Propositional Logic

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We build AI systems that simulate the process of reasoning by using Logic Rules for Representation and Reasoning.

The Basics of Logic

A Knowledge Base contains sentences expressed using a **representation language**

A Syntax

$x+y=4$
Acceptable syntax in arithmetic representation.

$x4y+=$
Nonacceptable syntax in arithmetic representation.

Semantics

$x+y=4$
means that when x and y are added, the result is 4.

M(α): All models in which sentence α is true ➔ *All (x,y) values that satisfy $x+y=4$*

Entailment

$$\alpha \vDash \beta$$

α being true entails that β is also true.

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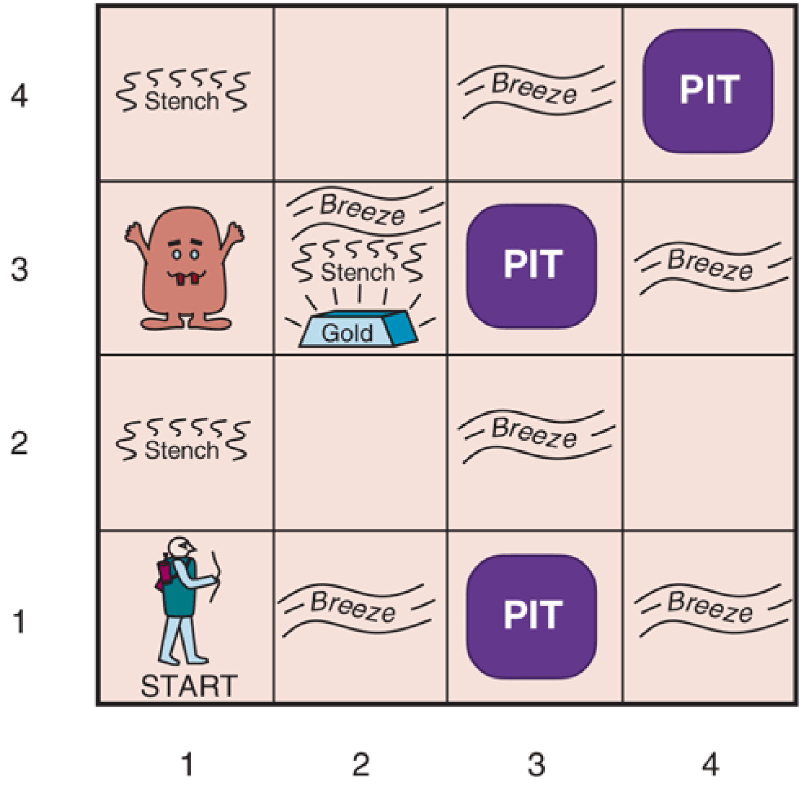

α = No breeze in square [1,1]
β = No pit in square [1,2]

No breeze in [1,1] entails no pit in [1,2]
Also,
No breeze in [1,1] entails no pit in [2,1]

Propositional Logic

Propositional Logic is a representation language. It is used to represent sentences in a Knowledge Base. It has syntax and semantics.

Propositional Logic Syntax

Proposition Symbols

Represents one fact that can be True or False.

Symbol	Meaning
$W_{1,3}$	Wumpus in cell [1,3]
$P_{3,1}$	Pit in cell [3,1]
$B_{2,1}$	Breeze in cell [2,1]
$G_{2,3}$	Gold in cell [2,3]

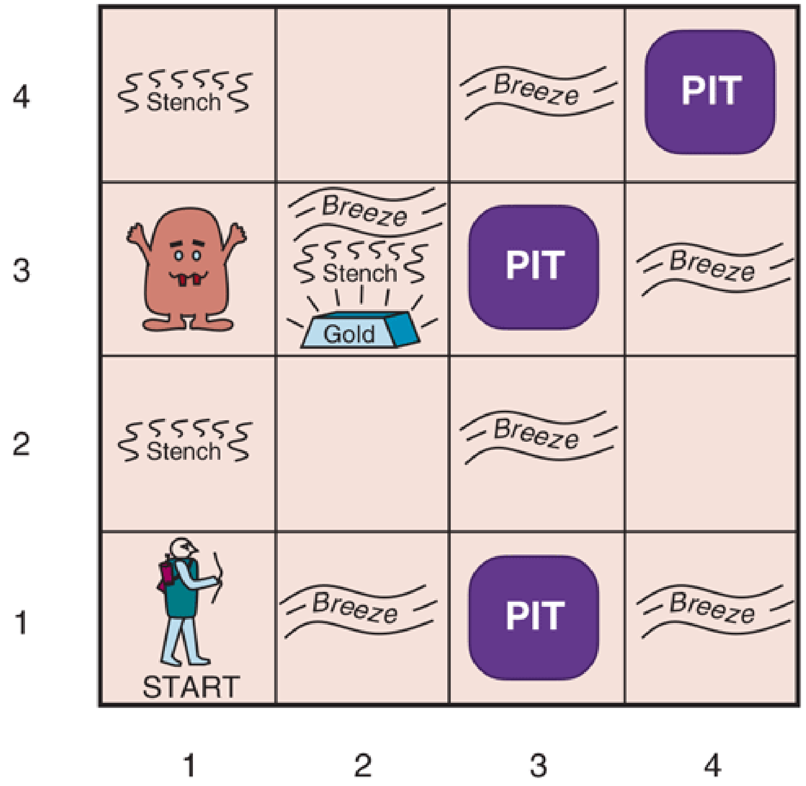

Fig 7.2, Russell & Norvig's Textbook

Propositional Logic Syntax

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One Propositional Symbol represents an atomic sentence. That's a simple fact about the world.

Propositional Logic Syntax

Logical Operators (Connectives):

Used to construct complex sentences from atomic sentences.

