Introduction to Stan

Harriet Ware

February 12 2021

Contents

1 Introduction

Today we will be starting off using Stan, looking at the kid's test score data set (available in resources for the Gelman Hill textbook).

```
library(tidyverse)
library(rstan)
library(tidybayes)
library(here)
```

The data look like this:

```
kidiq <- read_rds(here("data","kidiq.RDS"))
kidiq
```

```
## # A tibble: 434 x 4
##    kid_score mom_hs mom_iq mom_age
##        <int>  <dbl>  <dbl>   <int>
##  1        65      1  121.       27
##  2        98      1   89.4      25
##  3        85      1  115.       27
##  4        83      1   99.4      25
##  5       115      1   92.7      27
##  6        98      0  108.       18
##  7        69      1  139.       20
##  8       106      1  125.       23
##  9       102      1   81.6      24
## 10        95      1   95.1      19
## # ... with 424 more rows
```

As well as the kid's test scores, we have a binary variable indicating whether or not the mother completed high school, the mother's IQ and age.

2 Descriptives

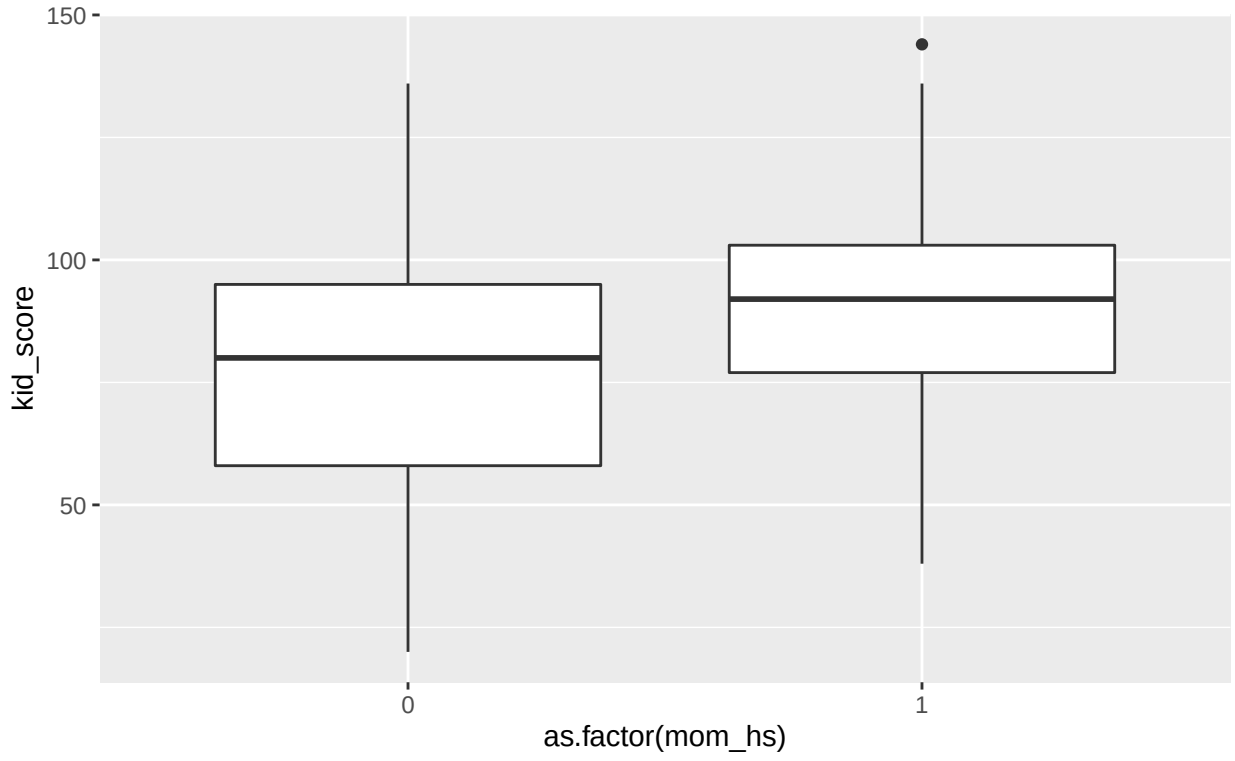
2.1 Question 1

Use plots or tables to show three interesting observations about the data. Remember:

- Explain what your graph/ tables show
- Choose a graph type that's appropriate to the data type

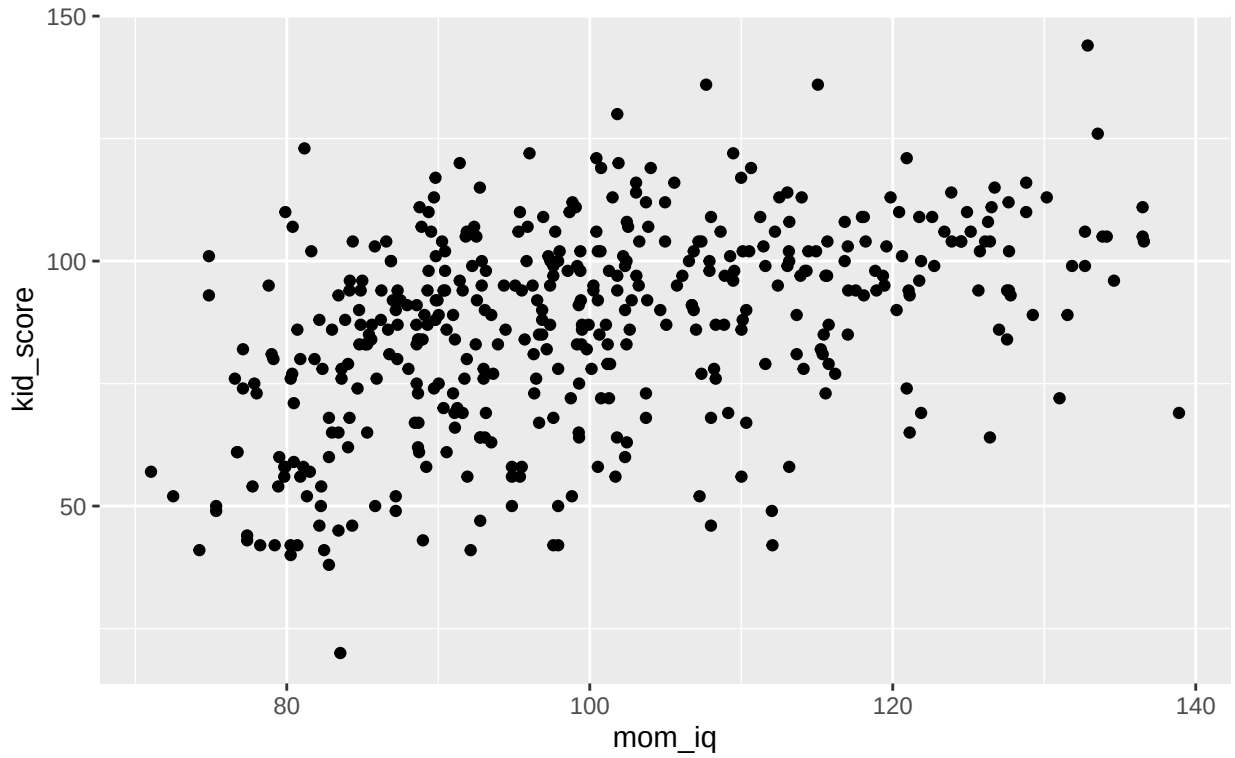
The first plot shows the kid's test score vs. an indicator for whether the mother completed high school. We see that, on average, children of mothers who have completed high school score higher than children of mothers who have not completed high school. Also, the range of test scores is larger for children of mothers who have not completed high school.

```
ggplot(kidiq,aes(x=as.factor(mom_hs),y=kid_score))+
  geom_boxplot()
```



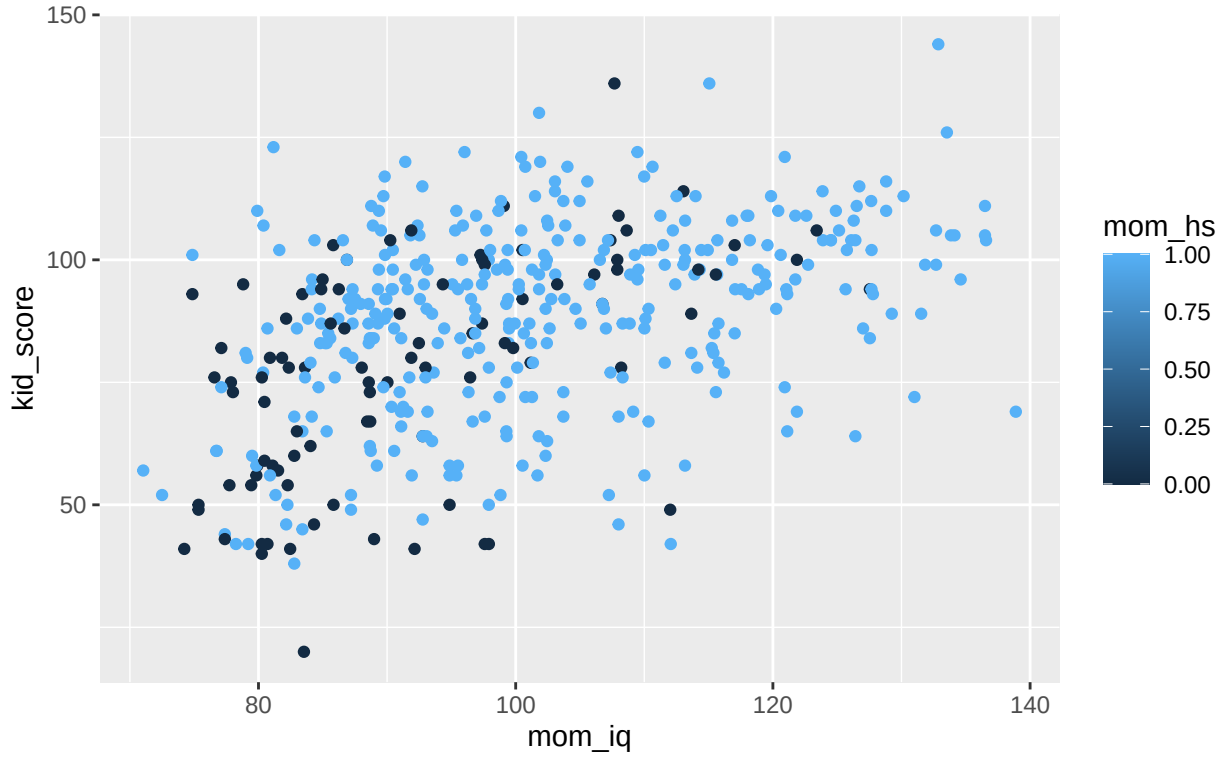
The second plot shows the kid's test score vs. mother's IQ. We see that, on average, higher maternal IQ is associated with higher test scores.

```
ggplot(kidiq,aes(x=mom_iq,y=kid_score))+
  geom_point()
```



