The Noisy Work of Uncertainty Visualisation Research: A Review

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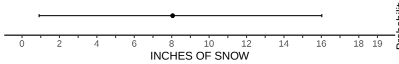
1 Background

From entertainment choices to news articles to insurance plans, the modern citizen is so over run with information in every aspect of their life it can be overwhelming. In this overflow of information, tools that can effectively summarize information down into simple and clear ideas become more valuable. Information visualisations remain one of the most powerful tools for fast and reliable science communication.

Visualization is an important step in exploratory data analysis and it is often utilised to **learn** what is important about a data set. Datasets such as Anscombe's quartet (Anscombe 1973) or the Datasaurus Dozen (Locke and D'Agostino McGowan 2018) highlight this power in visualisation. Additionally, visualisations allow for efficient and memorable communication. Even something as simple as sketching a distribution before recalling statistics or making predictions can greatly increase the accuracy of those measures (Hullman et al. 2018; Goldstein and Rothschild 2014).

Uncertainty visualisation is a relatively new field in research. Early papers that specifically reference "uncertainty visualisation" appear in the late 80s (Ibrekk and Morgan 1987), with geospatial information visualisation literature in the early 90s declaring this to be essential aspect of information display (MacEachren 1992; Carr, Olsen, and White 1992). These early experiments typically involved showing participants a distribution, such as those depicted in Figure 1, and asking the viewers to extract a probability or average. Despite the new terminology visualisation of uncertainty has been present since the earliest times. For example, box plots or histograms can be considered to be displaying uncertainty in the sense of variability in observations sampled from a population distribution. Today, there is an abundance of publications on the topic which makes it is timely to construct a review of the field. In fact, there have already been several reviews published.

Reviews on uncertainty visualisation rarely offer tried and tested rules for effective uncertainty visualisation, but rather comment on the *difficulties* faced when trying to summarize the field. Kinkeldey, MacEachren, and Schiewe (2014) found most experiments on the methods for uncertainty visualisation evaluation to be ad hoc, with no commonly agreed upon methodology or formalisation and no greater goal of describing general principals. Hullman (2016) commented on the difficulty in taking overarching themes from uncertainty visualisation, as several conflated issues make it unclear if subjects did poorly in an experiment because they misunderstood a visualisation, because the question was misinterpreted, or because they used a specific heuristic. Spiegelhalter (2017) commented that different plots are good for different things, and disagreed with the goal of identifying a universal "best" plot for all people and circumstances. Griethe and Schumann (2006) was unable to find common themes, but instead listed the findings and opinions of a collection of papers. L. Padilla, Kay, and Hullman (2022) summarized several cognitive effects that have repeatedly arised in uncertainty literature, however these effects were each discussed in isolation as a list of considerations an author might make. While these reviews are thorough in scope, none discuss how the existing literature con-



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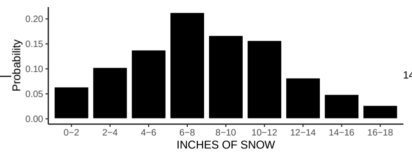

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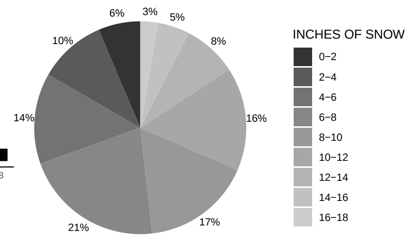

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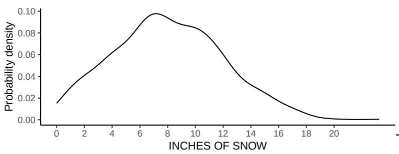

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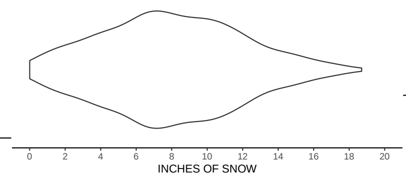

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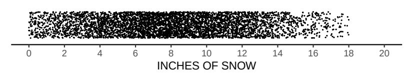

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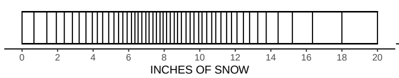

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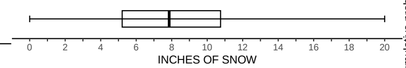

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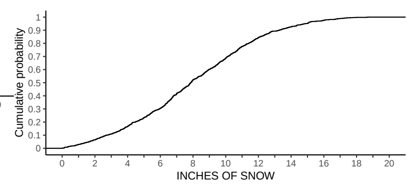


(i) Picture 9

Figure 1: A replication of the the uncertainty visualisations shown by Ibrekk and Morgan (1987) in one of the earliest uncertainty visualisation experiments. This early experiment is a good example of many of the issues that are still common in uncertainty visualisation today. For example, the '95% confidence interval' is more accurately a '95% prediction interval'. Additionally graphics that depict different mathematical objects that are also different on their visual components are compared because of a percieved relation to uncertainty. So visualisations of the mean, PDF and CDF, are all discussed as though they all contain the relevant statistical information. The axis also have different scales, and visualisation methods that are now unpopular for displaying proportions, such as a pie chart, are used.

tribute to the commonly state goal of uncertainty visualisation, scientific transparency. The problem faced by the literature is easily summarized with a famous quote by Henri Poincaré.

> "Science is built up of facts, as a house is built of stones; but an accumulation of facts is no more a science than a heap of stones is a house." - Henri Poincaré (1905)

That is to say, despite the wealth of reviews, the field of uncertainty visualisation remains a heap of stones. This review attempts to address this issue by offering a novel perspective on the uncertainty visualisation problem, and hopefully laying the foundations on which we can build a house.

This review is broken into several parts that each reflect a different approach to uncertainty visualisation. First we look at graphics that ignore uncertainty entirely and discuss why uncertainty should be included at all. Second, we look at methods that consider uncertainty to be just another variable and discuss the characteristics of uncertainty that make it a unique visualisation problem. Third, we look at methods that explicitly combine our estimate and its uncertainty and discuss if the visualisations created by these transformations are still "uncertainty visualisations". Fourth, we will discuss methods that implicitly include uncertainty by depicting a sample or original data in place of an estimate. Finally, we discuss how uncertainty visualisations can be effectively evaluated. When discussing each of these methods, we consider the *purpose* of uncertainty visualisation and comment on how effective each visualisation is at fulfilling that purpose.

Due to the fields origins and focus in geospatial information visualisation, there have been a large number of suggested variations on the choropleth map that allow authors to include uncertainty. We will use these maps to provide simple examples for each approach that can be easily compared to the "no uncertainty" choropleth map to better understand the costs and benefits of each approach. Despite the example of each method focusing on variations of the choropleth map, it is important to understand that the approaches we are discussing are universal and are not unique to maps.

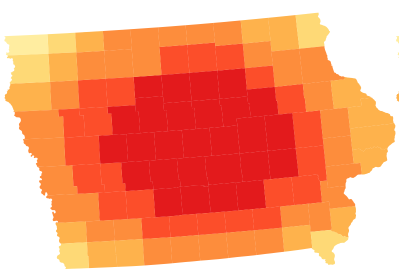
2 Ignoring uncertainty

A good place to start might be at deceptively straight forward question, why should we include uncertainty at all?