Symbol	Meaning
∨	OR (Disjunction)
∧	AND (Conjunction)
¬	Not
⇒	Implies
⇔	If an only If

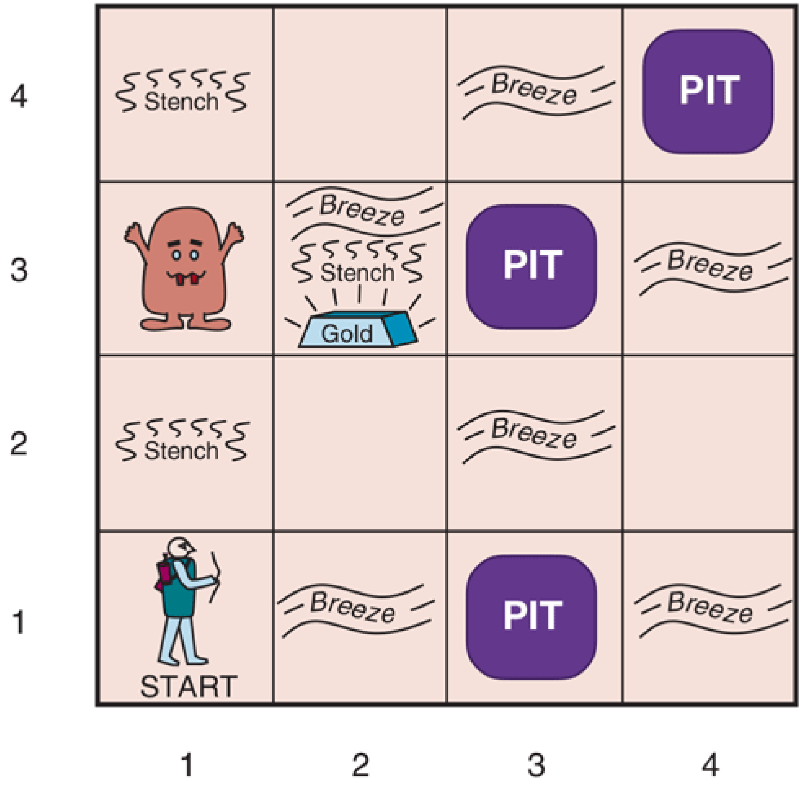


Fig 7.2, Russell & Norvig's Textbook

Propositional Logic Syntax

Logical Operators (Connectives):

Used to construct complex sentences from atomic sentences.

Sentence	Meaning
$W_{1,3} \lor P_{3,1}$	Wumpus in cell [1,3] **OR** Pit in cell [3,1]
$W_{1,3} \land P_{3,1}$	Wumpus in cell [1,3] **AND** Pit in cell [3,1]
$\neg W_{1,3}$	**Not** Wumpus in cell [1,3]

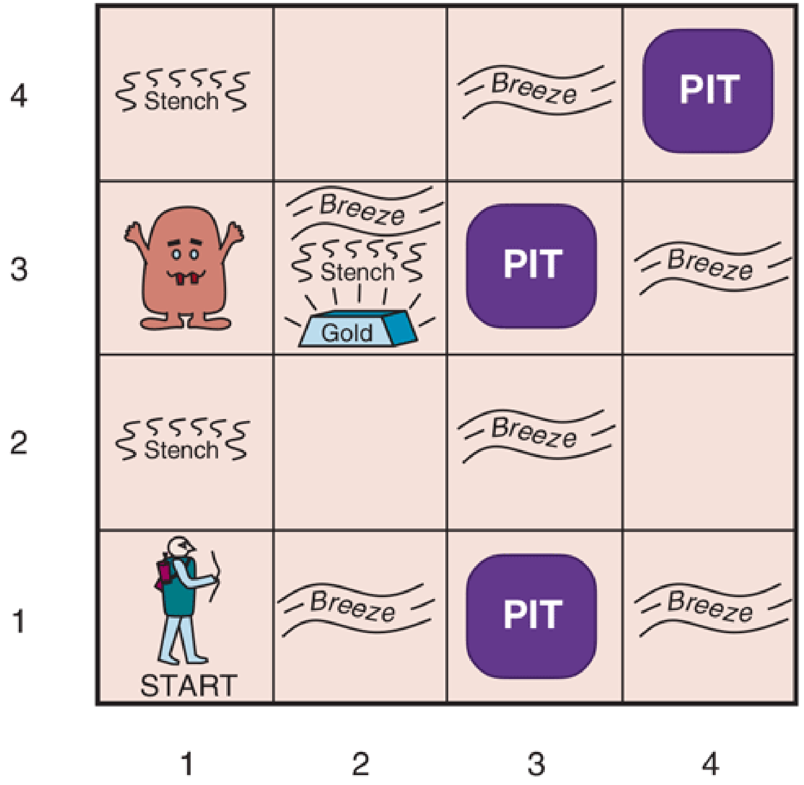


Fig 7.2, Russell & Norvig's Textbook

Propositional Logic Syntax

Logical Operators (Connectives):

⇒ Implies

Premise ⇒ Conclusion

Antecedent ⇒ Consequence

$$W_{1,3} \Rightarrow \neg W_{2,2}$$

Wumpus in [1,3] *Implies No Wumpus in* [2,2]

