Exam.Code:906
Sub. Code: 7844

1016

B.Engg. Second Semester

AS-201: Engineering Mathematics – II

(Common to all)

Time allowed: 3 Hours Max. Marks: 50

NOTE: Attempt five questions in all, selecting atleast two questions from each Part. All questions carry equal marks.

X-X-X

Part A

Question I Solve following the differential equations:

(i) $(x^3 + xy^2 + y)\,dx + (y^3 + x^2y + x)\,dy = 0$

(ii) $2xy\,dy - (x^2 + y^2 + 1)\,dx = 0$

(iii) $(D^2 - a^2)y = e^{2x}$

Question II (a) Find a power series solution in powers of x of the differential equation $y'' - 3y' + 2y = 0$

(b) Solve using method of Variation of Parameters $y'' - 2y' + y = \dfrac{e^x}{x^3}$

Question III (a) Given that $y = e^{2x}$ is a solution of $(2x+1)\dfrac{d^2y}{dx^2} - 4(x+1)\dfrac{dy}{dx} + 4y = 0$, find the complete solution by reducing the order of the given differential equation.

(b) Find the Laplace transform of (i) $t^2e^{-3t}\sin 2t$ (ii) $e^t\displaystyle\int_0^t \frac{\sin^2 t}{t}dt$

Question IV (a) Find the Inverse Laplace Transforms of (i) $\dfrac{1}{s(s+a)^3}$ (ii) $\ln\dfrac{s^2+1}{s(s+1)}$

(b) State and prove the second shifting theorem for Laplace transforms. Use it to find the Laplace transform of unit step function.

Part B

Question V (a) Prove the Euler formula for the Fourier coefficients of a periodic function with period 2π.

(b) Find the fourier series of the function $f(x) = x^2$ $(-1 < x < 1)$, with period $p = 2$. Use it to show that

$$1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \ldots = \frac{\pi^2}{12}.$$

Question VI (a) Find the Fourier Cosine and Sine integrals of

$$f(x) = e^{-kx} \quad (x > 0,\ k > 0)$$

(b) Show that the given integral represents the indicated function:

$$\int_0^\infty \frac{\sin w \cos xw}{w}dw = \begin{cases} \pi/2 & \text{if } 0 \le x < 1 \\ \pi/4 & \text{if } x = 1 \\ 0 & \text{if } x > 1 \end{cases}$$

Question VII (a) (i) Eliminate the constants a and b from the equation $2z = (ax + y)^2 + b$

(ii) Eliminate the arbitrary function f from the equation $f(x^2 + y^2 + z^2, z^2 - 2xy) = 0$

(b) Find the general integrals of the linear partial differential equation $px(z - 2y^2) = (z - qy)(z - y^2 - 2x^3)$

Question VIII Solve the one-dimensional wave equation $\dfrac{\partial^2 y}{\partial x^2} = \dfrac{1}{c^2}\dfrac{\partial^2 y}{\partial t^2}$ subject to the boundary conditions $y(0,t) = y(L,t) = 0$ and initial conditions $y(x,0) = f(x)$, $\dfrac{\partial y}{\partial t}(x,0) = g(x)$ where $f(x)$ is the initial deflection and $g(x)$ is the initial velocity. Here $y = y(x,t)$.

X-X-X