s^{10}\,t^4+93\,s^8\,t^4-124\,s^6\,t^4+105\,s^4\,t^4-91\,s^2\,t^4-\frac{124\,t^4}{s^2}+\frac{93\,t^4}{s^4}$$
$$-\frac{36\,t^4}{s^6}+\frac{5\,t^4}{s^8}+105\,t^4-3\,s^{12}\,t^2+24\,s^{10}\,t^2-79\,s^8\,t^2+124\,s^6\,t^2-91\,s^4\,t^2+24\,s^2\,t^2$$

$$+\frac{91\,t^2}{s^2}-\frac{124\,t^2}{s^4}+\frac{79\,t^2}{s^6}-\frac{24\,t^2}{s^8}+\frac{3\,t^2}{s^{10}}-24\,t^2+\frac{3\,s^{10}}{t^2}-\frac{24\,s^8}{t^2}+\frac{79\,s^6}{t^2}$$
$$-\frac{124\,s^4}{t^2}+\frac{91\,s^2}{t^2}+\frac{24}{s^2\,t^2}-\frac{91}{s^4\,t^2}+\frac{124}{s^6\,t^2}-\frac{79}{s^8\,t^2}+\frac{24}{s^{10}\,t^2}-\frac{3}{s^{12}\,t^2}-\frac{24}{t^2}+\frac{5\,s^8}{t^4}$$
$$-\frac{36\,s^6}{t^4}+\frac{93\,s^4}{t^4}-\frac{124\,s^2}{t^4}-\frac{91}{s^2\,t^4}+\frac{105}{s^4\,t^4}-\frac{124}{s^6\,t^4}+\frac{93}{s^8\,t^4}-\frac{36}{s^{10}\,t^4}+\frac{5}{s^{12}\,t^4}$$
$$+\frac{105}{t^4}+\frac{7\,s^6}{t^6}-\frac{36\,s^4}{t^6}+\frac{79\,s^2}{t^6}+\frac{124}{s^2\,t^6}-\frac{124}{s^4\,t^6}+\frac{108}{s^6\,t^6}-\frac{79}{s^8\,t^6}+\frac{36}{s^{10}\,t^6}-\frac{7}{s^{12}\,t^6}$$
$$-\frac{108}{t^6}+\frac{5\,s^4}{t^8}-\frac{24\,s^2}{t^8}-\frac{79}{s^2\,t^8}+\frac{93}{s^4\,t^8}-\frac{79}{s^6\,t^8}+\frac{53}{s^8\,t^8}-\frac{24}{s^{10}\,t^8}+\frac{5}{s^{12}\,t^8}+\frac{53}{t^8}$$
$$+\frac{3\,s^2}{t^{10}}+\frac{24}{s^2\,t^{10}}-\frac{36}{s^4\,t^{10}}+\frac{36}{s^6\,t^{10}}-\frac{24}{s^8\,t^{10}}+\frac{12}{s^{10}\,t^{10}}-\frac{3}{s^{12}\,t^{10}}-\frac{12}{t^{10}}-\frac{3}{s^2\,t^{12}}$$
$$+\frac{5}{s^4\,t^{12}}-\frac{7}{s^6\,t^{12}}+\frac{5}{s^8\,t^{12}}-\frac{3}{s^{10}\,t^{12}}+\frac{1}{s^{12}\,t^{12}}+\frac{1}{t^{12}}+s^{12}-12\,s^{10}+53\,s^8-108\,s^6$$
$$+105\,s^4-24\,s^2-\frac{24}{s^2}+\frac{105}{s^4}-\frac{108}{s^6}+\frac{53}{s^8}-\frac{12}{s^{10}}+\frac{1}{s^{12}}-23$$

$$true,\ true,\ true,\ \frac{-t^{24}+3\,t^{20}-5\,t^{16}+7\,t^{12}-5\,t^8+3\,t^4-1}{t^{12}} \qquad \textbf{(2.21)}$$

