First-Order Logic

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We explored using Propositional Logic for Knowledge Representation and Reasoning.

Propositional Logic has **limitations** in representing knowledge. **First-order logic (FOL)** have more representation power.

FOL represents the world as

- Objects
- Relationships between Objects
- Attributes of objects

Syntax & Semantics of FOL

Sentences in natural languages have ***nouns***, which refer to ***objects***.

Sentences in FOL have ***objects***.

Objects can be People, Places, Numbers, etc.

Sentences in natural languages have ***verbs***, which represent, along with adjectives & verbs, ***relations*** between objects.

Sentences in FOL have ***functions***.

Relations can "Father of", "Part of", etc.

Relations can also describe object properties!

Relations describing properties: red, round, large, etc.

Example: *Squares neighboring the Wumpus are smelly!*

Objects:

- *Squares*
- *Wumpus*

Relations:

- *Neighboring*
- *Smelly (property of square)*

Atomic Sentence

Represents an object, relation or property.

- $Richard$
- $John$
- $Crown$

- $Person(Richard)$
- $Person(John)$
- $King(John)$
- $LeftLeg(Richard)$
- $LeftLeg(John)$

- $Brother(Richard, John)$
- $Brother(John, Richard)$
- $OnHead(Crown, John)$

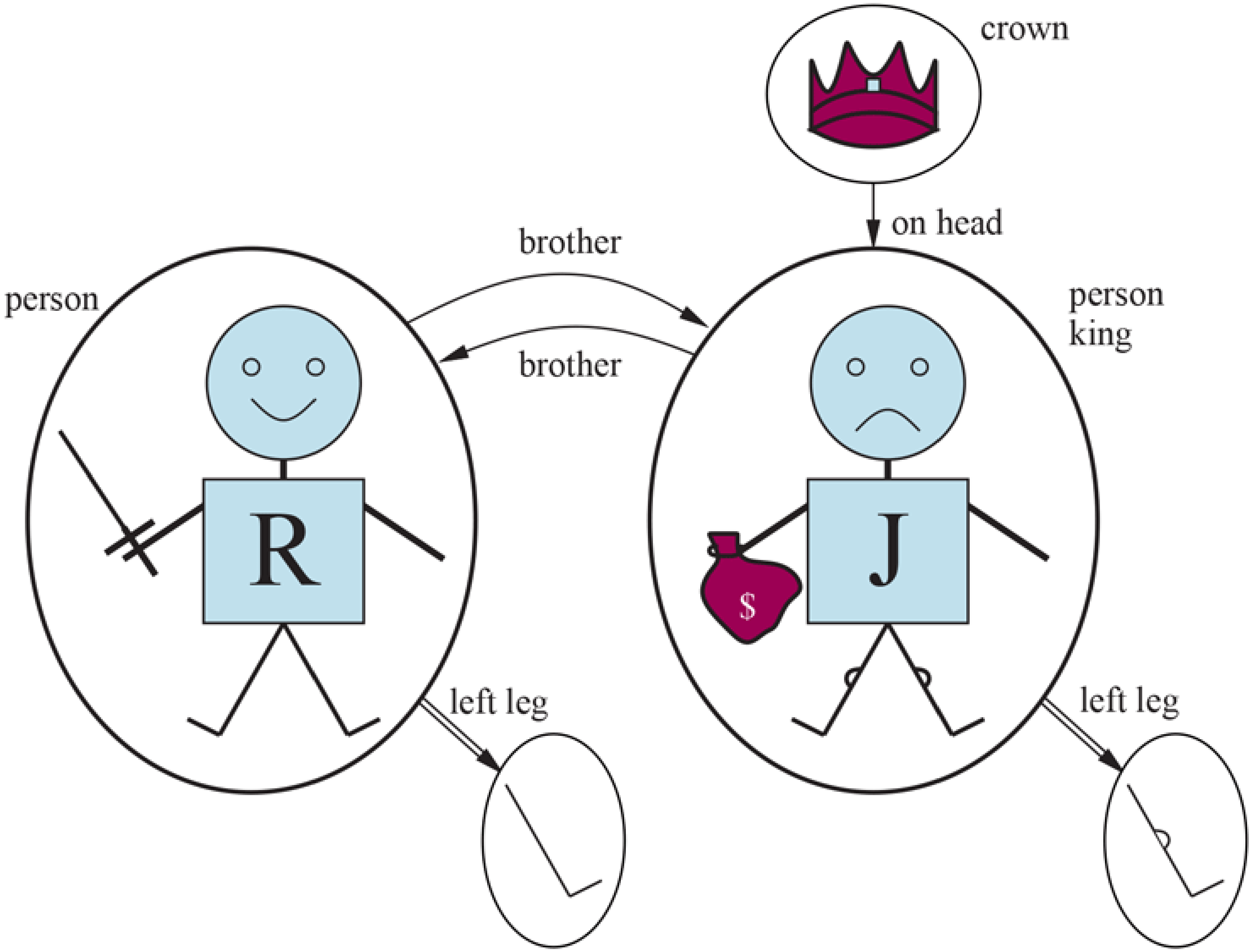

Fig 8.2, Russell & Norvig’s Textbook

Complex Sentences

Contain Logical Connectives

- $King(Richard) \lor King(John)$
- $\neg King(Richard) \Rightarrow King(John)$

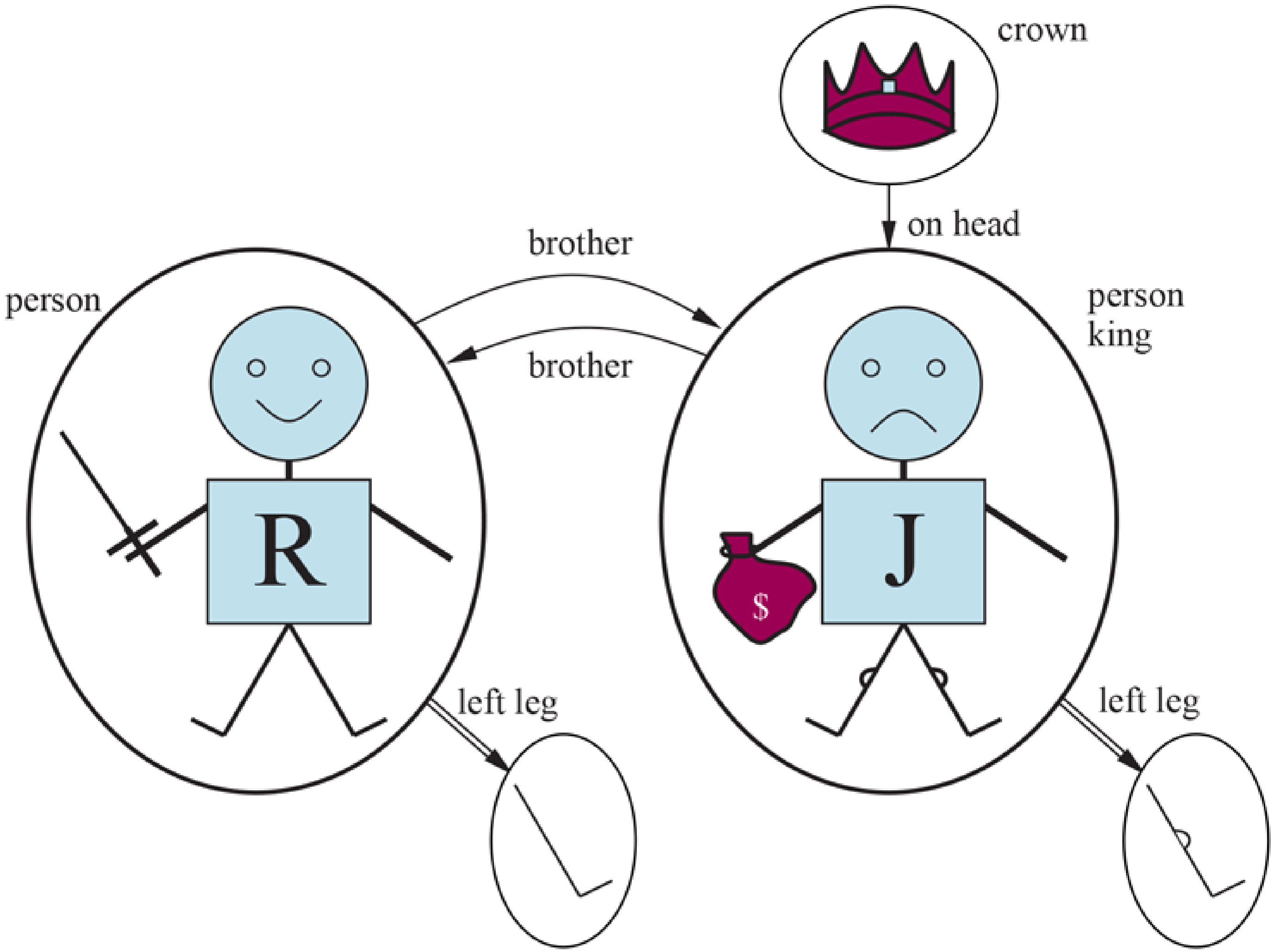

Fig 8.2, Russell & Norvig's Textbook

Complex Sentences

Quantifiers

Very powerful feature of FOL. Allows for expressing general rules.

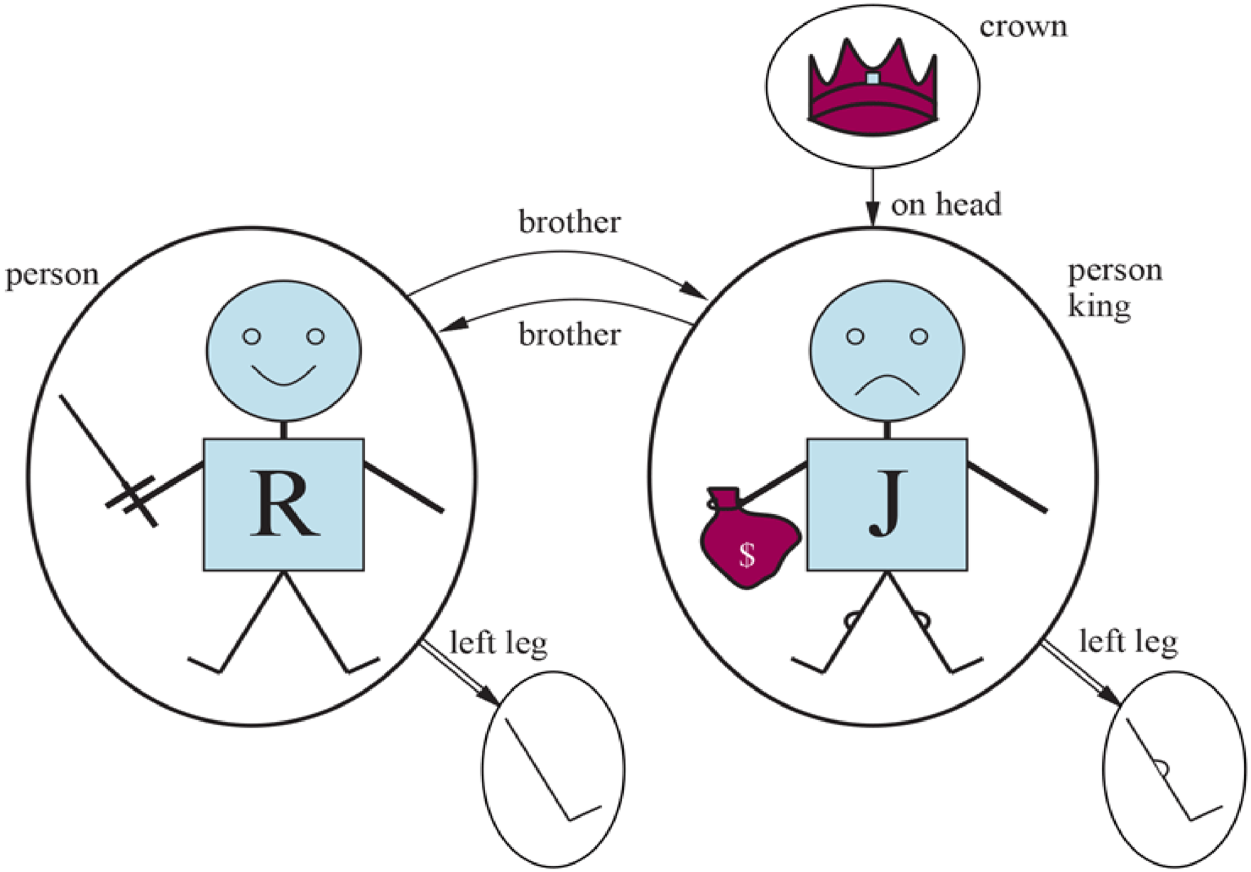

Fig 8.2, Russell & Norvig's Textbook

Complex Sentences

Quantifiers

Universal Quantifier
$\forall$: *For All*

Every King is a Person

$$\forall x\ King(x) \Rightarrow Person(x)$$

For all x, if x is a king, then x is a person.