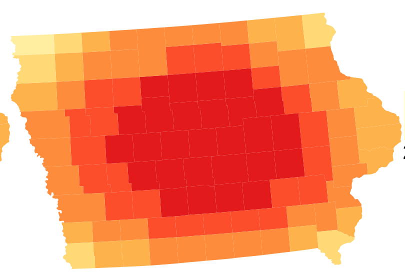
2.1 What is uncertainty

2.2 The choropleth map

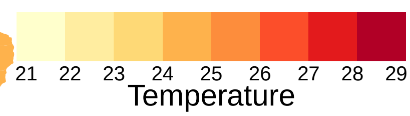
Figure 2 depicts a choropleth map of the counties of Iowa. Each of these counties are colored according to an estimate of average daily temperature that was simulated so that the values followed a clear spatial trend (hot in the middle of the map, and cold on the outside). The variance of these estimates were simulated such that a hypothesis test would indicate the existence of a spatial trend in the low variance map, while the trend in the high variance map should be indistinguishable from noise. Is this distinction in validity of the spatial trend clear in in the map? Is the validity of the trend communicated through the visualisation?



(a) Low Variance Data



(b) High Variance Data



(c) Choropleth Palette

Figure 2: Two choropleth maps that depict the counties of Iowa where each country coloured acording to a simulated average temperature. Both maps depict a spatial trend, where counties closer to the center of the map are hotter than counties on the edge of the map. This trend is only technically there in the low variance condition, i.e. if we performed a hypothesis test we would conclude the trend exists. The high variance condition displays a spatial trend that could simply be the result of noise, which means the plot is displaying a false conclusion.

2.3 Signal-supression

The two choropleth maps appearing to be identical in Figure 2 highlights the need for uncertainty visualisations. Uncertainty visualisation is is required for transparency and this sentiment has been repeated. Some authors suggest uncertainty is important to include as it communicates the legitimacy (or illegitimacy) of the conclusion drawn from visual inference

(Correll and Gleicher 2014; Kale et al. 2018; Griethe and Schumann 2006). Some authors have said that uncertainty should be included to degree of confidence or trust in the data (Boukhelifa et al. 2012; Zhao et al. 2023). Some authors directly connect uncertainty visualisation to hypothesis testing as it ensures the "validity" of a statement (Hullman 2020; Griethe and Schumann 2006), but allows for a proportional level of trust that is more detailed than the binary results of a hypothesis test (Correll and Gleicher 2014; Correll, Moritz, and Heer 2018). Some authors even go so far as to claim that failing to include uncertainty is akin to fraud or lying (Hullman 2020; Manski 2020).

This consensus leads us to understand that uncertainty visualisation is motivated by the need for a sort of "visual hypothesis test". A successful uncertainty visualisation would act as a "statistical hedge" for any inference me make using the graphic. Additionally, since the purpose of a visualisation is to give a quick "gist" of the information (Spiegelhalter 2017), this hedging needs to be communicated visually without any need for computation from the viewer. Additionally, Ndlovu, Shrestha, and Harrison (2023) found that participants applied the same methods they used for simple choropleth maps to complicated uncertainty maps even if that take away was invalid. Therefore, this hedging effect needs to be communicated simply through the visualisation. If we refer to the conclusion we draw from a graphic to be its "signal" and the variance that makes this signal harder to identify as the "noise", we can summaries this information into three key requirements. A good uncertainty visualisation needs to:

1) Reinforce justified signals to encourage confidence in results
2) Hide signals that are just noise to prevent unjustified conclusions
3) Perform tasks 1) and 2) in a way that is proportional to the level of confidence in those conclusions.

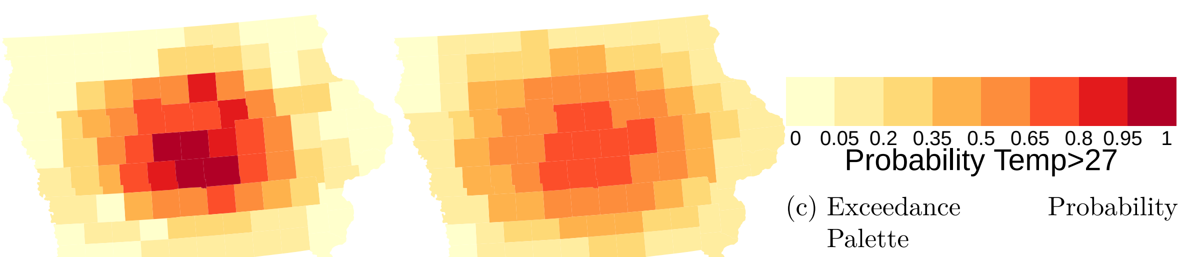
As Figure 2 showed, visualisations that are unconcerned with uncertainty have no issue showing justified signals, but struggle with the display of unjustified signals. Therefore, we coin this approach to uncertainty visualisation as "signal-suppression" since it primarily differentiates itself from from the "noiseless" visualisation approach through criteria (2). That is, the main difference between an uncertainty visualisation and a "normal" visualisation is that an uncertainty visualisation should prevent us from drawing unjustified conclusions.

2.4 Uncertainty as a signal

Uncertainty visualisation is not only motivated by signal-suppression, and we would be remiss to ignore these alternative motivations. Some authors claim the purpose of uncertainty is to improve decision making (Ibrekk and Morgan 1987; L. Padilla, Kay, and Hullman 2022; Hullman 2016; Cheong et al. 2016; Boone, Gunalp, and Hegarty 2018; L. M. Padilla, Ruginski, and Creem-Regehr 2017). Other authors do not describe uncertainty as important for decision making, but rather explicitly state it as variable of importance in of itself (Blenkinsop et al. 2000). While uncertainty can provide useful information in decision making, it is important to recognize the “uncertainty” in these cases is not acting as “uncertainty” at all. It is acting as signal.

This is obvious for the cases where we are explicitly interested in the variance or error, as we are literally trying to draw conclusions about an “uncertainty” statistic. The same is true for “decision making” experiments, but it is less overt. This is easiest to understand with an example. Imagine you are trying to decide if you want to bring an umbrella with you to work. An umbrella is annoying to bring with you, so you only want to pack it if the chance of rain is greater than 10%. Unfortunately, your weather prediction app only provides you with the predicted daily rainfall. Therefore, your decision will be improved with the inclusion of uncertainty, *not* because uncertainty is important for your decision, but because it gives you the tools required to calculate the *actual* statistic you are basing your decision on. In this sense, uncertainty is no more “special” to decision making than weight is in a BMI calculation.

Uncertainty visualisation’s made for these purposes should simply display the uncertainty statistic we are interested in, such as the variance, or probability of an event. This is precisely what we observe. Figure 3 depicts an exceedance probability map that has was designed as an alternative to the choropleth map to improve decision making under uncertainty (Kuhnert et al. 2018; Lucchesi, Kuhnert, and Wikle 2021). A keen viewer may notice that the “exceedance probability map” is actually just a choropleth map, only the statistic being displayed has changed. We do not believe this graphic be considered an uncertainty visualisation.



(a) Low Variance Data (b) High Variance Data (c) Exceedance Probability Palette

Figure 3: An exceedance probability map that depict the counties of Iowa where each country coloured acording to the probability that the average temperature exceeds 27. This map is a choropleth map where the variable of interest is a probability.