#Computed Externally
#UiiRUi:=ssimplify(spmm(Ui, spmm(iR, Ui))):
#A1134:=spmm(spmm(KP(id, KP(h, KP(h, h)))), KP(U^2, R^2)), KP(id, KP(R^2, id))):
B1134 := spmm(KP(U, R), KP(id, UiiRUi)) : K1134 := sort(sort(expand(sl3(A1134, B1134)), s), t);

$$K1134 := 2\,s^8\,t^{12}-4\,s^6\,t^{12}+2\,s^4\,t^{12}-4\,s^{10}\,t^{10}+4\,s^8\,t^{10}+8\,s^6\,t^{10}-8\,s^4\,t^{10}-4\,s^2\,t^{10}+4\,t^{10}$$
$$+2\,s^{12}\,t^8+4\,s^{10}\,t^8-20\,s^8\,t^8+8\,s^6\,t^8+12\,s^4\,t^8+8\,s^2\,t^8+\frac{4\,t^8}{s^2}+\frac{2\,t^8}{s^4}-20\,t^8-4\,s^{12}\,t^6$$
$$+8\,s^{10}\,t^6+8\,s^8\,t^6-4\,s^6\,t^6-46\,s^4\,t^6+46\,s^2\,t^6-\frac{8\,t^6}{s^2}-\frac{8\,t^6}{s^4}+\frac{4\,t^6}{s^6}+4\,t^6+2\,s^{12}\,t^4$$
$$-8\,s^{10}\,t^4+12\,s^8\,t^4-46\,s^6\,t^4+164\,s^4\,t^4-248\,s^2\,t^4-\frac{46\,t^4}{s^2}+\frac{12\,t^4}{s^4}-\frac{8\,t^4}{s^6}+\frac{2\,t^4}{s^8}+164\,t^4$$
$$-4\,s^{10}\,t^2+8\,s^8\,t^2+46\,s^6\,t^2-248\,s^4\,t^2+476\,s^2\,t^2+\frac{248\,t^2}{s^2}-\frac{46\,t^2}{s^4}-\frac{8\,t^2}{s^6}+\frac{4\,t^2}{s^8}-476\,t^2$$
$$+\frac{4\,s^8}{t^2}-\frac{8\,s^6}{t^2}-\frac{46\,s^4}{t^2}+\frac{248\,s^2}{t^2}+\frac{476}{s^2\,t^2}-\frac{248}{s^4\,t^2}+\frac{46}{s^6\,t^2}+\frac{8}{s^8\,t^2}-\frac{4}{s^{10}\,t^2}-\frac{476}{t^2}$$
$$+\frac{2\,s^8}{t^4}-\frac{8\,s^6}{t^4}+\frac{12\,s^4}{t^4}-\frac{46\,s^2}{t^4}-\frac{248}{s^2\,t^4}+\frac{164}{s^4\,t^4}-\frac{46}{s^6\,t^4}+\frac{12}{s^8\,t^4}-\frac{8}{s^{10}\,t^4}+\frac{2}{s^{12}\,t^4}$$
$$+\frac{164}{t^4}+\frac{4\,s^6}{t^6}-\frac{8\,s^4}{t^6}-\frac{8\,s^2}{t^6}+\frac{46}{s^2\,t^6}-\frac{46}{s^4\,t^6}-\frac{4}{s^6\,t^6}+\frac{8}{s^8\,t^6}+\frac{8}{s^{10}\,t^6}-\frac{4}{s^{12}\,t^6}+\frac{4}{t^6}$$

$$+\frac{2\,s^4}{t^8}+\frac{4\,s^2}{t^8}+\frac{8}{t^8}+\frac{12}{s^4\,t^8}+\frac{8}{s^6\,t^8}-\frac{20}{s^8\,t^8}+\frac{4}{s^{10}\,t^8}+\frac{2}{s^{12}\,t^8}-\frac{20}{t^8}-\frac{4}{s^2\,t^{10}}-\frac{8}{s^4\,t^{10}}$$
$$+\frac{8}{s^6\,t^{10}}+\frac{4}{s^8\,t^{10}}-\frac{4}{s^{10}\,t^{10}}+\frac{4}{t^{10}}+\frac{2}{s^4\,t^{12}}-\frac{4}{s^6\,t^{12}}+\frac{2}{s^8\,t^{12}}+4\,s^{10}-20\,s^8+4\,s^6+164\,s^4$$
$$-476\,s^2-\frac{476}{s^2}+\frac{164}{s^4}+\frac{4}{s^6}-\frac{20}{s^8}+\frac{4}{s^{10}}+649\,;\,test(K1134);$$