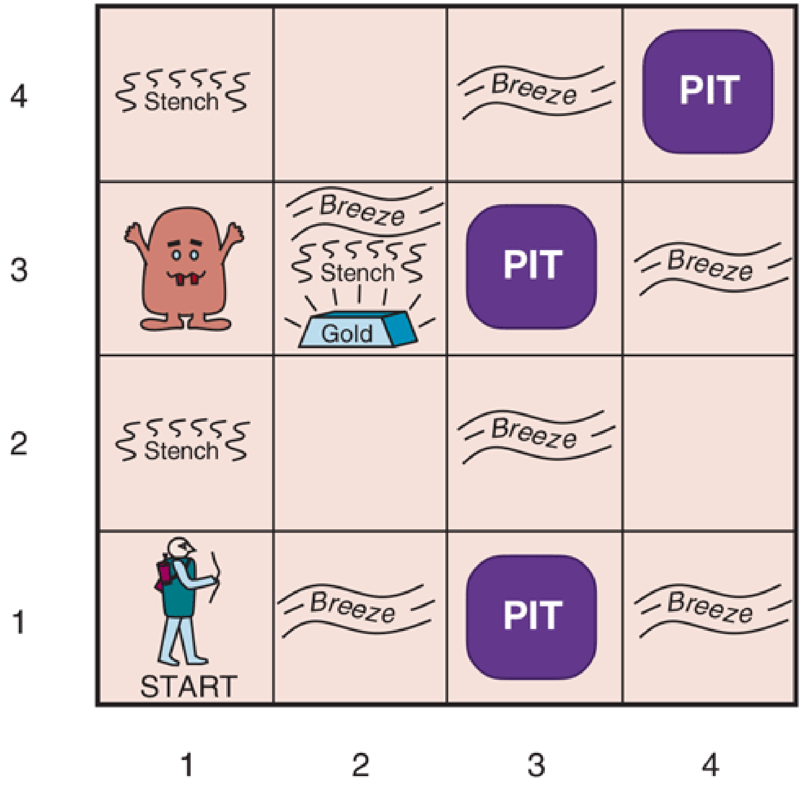


Fig 7.2, Russell & Norvig's Textbook

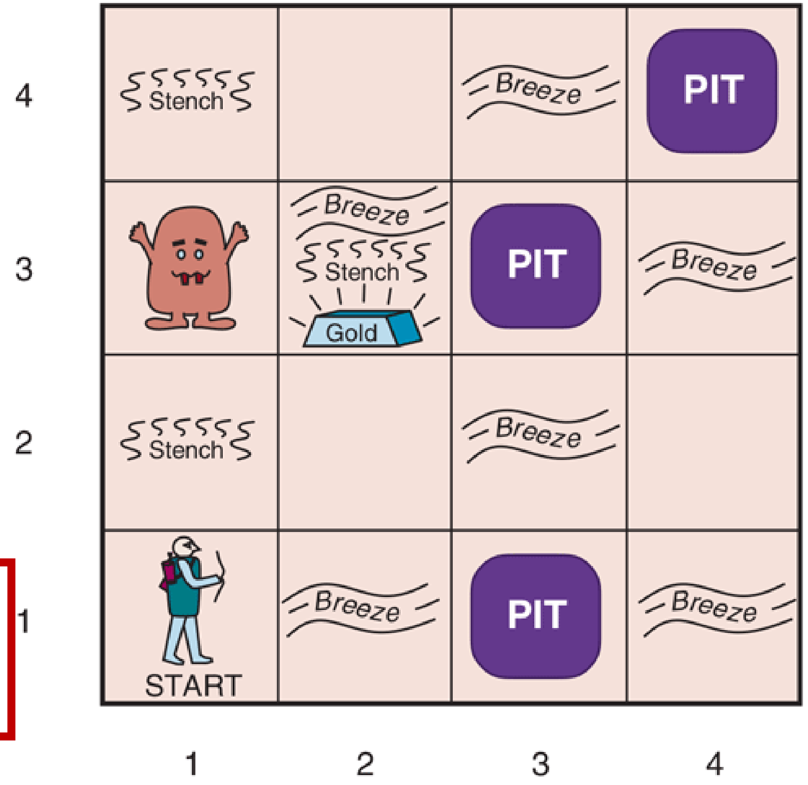
Propositional Logic Syntax

Logical Operators (Connectives):

$\Leftrightarrow$ If and Only If

$$B_{1,1} \Leftrightarrow P_{1,2} \lor P_{2,1}$$

There's a Breeze in $[1,1]$ *If and Only If* *there's a Pit in [1,2] or there's a pit in [2,1]*.



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Propositional Logic Semantics

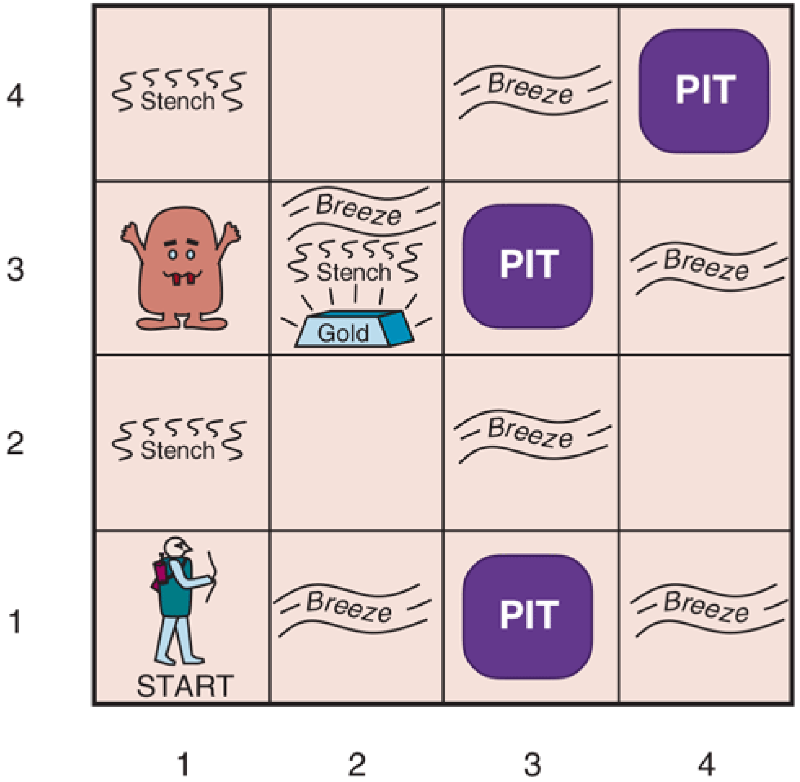


Fig 7.2, Russell & Norvig's Textbook

Specify the truth of a sentence for a given model (values for the proposition symbols).

