Exam Code: 0921
Sub. Code: 6831

1127

B.E. (Information Technology)

Third Semester

MATHS-303: Linear Algebra and Probability Theory

Time allowed: 3 Hours | Max. Marks: 50

NOTE: Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Part.

x-x-x

Question I (a) Find the rank of the following matrix by reducing it to its row echelon form

$$A = \begin{bmatrix} 1 & -2 & 0 & -4 \\ 3 & 2 & 1 & 4 \\ 2 & 3 & 7 & 2 \\ -1 & 2 & 0 & -3 \end{bmatrix}$$

(b) Let $P_2(t)$ be the vector spaces of all polynomials of degree ≤ 2 in a single variable t. Show that the polynomials $p_1 = t+1$, $p_2 = t-1$ and $p_3 = (t-1)^2$ form a basis of $P_2(t)$.

(c) Determine whether or not each of the following form a basis of $\mathbf{R}^3$:
(i) (1, 1, 1), (1, 2, 3), (2, -1, 1)
(ii) (1, 2, 3), (1, 3, 5), (1, 0, 1), (2, 3, 0);

(d) Let X be a continuous random variable with distribution

$$f(x) = \begin{cases} kx & \text{if } 0 \leq x \leq 5 \\ 0 & \text{elsewhere} \end{cases}$$

Evaluate k. Also find $P(2 \leq X \leq 5)$.

(e) Let X and Y be independent random variables. Then prove that $E(XY) = E(X)E(Y)$;

$(2 \times 5 = 10)$

Part A

Question II (a) Solve the following system of linear equations using Gauss elimination method.

$$7x_1 + 2x_2 - 2x_3 - 4x_4 + 3x_5 = 8$$
$$-3x_1 - 3x_2 + 2x_4 + x_5 = -1$$
$$4x_1 - x_2 - 8x_3 + 20x_5 = 1$$

(b) Find the dimension and a basis for the general solution W of the homogeneous system

$$x_1 + 2x_2 - 3x_3 + 2x_4 - 4x_5 = 0$$
$$2x_1 + 4x_2 - 5x_3 + x_4 - 6x_5 = 0$$
$$5x_1 + 10x_2 - 13x_3 + 4x_4 - 16x_5 = 0$$

(5+5=10)

Question III (a) Find the eigen values and eigen vectors of the following matrix A.

$$A = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 1 & 2 & 0 & -2 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Is this matrix diagonalizable?

P.T.O.

(b) Let $G : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear mapping defined by $G(x, y, z)) = (x + 2y - z, y + z, x + y - 2z)$ Find a basis and the dimension of (a) the image of G, (b) the kernel of G.

(5+5=10)

Question IV (a) State the Cayley-Hamilton theorem. Using it, invert the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 1 & 2 \\ 2 & 0 & -1 \end{bmatrix}.$$

(b) Consider the following two bases of $\mathbb{R}^2$:
$S = \{u_1, u_2\} = \{(1, -2), (3, -4)\}$ and $S' = \{v_1, v_2\} = \{(1, 3), (3, 8)\}$

(i) Find the change of basis matrix P from S to S'.
(ii) Find the change of basis matrix Q from S' to S.
(iii) Verify that $Q = P^{-1}$

(5+5=10)

Part B

Question V (a) Three machines A, B and C produce respectively 50%, 30% and 20% of the total number of items of a factory. The percentages of defective output of these machines are 3%, 4% and 5%. If an item is selected at random, find the probability that the item is defective. Further suppose an item is selected at random and is found to be defective. Find the probability that the item was produced by machine A.

(b) Two unbiased dice are thrown. If X is the sum of the numbers, then using Chebyshev's inequaltiy prove that

$$P\{|X - 7| \geq 3\} \leq \frac{35}{54}.$$

Compare this with actual probability.

(5+5=10)

Question VI (a) Two cards are selected at random from a box which contains five cards numbered 1, 1, 2, 2 and 3. Let X denote the sum and Y the maximum of the two numbers drawn. (i) Determine the joint distribution of X and Y. (ii) Find $Cov(X, Y)$ and $Correlation(X, Y)$.

(b) Show that if p is small and n is large, then the binomial distribution is approximated by the Poisson distribution.