The final plot shows the kid's test score vs. mother's IQ with different colours for children of mothers who completed high school (light blue) and those whose mothers did not complete high school (dark blue). If we were to draw a line through the points, it looks like the "did not complete high school" group would have a lower intercept and steeper slope than the "completed high school" group.

```
ggplot(kidiq,aes(x=mom_iq,y=kid_score,color=mom_hs))+
  geom_point()
```



3 Estimating mean, no covariates

In class we were trying to estimate the mean and standard deviation of the kid's test scores. The `kids2.stan` file contains a Stan model to do this. If you look at it, you will notice the first `data` chunk lists some inputs that we have to define: the outcome variable `y`, number of observations `N`, and the mean and standard deviation of the prior on `mu`. Let's define all these values in a `data` list.

```
y <- kidiq$kid_score
mu0 <- 80
sigma0 <- 10

data <- list(y = y,
             N = length(y),
             mu0 = mu0,
             sigma0 = sigma0)
```

Now we can run the model:

```
fit <- stan(file = "code/models/kids2.stan",
            data = data)
```

```
##
## SAMPLING FOR MODEL 'kids2' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 0 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 1: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 1: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 1: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 1: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 1: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 1: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 1: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 1: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 1: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 1: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 1: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 0.098 seconds (Warm-up)
## Chain 1:                0.089 seconds (Sampling)
## Chain 1:                0.187 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL 'kids2' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 0 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 2: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 2: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 2: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 2: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 2: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 2: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 2: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 2: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 2: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 2: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 2: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 2:
## Chain 2:  Elapsed Time: 0.098 seconds (Warm-up)
## Chain 2:                0.064 seconds (Sampling)
## Chain 2:                0.162 seconds (Total)
## Chain 2:
```

```
##
## SAMPLING FOR MODEL 'kids2' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 0 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0 seconds.
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:
## Chain 3: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 3: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 3: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 3: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 3: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 3: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 3: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 3: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 3: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 3: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 3: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 3: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 3:
## Chain 3:  Elapsed Time: 0.083 seconds (Warm-up)
## Chain 3:                0.069 seconds (Sampling)
## Chain 3:                0.152 seconds (Total)
## Chain 3:
##
## SAMPLING FOR MODEL 'kids2' NOW (CHAIN 4).
## Chain 4:
## Chain 4: Gradient evaluation took 0 seconds
## Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0 seconds.
## Chain 4: Adjust your expectations accordingly!
## Chain 4:
## Chain 4:
## Chain 4: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 4: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 4: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 4: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 4: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 4: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 4: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 4: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 4: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 4: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 4: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 4: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 4:
## Chain 4:  Elapsed Time: 0.122 seconds (Warm-up)
## Chain 4:                0.071 seconds (Sampling)
## Chain 4:                0.193 seconds (Total)
## Chain 4:
```

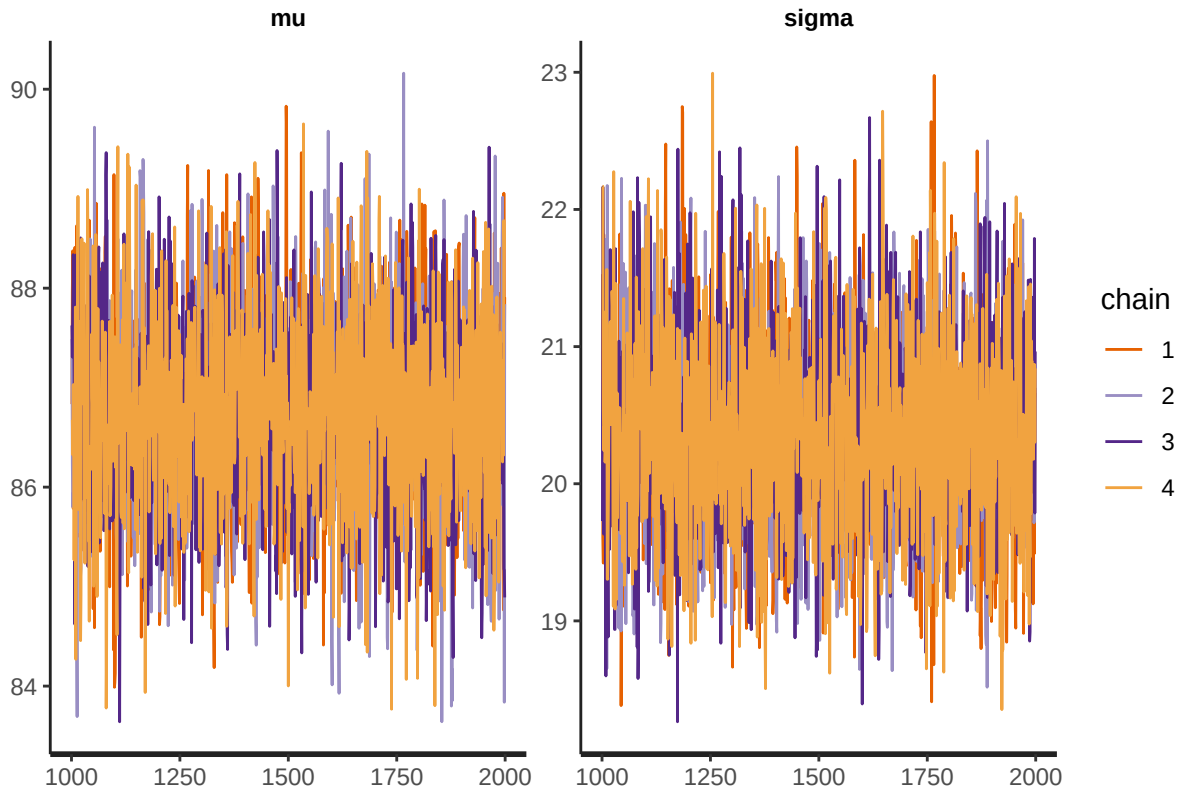
Look at the summary

```
fit
```

```
## Inference for Stan model: kids2.
## 4 chains, each with iter=2000; warmup=1000; thin=1;
## post-warmup draws per chain=1000, total post-warmup draws=4000.
##
##           mean se_mean   sd     2.5%      25%      50%      75%    97.5% n_eff
## mu       86.74    0.02 0.96    84.86    86.09    86.75    87.40    88.57  3177
## sigma    20.35    0.01 0.70    19.07    19.86    20.32    20.83    21.79  3828
## lp__  -1525.76    0.02 0.98 -1528.45 -1526.16 -1525.46 -1525.06 -1524.78  1966
##       Rhat
## mu       1
## sigma    1
## lp__     1
##
## Samples were drawn using NUTS(diag_e) at Fri Feb 12 09:32:44 2021.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).
```

Traceplot

```
traceplot(fit)
```



All looks fine.

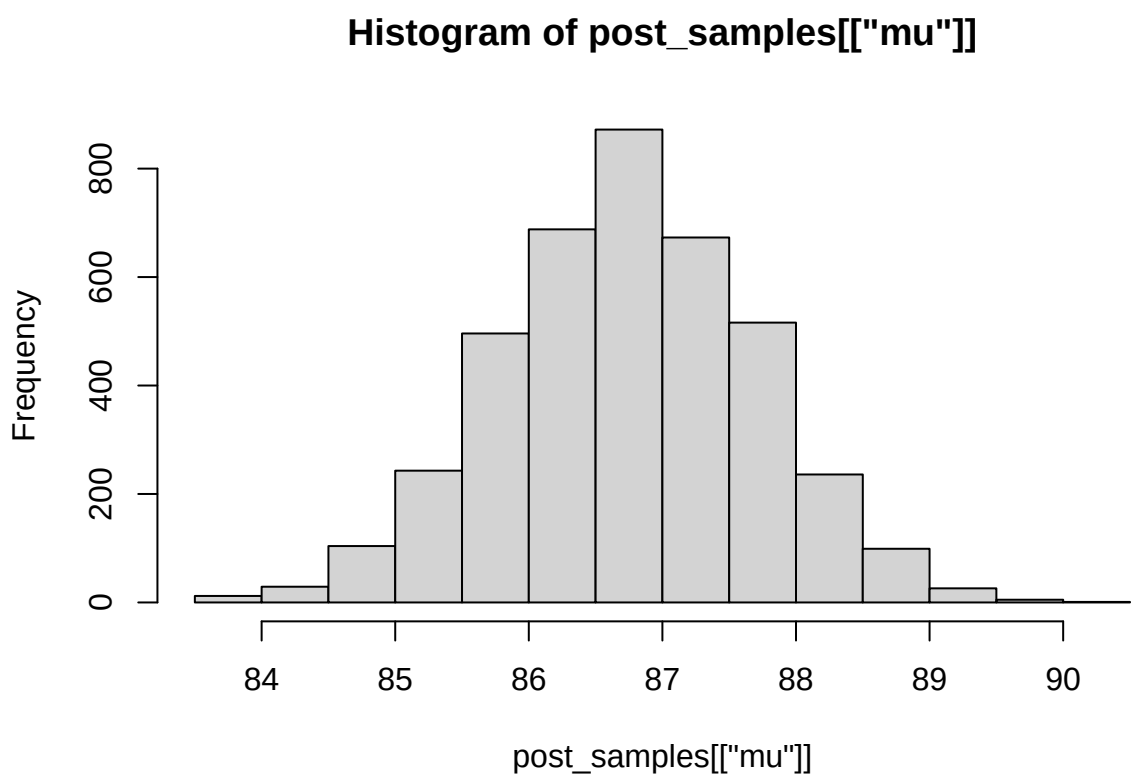
3.1 Understanding output

What does the model actually give us? A number of samples from the posteriors. To see this, we can use `extract` to get the samples.

```
post_samples <- extract(fit)
```

This is a list, and in this case, each element of the list has 4000 samples. E.g. quickly plot a histogram of mu

```
hist(post_samples[["mu"]])
```




```
median(post_samples[["mu"]])
```

```
## [1] 86.74552
```

```
quantile(post_samples[["mu"]], 0.025)
```

```
##     2.5%
## 84.85787
```

```
quantile(post_samples[["mu"]], 0.975)
```

```
##   97.5%
## 88.5699
```

3.2 Plot estimates

There are a bunch of packages, built-in functions that let you plot the estimates from the model, and I encourage you to explore these options (particularly in `bayesplot`, which we will most likely be using later on). I like using the `tidybayes` package, which allows us to easily get the posterior samples in a tidy format (e.g. using gather draws to get in long format). Once we have that, it's easy to just pipe and do ggplots as usual. `tidybayes` also has a bunch of fun visualizations, see more info here: https://mjskay.github.io/tidybayes/articles/tidybayes.html#introduction