$$\boldsymbol{Sentence = P_{1,2} \lor (P_{2,2} \land P_{3,1})}$$

$$\boldsymbol{m1 = \{P_{1,2} = False, P_{2,2} = False, P_{3,1} = True\}}$$

$$\boldsymbol{m1 = False\ OR\ (False\ AND\ True)}$$

$$\boldsymbol{= False\ OR\ False}$$

$$\boldsymbol{= False}$$

Propositional Logic Semantics

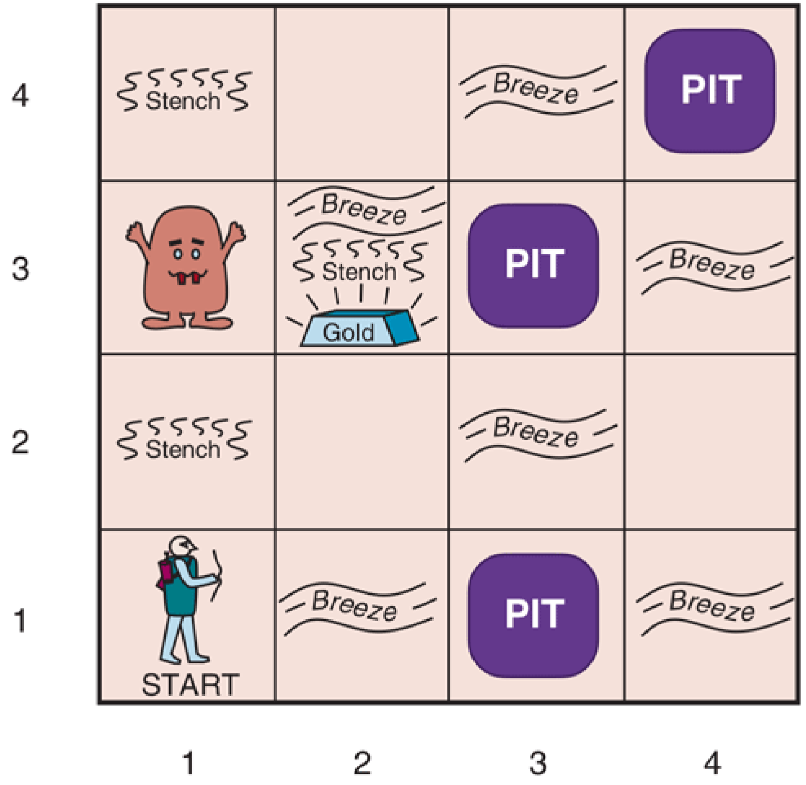


Fig 7.2, Russell & Norvig's Textbook

Five Semantic Rules for Complex Sentences:

- $\neg P$ is true iff P is false in m.
- $P \wedge Q$ is true iff both P and Q are true in m.
- $P \vee Q$ is true iff either P or Q is true in m.
- $P \Rightarrow Q$ is true unless P is true and Q is false in m.
- $P \Leftrightarrow Q$ is true iff P and Q are both true or both false in m.

Create a Knowledge Base for the Wumpus World using Propositional Logic

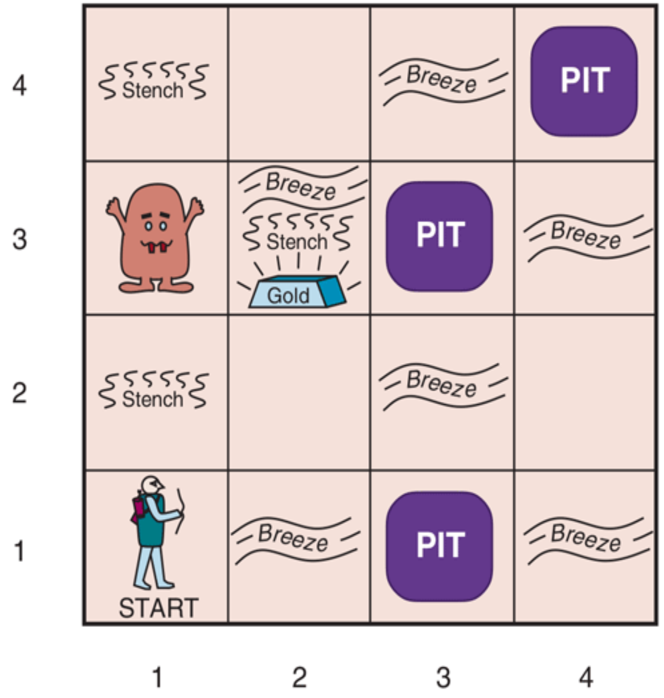


Fig 7.2, Russell & Norvig's Textbook

Step 1: Define Symbols

$P_{x,y}$ is true if there is a pit in $[x, y]$.

$W_{x,y}$ is true if there is a wumpus in $[x, y]$, dead or alive.

$B_{x,y}$ is true if there is a breeze in $[x, y]$.

$S_{x,y}$ is true if there is a stench in $[x, y]$.

$L_{x,y}$ is true if the agent is in location $[x, y]$.

