▶ Preamble

$w0 := ssimplify(E2 \cdot E1)[\,..,-1\,] : W := simplify(\langle w0|F1 \cdot w0|F2 \cdot w0|F1 \cdot F2 \cdot w0\rangle) : Wplus := subs{\sim}\left(t2 = I \cdot t1^{-1}, W\right) : Wminus := subs{\sim}\left(t2 = -I \cdot t1^{-1}, W\right) :$

$$f1 := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} : f2 := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & \frac{-\mathrm{I}\left(t1^2+1\right)}{t1^2-1} & 0 & 0 \end{bmatrix} : e1 :=$$

$$\begin{bmatrix} 0 & -\frac{t1^2+1}{2\,t1} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{-\frac{\mathrm{I}}{2}\left(t1^2-1\right)}{t1} \\ 0 & 0 & 0 & 0 \end{bmatrix} : e2 := \begin{bmatrix} 0 & 0 & \frac{t2}{2}\left(t1^2-1\right) & 0 \\ 0 & 0 & 0 & \frac{t2}{2}\left(t1^2-1\right) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} :$$

$$k1 := \begin{bmatrix} -\mathrm{I}\,t1 & 0 & 0 & 0 \\ 0 & \mathrm{I}\,t1 & 0 & 0 \\ 0 & 0 & t1 & 0 \\ 0 & 0 & 0 & -t1 \end{bmatrix} : k2 := \begin{bmatrix} -I \cdot t2 & 0 & 0 & 0 \\ 0 & t2 & 0 & 0 \\ 0 & 0 & I \cdot t2 & 0 \\ 0 & 0 & 0 & -t2 \end{bmatrix} : h1 := \begin{bmatrix} \lambda-1 & 0 & 0 & 0 \\ 0 & \lambda-3 & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda-2 \end{bmatrix} :$$

$$h2 := \begin{bmatrix} \mu-1 & 0 & 0 & 0 \\ 0 & \mu & 0 & 0 \\ 0 & 0 & \mu-3 & 0 \\ 0 & 0 & 0 & \mu-2 \end{bmatrix} :$$

$e3 := simplify(-(e1 \cdot e2 + I \cdot e2 \cdot e1)) : f3 := simplify(-(f2.f1 - I \cdot f1.f2)) :$

$check(br(e1, f1) - fl(k1)), check(br(e2, f2) - fl(k2)), check(br(e1, f2)), check(br(e2, f1)), check(k1 \cdot f1 + f1 \cdot k1), check(k2 \cdot f2 + f2 \cdot k2), check(k1 \cdot e1 + e1 \cdot k1), check(k2 \cdot e2 + e2 \cdot k2);$

$check\left(e3^2\right), check\left(f3^2\right), check(k1 \cdot f2 - \mathrm{I} \cdot f2 \cdot k1), check(k2 \cdot f1 - \mathrm{I} \cdot f1 \cdot k2), check(k1 \cdot e2 + \mathrm{I} \cdot e2 \cdot k1), check(k2 \cdot e1 + \mathrm{I} \cdot e1 \cdot k2), check\left(e1^2\right), check\left(f1^2\right), check\left(e2^2\right), check\left(f2^2\right), check(br(h1, e1) - 2\,e1), check(br(h2, e1) + e1), check(br(h1, e2) + e2), check(br(h2, e2) - 2\,e2), check(br(h1, f1) + 2\,f1), check(br(h2, f1) - f1), check(br(h1, f2) - f2), check(br(h2, f2) + 2\,f2)$

$$Good, Bad, \begin{bmatrix} \frac{t1^2\,t2^2+1}{2\,t2} & 0 & 0 & 0 \\ 0 & \frac{-\frac{I}{2}\,(t1^2\,t2^2+1)}{t2} & 0 & 0 \\ 0 & 0 & \frac{-t1^2\,t2^2-1}{2\,t2} & 0 \\ 0 & 0 & 0 & \frac{\frac{I}{2}\,(t1^2\,t2^2+1)}{t2} \end{bmatrix}, Good,$$

$Good, Good, Good, Good, Good$

$Good, Good, Good, Good, Good, Good, Good, Good, Good, Good, Good, Good, Good, Good, Good, Good, Good, Good$ **(1)**

$R1 := simplify\Big(DiagonalMatrix\Big(`\sim`_q\Big(Diagonal\Big(\frac{2}{3}\,KP(H1, subs(\lambda = \Lambda, H1)) + \frac{2}{3}\,KP(H2, subs(\mu = \mathrm{M}, H2)) + \frac{1}{3}\,KP(H2, subs(\lambda = \Lambda, H1)) + \frac{1}{3}\,KP(H1, subs(\mu = \mathrm{M}, H2))\Big)\Big)\Big).$
$(IdentityMatrix(64) + 2\,I\,KP(E1, F1)) \cdot (IdentityMatrix(64) + 2\,I\,KP(E3, F3)) \cdot (IdentityMatrix(64) + 2\,I\,KP(E2, F2)), power\Big):$

$R := `\sim`_{simplify}\Big(`\sim`_{expand}\Big(subs\Big(\mu = -\frac{2\,I\ln(t2)}{\pi}, \lambda = -\frac{2\,I\ln(t1)}{\pi}, \mathrm{M} = -\frac{2\,I\ln(t2)}{\pi}, \Lambda = -\frac{2\,I\ln(t1)}{\pi}, \frac{P \cdot R1}{R1_{8,8}}\Big)\Big)\Big):$