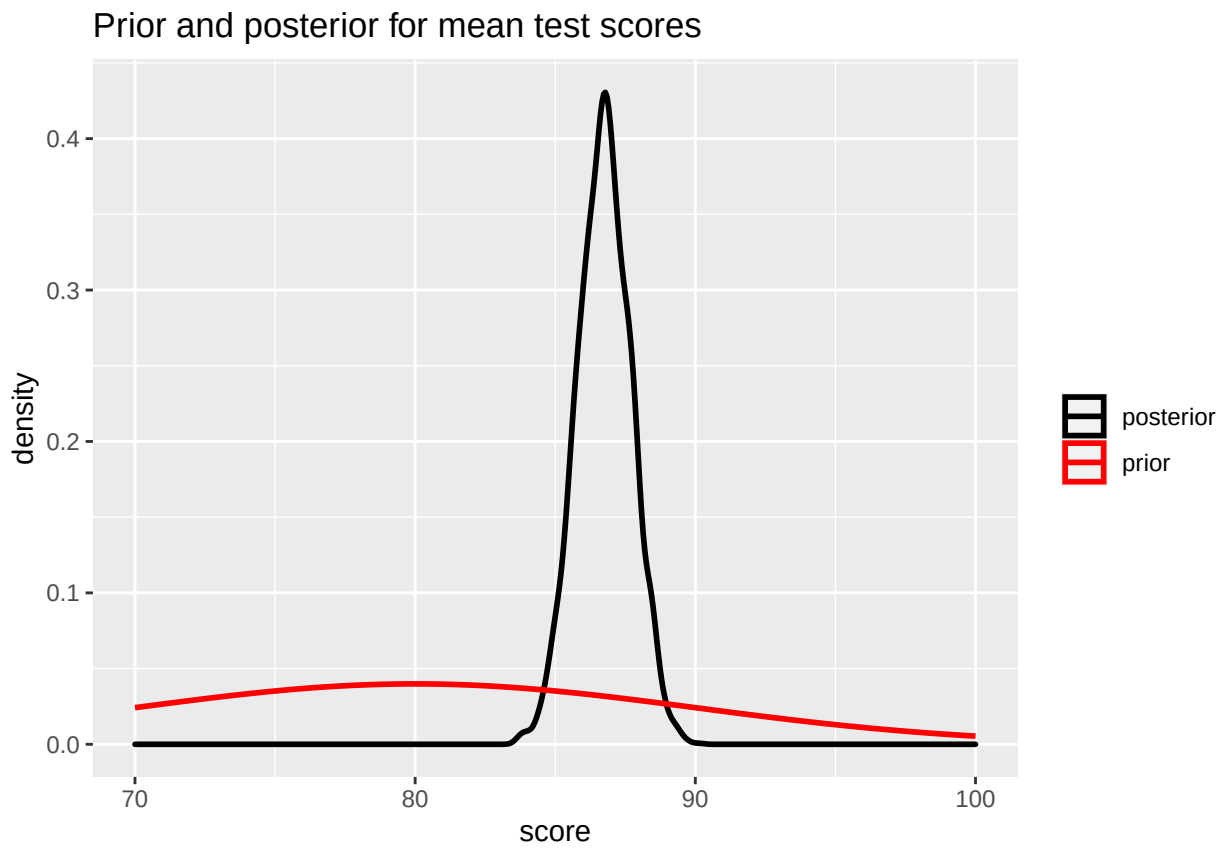
Get the posterior samples for mu and sigma in long format:

```
dsamples <- fit %>%
  gather_draws(mu, sigma)
dsamples
```

```
## # A tibble: 8,000 x 5
## # Groups:   .variable [2]
##    .chain .iteration .draw .variable .value
##     <int>      <int> <int> <chr>      <dbl>
##  1      1          1     1 mu          86.8
##  2      1          2     2 mu          88.0
##  3      1          3     3 mu          88.4
##  4      1          4     4 mu          85.8
##  5      1          5     5 mu          87.0
##  6      1          6     6 mu          88.0
##  7      1          7     7 mu          85.2
##  8      1          8     8 mu          88.4
##  9      1          9     9 mu          85.6
## 10      1         10    10 mu          85.4
## # ... with 7,990 more rows
```

Let's plot the density of the posterior samples for mu and add in the prior distribution

```
dsamples %>%
  filter(.variable == "mu") %>%
  ggplot(aes(.value, color = "posterior")) + geom_density(size = 1) +
  xlim(c(70, 100)) +
  stat_function(fun = dnorm,
        args = list(mean = mu0,
                    sd = sigma0),
        aes(colour = 'prior'), size = 1) +
  scale_color_manual(name = "", values = c("prior" = "red", "posterior" = "black")) +
  ggtitle("Prior and posterior for mean test scores") +
  xlab("score")
```



3.3 Question 2

Change the prior to be much more informative (by changing the standard deviation to be 0.1). Rerun the model. Do the estimates change? Plot the prior and posterior densities.

The estimates are similar to before, but $\hat{\mu}$ is closer to 80.

```
y <- kidiq$kid_score
mu0 <- 80
sigma0 <- 0.1

data <- list(y = y,
             N = length(y),
             mu0 = mu0,
             sigma0 = sigma0)

fit.1 <- stan(file = "code/models/kids2.stan",
            data = data)
```

```
##
## SAMPLING FOR MODEL 'kids2' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 0 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0 seconds.
## Chain 1: Adjust your expectations accordingly!
```

```
## Chain 1:
## Chain 1:
## Chain 1: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 1: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 1: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 1: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 1: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 1: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 1: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 1: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 1: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 1: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 1: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 1: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 0.084 seconds (Warm-up)
## Chain 1:                0.081 seconds (Sampling)
## Chain 1:                0.165 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL 'kids2' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 0 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 2: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 2: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 2: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 2: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 2: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 2: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 2: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 2: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 2: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 2: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 2: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 2:
## Chain 2:  Elapsed Time: 0.068 seconds (Warm-up)
## Chain 2:                0.065 seconds (Sampling)
## Chain 2:                0.133 seconds (Total)
## Chain 2:
##
## SAMPLING FOR MODEL 'kids2' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 0 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0 seconds.
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:
## Chain 3: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 3: Iteration:  200 / 2000 [ 10%]  (Warmup)
```

```
## Chain 3: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 3: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 3: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 3: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 3: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 3: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 3: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 3: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 3: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 3: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 3:
## Chain 3:  Elapsed Time: 0.06 seconds (Warm-up)
## Chain 3:                0.059 seconds (Sampling)
## Chain 3:                0.119 seconds (Total)
## Chain 3:
##
## SAMPLING FOR MODEL 'kids2' NOW (CHAIN 4).
## Chain 4:
## Chain 4: Gradient evaluation took 0 seconds
## Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0 seconds.
## Chain 4: Adjust your expectations accordingly!
## Chain 4:
## Chain 4:
## Chain 4: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 4: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 4: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 4: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 4: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 4: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 4: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 4: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 4: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 4: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 4: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 4: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 4:
## Chain 4:  Elapsed Time: 0.062 seconds (Warm-up)
## Chain 4:                0.079 seconds (Sampling)
## Chain 4:                0.141 seconds (Total)
## Chain 4:
```

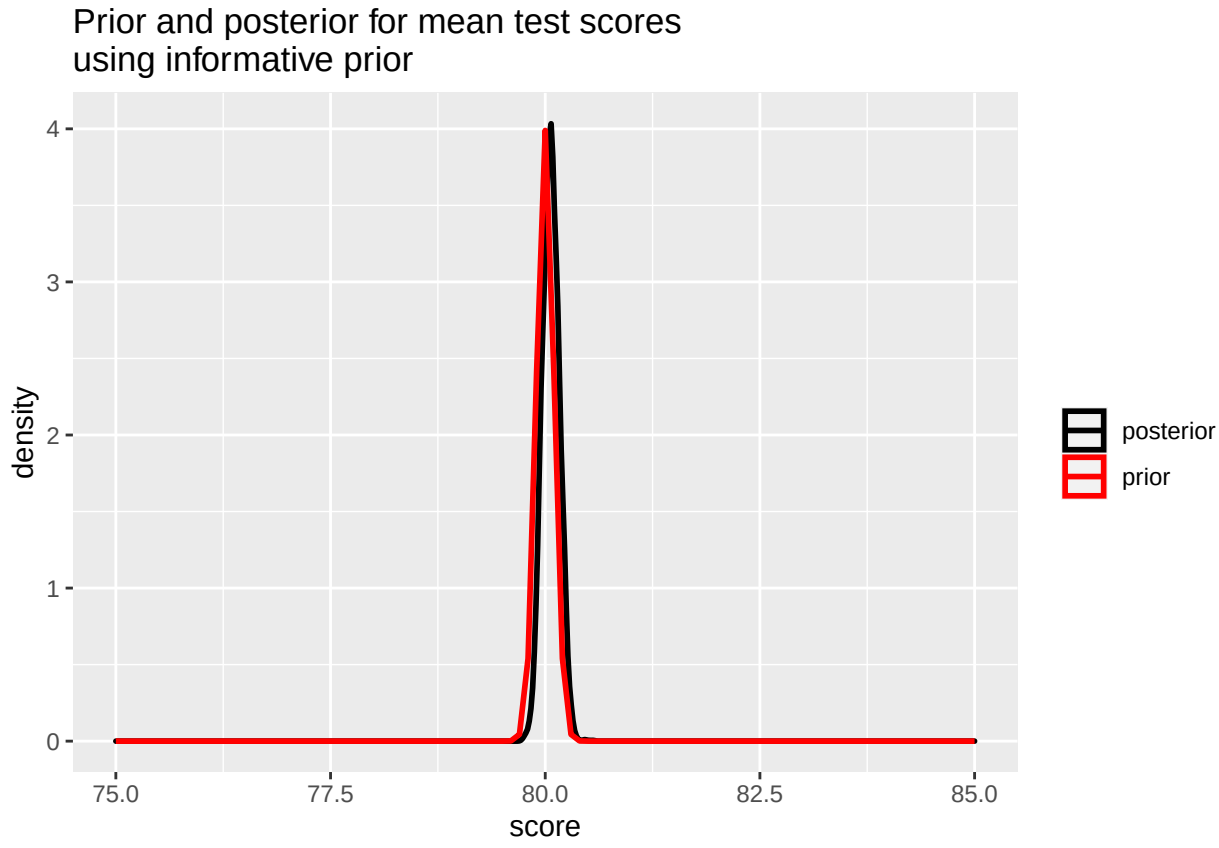
```
fit.1
```

```
## Inference for Stan model: kids2.
## 4 chains, each with iter=2000; warmup=1000; thin=1;
## post-warmup draws per chain=1000, total post-warmup draws=4000.
##
##           mean se_mean   sd     2.5%      25%      50%      75%    97.5% n_eff
## mu       80.07    0.00 0.10    79.88    80.00    80.06    80.13    80.25  3177
## sigma    21.43    0.01 0.72    20.07    20.96    21.41    21.89    22.95  3084
## lp__  -1548.36    0.03 1.00 -1551.08 -1548.74 -1548.07 -1547.65 -1547.39  1564
##       Rhat
## mu       1
## sigma    1
```

```
## lp__     1
##
## Samples were drawn using NUTS(diag_e) at Fri Feb 12 09:32:51 2021.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).
```

```r
dsamples <- fit.1 %>%
  gather_draws(mu, sigma)

dsamples %>%
  filter(.variable == "mu") %>%
  ggplot(aes(.value, color = "posterior")) + geom_density(size = 1) +
  xlim(c(75, 85)) +
  stat_function(fun = dnorm,
        args = list(mean = mu0,
                    sd = sigma0),
        aes(colour = 'prior'), size = 1) +
  scale_color_manual(name = "", values = c("prior" = "red", "posterior" = "black")) +
  ggtitle("Prior and posterior for mean test scores \nusing informative prior") +
  xlab("score")
```



4 Adding covariates

Now let's see how kid's test scores are related to mother's education. We want to run the simple linear regression

$$Score = \alpha + \beta X$$

where $X = 1$ if the mother finished high school and zero otherwise.