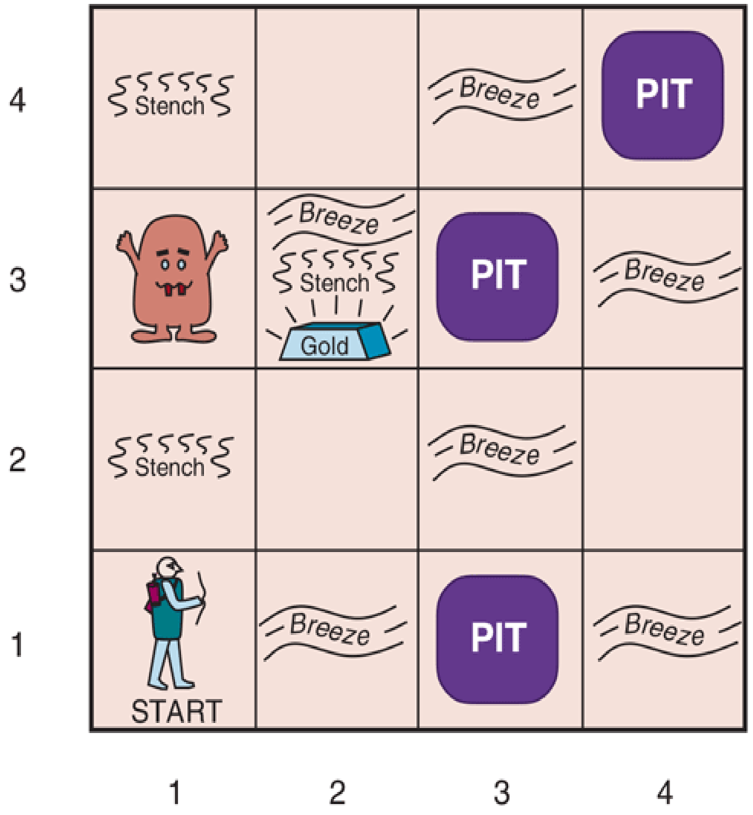


Fig 7.2, Russell & Norvig's Textbook

Step 2: Express Knowledge using Symbols

Each Rule is labeled as R_i:

- There is no pit in [1,1]: $R_1: \neg P_{1,1}.$
- Include breeze percept for the first two squares:

$$R_2: \neg B_{1,1}.$$

- A square is breezy IFF there is a pit in a neighboring cell: $R_3: B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1}).$

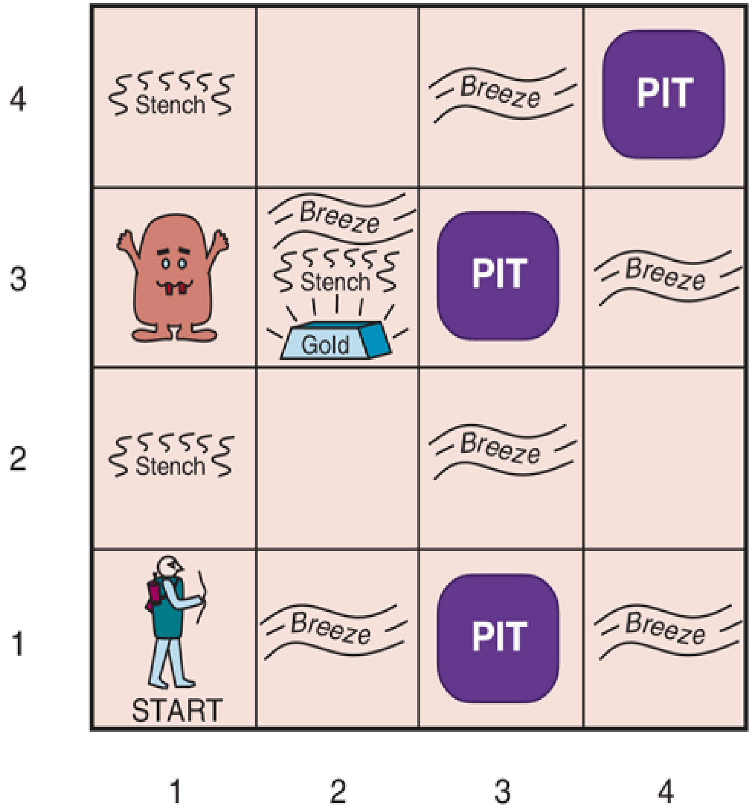


Fig 7.2, Russell & Norvig's Textbook

Step 3: Infer New Knowledge from Old One!

Check for entailments: $\boxed{KB \models \alpha}$

- These are the symbol in my KB:

$\boxed{B_{1,1}}$ $\boxed{P_{1,1}}$ $\boxed{P_{1,2}}$ $\boxed{P_{2,1}}$

- I would like to find if the KB entails that there is, or isn't, a pit in locations [1,2] or [2,1].

- Check if the KB entails α, where α could refer to $P_{1,2}$ or $P_{2,1}$.

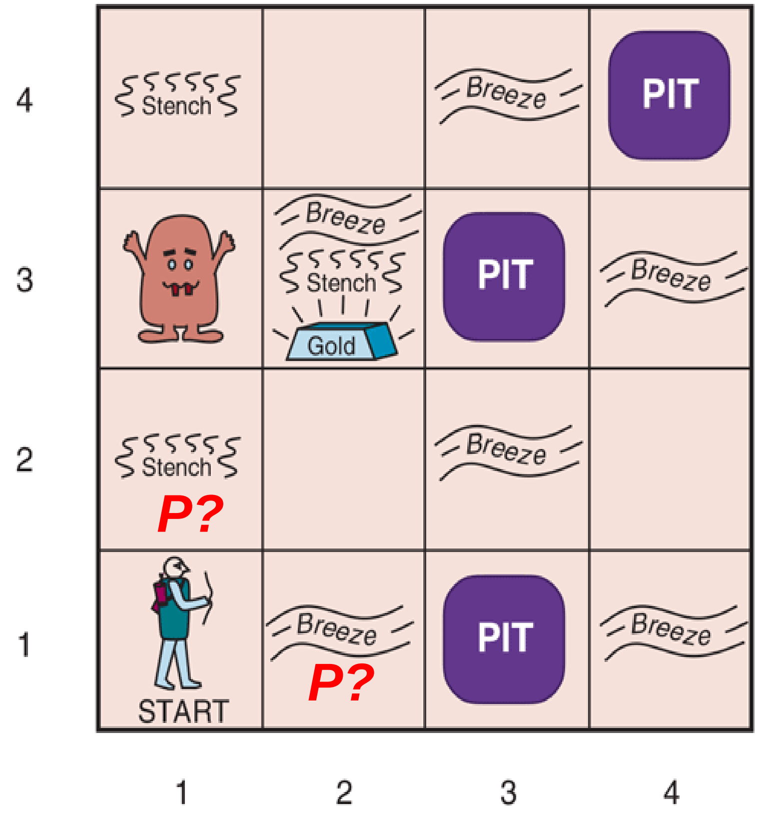


Fig 7.2, Russell & Norvig's Textbook

Step 3: Infer New Knowledge from Old One!