$$K1134 := 649-476\,s^2-476\,t^2-\frac{20}{t^8}+\frac{4}{t^6}+\frac{2}{s^{12}\,t^4}-\frac{20}{s^8\,t^8}-\frac{4}{s^6\,t^6}+\frac{8}{s^8\,t^6}-\frac{8\,s^6}{t^2}$$
$$-\frac{8\,t^6}{s^2}+\frac{12\,s^4}{t^4}+\frac{12\,t^4}{s^4}-\frac{46\,s^2}{t^4}-\frac{46\,t^2}{s^4}-\frac{8\,s^2}{t^6}-\frac{8\,t^2}{s^6}+\frac{12}{s^4\,t^8}+\frac{12}{s^8\,t^4}+\frac{8}{s^6\,t^8}$$
$$-\frac{46}{s^6\,t^4}-\frac{248}{s^2\,t^4}-20\,t^8+4\,t^6-20\,s^8+4\,s^6+\frac{46}{s^6\,t^2}-248\,s^4\,t^2-248\,s^2\,t^4+476\,s^2\,t^2$$
$$+\frac{46}{s^2\,t^6}-\frac{476}{t^2}-\frac{476}{s^2}-\frac{46}{s^4\,t^6}+\frac{164}{s^4}+\frac{164}{t^4}+\frac{476}{s^2\,t^2}-\frac{46\,s^4}{t^2}-20\,s^8\,t^8-\frac{46\,t^4}{s^2}$$
$$+2\,s^4\,t^{12}-4\,s^2\,t^{10}-\frac{248}{s^4\,t^2}+\frac{164}{s^4\,t^4}-46\,s^4\,t^6-46\,s^6\,t^4+\frac{8}{s^2\,t^8}+\frac{8}{s^8\,t^2}+164\,s^4$$
$$+\frac{248\,s^2}{t^2}+\frac{4\,t^8}{s^2}+\frac{4\,s^8}{t^2}+164\,t^4+\frac{248\,t^2}{s^2}-\frac{20}{s^8}+\frac{4}{s^6}+164\,s^4\,t^4+\frac{4}{s^{10}}-4\,s^6\,t^6$$
$$+4\,s^{10}+\frac{2}{s^4\,t^{12}}+4\,t^{10}+\frac{4}{t^{10}}+\frac{2\,s^8}{t^4}+\frac{2\,t^8}{s^4}-\frac{8\,s^6}{t^4}-\frac{8\,t^6}{s^4}+\frac{4\,s^6}{t^6}+\frac{4\,t^6}{s^6}-\frac{8\,s^4}{t^6}$$
$$-\frac{8\,t^4}{s^6}+\frac{2\,s^4}{t^8}+\frac{2\,t^4}{s^8}+\frac{4\,s^2}{t^8}+\frac{4\,t^2}{s^8}-\frac{4}{s^2\,t^{10}}-\frac{4}{s^{10}\,t^2}-\frac{8}{s^4\,t^{10}}-\frac{8}{s^{10}\,t^4}+\frac{8}{s^6\,t^{10}}$$
$$+\frac{8}{s^{10}\,t^6}-\frac{4}{s^6\,t^{12}}+\frac{4}{s^8\,t^{10}}+\frac{4}{s^{10}\,t^8}-\frac{4}{s^{12}\,t^6}+\frac{2}{s^8\,t^{12}}+\frac{2}{s^{12}\,t^8}-\frac{4}{s^{10}\,t^{10}}+2\,s^{12}\,t^4$$
$$-8\,s^{10}\,t^4-4\,s^{10}\,t^2-4\,s^{10}\,t^{10}-8\,s^4\,t^{10}+8\,s^8\,t^2+8\,s^2\,t^8+2\,s^8\,t^{12}-4\,s^{12}\,t^6+4\,s^{10}\,t^8$$
$$+4\,s^8\,t^{10}-4\,s^6\,t^{12}+8\,s^{10}\,t^6+8\,s^6\,t^{10}+2\,s^{12}\,t^8+12\,s^4\,t^8+12\,s^8\,t^4+8\,s^6\,t^8+8\,s^8\,t^6$$
$$+46\,s^2\,t^6+46\,s^6\,t^2$$

