SG18333

Exam. Code: 0905
Sub. Code: 6641

1128

B.E. (Biotechnology) First Semester
MATHS-101: Calculus
(Common to all Streams)

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Part. Use of non-programmable calculator is allowed.

x-x-x

1. (a) Find the limit of the sequence: $a_n = \frac{n!}{n^n}$.

   (b) Find the limit if it exists: $\lim_{(x,y)\to(0,0)} \frac{2x}{x^2+y^2+x}$.

   (c) Define domain, range and level curves of the function $f(x,y) = 1-|x|-|y|$. Draw its graph also.

   (d) Find the direction derivetive of $f(x,y) = xy + cos(y)$ at the point (3,0) in the direction of $\mathbf{A} = 3\mathrm{i} - 4\,\mathrm{j}$.

   (e) If $|a|$ is much greater than $|b|, |c|, |d|$, to which of a, b, c, d is the value of the function $f(a,b,c,d) = ad - bc$ is most sensitive? Justify your answer. $(5 \times 2 = 10)$

PART A

2. (a) Check the convergence of the following series: (3+3)

   (i) $\sum_{n=1}^{\infty}(-1)^n \frac{n!}{2^n}$

   (ii) $\sum_{n=2}^{\infty} \frac{n}{(\ln\ n)^{(n/2)}}$

   (b) Check the convergence of the series: (4)

$$\sum_{n=1}^{\infty} \frac{nx^n}{4^n(n^2+1)}$$

3. (a) Find $\frac{\partial w}{\partial x}$ if $w = x^2+y^2+z^2$ and $z = x^2+y^2$. (4)

   (b) Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the line $y = 2$ and $x = 0$ about the line $x = 4$. (3)

   (c) Find the length of the curve $x = \frac{y^{3/2}}{3} - \sqrt{y}$ from $y = 1$ to $y = 9$. (3)

4. (a) Show that $w = f(u,v)$ satisfies the Laplace equation $f_{uu} + f_{vv} = 0$, and if $u = (x^2-y^2)/2$ and $v = xy$, then w satisfies the Laplace equation $w_{xx} + w_{yy} = 0$. (5)

   (b) The temperature at a point (x,y) on a metal plate is $T(x,y) = 4x^2 - 4xy + y^2$. An ant on the plate walks around the circle of radius 5 centered at origin. What are the highest and lowest temperatures encoutered by the ant? (5)

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PART B

5. (a) Find the volume of the region in the first octant bounded by coordinates planes, the plane $x + y = 4$, and the cylinder $y^2 + 4z^2 = 16$. (6)

   (b) Find the area of the region that lies inside the cardioid $r = 1 + \cos\theta$ and outside the circle $r = 1$. (4)

6. (a) Find the outward flux of the field $\mathbf{F} = xz\ \mathbf{i} + yz\mathbf{j} + \mathbf{k}$ across the boundary of the region D: The entire surface of the upper cap cut from the solid sphere $x^2 + y^2 + z^2 \leq 25$ by the plane $z = 3$. (5)

   (b) Show that the differential form in the integral is exact and hence evaluate the integral: (5)

$$\int_{(1,1,1)}^{(1,2,3)} 3x^2\ dx + z^2/y\ dy + 2z\ \ln y\ dz$$

7. (a) Find $\mathbf{T}, \mathbf{N}, \mathbf{B}$ curvature and torsion for the space curve: (5)

$$\mathrm{r(t)} = \cosh t\ \mathbf{i} - \sinh t\ \mathbf{j} + t\ \mathbf{k}$$

   (b) A particle moves along the top of the parabola $y^2 = 2x$ from left to right at a constant speed of 5 units per second. Find the velocity of the particle as it moves through the point (2,2). (3)

   (c) Find the equation of the tangent plane to the surface $z = \sqrt{y - x}$ at the point (1,2,1). (2)

x–x–x

5th Semester

Mech.

de : 7463

Exam.Code:905
Sub. Code: 7836

1016

B.Engg. First Semester

AS-101: Engineering Mathematics – I

(Common to all)