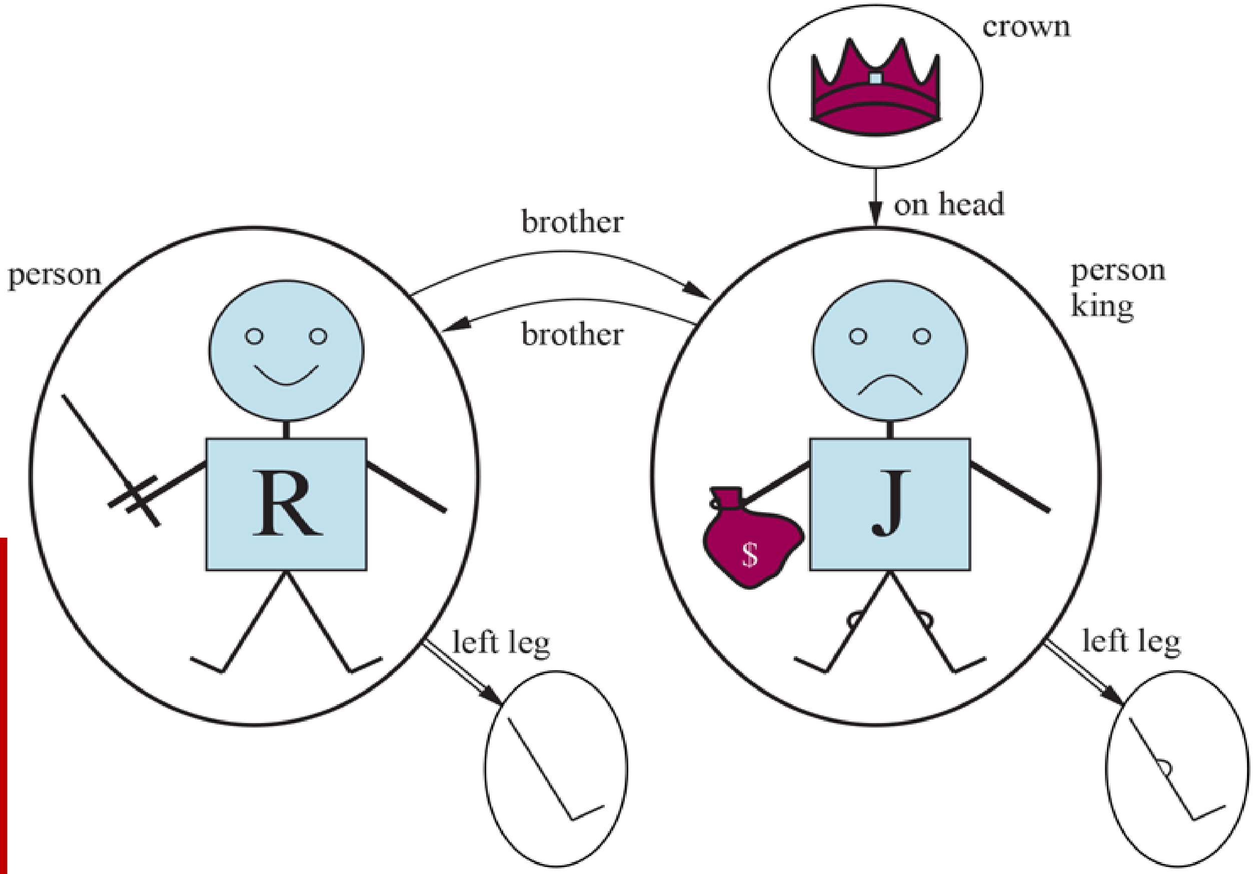

Fig 8.2, Russell & Norvig's Textbook

Complex Sentences

Quantifiers

Existential Quantifier

$\exists$: *There Exists*

There is a crown on King John's head

$$\exists x\ Crown(x) \wedge OnHead(x, John)$$

There exists an x, such that x is a crown, and it is on John's head.