There seem to be two different definitions of "uncertainty visualisation" floating around in the literature. The first considers *any* visualisation of error, variance, or probability to be an uncertainty visualisation. The second believes an "uncertainty visualisation" is the output of a function that takes a normal visualisation as an input, and transforms it to include the uncertainty information. The former group believe the purpose of uncertainty visualisation to provide signal about a distribution, while the later believe it should act as noise to obfuscate a signal. The lack of explicit distinction between these two motivations leaves the literature muddled and reviewers struggle to understand if uncertainty should be treated as a variable, as metadata, or as something else entirely (Kinkeldey, MacEachren, and Schiewe 2014). This disagreement creates constant contradictions in what the literature considers to be an "uncertainty visualisation". For example Wilkinson (2005) mentions that popular graphics, such as pie charts and bar charts omit uncertainty, and Wickham and Hofmann (2011) suggests their product plot framework, which includes histograms and bar charts, should be extended to include uncertainty however at least one or both of these charts are used in a significant number of uncertainty visualisation experiments (Ibrekk and Morgan 1987; Olston and Mackinlay 2002; Zhao et al. 2023; Hofmann et al. 2012). If you view uncertainty as a function applied to an existing graphic, then you would believe a pie chart and bar chart are not uncertainty visualisations, as they are yet to have the "uncertainty visualisation function" applied to them. If you view uncertainty as any graphic that depicts an "uncertainty statistic" then there are no limitations on which graphics can or cannot be uncertainty visualisations.

When we use "uncertainty visualization" to refer to graphics that simply communicate a variance or probability, we are classifying visualisations by the data they display, not their visual features. Graphics, just like statistics, are not defined by their input data. A scatter plot that compares mean and a scatter plot that compares variances are both scatter plots. Given that there is no special class of visualisation for *other* statistics (such the median or maximum) there is no reason to assume visualisations that simply depict a variance, error, or probability to be special. Some authors implicitly suggest that that visualisations of variance or probability are differentiated due to the psychological heuristics involved in interpreting uncertainty (Hullman et al. 2019). While it is true that heuristics lead people to avoid uncertainty (Spiegelhalter 2017) there is no evidence that this psychological effect translates to issues with the visual representation of uncertainty. Again, given that we do not make these same visual considerations for other variables that elicit distaste or irrational behavior, there is no reason to assume this is what makes uncertainty visualisation so special.

This leads us to the conclusion that the visualisations made for the purpose of displaying information about uncertainty statistics are not uncertainty visualisations. These graphics are just normal information visualisations, and authors can follow existing principles of graphical design. We will focus on the perspective that uncertainty visualisation serves to obfuscate signal, and an uncertainty visualisation is a variation on an existing graphic that gives it the ability to suppress false signals.

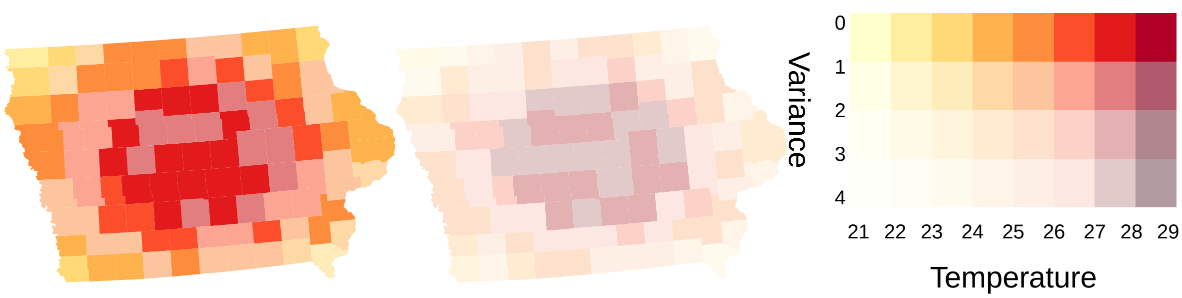
Of course, we do not believe there is anything wrong with explicitly visualizing variance, error, bias, or any other statistic used to depict uncertainty as a signal. Just like any other statistic, these metrics provide important and useful information for analysis and decisions. However, there is no interesting visualisation challenge associated with these graphics, and they do not require any special visualisation. The uncertainty in these graphics are acting as a signal variable, and they should be treated as such.

3 Visualising uncertainty as a variable

Upon hearing that uncertainty needs to be included for transparency, the solutions may seem obvious. You may think “well, I will just add a dimension to my plot that includes uncertainty”. This makes sense, as this is the simplest way to add uncertainty to an existing graphic is to simply map uncertainty to an unused visual channel.

3.1 The bivariate map

Figure 4 a variation of the choropleth map, where we have a two dimensional color palette. Not only is temperature mapped to hue, but variance is also mapped to saturation. While these two maps *do* look visually different (which was not the case in the choropleth map) the spatial trend is still clearly visible in both graphics. This means the uncertainty *is technically* being communicated, however the primary take away in the graphic is the spatial trend (that does not exist). The graphic did not hide the invalid signal, so it is not performing signal-suppression as we would like. At this point, it might be reasonable to ask, why? Why is including the uncertainty as a variable insufficient to achieve signal-suppression, and what changes should we make to ensure signal-suppression occurs?



(a) Low Variance Data (b) High Variance Data (c) Bivariate Palette

Figure 4: A bivariate map that depict the counties of Iowa where each county is coloured according to it’s average daily temperature and the variance in temperature. This map is a choropleth map with a two dimensional colour palette where temperature is represented by colour hue, and variance is represented by colour saturation. Even though uncertainty has been added to the graphic the spatial trend is still clearly visible in the case where the spatial trend could be attributed to noise.

3.2 Why this approach may (or may not) work

The difficulty in incorporating uncertainty into a visualisation is frequently mentioned but seldom explained. For example Hullman (2016) commented that it is straightforward to show a value but it is much more complex to show uncertainty but did not explain why. Many authors seem to believe uncertainty visualisation is a simple high-dimensional visualisation problem, as the difficulty comes from working out how to add uncertainty into already existing graphics (Griethe and Schumann 2006). The problem with this approach to uncertainty visualisation is that it treats uncertainty the same as we would any other variable. However, Figure 4 makes it clear that simply including uncertainty as a variable is insufficient to perform signal-suppresion. If we cannot treat uncertainty as any other variable, what should we treat it as? We need to understand what uncertainty actually *is*, in order to understand how to integrate it into a visualisation.

3.2.1 It's a variable... it's a metadata... it's uncertainty?

Describing what uncertainty actually is, is surprisingly hard. Most authors simply avoid the problem and describe the characteristics of uncertainty, of which there are plenty. Often, uncertainty is split using an endless stream of ever changing boundaries, such as whether the uncertainty is due to true randomness or a lack of knowledge (Spiegelhalter 2017; Hullman 2016; Walker et al. 2003), if the uncertainty is in the attribute, spatial elements, or temporal element of the data (Kinkeldey, MacEachren, and Schiewe 2014), whether the uncertainty is scientific (e.g. error) or human (e.g. disagreement among parties) (Benjamin and Budescu 2018), if the uncertainty is random or systematic (Sanyal et al. 2009), statistical or bounded (Gschwandtnei et al. 2016; Olston and Mackinlay 2002), recorded as accuracy or precision (Griethe and Schumann 2006; Benjamin and Budescu 2018), which stage of the data analysis pipeline the uncertainty comes from (Walker et al. 2003), how quantifiable the uncertainty is (Spiegelhalter 2017; Walker et al. 2003), etc. There are enough qualitative descriptors of uncertainty to fill a paper, but, none of this is particularly helpful in understanding how to integrate it into a visualisation.