Time allowed: 3 Hours

Max. Marks: 50

NOTE: *Attempt five questions in all, selecting atleast two questions from each Unit.*

x-x-x

UNIT- I

I. a) Find $\left(\frac{\partial w}{\partial y}\right)_x$, $\left(\frac{\partial w}{\partial y}\right)_z$ at the point (w, x, y, z) = (4, 2, 1, -1) if $w = x^2y^2 + yz - z^3$ and $x^2 + y^2 + z^2 = 6$.

b) Obtain Taylor's expansion of $\tan^{-1}\left(\frac{y}{x}\right)$ about (1, 1) upto and including second order terms. Hence compute f (1.1, 0.7). (5,5)

II. a) Use the method of Lagrange's multipliers to find the volume of the largest rectangular parallelotriped that can be inscribed in the ellipsoid: $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

b) Find the value of 'n' so that the equation $v = r^n(3\cos\theta - 1)$ satisfies the relation.

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial v}{\partial r}\right)+\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial v}{\partial\theta}\right)=0$$

(5,5)

III. Find the absolute maximum and minimum values of

$f(x, y) = 2 + 2x + 2y - x^2 - y^2$ on the triangular plate in the first quadrant, bounded by the lines x = 0, y = 0 and y = 9 – x. (10)

IV. a) Find the area of the surface generated by revolving the curve $y = \sqrt{2x - x^2}$, $0.5 \le x \le 1.5$ about x-axis.

b) Evaluate $\int_0^4\int_{y/2}^{\frac{y}{2}+1}\frac{2x-y}{2}\,dxdy$ by applying the transformation $u = \frac{2x-y}{2}$, $v = \frac{y}{2}$ and integrating over an approximate region in the uv-plane. (5,5)

UNIT- II

V. Show that the curvature of a smooth curve $\vec{r}(t) = f(t)\hat{i} + g(t)\hat{j}$ defined by twice differentiable functions x = f(t) and y = g(t) is given by the formula:

$$K = \frac{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|}{(\dot{x}^2+\dot{y}^2)^{3/2}}$$

(10)

P.T.O.

VI. a) Show that the parabola $y = ax^2$, $a \neq 0$, has its largest curvature at its vertex and has no minimum curvature.

b) Find T, N, B curvature (κ) and torsion (τ) for the curve

$\vec{r}(t) = \ln(\sec t)\hat{i} + t\,\hat{j}, \ -\frac{\pi}{2} < t < \frac{\pi}{2}$ (5,5)

VII. a) Show that $F = (e^x \cos y + yz)\hat{i} + (xz - e^x \sin y)\hat{j} + (xy + z)\hat{k}$, is conservative and find a potential function for it.

b) Show that $y\,dx + x\,dy + 4\,dz$ is exact and evaluate the line integral

$\int_{(1,1,1)}^{(2,3,-1)} y\,dx + x\,dy + 4\,dz$ over the line segment from (1,1,1) to (2, 3, -1). (5,5)

VIII. a) Apply Green's theorem to evaluate the integral $\oint (6y + x)\,dx + (y + 2x)\,dy$, where the curve C is defined by C : the circle $(x - 2)^2 + (y - 3)^2 = 4$.

b) State Stoke's theorem. Apply it to calculate the circulation of the field

$\vec{F} = x^2y^3\hat{i} + \hat{j} + z\hat{k}$ around the curve C defined by

C: The intersection of the cylinder $x^2 + y^2 = 4$ and the hemisphere $x^2 + y^2 + z^2 = 16$, $z \geq 0$, counter clockwise when viewed from above.

(5,5)

x-x-x

2nd Semester No. ....

(i) Printed Pages : 4

Roll No. ....

(ii) Questions : 7

Sub. Code : | 6 | 6 | 4 | 1 |

Exam. Code : | 9 | 0 | 5 |

B.E. Mechanical Engg. 1st Semester

1124

CALCULUS (Common to all Streams)

Paper : MATHS-101

Time Allowed : Three Hours] **[Maximum Marks : 50**

Note :– **First** question is compulsory. Attempt any **four** questions from Part–A and Part B with at least **two** questions from each Part. Use of non-programmable calculator is allowed.

I. (a) Define absolute convergence of a series. Give an example of a convergent series which is not absolutely convergent.