`kid3.stan` has the stan model to do this. Notice now we have some inputs related to the design matrix X and the number of covariates (in this case, it's just 1).

Let's get the data we need and run the model.

```
X <- as.matrix(kidiq$mom_hs, ncol = 1)
K <- 1

data <- list(y = y, N = length(y),
             X =X, K = K)
fit2 <- stan(file = "code/models/kids3.stan",
            data = data,
            iter = 1000)
```

```
##
## SAMPLING FOR MODEL 'kids3' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 0 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: Iteration:   1 / 1000 [  0%]  (Warmup)
## Chain 1: Iteration: 100 / 1000 [ 10%]  (Warmup)
## Chain 1: Iteration: 200 / 1000 [ 20%]  (Warmup)
## Chain 1: Iteration: 300 / 1000 [ 30%]  (Warmup)
## Chain 1: Iteration: 400 / 1000 [ 40%]  (Warmup)
## Chain 1: Iteration: 500 / 1000 [ 50%]  (Warmup)
## Chain 1: Iteration: 501 / 1000 [ 50%]  (Sampling)
## Chain 1: Iteration: 600 / 1000 [ 60%]  (Sampling)
## Chain 1: Iteration: 700 / 1000 [ 70%]  (Sampling)
## Chain 1: Iteration: 800 / 1000 [ 80%]  (Sampling)
## Chain 1: Iteration: 900 / 1000 [ 90%]  (Sampling)
## Chain 1: Iteration: 1000 / 1000 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 0.662 seconds (Warm-up)
## Chain 1:                0.382 seconds (Sampling)
## Chain 1:                1.044 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL 'kids3' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 0 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0 seconds.
## Chain 2: Adjust your expectations accordingly!
```

```
## Chain 2:
## Chain 2:
## Chain 2: Iteration:   1 / 1000 [  0%]  (Warmup)
## Chain 2: Iteration: 100 / 1000 [ 10%]  (Warmup)
## Chain 2: Iteration: 200 / 1000 [ 20%]  (Warmup)
## Chain 2: Iteration: 300 / 1000 [ 30%]  (Warmup)
## Chain 2: Iteration: 400 / 1000 [ 40%]  (Warmup)
## Chain 2: Iteration: 500 / 1000 [ 50%]  (Warmup)
## Chain 2: Iteration: 501 / 1000 [ 50%]  (Sampling)
## Chain 2: Iteration: 600 / 1000 [ 60%]  (Sampling)
## Chain 2: Iteration: 700 / 1000 [ 70%]  (Sampling)
## Chain 2: Iteration: 800 / 1000 [ 80%]  (Sampling)
## Chain 2: Iteration: 900 / 1000 [ 90%]  (Sampling)
## Chain 2: Iteration: 1000 / 1000 [100%]  (Sampling)
## Chain 2:
## Chain 2:  Elapsed Time: 0.648 seconds (Warm-up)
## Chain 2:                0.349 seconds (Sampling)
## Chain 2:                0.997 seconds (Total)
## Chain 2:
##
## SAMPLING FOR MODEL 'kids3' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 0 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0 seconds.
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:
## Chain 3: Iteration:   1 / 1000 [  0%]  (Warmup)
## Chain 3: Iteration: 100 / 1000 [ 10%]  (Warmup)
## Chain 3: Iteration: 200 / 1000 [ 20%]  (Warmup)
## Chain 3: Iteration: 300 / 1000 [ 30%]  (Warmup)
## Chain 3: Iteration: 400 / 1000 [ 40%]  (Warmup)
## Chain 3: Iteration: 500 / 1000 [ 50%]  (Warmup)
## Chain 3: Iteration: 501 / 1000 [ 50%]  (Sampling)
## Chain 3: Iteration: 600 / 1000 [ 60%]  (Sampling)
## Chain 3: Iteration: 700 / 1000 [ 70%]  (Sampling)
## Chain 3: Iteration: 800 / 1000 [ 80%]  (Sampling)
## Chain 3: Iteration: 900 / 1000 [ 90%]  (Sampling)
## Chain 3: Iteration: 1000 / 1000 [100%]  (Sampling)
## Chain 3:
## Chain 3:  Elapsed Time: 0.511 seconds (Warm-up)
## Chain 3:                0.39 seconds (Sampling)
## Chain 3:                0.901 seconds (Total)
## Chain 3:
##
## SAMPLING FOR MODEL 'kids3' NOW (CHAIN 4).
## Chain 4:
## Chain 4: Gradient evaluation took 0 seconds
## Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0 seconds.
## Chain 4: Adjust your expectations accordingly!
## Chain 4:
## Chain 4:
## Chain 4: Iteration:   1 / 1000 [  0%]  (Warmup)
## Chain 4: Iteration: 100 / 1000 [ 10%]  (Warmup)
```

```
## Chain 4: Iteration: 200 / 1000 [ 20%]  (Warmup)
## Chain 4: Iteration: 300 / 1000 [ 30%]  (Warmup)
## Chain 4: Iteration: 400 / 1000 [ 40%]  (Warmup)
## Chain 4: Iteration: 500 / 1000 [ 50%]  (Warmup)
## Chain 4: Iteration: 501 / 1000 [ 50%]  (Sampling)
## Chain 4: Iteration: 600 / 1000 [ 60%]  (Sampling)
## Chain 4: Iteration: 700 / 1000 [ 70%]  (Sampling)
## Chain 4: Iteration: 800 / 1000 [ 80%]  (Sampling)
## Chain 4: Iteration: 900 / 1000 [ 90%]  (Sampling)
## Chain 4: Iteration: 1000 / 1000 [100%]  (Sampling)
## Chain 4:
## Chain 4:  Elapsed Time: 0.511 seconds (Warm-up)
## Chain 4:                0.391 seconds (Sampling)
## Chain 4:                0.902 seconds (Total)
## Chain 4:
```

```
fit2
```

```
## Inference for Stan model: kids3.
## 4 chains, each with iter=1000; warmup=500; thin=1;
## post-warmup draws per chain=500, total post-warmup draws=2000.
##
##             mean se_mean   sd     2.5%      25%      50%      75%    97.5%
## alpha      77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## beta[1]    11.39    0.09 2.23     7.04     9.90    11.33    12.82    15.97
## sigma      19.81    0.03 0.69    18.47    19.32    19.81    20.30    21.16
## mu[1]      89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[2]      89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[3]      89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[4]      89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[5]      89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[6]      77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[7]      89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[8]      89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[9]      89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[10]     89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[11]     89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[12]     89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[13]     89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[14]     89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[15]     77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[16]     89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[17]     89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[18]     89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[19]     89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[20]     77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[21]     89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[22]     89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[23]     89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[24]     89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[25]     77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[26]     89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[27]     89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[28]     89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
```

```
## mu[29]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[30]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[31]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[32]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[33]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[34]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[35]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[36]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[37]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[38]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[39]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[40]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[41]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[42]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[43]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[44]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[45]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[46]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[47]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[48]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[49]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[50]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[51]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[52]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[53]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[54]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[55]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[56]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[57]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[58]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[59]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[60]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[61]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[62]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[63]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[64]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[65]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[66]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[67]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[68]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[69]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[70]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[71]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[72]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[73]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[74]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[75]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[76]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[77]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[78]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[79]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[80]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[81]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[82]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
```

```
## mu[83]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[84]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[85]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[86]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[87]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[88]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[89]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[90]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[91]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[92]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[93]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[94]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[95]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[96]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[97]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[98]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[99]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[100]   77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[101]   89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[102]   89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[103]   89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[104]   89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[105]   89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[106]   89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[107]   89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[108]   89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[109]   89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[110]   89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[111]   77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[112]   89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[113]   89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[114]   89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[115]   89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[116]   89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[117]   89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[118]   89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[119]   77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[120]   89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[121]   89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[122]   89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[123]   89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[124]   89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[125]   89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[126]   89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[127]   77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[128]   89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[129]   89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[130]   89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[131]   89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[132]   89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[133]   89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[134]   89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[135]   77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[136]   89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
```

```
## mu[137]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[138]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[139]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[140]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[141]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[142]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[143]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[144]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[145]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[146]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[147]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[148]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[149]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[150]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[151]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[152]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[153]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[154]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[155]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[156]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[157]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[158]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[159]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[160]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[161]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[162]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[163]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[164]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[165]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[166]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[167]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[168]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[169]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[170]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[171]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[172]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[173]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[174]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[175]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[176]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[177]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[178]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[179]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[180]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[181]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[182]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[183]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[184]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[185]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[186]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[187]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[188]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[189]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[190]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
```

```
## mu[191]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[192]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[193]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[194]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[195]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[196]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[197]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[198]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[199]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[200]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[201]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[202]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[203]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[204]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[205]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[206]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[207]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[208]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[209]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[210]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[211]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[212]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[213]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[214]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[215]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[216]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[217]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[218]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[219]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[220]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[221]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[222]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[223]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[224]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[225]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[226]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[227]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[228]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[229]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[230]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[231]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[232]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[233]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[234]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[235]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[236]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[237]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[238]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[239]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[240]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[241]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[242]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[243]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[244]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
```

```