(5+5=10)

Question VII (a) A box contains 5 red, 3 white and 2 blue marbles. A sample of six marbles is drawn with replacement, i.e. each marble is replaced before the next one is drawn. Find the probability that(i) 3 are red, 2 are white and 1 is blue (ii) 2 of each colour appears.

(b) Suppose 220 misprints are distributed randomly throughout a book of 200 pages. Find the probability that a given page contains (i) no misprints (ii) 2 or more misprints.

(5+5=10)

x–x–x

Exam. Code: 0921
Sub. Code: 6831

1128

B. Engg. (Information Technology)

3rd Semester

MATHS-303: Linear Algebra and Probability Theory

Time allowed: 3 Hours Max. Marks: 50

NOTE: *Attempt* ***five*** *questions in all, including Q. No. I (Unit-I) which is compulsory and selecting atleast two questions each from Unit II-III.*

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UNIT-I

I. (a) Find the conditions on a,b,c so that $\vec{v}=(a,b,c)$ *in* IR^3 belongs to linear span of vectors $(1,2,0)\ \ (-1,1,2)\ \ (3,0,-4)$.

(b) Let $F: IR^2 \to IR^2$ be defined by $F(x,y)=(2x+3y,4x-5y)$. Find the matrix representation of F relative to the basis $\{(1,-2),(2,-5)\}$ *of* IR^2.

(c) A speaks truth 4 out of 5 times. A die is tossed. He reports that there is a six. What is the probability that actually there was six?

(d) A box contains 'a' white and 'b' black balls. 'c' balls are drawn at random. Find the expected value of the number of white balls drawn. (3+2+3+2)

UNIT-II

II. (a) Find the coordinate vector of $3t^3-4t^2+2t-5$ relative to the basis $\{(t-1)^3,(t-1)^2,(t-1),1\}$ *of* $P_3(t)$.

(b) Find a linear mapping $F: IR^2 \to IR^2$ whose image is spanned by $(1,2,3)$ *and* $(4,5,6)$. (5+5)

III. (a) Find the characteristics polynomial of matrix

$$\begin{bmatrix} 2 & 5 & 1 & 1 \\ 1 & 4 & 2 & 2 \\ 0 & 0 & 6 & -5 \\ 0 & 0 & 2 & 3 \end{bmatrix}$$

(b) Reduce the matrix $\begin{bmatrix} 1 & -2 & 3 & 1 & 2 \\ 1 & 1 & 4 & -1 & 3 \\ 2 & 5 & 9 & -2 & 8 \end{bmatrix}$ to echelon form. (5+5)

P.T.O.

IV. (a) Verify the characteristic equation for the matrix $\begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$.

(b) State Rank-Nullity theorem and verify it form the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x, y, z) = (2x + y - 2z,\ 2x + 3y - 4z\ \ x + y - z)$. (5+5)

UNIT-III

V. (a) Two random variables X and Y have the following probability density function $f(x, y) = \begin{cases} 2 - x - y & ; \ 0 \le x \le 1 \quad 0 \le y \le 1 \\ 0 & ; \ otherwise \end{cases}$,

Find Var(X) and Var(Y). Also find covariance between X and Y.

(b) After correcting 50 pages of the proof of a book, the proof reader finds that there are, on the average, 2 errors per 5 pages. How many pages would one expect to find with 0,1,2,3 and 4 errors, in 1000 pages of the first print of the book? (5+5)

VI. (a) In a binomial distribution consisting of 5 independent trials, probabilities of 1 and 2 successes are 0.4096 and 0.2048 respectively. Find the parameter p of the distribution.

(b) For the joint probability distribution of two random variables X and Y given below:

	1	2	3	4	Total
1	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{10}{36}$
2	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{9}{36}$
3	$5/36$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{8}{36}$
4	$\frac{1}{36}$	$2/36$	$\frac{1}{36}$	$5/36$	$\frac{9}{36}$
Total	$\frac{11}{36}$	$\frac{9}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	1

Find conditional distribution of X given the value of Y=1 and that of Y given the value of X=2.

(5+5)

VII. (a) A symmetric dic is thrown 600 times. Find the lower bound for the probability of getting 80 to 120 sixes.

(b) What is the probability that atleast two out of n people have the same birthday? Assume 365 days in a year and all days are equally likely.