Rather than trying to define uncertainty by what it *is* it may be easier to try and describe what uncertainty *is not*. Descriptive statistics describe our sample as it is and summarizes large data down into an easy to swallow format. Descriptive statistics are not seen as the primary goal of modern statistics, however, this was not always the case. In 19th century England, *positivism* was the popular philosophical approach to science (positivists included famous statisticians such as Francis Galton and Karl Pearson). Practitioners of the approach believed statistics ended with descriptive statistics as science must be based on actual experience and observations (Otsuka 2023). In order to make statements about population statistics, future values, or new observations we need to perform inference, which requires the assumption of the "uniformity of nature", i.e. we need to assume that unobserved phenomena should be similar to observed phenomena (Otsuka 2023). Positivists abhore the assumption of the "uniformity of nature" as they believed referencing the unobservable is bad science. In other words, these scientists embraces descriptive statistics and shunned inferential statistics due to the inherent uncertainty that came with them. Uncertainty is a by-product of inference.

This history lesson illustrates what uncertainty actually is. At several stages in a statistical analysis, we will violate the "uniformity of nature" assumption. Each of these violations will impact the statistic we have calculated and push it further from the population parameter we wish to draw inference on. Uncertainty is the amalgamation of these impacts. If we do not violate the uniformity of nature assumption at any point in our analysis, we do not have any uncertainty.

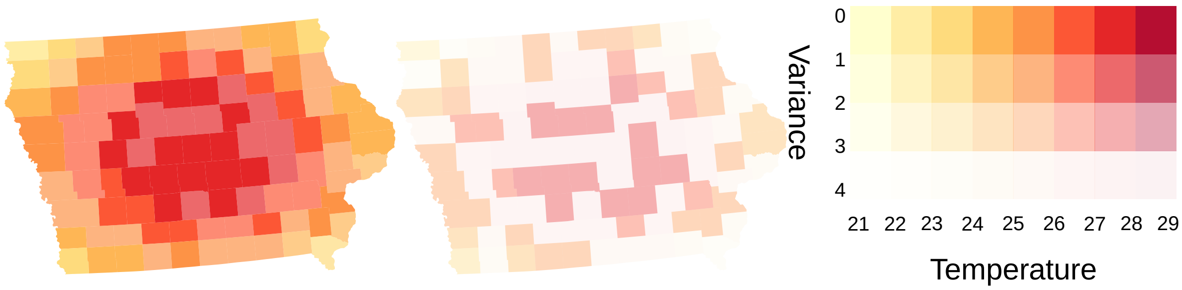
This interpretation of uncertainty indicates that the uncertainty on a statistic is not of value in of itself. Uncertainty is metadata about our statistic that is required for valid inference. This means uncertainty should not be visualised by itself and we should seek to display signal and uncertainty together as a "single integrated uncertain value" (Kinkeldey, MacEachren, and Schiewe 2014). This aspect of uncertainty visualization makes it a uniquely difficult problem. This is something frequently mentioned

3.2.2 Visualising the "single integrated uncertain value"

Typically, when making visualisations, we want the visual channels to be separable, that is, we don't want the data represented through one visual channel to interfere with the others (Smart and Szafir 2019). Mapping uncertainty and signal to separable channels allows them to be read separately, which does not align with the goal of communicating them as a "single integrated channel". Additionally, visualizing uncertainty and signal separately allows the uncertainty information to simply be ignored, which is a pervasive issue in current uncertainty visualisation methods (L. Padilla, Kay, and Hullman 2022). We can see this problem in Figure 4, as it sends the message "this data has a spatial trend and the estimates have a large variance" as we read the signal and the uncertainty separately.

This means effective uncertainty visualisation should be leveraging integrability. That is, the visual channels of the uncertainty and the signal would need to be separately manipulable, but read as a single channel by the human brain. While most visual aesthetics *are* separable, there are some variables that have been shown to be integrable, such as color hue and brightness (Vanderplas, Cook, and Hofmann 2020). When visualizing uncertainty using its own visual channel, we can also consider visual semiotics and make sure to map uncertainty to intuitive visual channels, such as mapping more uncertain values to lighter colors (Maceachren et al. 2012).

Figure 5 is an example of a variations of Figure 4 where uncertainty is mapped to transparency, and temperature is mapped to color hue to leverage these visualisation concepts. This method achieves signal suppression quite well. The spatial trend is clearly visible in the low variance case and that trend it becomes much harder to identify in the high variance case. While this was an effective approach for this graphic, relying on integrability may not give us the amount of control we want over our signal-supresison. Without a strong understanding of how these visual channels collapse down into a single channel, relying on integrability could create unintended consequences such as displaying phantom signals or hiding justified signals.



(a) Low Variance Data (b) High Variance Data (c) Transparency Palette

Figure 5: A bivariate map that depict the counties of Iowa where each county is coloured acording to it's average daily temperature and variance. This map is a variation on the previous bivariate map where instead of variance being mapped to colour saturation, it is mapped to transparency.

Additionally, multi-dimensional colour palettes can make the graphics harder to read and hurt the accessibility of the plots (Vanderplas and Hofmann 2015).

There is another reason Figure 5 is better at signal-suppression than Figure 4, and it may not be due to integrability. Colour value has a second desirable quality for signal-supression, which is that the colours become harder to distinguish as the value decreases. This means high uncertainty values are harder to differentiate than low uncertainty values. This implicit feature of colour value can generalised to other aesthetics by transforming the visual feature space ourselves.

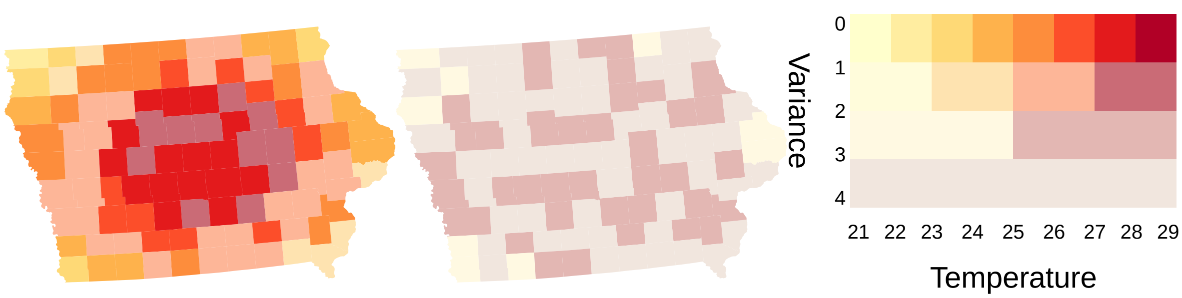
4 Visualising uncertainty and signal in a new space

Instead of hoping that uncertainty might collapse signal values into a single dimension, we can do some of that work ourselves, and uncertainty visualisation authors already have.

4.1 Value Supressing Uncertainty Palettes

The Value Suppressing Uncertainty Palette (VSUP) (Correll, Moritz, and Heer 2018), was designed with the intention of preventing high uncertainty values from being extracted from a map. Since the palette was designed with the extraction of individual values in mind and it has only been tested on simple value extraction tasks (Correll, Moritz, and Heer 2018) or search tasks (Ndlovu, Shrestha, and Harrison 2023), it is unclear how effective the method is at suppressing broader insights such as spatial trends.

Figure 6 is a visualisation of the Iowa temperature data using a VSUP to color the counties. The low uncertainty case still has a visible spatial trend, while the spatial trend in the high uncertainty map has functionally disappeared. This means the VSUP has successfully suppressed the spatial trend in the data. However the spatial trend may not be the only signal of concern in our graphic. Now we must return to the original signal-suppression criteria and ask ourselves if they have all been met. Are all the justified signals reinforced, while all the unjustified signals are suppressed? Is a graphic that performs perfect signal-suppression even possible?