Check for entailments: $\boxed{KB \models \alpha}$

- Create a Truth-Table for all symbols in KB.
- Set the values we know $B_{1,1} = False, P_{1,1} = False$.
- Consider all possible values for the symbols we don't know that we do not know: $P_{1,2}, P_{2,1}$

$B_{1,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	R_1	R_2	R_3
False	False	True	True			
False	False	True	False			
False	False	False	True			
False	False	False	False			

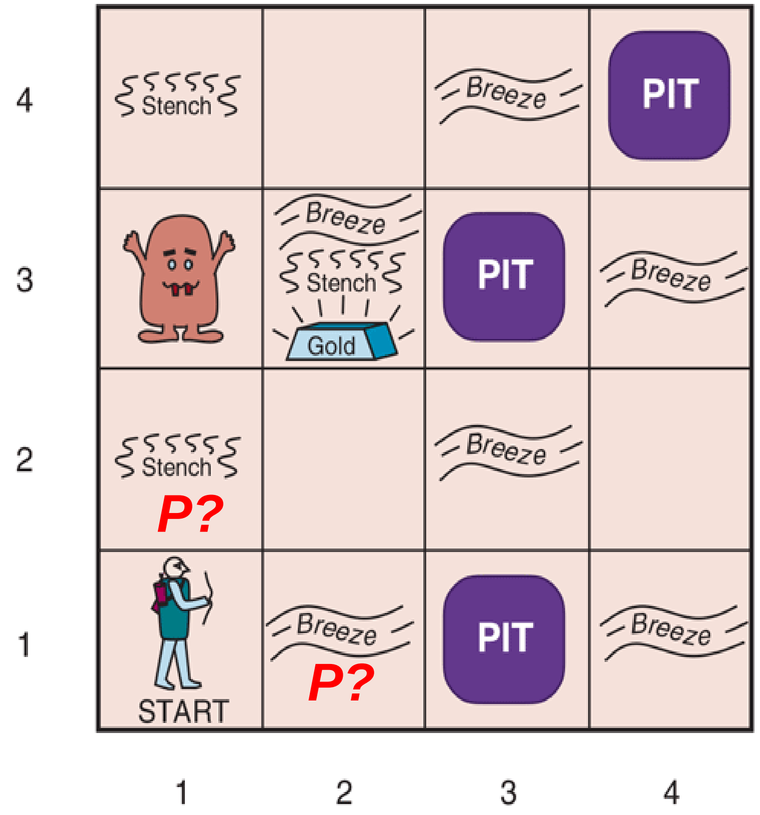


Fig 7.2, Russell & Norvig's Textbook

Step 3: Infer New Knowledge from Old One!

Check for entailments: $\boxed{KB \models \alpha}$

- Check if some values of $P_{1,2}, P_{2,1}$ satisfy all rules:

$\boxed{R_1 : \quad \neg P_{1,1}.}$ $\boxed{R_2 : \quad \neg B_{1,1}.}$ $\boxed{R_3 : \quad B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1}).}$

$B_{1,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	R_1	R_2	R_3
False	False	True	True	True	True	False
False	False	True	False	True	True	False
False	False	False	True	True	True	False
False	False	False	False	True	True	True

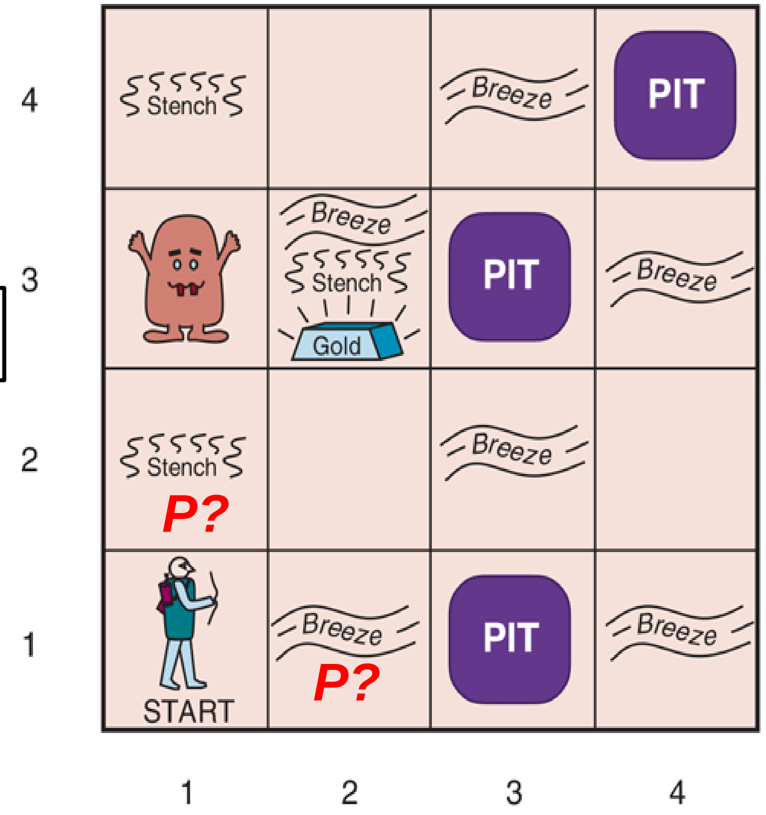


Fig 7.2, Russell & Norvig's Textbook

There is one case in which All Rules are Satisfied: $P_{1,2} = False, P_{2,1} = False$.
Therefore, we infer that there is no Pit in either locations.

Step 3: Infer New Knowledge from Old One!

Let the agent move to [2,1].

Now repeat the same inference exercise to find out if there's a Pit in locations [2,2] and [3,1].