## mu[245]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[246]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[247]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[248]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[249]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[250]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[251]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[252]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[253]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[254]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[255]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[256]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[257]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[258]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[259]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[260]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[261]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[262]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[263]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[264]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[265]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[266]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[267]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[268]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[269]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[270]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[271]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[272]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[273]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[274]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[275]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[276]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[277]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[278]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[279]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[280]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[281]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[282]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[283]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[284]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[285]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[286]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[287]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[288]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[289]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[290]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[291]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[292]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[293]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[294]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[295]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[296]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[297]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[298]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
```

```
## mu[299]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[300]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[301]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[302]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[303]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[304]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[305]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[306]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[307]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[308]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[309]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[310]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[311]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[312]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[313]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[314]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[315]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[316]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[317]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[318]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[319]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[320]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[321]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[322]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[323]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[324]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[325]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[326]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[327]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[328]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[329]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[330]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[331]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[332]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[333]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[334]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[335]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[336]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[337]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[338]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[339]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[340]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[341]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[342]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[343]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[344]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[345]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[346]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[347]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[348]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[349]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[350]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[351]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[352]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
```

```
## mu[353]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[354]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[355]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[356]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[357]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[358]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[359]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[360]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[361]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[362]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[363]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[364]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[365]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[366]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[367]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[368]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[369]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[370]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[371]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[372]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[373]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[374]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[375]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[376]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[377]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[378]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[379]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[380]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[381]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[382]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[383]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[384]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[385]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[386]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[387]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[388]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[389]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[390]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[391]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[392]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[393]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[394]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[395]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[396]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[397]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[398]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[399]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[400]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[401]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[402]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[403]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[404]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[405]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[406]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
```

```
## mu[407]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[408]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[409]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[410]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[411]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[412]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[413]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[414]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[415]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[416]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[417]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[418]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[419]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[420]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[421]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[422]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[423]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[424]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[425]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[426]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[427]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[428]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[429]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[430]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[431]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[432]    77.86    0.08 2.03    73.72    76.49    77.93    79.25    81.72
## mu[433]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## mu[434]    89.25    0.02 1.08    87.17    88.48    89.27    90.01    91.27
## lp__    -1514.13    0.05 1.23 -1517.39 -1514.66 -1513.83 -1513.25 -1512.69
##         n_eff Rhat
## alpha     629    1
## beta[1]   554    1
## sigma     738    1
## mu[1]    2277    1
## mu[2]    2277    1
## mu[3]    2277    1
## mu[4]    2277    1
## mu[5]    2277    1
## mu[6]     629    1
## mu[7]    2277    1
## mu[8]    2277    1
## mu[9]    2277    1
## mu[10]   2277    1
## mu[11]   2277    1
## mu[12]   2277    1
## mu[13]   2277    1
## mu[14]   2277    1
## mu[15]    629    1
## mu[16]   2277    1
## mu[17]   2277    1
## mu[18]   2277    1
## mu[19]   2277    1
## mu[20]    629    1
## mu[21]   2277    1
```

```
## mu[22]   2277    1
## mu[23]   2277    1
## mu[24]   2277    1
## mu[25]    629    1
## mu[26]   2277    1
## mu[27]   2277    1
## mu[28]   2277    1
## mu[29]   2277    1
## mu[30]   2277    1
## mu[31]   2277    1
## mu[32]   2277    1
## mu[33]   2277    1
## mu[34]    629    1
## mu[35]   2277    1
## mu[36]   2277    1
## mu[37]   2277    1
## mu[38]   2277    1
## mu[39]   2277    1
## mu[40]   2277    1
## mu[41]   2277    1
## mu[42]   2277    1
## mu[43]   2277    1
## mu[44]   2277    1
## mu[45]    629    1
## mu[46]   2277    1
## mu[47]   2277    1
## mu[48]   2277    1
## mu[49]   2277    1
## mu[50]   2277    1
## mu[51]   2277    1
## mu[52]   2277    1
## mu[53]   2277    1
## mu[54]   2277    1
## mu[55]   2277    1
## mu[56]    629    1
## mu[57]   2277    1
## mu[58]    629    1
## mu[59]    629    1
## mu[60]   2277    1
## mu[61]   2277    1
## mu[62]   2277    1
## mu[63]    629    1
## mu[64]   2277    1
## mu[65]   2277    1
## mu[66]   2277    1
## mu[67]   2277    1
## mu[68]   2277    1
## mu[69]   2277    1
## mu[70]   2277    1
## mu[71]   2277    1
## mu[72]    629    1
## mu[73]    629    1
## mu[74]   2277    1
## mu[75]   2277    1
```

```
## mu[76]   2277    1
## mu[77]   2277    1
## mu[78]    629    1
## mu[79]   2277    1
## mu[80]    629    1
## mu[81]    629    1
## mu[82]   2277    1
## mu[83]   2277    1
## mu[84]   2277    1
## mu[85]   2277    1
## mu[86]   2277    1
## mu[87]   2277    1
## mu[88]   2277    1
## mu[89]   2277    1
## mu[90]   2277    1
## mu[91]   2277    1
## mu[92]   2277    1
## mu[93]   2277    1
## mu[94]   2277    1
## mu[95]   2277    1
## mu[96]    629    1
## mu[97]   2277    1
## mu[98]    629    1
## mu[99]    629    1
## mu[100]   629    1
## mu[101]  2277    1
## mu[102]  2277    1
## mu[103]  2277    1
## mu[104]  2277    1
## mu[105]  2277    1
## mu[106]  2277    1
## mu[107]  2277    1
## mu[108]  2277    1
## mu[109]  2277    1
## mu[110]  2277    1
## mu[111]   629    1
## mu[112]  2277    1
## mu[113]  2277    1
## mu[114]  2277    1
## mu[115]  2277    1
## mu[116]  2277    1
## mu[117]  2277    1
## mu[118]  2277    1
## mu[119]   629    1
## mu[120]  2277    1
## mu[121]  2277    1
## mu[122]  2277    1
## mu[123]  2277    1
## mu[124]  2277    1
## mu[125]  2277    1
## mu[126]  2277    1
## mu[127]   629    1
## mu[128]  2277    1
## mu[129]  2277    1
```

```
## mu[130]  2277    1
## mu[131]  2277    1
## mu[132]  2277    1
## mu[133]  2277    1
## mu[134]  2277    1
## mu[135]   629    1
## mu[136]  2277    1
## mu[137]  2277    1
## mu[138]  2277    1
## mu[139]  2277    1
## mu[140]  2277    1
## mu[141]  2277    1
## mu[142]  2277    1
## mu[143]  2277    1
## mu[144]  2277    1
## mu[145]  2277    1
## mu[146]  2277    1
## mu[147]  2277    1
## mu[148]  2277    1
## mu[149]  2277    1
## mu[150]  2277    1
## mu[151]  2277    1
## mu[152]  2277    1
## mu[153]  2277    1
## mu[154]  2277    1
## mu[155]  2277    1
## mu[156]  2277    1
## mu[157]  2277    1
## mu[158]  2277    1
## mu[159]  2277    1
## mu[160]  2277    1
## mu[161]   629    1
## mu[162]   629    1
## mu[163]  2277    1
## mu[164]  2277    1
## mu[165]  2277    1
## mu[166]  2277    1
## mu[167]  2277    1
## mu[168]  2277    1
## mu[169]   629    1
## mu[170]  2277    1
## mu[171]  2277    1
## mu[172]  2277    1
## mu[173]  2277    1
## mu[174]  2277    1
## mu[175]   629    1
## mu[176]  2277    1
## mu[177]  2277    1
## mu[178]  2277    1
## mu[179]  2277    1
## mu[180]  2277    1
## mu[181]   629    1
## mu[182]  2277    1
## mu[183]  2277    1
```

```
## mu[184]  2277    1
## mu[185]   629    1
## mu[186]  2277    1
## mu[187]  2277    1
## mu[188]  2277    1
## mu[189]  2277    1
## mu[190]  2277    1
## mu[191]   629    1
## mu[192]   629    1
## mu[193]   629    1
## mu[194]   629    1
## mu[195]  2277    1
## mu[196]   629    1
## mu[197]  2277    1
## mu[198]  2277    1
## mu[199]  2277    1
## mu[200]  2277    1
## mu[201]  2277    1
## mu[202]   629    1
## mu[203]  2277    1
## mu[204]  2277    1
## mu[205]  2277    1
## mu[206]   629    1
## mu[207]  2277    1
## mu[208]  2277    1
## mu[209]  2277    1
## mu[210]  2277    1
## mu[211]   629    1
## mu[212]  2277    1
## mu[213]   629    1
## mu[214]  2277    1
## mu[215]  2277    1
## mu[216]  2277    1
## mu[217]  2277    1
## mu[218]  2277    1
## mu[219]  2277    1
## mu[220]  2277    1
## mu[221]  2277    1
## mu[222]  2277    1
## mu[223]  2277    1
## mu[224]  2277    1
## mu[225]  2277    1
## mu[226]  2277    1
## mu[227]  2277    1
## mu[228]  2277    1
## mu[229]  2277    1
## mu[230]   629    1
## mu[231]  2277    1
## mu[232]  2277    1
## mu[233]  2277    1
## mu[234]  2277    1
## mu[235]  2277    1
## mu[236]  2277    1
## mu[237]  2277    1
```

```
## mu[238]  2277    1
## mu[239]  2277    1
## mu[240]   629    1
## mu[241]  2277    1
## mu[242]  2277    1
## mu[243]   629    1
## mu[244]  2277    1
## mu[245]  2277    1
## mu[246]  2277    1
## mu[247]  2277    1
## mu[248]  2277    1
## mu[249]  2277    1
## mu[250]  2277    1
## mu[251]  2277    1
## mu[252]  2277    1
## mu[253]  2277    1
## mu[254]  2277    1
## mu[255]   629    1
## mu[256]   629    1
## mu[257]  2277    1
## mu[258]   629    1
## mu[259]   629    1
## mu[260]  2277    1
## mu[261]  2277    1
## mu[262]  2277    1
## mu[263]   629    1
## mu[264]  2277    1
## mu[265]  2277    1
## mu[266]  2277    1
## mu[267]  2277    1
## mu[268]   629    1
## mu[269]   629    1
## mu[270]  2277    1
## mu[271]  2277    1
## mu[272]  2277    1
## mu[273]  2277    1
## mu[274]  2277    1
## mu[275]   629    1
## mu[276]   629    1
## mu[277]  2277    1
## mu[278]  2277    1
## mu[279]  2277    1
## mu[280]  2277    1
## mu[281]  2277    1
## mu[282]  2277    1
## mu[283]  2277    1
## mu[284]   629    1
## mu[285]   629    1
## mu[286]   629    1
## mu[287]  2277    1
## mu[288]  2277    1
## mu[289]   629    1
## mu[290]   629    1
## mu[291]   629    1
```

```
## mu[292]   629    1
## mu[293]  2277    1
## mu[294]  2277    1
## mu[295]   629    1
## mu[296]  2277    1
## mu[297]   629    1
## mu[298]   629    1
## mu[299]   629    1
## mu[300]  2277    1
## mu[301]  2277    1
## mu[302]   629    1
## mu[303]  2277    1
## mu[304]  2277    1
## mu[305]  2277    1
## mu[306]  2277    1
## mu[307]  2277    1
## mu[308]  2277    1
## mu[309]  2277    1
## mu[310]   629    1
## mu[311]  2277    1
## mu[312]  2277    1
## mu[313]  2277    1
## mu[314]  2277    1
## mu[315]  2277    1
## mu[316]  2277    1
## mu[317]  2277    1
## mu[318]  2277    1
## mu[319]  2277    1
## mu[320]  2277    1
## mu[321]   629    1
## mu[322]  2277    1
## mu[323]  2277    1
## mu[324]  2277    1
## mu[325]  2277    1
## mu[326]  2277    1
## mu[327]  2277    1
## mu[328]  2277    1
## mu[329]  2277    1
## mu[330]  2277    1
## mu[331]  2277    1
## mu[332]  2277    1
## mu[333]  2277    1
## mu[334]  2277    1
## mu[335]   629    1
## mu[336]  2277    1
## mu[337]  2277    1
## mu[338]  2277    1
## mu[339]  2277    1
## mu[340]  2277    1
## mu[341]   629    1
## mu[342]  2277    1
## mu[343]  2277    1
## mu[344]  2277    1
## mu[345]  2277    1
```

```
## mu[346]   629    1
## mu[347]   629    1
## mu[348]  2277    1
## mu[349]  2277    1
## mu[350]  2277    1
## mu[351]  2277    1
## mu[352]  2277    1
## mu[353]  2277    1
## mu[354]  2277    1
## mu[355]  2277    1
## mu[356]   629    1
## mu[357]  2277    1
## mu[358]  2277    1
## mu[359]   629    1
## mu[360]  2277    1
## mu[361]  2277    1
## mu[362]  2277    1
## mu[363]  2277    1
## mu[364]  2277    1
## mu[365]  2277    1
## mu[366]  2277    1
## mu[367]  2277    1
## mu[368]  2277    1
## mu[369]  2277    1
## mu[370]  2277    1
## mu[371]  2277    1
## mu[372]  2277    1
## mu[373]  2277    1
## mu[374]  2277    1
## mu[375]   629    1
## mu[376]  2277    1
## mu[377]  2277    1
## mu[378]  2277    1
## mu[379]  2277    1
## mu[380]  2277    1
## mu[381]  2277    1
## mu[382]   629    1
## mu[383]   629    1
## mu[384]   629    1
## mu[385]   629    1
## mu[386]  2277    1
## mu[387]  2277    1
## mu[388]  2277    1
## mu[389]   629    1
## mu[390]  2277    1
## mu[391]  2277    1
## mu[392]  2277    1
## mu[393]   629    1
## mu[394]  2277    1
## mu[395]  2277    1
## mu[396]  2277    1
## mu[397]  2277    1
## mu[398]  2277    1
## mu[399]  2277    1
```

```
## mu[400]  2277    1
## mu[401]   629    1
## mu[402]   629    1
## mu[403]   629    1
## mu[404]  2277    1
## mu[405]   629    1
## mu[406]   629    1
## mu[407]  2277    1
## mu[408]  2277    1
## mu[409]  2277    1
## mu[410]   629    1
## mu[411]  2277    1
## mu[412]  2277    1
## mu[413]  2277    1
## mu[414]  2277    1
## mu[415]  2277    1
## mu[416]   629    1
## mu[417]  2277    1
## mu[418]   629    1
## mu[419]  2277    1
## mu[420]   629    1
## mu[421]   629    1
## mu[422]   629    1
## mu[423]   629    1
## mu[424]  2277    1
## mu[425]  2277    1
## mu[426]  2277    1
## mu[427]  2277    1
## mu[428]   629    1
## mu[429]   629    1
## mu[430]   629    1
## mu[431]  2277    1
## mu[432]   629    1
## mu[433]  2277    1
## mu[434]  2277    1
## lp__      730    1
##
## Samples were drawn using NUTS(diag_e) at Fri Feb 12 09:35:08 2021.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).
```