$Rplus := LinearSolve(KP(Wplus, Wplus), subs{\sim}(t2 = I \cdot t1^{-1}, R).KP(Wplus, Wplus)) : Rminus := LinearSolve(KP(Wplus, Wplus), subs{\sim}(t2 = -I \cdot t1^{-1}, R).KP(Wplus, Wplus)) :$
$de1 := KroneckerProduct(e1, k1) + KroneckerProduct(id, e1) :$
$de2 := KroneckerProduct(e2, k2) + KroneckerProduct(id, e2) : df1 := KroneckerProduct(f1, id) + KroneckerProduct\Big(\frac{1}{k1}, f1\Big) : df2 := KroneckerProduct(f2, id) + KroneckerProduct\Big(\frac{1}{k2}, f2\Big) : dk1 := KroneckerProduct(k1, k1) : dk2 := KroneckerProduct(k2, k2) : de3 := -de1 \cdot de2 - (I\,de2).de1 : df3 := -df2 \cdot df1 + (I\,df1).df2 :$
$check(br(de1, df1) - fl(dk1)), check(subs(t2 = I \cdot t1^{-1}, br(de2, df2) - fl(dk2))), check(subs(t2 = -I \cdot t1^{-1}, br(de2, df2) - fl(dk2))), check(br(de1, df2)), check(br(de2, df1)), check(dk1 \cdot df1 + df1 \cdot dk1), check(dk2 \cdot df2 + df2 \cdot dk2), check(dk1 \cdot de1 + de1 \cdot dk1),$
$check(dk2 \cdot de2 + de2 \cdot dk2), check(dk1 \cdot df2 - I \cdot df2 \cdot dk1), check(dk2 \cdot df1 - I \cdot df1 \cdot dk2), check(dk1 \cdot de2 + I \cdot de2 \cdot dk1), check(dk2 \cdot de1 + I \cdot de1 \cdot dk2), check(de1^2), check(df1^2), check(de2^2), check(df2^2), check(de3^2), check(df3^2);$

$Good, Good, Good, Good, Good, Good, Good, Good, Good, Good, Good, Good, Good, Good, Good, Good, Good, Good, Good$ **(2)**

$de21 := de2 \cdot de1 : de1212 := de21^2 : dflist := [idd, df1, df2, df1 \cdot df2] :$

$longdflist := [idd, df1, df2, df1 \cdot df2, df2 \cdot df1, df1 \cdot df2 \cdot df1, df2 \cdot df1 \cdot df2, df2 \cdot df1 \cdot df2 \cdot df1] : v1 := de1 \cdot KP(f1, f1)_{.., 1} : v2 := de2 \cdot KP(f2, f2)_{.., 1} : v3 := IdentityMatrix(16)_{.., 1} :$

$GammaW := Matrix(16)$: **for** j **to** 4 **do**

$GammaW_{.., j} := dflist_j \cdot v1;$

$GammaW_{.., j+4} := dflist_j \cdot v2$

end do: **for** j **to** 8 **do**

$GammaW_{.., j+8} := longdflist_j \cdot v3$

end do: $GammaW := simplify(GammaW) : GammaWinv := \frac{1}{GammaW} :$

$signsred := simplify\left(subs\left(t2 = I \cdot t1^{-1}, tr2\left(GammaWinv \cdot KP\left(id, \left(\frac{1}{k1^2}\right) \cdot \left(\frac{1}{k2^2}\right)\right) \cdot GammaW\right)\right)\right);$

$$signsred := \begin{bmatrix} -\frac{t1^2}{t1^4+1} & 0 & 0 & 0 \\ 0 & \frac{t1^2}{t1^4+1} & 0 & 0 \\ 0 & 0 & -\frac{t1^2}{t1^4-1} & 0 \\ 0 & 0 & 0 & \frac{t1^2}{t1^4-1} \end{bmatrix} \quad \mathbf{(3)}$$

$Rplusredrel := simplify\left(subs\left(t2 = I \cdot t1^{-1}, tr2\left(GammaWinv.\left(Rplus - \frac{1}{Rplus} - \left(t1^2 - \frac{1}{t1^2}\right) IdentityMatrix(16)\right).GammaW\right)\right)\right);$

$$Rplusredrel := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-t1^4+1}{t1^2} & 0 \\ 0 & 0 & 0 & \frac{-t1^4+1}{t1^2} \end{bmatrix} \quad \mathbf{(4)}$$

$Rminusredrel := simplify\left(subs\left(t2 = -I \cdot t1^{-1}, tr2\left(GammaWinv.\left(Rminus - \frac{1}{Rminus} - \left(t1^2 - \frac{1}{t1^2}\right) IdentityMatrix(16)\right) \cdot GammaW\right)\right)\right);$

$$Rminusredrel := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-t1^4+1}{t1^2} & 0 \\ 0 & 0 & 0 & \frac{-t1^4+1}{t1^2} \end{bmatrix} \tag{5}$$

simplify(*subs*~(*t2* = *I*·*t1*$^{-1}$, *signsred*) .*Rplusredrel*), *simplify*(*subs*~(*t2* =−*I*·*t1*$^{-1}$, *signsred*
 · *Rminusredrel*));

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \tag{6}$$

#any intertwiner multiplied then traced with this gives zero, so it satisfies Conway relation