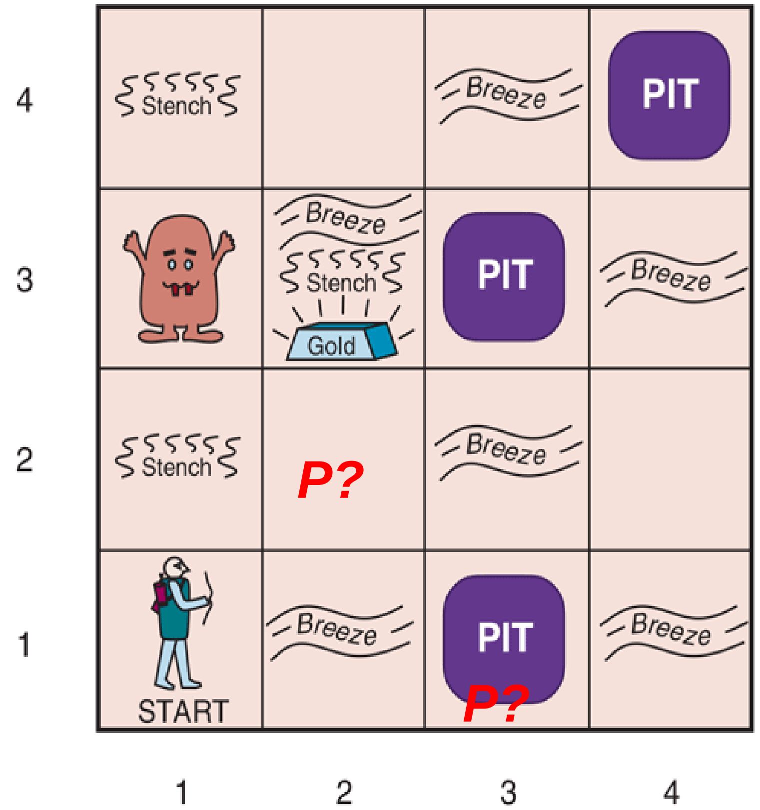


Fig 7.2, Russell & Norvig's Textbook

Step 2: Express Knowledge using Symbols

Each Rule is labeled as R_i:

- There are no pits in [1,1], [2,1]:

$$R_1 : \quad \neg P_{1,1} .$$

$$R_2 : \quad \neg \ P_{2,1} .$$

- Include breeze percept for the first two squares:

$$R_4 : \quad \neg B_{1,1} .$$
$$R_5 : \quad B_{2,1} .$$

- A square is breezy IFF there is a pit in a neighboring cell:

$$R_3 : \quad B_{2,1} \Leftrightarrow (P_{1,1} \lor P_{2,2} \lor P_{3,1}) .$$

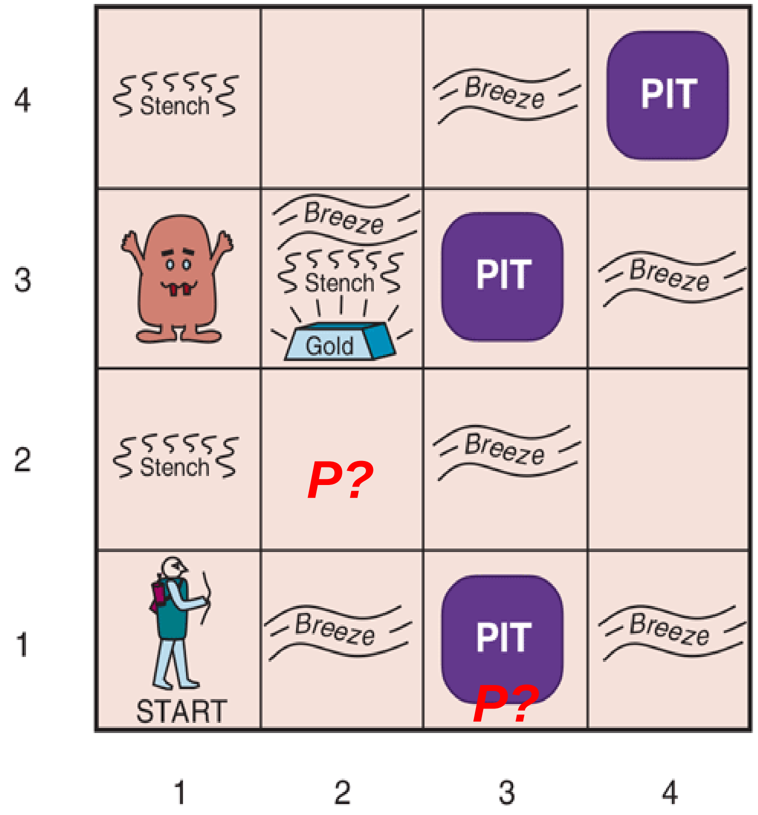


Fig 7.2, Russell & Norvig's Textbook

Step 3: Infer New Knowledge from Old One!

Check for entailments: $\boxed{KB \models \alpha}$

- Create a Truth-Table for all symbols in KB.
- Set the values we know
  - $B_{1,1} = False,$
  - $B_{2,1} = True,$
  - $P_{1,1} = False,$
  - $\mathrm{P}_{2,1} = \mathrm{False}.$
- Consider all possible values for the symbols we do not know: $P_{3,1}, P_{2,2}$