4.1 Question 3

a) Confirm that the estimates of the intercept and slope are comparable to results from `lm()`

```
fitlm<-lm(kid_score~mom_hs,data=kidiq)
summary(fitlm)
```

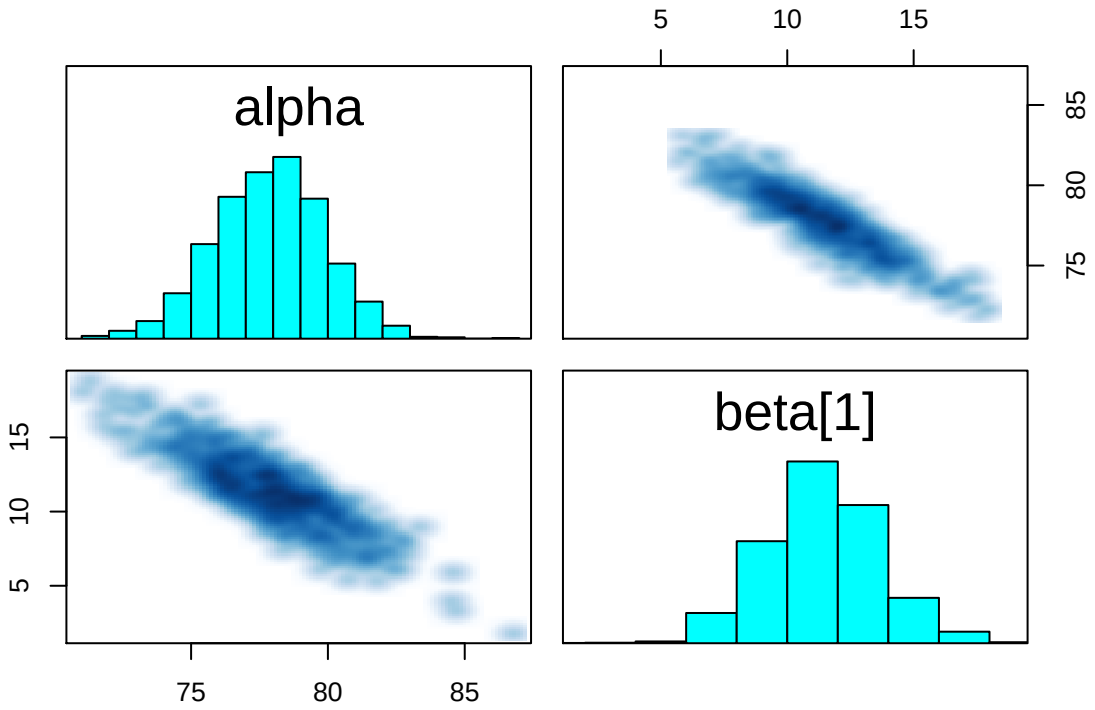
```
##
## Call:
## lm(formula = kid_score ~ mom_hs, data = kidiq)
##
```

```
## Residuals:
##     Min      1Q  Median      3Q     Max
## -57.55 -13.32   2.68  14.68  58.45
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   77.548      2.059  37.670  < 2e-16 ***
## mom_hs        11.771      2.322   5.069 5.96e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 19.85 on 432 degrees of freedom
## Multiple R-squared:  0.05613,    Adjusted R-squared:  0.05394
## F-statistic: 25.69 on 1 and 432 DF,  p-value: 5.957e-07
```

b) Do a `pairs` plot to investigate the joint sample distributions of the slope and intercept. Comment briefly on what you see. Is this potentially a problem?

Ideally we would see a cloud of points with most of the density in the middle, indicating that the slope and intercept sample distributions are not correlated. In this case, the points form a straight line, which implies the slope and intercept sample distributions are correlated and is potentially a problem.

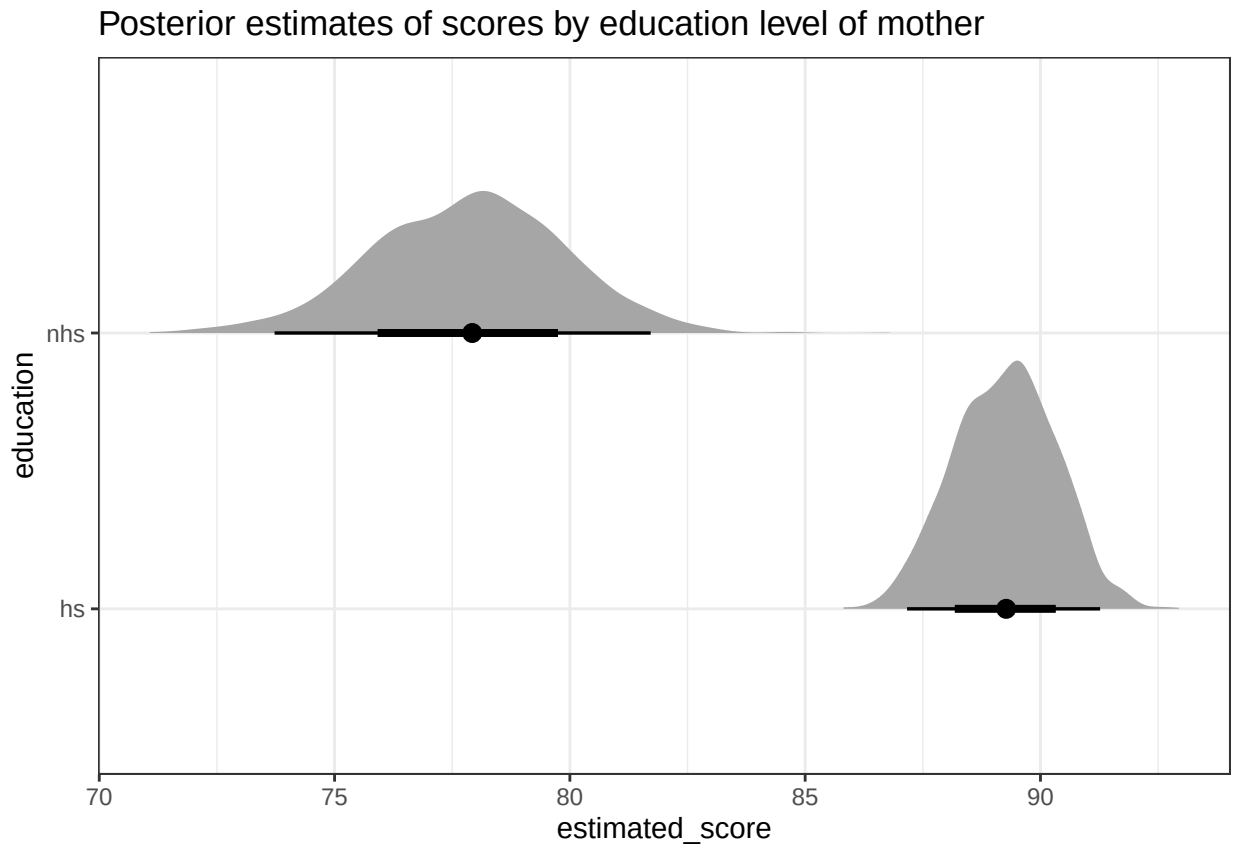
```
pars=c("alpha","beta[1]")
pairs(fit2,pars=pars)
```



4.2 Plotting results

It might be nice to plot the posterior samples of the estimates for the non-high-school and high-school mothered kids. Here's some code that does this: notice the `beta[condition]` syntax. Also notice I'm using `spread_draws`, because it's easier to calculate the estimated effects in wide format

```
fit2 %>%
  spread_draws(alpha, beta[condition], sigma) %>%
     mutate(nhs = alpha, # no high school is just the intercept
            hs = alpha + beta) %>%
  pivot_longer(nhs:hs, names_to = "education", values_to = "estimated_score") %>%
  ggplot(aes(y = education, x = estimated_score)) +
  stat_halfeye() +
  theme_bw() +
  ggtitle("Posterior estimates of scores by education level of mother")
```



4.3 Question 4

Add in mother's IQ as a covariate and rerun the model. Please mean center the covariate before putting it into the model. Interpret the coefficient on the (centered) mum's IQ.