(a) Low Variance Data

(b) High Variance Data

(c) VSUP Palette

Figure 6: A map made with a VSUP. The counties of Iowa are coloured acording to it’s average daily temperature and the variance in temperature, although the space they have been mapped to is slightly warped. Similar to the bivariate map, temperature is mapped to hue while variance is mapped to saturation. This map successfully reduces the visibility of the spatial trend in the high uncertainty case while maintaining the visibility of the spatial trend in the low uncertainty case.

4.2 What can and cannot be suppressed?

The methods used by the VSUP bring to light a slight problem with uncertainty visualisation. Specifically that uncertainty and the purpose of visualisation are somewhat at odds with one another. There are two primary motivations behind visualisation, communication and exploratory data analysis (EDA). Communication involves identifying a signal we want to communicate and designing a visualisation that best conveys that, while EDA involves creating a versatile visualisation using it to extract several signals. If we are designing an uncertainty visualisation for communication then we can just suppress the specific signal we are seeking to communicate. In the map example, we would consider Figure 6 to be a success as the only signal we are concerned with is the spatial trend. However, it is not uncommon for authors to express a desire for uncertainty visualisations that perform signal-suppression in visualisations made for EDA (Sarma et al. 2024; Griethe and Schumann 2006). Unfortunately it is not clear whether or not uncertainty visualisations for EDA are possible.

4.2.1 There is no uncertainty in EDA

Earlier we established that uncertainty is a by-product of inference, which means without inference, there is no uncertainty. Often EDA is used to give us an understanding of our data and identify which signals are worth pursuing. In this sense, EDA is the visual parallel to descriptive statistics, as it is performed without an explicit hypothesis which means there is no inference, and by extension, there is no uncertainty.

Some authors recognize inference will occur (in some shape or form) and believe uncertainty *should* be visualised but do not recognize *how* uncertainty would be visualised. Hullman and Gelman (2021) argued that there is no such thing as a "model-free" visualisation, therefore all visualisations require uncertainty as we are always performing inference. However, even something as simple as calculating the uncertainty that is used to suppress our visualisation is not model free, as we need to identify if the sampling variance or the sample variance is more appropriate (Hofman, Goldstein, and Hullman 2020). While we agree that people cannot prevent themselves from performing inference, this does not mean it is possible to include uncertainty in a visualisation designed for EDA. However, this does mean we should endeavor for a versatile uncertainty visualisation method that is able to perform signal-suppression on all the signals displayed in the visualisation. It is unlikely that the methods discussed thus far are able to achieve this.

4.2.2 The limitations of explicitly visualising uncertainty and signal

The lack of versatility of the VSUP is easy to see with a simple example. Let's say we have a graphic that depicts a set of coefficients from a linear regression and the value of the coefficient is shown using a single color. We want to know "Which of these coefficients are different from

0?” as well as “Which of these coefficients are different from each other?”. To answer this question we do a series of t-tests on these estimates.

All of the individual t-tests of fail to reject the null hypothesis that the coefficients are different from 0. We then make a visualisation that suppresses this signal and ensures that all of the estimates are visually indistinguishable from 0. We then do a comparison of two means t-test and find that several of the values need to be visually distinguishable from each other. The VSUP method must pick a single color for each estimate, and these colours must be *either* visually distinguishable or indistinguishable from each other. We cannot perform signal-suppression on both these signals simultaneously.

This example highlights a fundamental problem with the VSUP that extends to the bivariate map as well. When we blend these colors, we need to decide at what level of *uncertainty* to blend these colors together. Even though the bivariate map does not explicitly combine color values at certain variance levels, the mapping of variance to color saturation does this implicitly. That is, at certain saturation values the colors in a bivariate map are imperceptibly different to the human brain and appear as though they are mapped to the same value. At this point, it is irrelevant whether or not the colours are technically different, they are the same color in the human brain. Which hypothesis are suppressed and which are not largely depends on the method used to combining colors in the palette (Kay 2019). The VSUP here used a tree based method as that is what was used by Correll, Moritz, and Heer (2018), but there are alternatives that are more appropriate for different hypothesis.

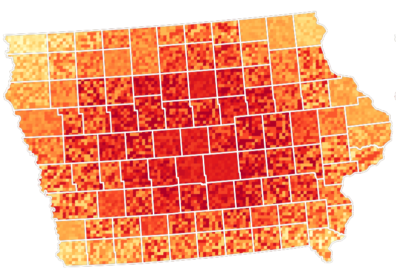
If we only use a single value to express each signal-suppressed statistic, we will always need to decide which signals we suppress and which we do not. However, If we could express the statistic of a cell using multiple colors, this limitation may disappear entirely.

5 Implicitly Combining Uncertainty and Signal

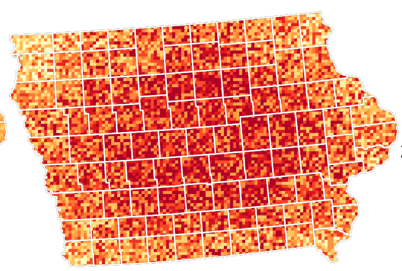
There is technically a stage of our analysis where the estimate and variance are not separate, when we only have a sample. Rather than trying to figure out how to combine signal and uncertainty into a single color, we can just display a sample instead and allow the viewer to extract *both* the estimate and the variance.

5.1 Pixel map

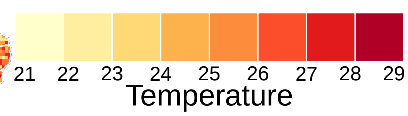
Figure 7 displays a pixel map (Lucchesi, Kuhnert, and Wikle 2021), which is a variation of the choropleth map where each area is divided up into several smaller areas, each colored using outcomes from the larger area's temperature sampling distribution. The spatial trend is clearly visible in the low variance case, but functionally disappears in the low variance case. While the spatial trend is just barely visible in the high uncertainty case, it is much harder to see. This means the graphic also achieves the third criteria for signal-suppression, i.e. our difficulty in seeing the distribution is proportional to the level of uncertainty in the graphic.



(a) Low Variance Data



(b) High Variance Data



(c) Pixel Map Palette

Figure 7: A pixel map of the counties of Iowa. In this map, each county is broken up into several smaller regions and coloured according to a potential daily temperature, given the its average daily temperature and its sampling distribution. This results in each county being represented by a sample rather than a single value. In this graphic, we can clearly see the spatial trend in the low variance case, while the spatial trend is much harder to identify in the high variance case.

It is clear that the pixel-map is not only "suppressing" the false information, but it is doing so by simulating *more* information. The efficacy of this method means that visualisations of simulated samples pop up repeatedly in the literature, with examples including samples that are animated over time (Hullman, Resnick, and Adar 2015; Blenkinsop et al. 2000), pixel-maps, and spaghetti time series plots. Not only does this method help readers understand the plot level "gist", it is also unlikely to damage the viewers ability to extract individual estimates. Extracting global statistics, such as the mean or variance, from a sample can be done with relative each, especially when those values are mapped to color (Franconeri 2021). Therefore, the pixel map performs signal suppression, without sacrificing the viewers ability to extract general statistics, unless those statistics *should* be harder to extract due to the uncertainty in the value. So is this the best uncertainty visualisation? And if so, why?