$$true,\ true,\ true,\ 1 \qquad \textbf{(2.22)}$$

#Computed Externally

#U2iiR2Ui:=spmm(KP(U^2, id), spmm(KP(id, R^2), Ui)) : R2iiURiiURi := spmm(KP(R^2, id), spmm(iU, spmm(Ri, spmm(iU, Ri)))) :

 # A1142:=spmm(KP(id, KP(h, KP(h, h)))), spmm(KP(U, ID), KP(id, U2iiR2Ui))) : B1142 := spmm(KP(R, ID), KP(id, R2iiURiiURi)) : K1142 := sort(sort(expand(sl3(A1142, B1142)), s), t);

$$K1142 := \frac{1}{s^6\,t^6}\left(\left(12\,t^{12}-34\,t^{10}+34\,t^8-12\,t^6\right)s^{12}+\left(-34\,t^{12}+148\,t^{10}-228\,t^8+148\,t^6\right.\right.$$
$$\left.-34\,t^4\right)s^{10}+\left(34\,t^{12}-228\,t^{10}+496\,t^8-496\,t^6+228\,t^4-34\,t^2\right)s^8+\left(-12\,t^{12}+148\,t^{10}\right.$$
$$\left.-496\,t^8+721\,t^6-496\,t^4+148\,t^2-12\right)s^6+\left(-34\,t^{10}+228\,t^8-496\,t^6+496\,t^4-228\,t^2\right.$$
$$\left.+34\right)s^4+\left(-34\,t^8+148\,t^6-228\,t^4+148\,t^2-34\right)s^2-12\,t^6+34\,t^4-34\,t^2+12\Big)\,;$$
test(K1142);

$$K1142 := \frac{1}{s^6 t^6}\left(\left(12 t^{12} - 34 t^{10} + 34 t^8 - 12 t^6\right) s^{12} + \left(-34 t^{12} + 148 t^{10} - 228 t^8 + 148 t^6 - 34 t^4\right) s^{10} + \left(34 t^{12} - 228 t^{10} + 496 t^8 - 496 t^6 + 228 t^4 - 34 t^2\right) s^8 + \left(-12 t^{12} + 148 t^{10} - 496 t^8 + 721 t^6 - 496 t^4 + 148 t^2 - 12\right) s^6 + \left(-34 t^{10} + 228 t^8 - 496 t^6 + 496 t^4 - 228 t^2 + 34\right) s^4 + \left(-34 t^8 + 148 t^6 - 228 t^4 + 148 t^2 - 34\right) s^2 - 12 t^6 + 34 t^4 - 34 t^2 + 12\right)$$

$$true, true, true, 1 \qquad \textbf{(2.23)}$$

$$T22 := sl3\left(KP(id, h).R, R\right); test(T22);$$

$$T22 := \frac{(t-1)\,(t+1)\left(s^2 t^2 + 1\right)(s-1)\,(s+1)}{t^2 s^2}$$

$$true, true, true, 0 \qquad \textbf{(2.24)}$$

$$T24 := sl3\left(KP(id, h).R^3, R\right); test(T24); expand\left(simplify\left(\frac{T24}{T22}\right)\right);$$

$$T24 := \frac{(t+1)\left(\left(t^8 + t^4\right) s^8 + \left(t^8 + 1\right) s^4 + t^4 + 1\right)(t-1)\left(s^2 t^2 + 1\right)(s+1)\,(s-1)}{s^6 t^6}$$

$$true, true, true, 0$$

$$s^4 t^4 + s^4 + t^4 + \frac{1}{t^4} + \frac{1}{s^4} + \frac{1}{s^4 t^4} \qquad \textbf{(2.25)}$$