The coefficient on the centered mom's IQ is 0.56. This means that comparing children with the same value of `mom.hs`, but whose mothers differ by 1 point in IQ, we would expect to see a difference of 0.6 points in the child's test score.

```r
# what is the mean value of the time variable week?
mean_mom_iq <- mean(kidiq$mom_iq)

# add new variable, weekc, that is centered around the mean
kidiq <- kidiq %>%
  mutate(mom_iqc = mom_iq - mean_mom_iq)

X <- cbind(kidiq$mom_hs,kidiq$mom_iqc)
K <- 2

data <- list(y = y, N = length(y),
             X =X, K = K)
fit3 <- stan(file = "code/models/kids3.stan",
            data = data,
            iter = 1000)
```

```
##
## SAMPLING FOR MODEL 'kids3' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 0 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: Iteration:   1 / 1000 [  0%]  (Warmup)
## Chain 1: Iteration: 100 / 1000 [ 10%]  (Warmup)
## Chain 1: Iteration: 200 / 1000 [ 20%]  (Warmup)
## Chain 1: Iteration: 300 / 1000 [ 30%]  (Warmup)
## Chain 1: Iteration: 400 / 1000 [ 40%]  (Warmup)
## Chain 1: Iteration: 500 / 1000 [ 50%]  (Warmup)
## Chain 1: Iteration: 501 / 1000 [ 50%]  (Sampling)
## Chain 1: Iteration: 600 / 1000 [ 60%]  (Sampling)
## Chain 1: Iteration: 700 / 1000 [ 70%]  (Sampling)
## Chain 1: Iteration: 800 / 1000 [ 80%]  (Sampling)
## Chain 1: Iteration: 900 / 1000 [ 90%]  (Sampling)
## Chain 1: Iteration: 1000 / 1000 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 0.743 seconds (Warm-up)
## Chain 1:                0.417 seconds (Sampling)
## Chain 1:                1.16 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL 'kids3' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 0 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: Iteration:   1 / 1000 [  0%]  (Warmup)
## Chain 2: Iteration: 100 / 1000 [ 10%]  (Warmup)
## Chain 2: Iteration: 200 / 1000 [ 20%]  (Warmup)
## Chain 2: Iteration: 300 / 1000 [ 30%]  (Warmup)
```

```
## Chain 2: Iteration: 400 / 1000 [ 40%]  (Warmup)
## Chain 2: Iteration: 500 / 1000 [ 50%]  (Warmup)
## Chain 2: Iteration: 501 / 1000 [ 50%]  (Sampling)
## Chain 2: Iteration: 600 / 1000 [ 60%]  (Sampling)
## Chain 2: Iteration: 700 / 1000 [ 70%]  (Sampling)
## Chain 2: Iteration: 800 / 1000 [ 80%]  (Sampling)
## Chain 2: Iteration: 900 / 1000 [ 90%]  (Sampling)
## Chain 2: Iteration: 1000 / 1000 [100%]  (Sampling)
## Chain 2:
## Chain 2:  Elapsed Time: 0.622 seconds (Warm-up)
## Chain 2:                0.572 seconds (Sampling)
## Chain 2:                1.194 seconds (Total)
## Chain 2:
##
## SAMPLING FOR MODEL 'kids3' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 0 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0 seconds.
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:
## Chain 3: Iteration:   1 / 1000 [  0%]  (Warmup)
## Chain 3: Iteration: 100 / 1000 [ 10%]  (Warmup)
## Chain 3: Iteration: 200 / 1000 [ 20%]  (Warmup)
## Chain 3: Iteration: 300 / 1000 [ 30%]  (Warmup)
## Chain 3: Iteration: 400 / 1000 [ 40%]  (Warmup)
## Chain 3: Iteration: 500 / 1000 [ 50%]  (Warmup)
## Chain 3: Iteration: 501 / 1000 [ 50%]  (Sampling)
## Chain 3: Iteration: 600 / 1000 [ 60%]  (Sampling)
## Chain 3: Iteration: 700 / 1000 [ 70%]  (Sampling)
## Chain 3: Iteration: 800 / 1000 [ 80%]  (Sampling)
## Chain 3: Iteration: 900 / 1000 [ 90%]  (Sampling)
## Chain 3: Iteration: 1000 / 1000 [100%]  (Sampling)
## Chain 3:
## Chain 3:  Elapsed Time: 0.845 seconds (Warm-up)
## Chain 3:                0.509 seconds (Sampling)
## Chain 3:                1.354 seconds (Total)
## Chain 3:
##
## SAMPLING FOR MODEL 'kids3' NOW (CHAIN 4).
## Chain 4:
## Chain 4: Gradient evaluation took 0 seconds
## Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0 seconds.
## Chain 4: Adjust your expectations accordingly!
## Chain 4:
## Chain 4:
## Chain 4: Iteration:   1 / 1000 [  0%]  (Warmup)
## Chain 4: Iteration: 100 / 1000 [ 10%]  (Warmup)
## Chain 4: Iteration: 200 / 1000 [ 20%]  (Warmup)
## Chain 4: Iteration: 300 / 1000 [ 30%]  (Warmup)
## Chain 4: Iteration: 400 / 1000 [ 40%]  (Warmup)
## Chain 4: Iteration: 500 / 1000 [ 50%]  (Warmup)
## Chain 4: Iteration: 501 / 1000 [ 50%]  (Sampling)
## Chain 4: Iteration: 600 / 1000 [ 60%]  (Sampling)
```

```
## Chain 4: Iteration: 700 / 1000 [ 70%]  (Sampling)
## Chain 4: Iteration: 800 / 1000 [ 80%]  (Sampling)
## Chain 4: Iteration: 900 / 1000 [ 90%]  (Sampling)
## Chain 4: Iteration: 1000 / 1000 [100%]  (Sampling)
## Chain 4:
## Chain 4:  Elapsed Time: 0.801 seconds (Warm-up)
## Chain 4:                0.471 seconds (Sampling)
## Chain 4:                1.272 seconds (Total)
## Chain 4:
```

```
fit3
```

```
## Inference for Stan model: kids3.
## 4 chains, each with iter=1000; warmup=500; thin=1;
## post-warmup draws per chain=500, total post-warmup draws=2000.
##
##             mean se_mean   sd     2.5%      25%      50%      75%    97.5%
## alpha      82.34    0.06 1.90    78.45    81.08    82.39    83.60    86.07
## beta[1]     5.71    0.06 2.17     1.66     4.22     5.69     7.15    10.05
## beta[2]     0.57    0.00 0.06     0.45     0.53     0.57     0.61     0.68
## sigma      18.10    0.02 0.61    16.93    17.69    18.07    18.49    19.36
## mu[1]     100.02    0.04 1.43    97.13    99.08   100.02   100.94   102.79
## mu[2]      82.02    0.03 1.22    79.65    81.19    82.00    82.84    84.48
## mu[3]      96.80    0.03 1.20    94.44    96.01    96.81    97.58    99.17
## mu[4]      87.74    0.02 0.96    85.86    87.06    87.73    88.38    89.61
## mu[5]      83.94    0.02 1.11    81.75    83.20    83.92    84.69    86.17
## mu[6]      86.82    0.06 2.07    82.52    85.47    86.86    88.19    90.83
## mu[7]     110.09    0.06 2.31   105.43   108.58   110.12   111.56   114.52
## mu[8]     102.30    0.04 1.62    99.03   101.24   102.29   103.33   105.38
## mu[9]      77.63    0.04 1.55    74.61    76.60    77.62    78.66    80.70
## mu[10]     85.25    0.02 1.04    83.22    84.56    85.24    85.96    87.34
## mu[11]     81.57    0.03 1.25    79.14    80.74    81.56    82.42    84.09
## mu[12]     85.13    0.02 1.05    83.10    84.43    85.12    85.84    87.24
## mu[13]     81.79    0.03 1.24    79.40    80.96    81.78    82.63    84.29
## mu[14]     96.05    0.03 1.16    93.77    95.29    96.06    96.80    98.33
## mu[15]     82.64    0.06 1.91    78.77    81.38    82.69    83.92    86.41
## mu[16]     99.62    0.03 1.40    96.81    98.71    99.62   100.53   102.34
## mu[17]     96.22    0.03 1.17    93.92    95.46    96.24    96.99    98.52
## mu[18]     94.62    0.02 1.08    92.48    93.91    94.63    95.33    96.74
## mu[19]    107.23    0.05 2.05   103.15   105.86   107.25   108.55   111.15
## mu[20]     80.79    0.05 1.86    77.03    79.57    80.85    82.01    84.43
## mu[21]     93.77    0.02 1.04    91.66    93.08    93.77    94.46    95.84
## mu[22]    103.19    0.04 1.69    99.82   102.08   103.20   104.27   106.44
## mu[23]     86.86    0.02 0.99    84.95    86.20    86.87    87.52    88.81
## mu[24]     88.01    0.02 0.96    86.11    87.34    87.99    88.65    89.85
## mu[25]     80.98    0.05 1.87    77.22    79.75    81.03    82.21    84.62
## mu[26]    100.37    0.04 1.46    97.45    99.42   100.38   101.32   103.20
## mu[27]     87.34    0.02 0.97    85.48    86.66    87.33    87.98    89.24
## mu[28]     86.87    0.02 0.99    84.96    86.20    86.87    87.52    88.81
## mu[29]     76.92    0.04 1.61    73.76    75.84    76.90    77.97    80.09
## mu[30]     96.16    0.03 1.16    93.87    95.39    96.17    96.92    98.44
## mu[31]     93.23    0.02 1.02    91.16    92.56    93.23    93.90    95.25
## mu[32]     89.08    0.02 0.95    87.18    88.44    89.06    89.69    90.89
## mu[33]     98.23    0.03 1.30    95.66    97.37    98.24    99.06   100.77
```

```