5.2 Show me the data

The pixel map is not the best uncertainty visualisation, but it is trying to *imitate* the best uncertainty visualisation. The best uncertainty visualisation is the visualisation that best captures the limitations of of our raw data. As we discussed in previous sections, we can consider uncertainty to be "the amalgamations of the impacts of violations to the assumption of the uniformity of nature". It's not a definition that rolls off the tongue, but we can work with it.

Thankfully, this definition of uncertainty aligns nicely with all the concepts that are included in the uncertainty umbrella. Some works (Hullman et al. 2018; Maceachren et al. 2012; Thomson et al. 2005) focus narrowly on specific terms with mathematical definitions, such as probability, confidence intervals, variance, error, or precision. These works are only concerned with quantifying the final impact of uncertainty on our statistics. That is, how large should the bound around our statistic be, such that our "true" statistic can be inferred. Others (Griethe and Schumann 2006; Wilkinson 2005; Pang, Wittenbrink, and Lodha 1997; Pham, Streit, and Brown 2009; Boukhelifa et al. 2017) include broader loosely related elements, such as missing values, reliability, model validity, or source integrity. These broader and harder to quantify concepts are concerned about potential sources of uncertainty, that is, they describe violations to the assumption of the uniformity of nature.

What this means is that we have two types of uncertainty, but one is more "processed" than the other. Quantifiable uncertainties are just assumption violations expressed as an effect on our final statistic. The disconnect between these two expressions of uncertainty creates a huge problem for authors trying to visualise it. A survey of visualisation authors cited "not knowing how to calculate uncertainty" as one of the primary reasons they did not include it in visualisations (Hullman 2020).

There are two reasons we might leave our uncertainty as an assumption violation rather quantifying the effect. The first reason is that we may be unable to translate the assumption violation to a quantifiable uncertainty. There is no blanket rule that allows us to reliably quantify all uncertainty for every statistic, although some researchers have tangled with the idea. For example, Thomson et al. (2005) suggests a mathematical formula for *examples* of uncertainty, and information theory tries to quantify uncertainty using the idea of entropy, but they ignore the disconnect between the broad concept of uncertainty and what we can reliably quantify. Some authors don't believe that it is even possible to quantify all the assumption violations. Spiegelhalter (2017) mentioned that combining the uncertainties that appear at each stage of an analysis into a single "uncertainty" value is near impossible.

The second reason to leave uncertainty as a potential violation of our assumptions, is that we might not know the final statistic we are seeking to calculate. This is the case for visualisations made for EDA, and a large number of developments in EDA visualisation have been in displaying these difficult to quantify violations. For example, Tierney and Cook (2023) builds upon the tidy data principles to allow users to handle missing values. This includes data plots with a missing value "shadow" that allows visualisation authors to identify if the variables used in a plot have any structure in their missing values, which would contribute to uncertainty.

With this understanding it becomes clear to see why uncertainty is tied to an endless string of examples in the data analysis pipeline. Uncertainty examples include imputed data, model selection, inherent randomness, biased sampling, etc, not because these things *are* uncertainty, but because they *create* uncertainty when we perform inference. Whether or not these elements are relevant is highly dependent on what statistic you are trying to draw inference on, and by extension, the purpose of your visualisation.

This relationship between uncertainty and the "purpose" of our analysis is littered throughout the literature. Multiple authors have commented on the need to consider quantifying and expressing uncertainty at every stage of a project as the "goal" shapes every step of the analysis (Kinkeldey, MacEachren, and Schiewe 2014; Hullman 2016; Refsgaard et al. 2007). Otsuka (2023) suggested that the process of observing data to perform statistics is largely dependent on our goals, because the process of boiling real world entities down into probabilistic objects (or "probabilistic kind" as he puts it) depends on the relationship we seek to identify with our data. Meng (2014) commented what is kept as data and what is tossed away is determined by the motivation of an analysis and what was previously noise can be shown to become signal depending on the the question we seek to answer. Wallsten et al. (1997) argue that the best method for evaluating or combining subjective probabilities depends on the uncertainty the decision maker wants to represent and why it matters. Fischhoff and Davis (2014) looks at uncertainty visualisation for decision making decides that we should have different ways of communicating uncertainty based off what the user is supposed to do with it.

This makes it very difficult to move quantified uncertainty through the the layers of our analysis, especially when designing a visualisation for EDA. If we don't know what the final statistic is, we cannot quantify the effects of our assumptions. Therefore, often the best uncertainty visualisation is not an "uncertainty visualisation" at all, but simply the most accurate depiction of our raw data as it gives us a good idea of it's limitations.

This does not mean that visualizing raw data instead of implementing sampling techniques will always prevent insignificant signal from getting through. Buja et al. (2009) illustrated how groups that appear linearly separable in a linear discriminant analysis (LDA) visualisation of the data can actually be the result of a LDA performed on too many variables, something that was not clear from the visualisation until the line-up protocol was implemented. However just showing the data it is simple but effective option for uncertainty visualisation that that seems to be largely overlooked. While it is not always possible, it should always be considered as an effective uncertainty visualisation when the raw data is available.

6 Evaluating uncertainty visualisations

If we want to make conclusions about how effective any uncertainty visualisation method is we need to look at the results of evaluation experiments. Unfortunately the illustrative methods we have used thus far, i.e. showing a graphic and saying "wow look at this", are lacking if we want any generalizable results. However, despite the abundance of uncertainty visualisation evaluation experiments, existing literature reviews have struggled to synthesise them into any common rules (Kinkeldey, MacEachren, and Schiewe 2014; Hullman 2016).

Here we discuss common evaluation methods, why these methods might struggle to create a cohesive set of recommendations for uncertainty visualisations, and consier how we to best evaluate visualisations on their ability to perform signal-suppression.

6.1 Current methods

Including uncertainty in a visualisation comes with many secondary benefits. Examples of these benefits include better decisions, more trust in the results and the ability to extract additional statistics, such as the variance. Ultimately, these secondary benefits are not the primary goal of uncertainty visualisation, and evaluating uncertainty visualisations on these criteria often has unintended consequences.

6.1.1 Value extraction of uncertainty statistics

Uncertainty visualisations are most commonly evaluated on how accurately readers can extract information from them (Hullman et al. 2019). This most commonly comes in the form of separate questions about estimates variance (Kinkeldey, MacEachren, and Schiewe 2014). This means a significant chunk of evaluation studies boil down to showing a participant a visualisation and asking questions such as "what is the variance of X?", or "what is the mean of X?". Additionally, uncertainty visualisation experiments often compare graphics on the basis of being "uncertainty visualisations" (Ibrekk and Morgan 1987; Hullman, Resnick, and Adar 2015; Hofman, Goldstein, and Hullman 2020) regardless of what information can actually be extracted by the chart. This seems like a relatively straight forward approach, and it is similar to how non-uncertainty visualisations are evaluated, but is this appropriate for uncertainty visualisations? Uncertainty is not something we are trying to infer, uncertainty is a by-product of inference. By shifting the focus of our inference to $Var(X)$ or $P(X)$ we are just evaluating visualisations on their ability to convey uncertainty statistics. These graphics may not always set out to communicate uncertainty as a signal, but they are certainty evaluating uncertainty as a signal.