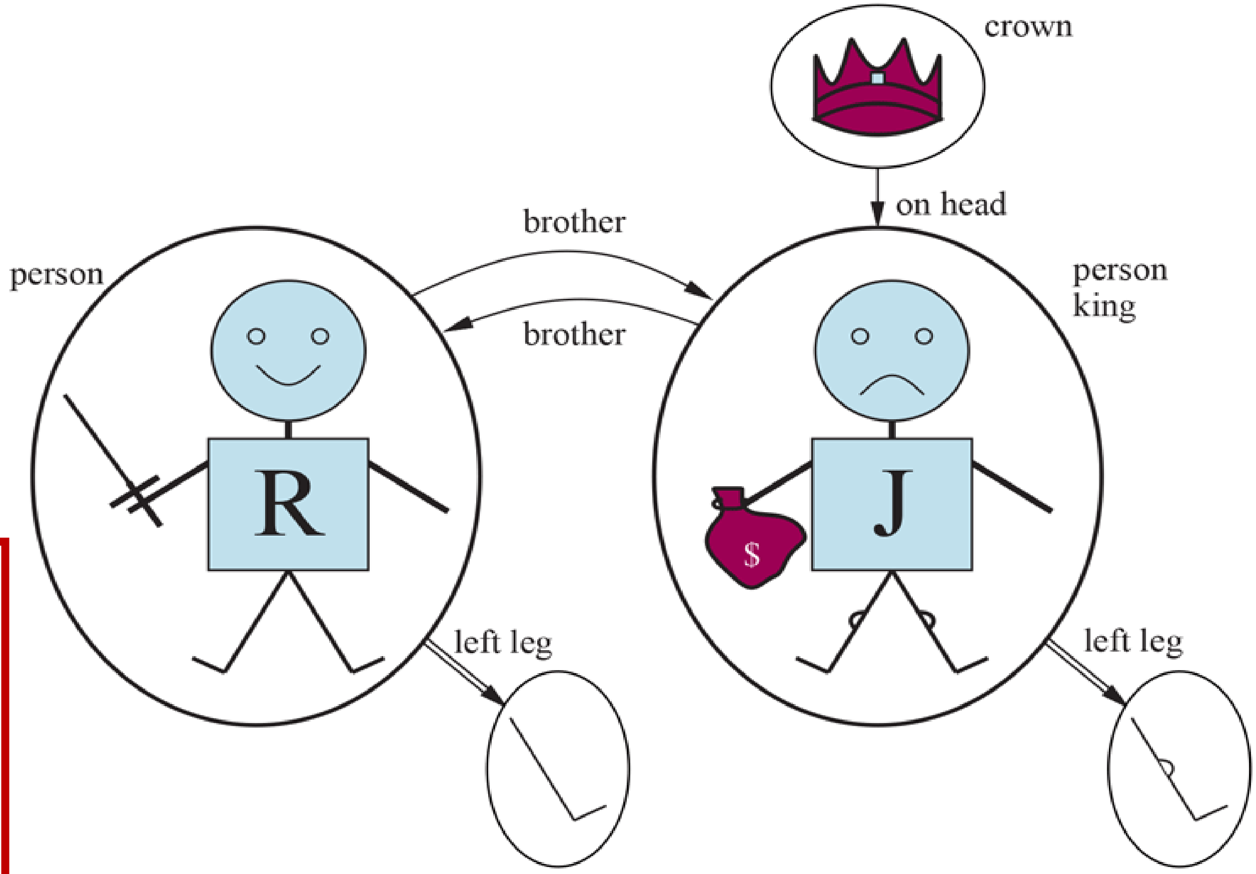

Fig 8.2, Russell & Norvig's Textbook

Nesting Quantifiers

Quantifier order changes the meaning

$\forall x \ \exists y \ Loves(x, y)$

For all x, there exists a y where x loves y.

Everybody Loves Somebody.

$\exists y \ \forall x \ Loves(x, y)$

There exists a y, such that all x loves y.

There is someone who is loved by everyone.

Equality

To imply that two symbols represent the same object

$$\boldsymbol{Father(John) = Henry}$$

Unique-Names Assumption

No two distinct objects are represented with the same symbol

By default, two different symbols represent two different objects; unless otherwise specified by an equality.

$$Brother(John, Richard) \land Brother(Geoffry, Richard)$$

Closed-World Assumption

Atomic sentences that are not known to be true are considered false.

All natural sciences are based on the closed-world assumption!

First-Order Logic provides means to represent the world as objects with properties and relations among them, as well as representing rules that apply to the objects.