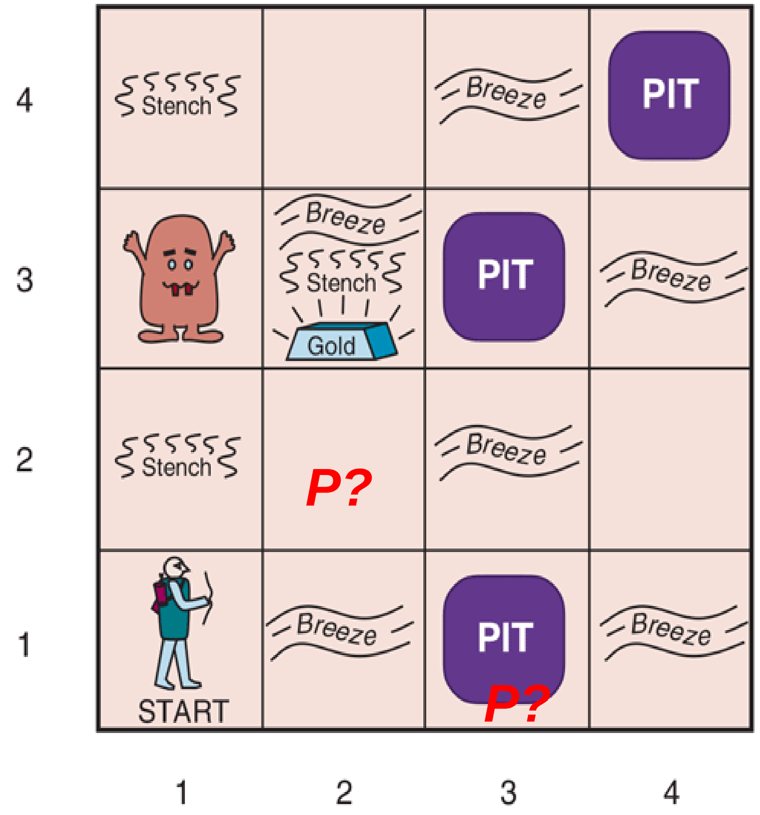


Fig 7.2, Russell & Norvig's Textbook

Step 3: Infer New Knowledge from Old One!

Check for

$R_1 : \quad \neg$

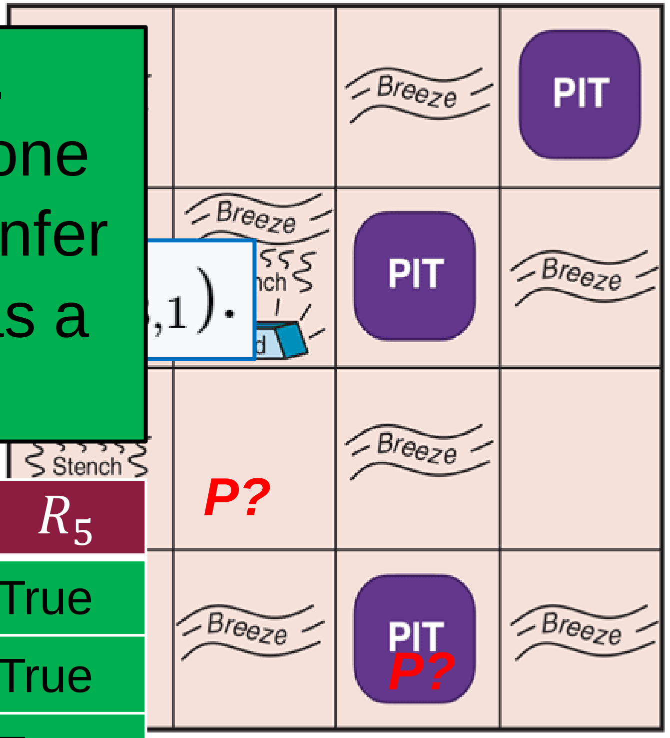
$R_2 : \ \neg$

$,1)$.

There are 3 cases in which the KB is all True. However, $P_{2,2}$ is True in two cases and False in one case. Same for $P_{3,1}$. This means that we cannot infer from current knowledge whether [3,1] or [2,2] has a Pit!

$B_{1,1}$	$P_{1,1}$	$P_{2,1}$	$B_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5
False	False	False	True	True	True	True	True	True	True	True
False	False	False	True	True	False	True	True	True	True	True
False	False	False	True	False	True	True	True	True	True	True
False	False	False	True	False	False	True	True	False	True	True

2



2 3 4

Russell & Norvig's Textbook

Step 3: Infer New Knowledge from Old One!

Check for entailments: $\boxed{KB \models \alpha}$

KB cannot entail whether [3,1] has a Pit or not,
and whether [2,2] has a Pit or not!

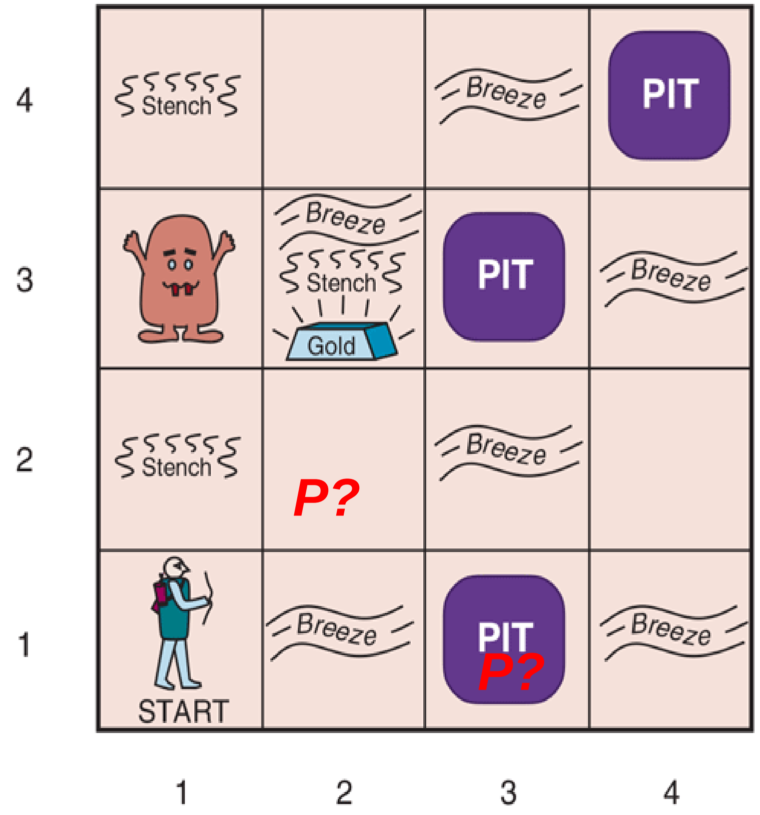


Fig 7.2, Russell & Norvig's Textbook

Propositional Logic

Inference tries to find if the **KB entails** a certain sentence α by checking if the value of α is True whenever the KB is **True**.

Propositional Logic provides means to **represent** logical facts about the world (KB), and inference methods to **reason** and **infer** new knowledge from current knowledge.