The problem with evaluating uncertainty as a signal are identical to the problems associated with displaying uncertainty as a signal. There is no reason to assume uncertainty would behave any differently to any other variable when we evaluate them in this way. For example, Ibrekk and Morgan (1987) found that participants were more accurate at extracting a statistic when it could be directly read off the graphic, than when it required an area estimate (which is the case if using the PDF), or when there was no visual indicator for the statistic at all (which is the case when using the CDF of an asymmetric function). Hullman, Resnick, and Adar (2015) found that a visualisation that allows viewers to count outcomes to estimate a probability outperformed one that required a complicated area calculation. Hofman, Goldstein, and Hullman (2020) and Zhang et al. (2022) found that participants were better at answering questions about a prediction intervals when shown a prediction interval instead of a sampling distribution. Gschwandtnei et al. (2016) found that graphics where the required statistic could be directly read off the plot outperformed those that involved guesswork due to a gradually decreasing line. Cheong et al. (2016) found that participants made better decisions when they were explicitly given the relevant probability in text rather than when they needed to read it off a map. It is well established that extracting information from a graphic using a perceptual task will always be less accurate than explicitly reading the value provided in text form (Cleveland and McGill 1984).

Directly asking about uncertainty evaluates uncertainty as a signal, not as noise, so it is not a reliable evaluation method when we are designing graphics for signal suppression. Many uncertainty visualisation researchers are already aware of this, so direct questions are not the only methods used to evaluate uncertainty visualisations. We will discuss these alternatives in the following section.

6.1.2 Trust and confidence

Trust is a by-product of displaying uncertainty and it commonly measured in uncertainty evaluation studies (Hullman et al. 2019). Considering trust, and not transparency, as the metric of importance in uncertainty communication can lead to a questionable subtext that argues against transparency, something that has been noticed by several other authors [Spiegelhalter (2017); ONeill2018]. Science communication should be primarily concerned with accuracy.

Setting trust as the variable of interest implicitly encourages statisticians to set trust and as the primary goal of communication. Hullman (2020) found that author simultaneously argued that failing to visualise uncertainty was akin to fraud, but also many avoided uncertainty visualisation because they didn't want their work to come across as "untrustworthy". This is a classic example of the negative impacts of placing direct importance on *trust* rather than *transparency*. In cases of high uncertainty, authors will opt to leave out uncertainty information because it decreases confidence in the authors conclusions.

The disconnect between trust and transparency extends to evaluation experiments as well. Zhao et al. (2023) displayed a several variations of a visualisation of a model prediction and its uncertainty and took participants using the model prediction as a sign of trust. They reported that visualising uncertainty information caused participants to trust the model in the low variance case, but the results in the high variance case were inconclusive. The discussion made it clear the authors thought the uncertainty information should make the visualisation more trustworthy. However, this perspective conflates transparency and trustworthiness. If a model has very high uncertainty because of a very low sample size, the heuristics a human might use to make the same decision could be more accurate.

Another metric that is similar to trust is the participants confidence in their decision or extracted value. Confidence has many of the same issues as trust, but it has an additional confounding factor. In non-uncertainty visualisation evaluation experiments, "confidence" is used as a proxy for the clarity of the visualisation. Confidence cannot simultaneously be a measure of clarity of visualisation *and* a way to capture the uncertainty expressed in a visualisation. Uncertainty visualisations conflate these two measures when they ask about confidence.

6.1.3 Questions that attempts to capture signal-suppression

There is also a swath of studies that are aware a question that boils down to a value extraction experiment, or a question that should be answered by literally ignoring uncertainty information is not what we want when we consider uncertainty visualisations. These papers often try to ask a question that should utilise both the uncertainty and signal in the response, however this is rarely what actually occurs. This method typically results in in cryptic or confusing questions that create a large amount of noise on the interpretation side of the analysis (Hullman 2016).

Some authors opt for asking slightly vague questions that imply a use of uncertainty, but compare it to a ground truth that is very specific. Ibrekk and Morgan (1987) asked participants for the "best estimate" which was evaluated in accuracy by comparing it to the mean, however the "best estimate" depends on the loss function we are using, and a loss function of minimised error was not implied by the question. Hofmann et al. (2012) showed two distributions in 20 different visualisations (a line-up protocol) using a jittered sample, a density plot, a histogram, and a box plot and asked participants. Participants were asked to report in which of the plots was "the blue group furthest to the right" The experiment set up is shown in fig-right. The participants answers were then compared to a ground truth where the correct plot had a blue distribution with a right shifted mean. By comparing the results to a ground truth statistic and marking participants as "wrong" or "right", the error from the participants that had an alternative interpretation to the concept of "furthest right" was conflated with the error from a the visualisation choice. These papers make it unclear if the participants got the answers wrong because they misunderstood the question or because of something related to the plot. Therefore, this method leads to inconclusive results about the plot design, and is not advised.

Another method used by authors is to ask a deterministic question about a random event. L. M. Padilla, Ruginski, and Creem-Regehr (2017) provided participants with a visualisation of the cone of uncertainty and asked then to "decide which oil rig will receive more damage based on the depicted forecast of the hurricane path". The cone of uncertainty provides a 60% confidence interval for the location of the eye of a hurricane, which allows us to know the area where the eye of the storm will go, it does not given any information about the intensity of a storm, the size of a storm, or even if a location will be hit. This inclusion of determinism seems to cause the authors to stumble themselves, as they are not consistent with their assumptions. In their first experiment L. M. Padilla, Ruginski, and Creem-Regehr (2017) indicated the correct answer was to assume that the storm was equally intense no matter how far from the centre of the distribution an oil rig was, however answering their third experiment correctly hinged on assuming the intensity of the storm at a particular point (which in this experiment they phrased as damage) *does* change in intensity as you move away from the centre of the distribution. Given these conflicting assumptions, it is unclear how the participants were supposed to adjust the probabilistic path information to answer a deterministic question about which oil rig would receive the most damage. Other authors have commented on the complexity of communicating hurricane risk because the path, storm surge and wind speed are all important and cannot be ignored (Spiegelhalter 2017). The flip side of this is asking participants for a deterministic answer to a probabilistic question. Correll and Gleicher (2014) asked participants "how likely is candidate B to win the election?" when the two distributions indicated voter preference. Participants were not able to answer the question about likelihood in term of probability, but were instead given seven options from 1=Outcome will be most in favour of A to 7=Outcome will be most in favour of B. The ground truth statistic for this question was a scalar multiple of Cohen's d, indicating participants were supposed to incorporate uncertainty information using a very specific formula that was likely unknown to them but assumed to be used implicitly.

6.1.4 Identifying heuristics

- Things that are true for both normal visualisation and uncertainty, but of particular interest for uncertainty

Additionally, these heuristics and biases can change depending on the larger scope of the graphic and the population we are communicating with (Spiegelhalter 2017; Kinkeldey, MacEachren, and Schiewe 2014).

Heuristic checks are useful because they look at unknown pitfalls that might exist in interpretation of current plots. Since the hypothesis for these experiments are usually quite specific, e.g. "do people perceive a an outcome that is within the bar as more likely than one outside it, even if both outcomes are the same distance from the mean?". This means they are less likely to fall into the trap of trying to answer questions that are *far* too broad to be answered with a single experiment (e.g. "is a scatter plot better at showing uncertainty than a box plot"). This work also provides useful insights for experiments by highlights pitfalls participants might fall into when they review the results of evaluation experiments (Hullman 2016). Newman and Scholl (2012) found that participants were more likely to view points within the bar as more likely than points outside of the bar in bar charts with error bars. Similar effects have been identified in other types of uncertainty displayed. L. M. Padilla, Ruginski, and Creem-Regehr (2017) found that points that were on an outcome of an ensemble display were perceived as more likely than points not on an outcome, even when the point that was not on a specific outcome of the ensemble was closer to the mean of the uncertainty distribution. The sine illusion can cause the confidence interval of a smoothed sine curve to seem wider at the peaks than the troughs, causing us to underestimate uncertainty associated with changing values (Vanderplas and Hofmann 2015).