(b) Find the value of the Jacobian $\frac{\partial(u, v)}{\partial(r, \theta)}$, where $u = x^2 - y^2$, $v = 2xy$ and $x = r \cos \theta$, $y = r \sin \theta$.

(c) What are quadric surfaces ? Give examples of ellipsoids and cones.

(d) Change the order of integration :

$$\int_0^{\infty}\int_0^{x} e^{-xy}\, y\, dy\, dx .$$

(e) What is a potential function ? Show by an example how to find a potential function for a conservative field ?

5×2=10

6. (a) Derive an expression for change in length

PART–A

II. (a) Which of the following series converge or diverge ? Justify your answer properly :

(i) $\sum_{n=1}^{\infty} \frac{(n!)^n}{(n^n)^2}$

(ii) $\sum_{n=1}^{\infty} \frac{(-1)^n \tan^{-1} n}{n^2+1}$

(iii) $\sum_{n=1}^{\infty} \frac{\tanh n}{n^2}$

(iv) $\sum_{n=1}^{\infty} (-1)^n \left(\sqrt{n+\sqrt{n}} - \sqrt{n}\right)$

(v) $\sum_{n=3}^{\infty} \frac{\left(\frac{1}{n}\right)}{(\ell n n)\sqrt{\ell n^2 n - 1}}$.

(b) Find the volume of the solid generated by revolving the region in first quadrant which is bounded on the left by the circle $x^2 + y^2 = 3$, on the right by the line $x = \sqrt{3}$, and above by the line $y = \sqrt{3}$ about y-axis. 5+5=10

III. (a) Find the absolute maxima and minima of the function $f(x, y) = 4x - 8xy + 2y + 1$ on the triangular plate bounded by the lines $x = 0$, $y = 0$, $x + y = 1$ in the first quadrant.

(b) Find the linearization $L(x, y, z)$ of the function $f(x, y, z) = x^2 - xy + 3 \sin z$ at the point $(x_0, y_0, z_0) = (2, 1, 0)$. Find an upper bound for the error incurred in replacing f by L on the rectangle : $R : |x-2| \le 0.01$, $|y-1| \le 0.02$, $|z| \le 0.01$. 5+5

IV. (a) Show that the volume of the segment cut from the paraboloid : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}$ by the plane $z = h$ equals half the segment's base times its altitude.

(b) Use the method of Lagrange's multipliers to find the value of the largest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2} = 1$. 5+5

PART–B

V. (a) Find the area of the region that lies inside the cardioid $r = 1 + \cos\theta$ and outside the circle $r = 1$.

(b) Evaluate the integral $\iint_R (3x^2 + 14xy + 8y^2)\, dx dy$ by applying the transformation : $u = 3x + 2y$, $v = x + 4y$, for the region R in the first quadrant bounded by the lines $y = \frac{-3}{2}x + 1$, $y = \frac{-3}{2}x + 3$ and $y = \frac{-1}{4}x + 1$. 5+5

VI. (a) Show that the curvature of a smooth curve $\bar{r}(t) = f(t)\hat{i} + g(t)\hat{j}$ defined by twice differentiable function $x = f(t)$ and $y = g(t)$ is given by $\rho = \frac{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|}{(\dot{x}^2 + \dot{y}^2)^{3/2}}$.

(b) Without finding T and N, write the acceleration of the motion $\bar{r}(t) = (e^t \cos t)\hat{i} + (e^t \sin t)\hat{j} + \sqrt{2}\, e^t \hat{k}$, $t=0$, in the form $\bar{a} = a_T T + a_N N$. 5+5

VII. (a) State Green's theorem. Use it to find the counter clockwise circulation and outward flux for the field $\vec{F}$ and curve C given by $\vec{F} = (x + y)\hat{i} - (x^2 + y^2)\hat{j}$,

C : The triangle bounded by $y = 0$, $x = 1$ and $y = x$.