(5+5)

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1018

B. Engg. (Computer Science and Engineering)
Sixth Semester
CS-602: Linear Algebra and Probability Theory

Exam. Code: 0918
Sub. Code: 6793

Time allowed: 3 Hours

Max. Marks: 50

Note: *Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Unit. Use of single calculator and statistical table is allowed.*

0-0-0

I. a) Define subspace of a vector space with suitable example. Prove that intersection of two sub-spaces of a vector space is again a subspace of a vector space.

b) Define rank of a matrix. Find the same for the matrix, $A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$

c) Define a linear transformation. Examine whether the linear map $T:R^2 \to R^2$ defined by $T(x,y) = (x-y,\ x-2y)$ is singular or non-singular? Also find the formala for T^{-1} (if exists).

d) Differentiate between discrete and continuous random variable with suitable examples.

e) If $P(x=2)=9P(x=4)+90P(6)$ in the Poisson distribution, then find $E(x)$. (5×2)

UNIT - I

II. a) Write $\vec{X} = (3,10,7)$ as a linear combination of $\vec{u} = (1,3,-2)$, $\vec{v} = (1,4,2)$ and $\vec{w} = (2,8,1)$.

b) Solve the system:-
$x + y - 2z = 3;\ 4x - 2y + z = 5,\ 3x - y + 3z = 8$ using Gauss elimination method with partial pivoting.

c) Find condition on a, b, c so that $\vec{v} = (a,b,c)$ in R^3 belongs to S = Span $(\vec{u}\ \vec{v}\ \vec{w})$ where $\vec{u} = (1,2,0)$, $\vec{v} = (-1,1,2)$, $\vec{w}\ (3,0,-4)$. (3,4,3)

III. a) Examine whether $A = \begin{pmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{pmatrix}$ is diagonalizable or not? If yes, obtain the matrix P such that $P^{-1}AP$ is a digonalizable.

b) State Cayey – Hamilton theorem. Verify the same for $A = \begin{pmatrix} 1 & 2 & 0 \\ -1 & 1 & 2 \\ 1 & 2 & 1 \end{pmatrix}$

Also compute A^{-1} using it.

(5,5)

P.T.O.

IV. a) Let $T: R^4 \rightarrow R^3$ be the linear mapping defined by : $T(x,y,z,t)=(x-y+z+t,\ x+2z-t,\ x+y+3z-3t)$. Find a basis and the dimension of:-

i) image of T, ii) the kernel of T.

b) Let T be the linear operator on R^3 defined by $T(x,y,z)=(2y+z,x-4y,3x)$. Find the matrix representation of T relative to the basis $S=\{(1,1,1),(1,1,0),(1,0,0)\}$

(5,5)

UNIT - II

V. a) An urn contains six white and ten black balls. Another urn contains four white and three black balls. Two balls are drawn from the first urn one by one and placed into the second urn. Now, one ball is drawn from the second urn. What is the probability that it is a white ball?

b) There are three bags: first containing one white, two red, three green balls; second two white, three red, one green ball and third three white, one red and two green balls. Two balls are drawn from a bag chosen at random. These are formed to be one white and one red. Find the probability that the balls so drawn came from the second ball.

(5,5)

VI. a) A car hire firm has three cars which it hires out day by day. The number of demands for a car on each day is distributed as a Poisson variate with mean 1.4. Calculate the proportion of days on which:

i) neither car is used ii) some of the demand is refused

b) Find the moment generating function of the exponential distribution

$$f(x)=\frac{1}{c}e^{\frac{-x}{c}},0\le x\le\infty,c>0$$

Hence, find its mean and standard deviation.

(5,5)

VII. a) For two random variables x and y with the same mean, the two regression equations are $y = ax+b$ and $x = \propto Y + \beta$. Prove that $\frac{b}{\beta} = \frac{1-a}{1-\propto}$. Also find the common mean.

b) The pdf of the random variable (X,Y) is given by:-

$$f(x)=\begin{cases}\frac{1}{4}e^{\frac{-(x+y)}{2}} & ,x>o,y>0\\ 0,elsewhere & \end{cases}$$. Find the distribution of $\left(\frac{X-Y}{4}\right)$.

(5,5)

0-0-0