In a similar vein, experiments that verify smaller aspects of plot design might be more useful to the field in the long run because it helps contribute to a larger working theory of "how do we see visualisations". Many visualisation experiments try to compare two plots with several differences, but do not seem to be interested in the mechanisms by which we extract information from visualisations. Small perceptual tasks that seek to answer small but highly relevant questions (for example, if colour hue and colour value can be perceived as a single signal suppressed variable) would be useful to the field.

6.2 Testing signal supression

If a graphic was performing signal suppression we would expect the values to be harder to read at higher levels of uncertainty, something that is often seen as a bad thing in current evaluation studies. - trust experiment - weird animation of grass

Since the primary goal of visualisation *is* developing insights (North 2006), you will probably extract several signals and use them to make a broader conclusion. A simple example is the spatial trend in

If asking direct questions about uncertainty causes us to treat it as a signal, how do we evaluate uncertainty as *noise*? When we ask the viewer of a plot to look at data and extract a value, we are asking them to perform inference on that value. There will be noise associated with that answer and that is uncertainty. If we ask direct question about some uncertainty metric, we have turned the uncertainty into signal because that is what the participants are drawing inference on.

So, the current methods of measuring or understanding the role of uncertainty in a visualisation is questionable at best, however this is not because visualisation authors are missing the mark, but rather uncertainty is *particularly* difficult to express in a visualisation. In simple estimates or verbal communication, the signal is often easy to identify because it is what we are explicitly saying. Unlike statistical models, visualisations are used in both data exploration and communication. This means what exactly is a *signal* in any particular visualisation is hard to identify, since we often let the visualisation *tell us* what the signal is. Additionally, you cannot add noise to *every single possible* signal one might take from a visualisation. Two people looking at the same visualisation might, just by chance, develop two entirely different insights and draw inference on two completely different statistics. These unique and fascinating challenges that are faced by uncertainty visualisation have been completely untouched by the literature. This section will cover some interesting research in uncertainty visualisation and suggestions for better ways to measure uncertainty.

6.2.1 Qualitative Studies

Alternatively visualisation research could shift away from the accuracy concept all together ask questions that allow for open ended responses. This method can enlighten authors as to *how* the uncertainty information was used by the participants. Hofmann et al. (2012) tried to capture this by asking participants why they considered a particular plot to be more "right shifted", however this qualitative assessment does not seem to have made it into the final paper. Daradkeh (2015) presented participants with ten investment alternatives and asked participants "from among available alternatives, which alternative do you prefer the most", and were asked to think aloud and consider the uncertainty in their decision making. The experimenters goal was to observe and organise the methods people use when making decisions in the face of uncertainty. This study was an excellent example in a useful experimental design. They highlighted the specific aspects of uncertainty that participants typically considered, such as the range of outcomes that are above/below a certain threshold, minimum and maximum values, the risk of a loss, etc, and mapped where in the decision making process participants made these considerations. Data visualisation is commonly utilised as a tool in data exploration, so it is not uncommon for a data analyst to make a plot with only a vague goal and pull out a large number of adjacent observations. This experimental framework could replicate this process.

6.2.2 Just noticeable signal

there are some papers that do identify signal suppress effects, but they discuss it as a failure rather than a success.

It could be argued that a well done uncertainty visualisations should have an imperceptible signal unless the signal would be identified with a hypothesis test, almost like a reverse line up protocol, but this idea also has some issues that should be considered. The reject or do not reject concepts in hypothesis testing do not offer a complete image of uncertainty, and exploration of uncertainty visualisation largely stems from a desire to move away from this binary framework.

Weihao Li and VanderPlas (2024) found that human viewers were less sensitive to deviations from the null hypothesis than the typical statistical tests.

What this all means, is that there will always be some trade off when we are designing a visualisation for signal-suppression. There will be a trade off in *which* signal we suppress, as well as *how much* we suppress it. This is true for any graphic that is comparable in some way to a statistical test or process. A good example is the line-up protocol, which is a visualisation tool that used to check if perceived patterns are real or merely the result of chance (Buja et al. 2009; Wickham et al. 2010). This motivation is similar to our signal-suppression goal. Weihao Li and VanderPlas (2024) compared standard statistical tests to the lineup-protocol, and evaluated the visualisations using the power curves that are typical for hypothesis testing. In a similar vein, Kim et al. (2019) investigated how different uncertainty visualisation methods influenced user's prior beliefs, and evaluated the graphics by comparing their results to those from Bayesian inference. While these methods apply different statistical philosophies, they both investigated the sensitivity of visualisation methods relative to existing evaluation methods. This approach may prove useful when considering

7 Future work

This paper has identified gaps in the uncertainty visualisation literature that could be filled to progress the field.

Each new development should be accompanied by a mathematical definition of the uncertainty being addressed. Ideally, a mathematically definition of uncertainty that allows us to combine these components would be developed, but in the absence of that, authors should be more specific about what aspect of “uncertainty” they are covering with their visualisation.

The concept of uncertainty should be formalised within the grammar of graphics. This formalisation would allow uncertainty visualisation authors to have a clear understanding of what is or is not an uncertainty visualisation. Additionally placing uncertainty visualisation in the framework that is used to understand existing information visualisation research would help authors understand when existing methods can be used to explain their results. incorporating uncertainty into the grammar of graphics will also give a more precise concept of the information contained within a plot. Other fields of science employ marginal changes when designing experiments to ensure it is well understood *what* aspect of their experiment is contributing to their results, and a better sense of what “marginal” is in the case of uncertainty visualisation would greatly help the field. (*XXX Is data pipeline connected with the grammar of graphics? Should this be a recommendation?*)

Experimental practices on uncertainty visualisation need to be standardised. If we are going to consider uncertainty as noise, not signal, there needs to be a way to identify this signal suppression in an experimental design. As the literature currently exists, there is no way to combine papers to get a meaningful sense of how uncertainty information is understood by a viewer. There is also the possibility that uncertainty visualisation evaluations will need to swap to a qualitative methodology where participants are allowed to freely comment on what they notice in graphics until we establish how the existence of noise can be observed.

If an uncertainty visualisation researcher would prefer to perform experiments rather than formalise methods, there are options there too. It would be interesting to know if any perceptual tasks that can be mapped to two different visual tasks condense into a single dimension when looking for overarching signal in a plot. Alternatively, the task dependency many authors in uncertainty visualisation mention would be a useful direction to consider. It is clear that the the number of potential tasks that can be performed on a visualisation increases with with the number of observations. A single observation is limited to value extraction, two observations can be compared, multiple observations allow for shapes or global statistics to be extracted. The interaction between sample size and task is of particular interest to the uncertainty visualisation community, as uncertainty can be expressed through multiple observations using a sample, or through a single value using an error. Of course, this is limited by the fact that there also isn’t a definition for what is a “task” and given the mess created by the lack of formalisation in uncertainty visualisation, it may be wise to formalise that concept before performing these experiments. Amar, Eagan, and Stasko (2005) suggested a taxonomy for information visualisation based on the types of tasks we use visualisations for and suggest 10 "analytical primitives" that we can then map to visualisations, which could be a good starting point. Regardless, these are directions of research would be fruitful to the uncertainty visualisation community even if it appears on the surface to be research that is only beneficial to the "normal" visualisation community. (*XXX Not sure what this paragraph is recommending?*)

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