(b) Find the flux of the field $\vec{F} = z^2\hat{i} + x\hat{j} - 3z\hat{k}$ upward through the surface cut from the parabolic cylinder $z = 4 - y^2$ by the planes $x = 0$, $x = 1$ and $z = 0$. 5+5

Exam. Code: 905
Sub. Code: 7889

1016

B. Engg. (Biotechnology Engineering) 1[st] Semester
MATHS-101: Calculus
(Common to all streams)

Time allowed: 3 hours

Max. Marks: 50

Note: Attempt *<u>Five</u>* *questions in all including Q. No. I which is compulsory and selecting atleast two questions from each Part. Use of simple calculator is allowed.*

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I. a) Define absolute and conditional convergence. Give an example of a series which converges conditionally but not absolutely.

b) Find the derivative of the function:
$f(x,y) = x - \frac{y^2}{x} + \sqrt{3}\sec^{-1}(2xy)$ at the point $(1,1)$ in the direction of $\vec{A} = 12\hat{\imath} = 5\hat{\imath}$.

c) State Euler's theorem for homogeneous functions.

d) How does the relation between first partial derivatives and continuity of functions of two variables differ from the relation between first derivatives and continuity for real-valued functions of single independent variable? Give an example.

e) What is a potential function? Show by an example how to find a potential function for a conservative field. (5x2)

PART-A

II. a) Find the lineachzation $L(x,y,z)$ of the function $f(x,y,z) = \sqrt{2}$ cas $x \sin(y+z)$ at the point $(0,0,\frac{\pi}{4})$ over the region R defined by R: $|x| \leq 0.01$, $|y| \leq 0.01$, $|z - \frac{\pi}{4}| \leq 0.01$. Also find an upper bound for the error in the approximation $f(x,y,z) \approx L(x,y,z)$ on the given region.

b) Check the convergence or divergence of the following series:

(i) $\sum_{n=1}^{\infty} \frac{1}{n(1+\ln^2 n)}$, (ii) $\sum_{n=3}^{\infty} \frac{5n^3-3n}{n^2(n-2)(n^2+5)}$,

(iii) $\sum_{n=1}^{\infty} \frac{n!\ln n}{n(n+2)!}$, (iv) $\sum_{n=1}^{\infty} (-1)^n\left(\sqrt{n+\sqrt{n}}-\sqrt{n}\right)$.

III. a) Find the radius and interval of convergence of the power series:

$$\sum_{n=1}^{\infty} \frac{(4x-5)^{2n+1}}{n^{3/2}}$$

For what values of 'x' does the series converge absolutely and conditionally?

b) Find the volume of the solid generated by revolving the region in the first quadrant bounded on the left by the circle: $x^2+y^2=3$, on the right by the line $x=\sqrt{3}$, and above by the line $y=\sqrt{3}$ about y-axis. (5,5)

IV. a) Use the method of Lagrange multipliers to find the dimensions of the rectangle of the greatest area that can be inscribed in the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ with sides parallel to the co-ordinate ones.

b) Find the equation of the circular cylinder having for its base the circle: $x^2+y^2+z^2=9$, $x-y+z=3$. (5,5)

<u>P.T.O.</u>

different parameters used to give 'light' effects.

b) How a graph can be exported into image file? What are the different image formats in

(4)

PART-B

V. a) Let D be the region in xyz – space defined by the inequalities: $|\leq x \leq 2, 0 \leq$ $\leq 2, 0 \leq z \leq |$. Evaluate: $\iiint_D (x^2y+3xyz)\, dxdydz$ by applying the transformation $u=x$, $\upsilon=xy$, $w=3z$ and integrating over an appropriate region G in u υw-space.

b) Integrate the function $f(u,\upsilon) = \upsilon-\sqrt{u}$ over the triangular region cut from th first quadrant of the u υ-plane by the line $u+\upsilon=1$. (5,5)

VI. a) Show that the curvature of a smooth curve $\vec{r}(t) = f(t)\,\hat{\imath}+g(t)\,\hat{\jmath}$ defined by twic differentiable functions $x=f(t)$ and $y=cj(t)$ is given by the formula.

b) Without finding $\vec{T}$ and $\vec{N}$, write the acceleration of the motion $\vec{r}(t) = (e^t\cos t)$ $\hat{\imath}+(e^t\sin t)\,j + \sqrt{2}e^t\,\hat{k}$, $t=0$ in the form $\vec{a} = a_T\,T + a_N\,\vec{N}$. (5,5)

VII. a) State Green's theorem. Apply it to evaluate the integral:

$$\oint y^2\,dx+x^2\,dy,$$

where C: the triangle bounded by $x=0$, $x+y=1$, $y=0$.

b) Use divergence theorem to find the outward flux of

$$\vec{F} = (5x^3+12xy^2)\,\hat{\imath} + (y^3+e^y\,smz)\,j + (5z^3+e^y\,casz)\,\hat{k},$$

across the boundary of the solid region between the spheres $x^2+y^2+z^2=1$ and $x^2+y^2+z^2=2$. (5,5)

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B.E. (Biotechnology) First Semester
MATHS-101: Calculus
(Common to all Streams)

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NOTE: Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Part. Use of non-programmable calculator is allowed.

x-x-x

1. (a) Find the limit of the sequence: $a_n = \frac{n!}{n^n}$.

(b) Find the limit if it exists: $\lim_{(x,y)\to(0,0)} \frac{2x}{x^2+y^2+x}$.

(c) Define domain, range and level curves of the function $f(x,y) = 1-|x|-|y|$. Draw its graph also.

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(e) If $|a|$ is much greater than $|b|, |c|, |d|$, to which of a, b, c, d is the value of the function $f(a,b,c,d) = ad - bc$ is most sensitive? Justify your answer. $(5 \times 2 = 10)$

PART A

2. (a) Check the convergence of the following series: (3+3)

(i) $\sum_{n=1}^{\infty}(-1)^n \frac{n!}{2^n}$

(ii) $\sum_{n=2}^{\infty} \frac{n}{(\ln\ n)^{(n/2)}}$

(b) Check the convergence of the series: (4)

$$\sum_{n=1}^{\infty} \frac{nx^n}{4^n(n^2+1)}$$

3. (a) Find $\frac{\partial w}{\partial x}$ if $w = x^2+y^2+z^2$ and $z = x^2+y^2$. (4)

(b) Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the line $y = 2$ and $x = 0$ about the line $x = 4$. (3)

(c) Find the length of the curve $x = \frac{y^{3/2}}{3} - \sqrt{y}$ from $y = 1$ to $y = 9$. (3)

4. (a) Show that $w = f(u,v)$ satisfies the Laplace equation $f_{uu} + f_{vv} = 0$, and if $u = (x^2-y^2)/2$ and $v = xy$, then w satisfies the Laplace equation $w_{xx} + w_{yy} = 0$. (5)

(b) The temperature at a point (x,y) on a metal plate is $T(x,y) = 4x^2 - 4xy + y^2$. An ant on the plate walks around the circle of radius 5 centered at origin. What are the highest and lowest temperatures encoutered by the ant? (5)

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PART B

5. (a) Find the volume of the region in the first octant bounded by coordinates planes, the plane $x+y=4$, and the cylinder $y^2+4z^2=16$. (6)

(b) Find the area of the region that lies inside the cardioid $r=1+\cos\theta$ and outside the circle $r=1$. (4)

6. (a) Find the outward flux of the field $\mathbf{F} = xz\ \mathbf{i} + yz\mathbf{j} + \mathbf{k}$ across the boundary of the region D: The entire surface of the upper cap cut from the solid sphere $x^2+y^2+z^2 \leq 25$ by the plane $z=3$. (5)

(b) Show that the differential form in the integral is exact and hence evaluate the integral: (5)

$$\int_{(1,1,1)}^{(1,2,3)} 3x^2\ dx + z^2/y\ dy + 2z\ \ln y\ dz$$

7. (a) Find $\mathbf{T}, \mathbf{N}, \mathbf{B}$ curvature and torsion for the space curve: (5)

$$\mathrm{r(t)} = \cosh t\ \mathrm{i} - \sinh t\ \mathrm{j} + t\ \mathrm{k}$$

(b) A particle moves along the top of the parabola $y^2=2x$ from left to right at a constant speed of 5 units per second. Find the velocity of the particle as it moves through the point (2,2). (3)

(c) Find the equation of the tangent plane to the surface $z=\sqrt{y-x}$ at the point (1,2,1). (2)

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