## mu[34]     87.23    0.06 2.09    82.89    85.87    87.28    88.62    91.31
## mu[35]     86.08    0.02 1.01    84.09    85.40    86.07    86.76    88.10
## mu[36]     84.01    0.02 1.11    81.83    83.27    83.99    84.76    86.22
## mu[37]     85.72    0.02 1.03    83.70    85.04    85.71    86.41    87.79
## mu[38]     92.02    0.02 0.98    90.01    91.38    92.04    92.68    93.96
## mu[39]     80.79    0.03 1.31    78.20    79.91    80.78    81.69    83.40
## mu[40]     82.02    0.03 1.22    79.65    81.19    82.00    82.84    84.48
## mu[41]     89.48    0.02 0.95    87.58    88.84    89.48    90.10    91.31
## mu[42]    105.15    0.05 1.86   101.43   103.90   105.16   106.34   108.67
## mu[43]     78.65    0.04 1.47    75.76    77.65    78.63    79.63    81.57
## mu[44]    102.64    0.04 1.64    99.31   101.56   102.64   103.70   105.77
## mu[45]     74.30    0.05 1.87    70.77    72.99    74.30    75.50    78.00
## mu[46]    103.08    0.04 1.68    99.71   101.98   103.08   104.14   106.30
## mu[47]     76.26    0.04 1.67    72.98    75.14    76.24    77.36    79.54
## mu[48]     95.51    0.03 1.13    93.29    94.77    95.52    96.26    97.72
## mu[49]     93.90    0.02 1.05    91.78    93.21    93.90    94.60    95.98
## mu[50]     87.71    0.02 0.96    85.83    87.04    87.70    88.35    89.58
## mu[51]     89.42    0.02 0.95    87.52    88.79    89.41    90.04    91.25
## mu[52]    102.16    0.04 1.60    98.92   101.11   102.16   103.19   105.23
## mu[53]     85.13    0.02 1.05    83.10    84.43    85.12    85.84    87.24
## mu[54]     83.72    0.02 1.12    81.53    82.97    83.71    84.48    85.99
## mu[55]     89.08    0.02 0.95    87.18    88.44    89.06    89.69    90.89
## mu[56]     70.91    0.05 1.96    67.16    69.51    70.88    72.21    74.80
## mu[57]     85.91    0.02 1.02    83.90    85.23    85.90    86.59    87.95
## mu[58]     72.34    0.05 1.91    68.66    70.99    72.31    73.61    76.12
## mu[59]     69.06    0.05 2.04    65.15    67.64    69.02    70.40    73.16
## mu[60]     93.72    0.02 1.04    91.62    93.04    93.72    94.41    95.79
## mu[61]     94.62    0.02 1.08    92.48    93.91    94.63    95.33    96.74
## mu[62]     95.79    0.03 1.14    93.53    95.04    95.80    96.53    98.03
## mu[63]     83.00    0.06 1.92    79.12    81.73    83.05    84.29    86.80
## mu[64]     88.81    0.02 0.95    86.91    88.17    88.79    89.42    90.63
## mu[65]     89.01    0.02 0.95    87.11    88.38    88.99    89.63    90.83
## mu[66]     87.38    0.02 0.97    85.53    86.71    87.37    88.02    89.28
## mu[67]     84.36    0.02 1.09    82.24    83.63    84.35    85.10    86.52
## mu[68]     80.85    0.03 1.30    78.27    79.97    80.83    81.74    83.44
## mu[69]     80.86    0.03 1.30    78.28    79.98    80.84    81.75    83.45
## mu[70]     80.23    0.03 1.35    77.55    79.33    80.23    81.15    82.92
## mu[71]     82.29    0.03 1.21    79.94    81.47    82.27    83.09    84.74
## mu[72]     94.74    0.08 2.54    89.47    93.08    94.80    96.36    99.66
## mu[73]     97.95    0.09 2.78    92.24    96.16    98.05    99.71   103.32
## mu[74]     93.71    0.02 1.04    91.61    93.02    93.71    94.40    95.78
## mu[75]     82.63    0.03 1.19    80.32    81.83    82.61    83.42    85.05
## mu[76]     81.75    0.03 1.24    79.36    80.92    81.74    82.60    84.25
## mu[77]     95.44    0.03 1.12    93.23    94.71    95.45    96.19    97.65
## mu[78]     69.88    0.05 2.00    66.01    68.48    69.83    71.20    73.90
## mu[79]     85.43    0.02 1.04    83.40    84.74    85.41    86.12    87.50
## mu[80]     73.05    0.05 1.89    69.42    71.74    73.04    74.30    76.78
## mu[81]     74.55    0.05 1.86    71.01    73.24    74.55    75.74    78.24
## mu[82]     85.43    0.02 1.04    83.40    84.74    85.41    86.12    87.50
## mu[83]     94.42    0.02 1.07    92.29    93.71    94.43    95.13    96.53
## mu[84]    103.36    0.04 1.71    99.96   102.23   103.37   104.44   106.62
## mu[85]     82.35    0.03 1.20    80.01    81.54    82.33    83.15    84.80
## mu[86]     87.64    0.02 0.97    85.77    86.96    87.63    88.28    89.52
## mu[87]     77.37    0.04 1.57    74.31    76.32    77.36    78.41    80.49
```

```
## mu[88]     94.01    0.02 1.05    91.89    93.32    94.01    94.71    96.10
## mu[89]    108.73    0.06 2.19   104.37   107.30   108.75   110.12   112.89
## mu[90]    101.57    0.04 1.56    98.44   100.57   101.57   102.58   104.56
## mu[91]    107.66    0.05 2.09   103.50   106.27   107.68   109.00   111.64
## mu[92]     83.00    0.03 1.16    80.73    82.22    82.98    83.78    85.39
## mu[93]     89.90    0.02 0.95    88.00    89.25    89.89    90.54    91.72
## mu[94]    102.95    0.04 1.67    99.60   101.85   102.95   104.01   106.15
## mu[95]    103.66    0.04 1.73   100.20   102.51   103.67   104.76   106.97
## mu[96]     78.08    0.05 1.84    74.54    76.84    78.10    79.27    81.76
## mu[97]     79.43    0.03 1.41    76.64    78.48    79.42    80.38    82.23
## mu[98]     82.64    0.06 1.91    78.77    81.38    82.69    83.92    86.41
## mu[99]     86.88    0.06 2.07    82.57    85.52    86.92    88.24    90.89
## mu[100]    70.33    0.05 1.98    66.52    68.93    70.30    71.63    74.29
## mu[101]   100.02    0.04 1.43    97.13    99.08   100.02   100.94   102.79
## mu[102]    89.78    0.02 0.95    87.87    89.14    89.78    90.41    91.60
## mu[103]    88.30    0.02 0.95    86.42    87.65    88.28    88.93    90.13
## mu[104]    81.56    0.03 1.25    79.13    80.72    81.55    82.41    84.08
## mu[105]    79.72    0.03 1.39    76.97    78.78    79.72    80.65    82.47
## mu[106]    99.07    0.03 1.36    96.33    98.17    99.07    99.95   101.71
## mu[107]    82.69    0.03 1.18    80.39    81.89    82.67    83.48    85.11
## mu[108]    85.97    0.02 1.02    83.97    85.29    85.95    86.65    88.00
## mu[109]   103.72    0.04 1.74   100.25   102.56   103.73   104.82   107.03
## mu[110]    99.30    0.03 1.38    96.53    98.39    99.30   100.20   101.98
## mu[111]    89.15    0.07 2.19    84.56    87.72    89.22    90.61    93.43
## mu[112]    83.80    0.02 1.12    81.61    83.06    83.79    84.56    86.06
## mu[113]    85.79    0.02 1.02    83.79    85.12    85.78    86.48    87.86
## mu[114]    83.18    0.03 1.15    80.93    82.40    83.15    83.95    85.54
## mu[115]    89.79    0.02 0.95    87.88    89.15    89.79    90.43    91.61
## mu[116]    88.19    0.02 0.96    86.30    87.52    88.17    88.82    90.03
## mu[117]    82.28    0.03 1.21    79.93    81.45    82.26    83.08    84.72
## mu[118]    96.58    0.03 1.19    94.24    95.81    96.59    97.36    98.93
## mu[119]    80.44    0.05 1.86    76.70    79.22    80.49    81.65    84.08
## mu[120]    91.21    0.02 0.96    89.22    90.57    91.22    91.84    93.07
## mu[121]    86.76    0.02 0.99    84.84    86.09    86.75    87.42    88.72
## mu[122]   106.09    0.05 1.95   102.17   104.80   106.11   107.34   109.79
## mu[123]    89.08    0.02 0.95    87.18    88.44    89.06    89.69    90.89
## mu[124]    84.36    0.02 1.09    82.24    83.63    84.35    85.10    86.52
## mu[125]    90.33    0.02 0.95    88.42    89.68    90.32    90.97    92.16
## mu[126]    90.15    0.02 0.95    88.24    89.51    90.14    90.79    91.98
## mu[127]    86.52    0.06 2.05    82.26    85.18    86.55    87.88    90.52
## mu[128]    83.83    0.02 1.12    81.64    83.08    83.81    84.58    86.08
## mu[129]    84.90    0.02 1.06    82.85    84.19    84.88    85.62    87.03
## mu[130]    84.16    0.02 1.10    82.00    83.43    84.14    84.91    86.34
## mu[131]    78.65    0.04 1.47    75.76    77.65    78.63    79.63    81.57
## mu[132]    71.63    0.05 2.08    67.45    70.27    71.62    72.99    75.72
## mu[133]   103.73    0.04 1.74   100.26   102.57   103.74   104.83   107.05
## mu[134]    96.94    0.03 1.21    94.56    96.15    96.94    97.73    99.32
## mu[135]    77.23    0.05 1.84    73.71    76.03    77.26    78.43    80.90
## mu[136]   106.67    0.05 2.00   102.67   105.34   106.69   107.96   110.49
## mu[137]    72.46    0.05 2.00    68.42    71.14    72.46    73.78    76.38
## mu[138]    88.45    0.02 0.95    86.57    87.81    88.43    89.08    90.26
## mu[139]   106.57    0.05 1.99   102.58   105.25   106.59   107.85   110.37
## mu[140]    87.22    0.02 0.98    85.36    86.55    87.22    87.88    89.14
## mu[141]    96.93    0.03 1.21    94.55    96.13    96.93    97.71    99.31
```

```
## mu[142]    97.72    0.03 1.26    95.23    96.90    97.73    98.54   100.20
## mu[143]   108.73    0.06 2.19   104.37   107.30   108.75   110.12   112.89
## mu[144]    91.76    0.02 0.97    89.74    91.11    91.78    92.40    93.67
## mu[145]   107.39    0.05 2.06   103.28   106.01   107.41   108.71   111.33
## mu[146]    90.69    0.02 0.95    88.76    90.05    90.68    91.32    92.52
## mu[147]    87.70    0.02 0.96    85.82    87.02    87.69    88.34    89.57
## mu[148]    88.30    0.02 0.95    86.42    87.65    88.28    88.93    90.13
## mu[149]    82.11    0.03 1.22    79.75    81.28    82.10    82.93    84.57
## mu[150]    99.91    0.04 1.42    97.04    98.98    99.90   100.83   102.66
## mu[151]    95.50    0.03 1.13    93.28    94.76    95.51    96.24    97.70
## mu[152]   103.02    0.04 1.68    99.66   101.92   103.02   104.09   106.24
## mu[153]   100.93    0.04 1.50    97.93    99.95   100.93   101.91   103.84
## mu[154]    96.76    0.03 1.20    94.41    95.98    96.77    97.54    99.12
## mu[155]    83.29    0.03 1.15    81.04    82.52    83.26    84.05    85.63
## mu[156]    81.62    0.03 1.25    79.20    80.78    81.61    82.47    84.13
## mu[157]    92.12    0.02 0.98    90.09    91.47    92.13    92.77    94.06
## mu[158]    90.86    0.02 0.95    88.93    90.23    90.87    91.50    92.70
## mu[159]    90.24    0.02 0.95    88.33    89.60    90.23    90.87    92.06
## mu[160]    93.77    0.02 1.04    91.66    93.08    93.77    94.46    95.84
## mu[161]    75.80    0.05 1.84    72.28    74.56    75.82    76.98    79.51
## mu[162]    91.99    0.07 2.36    87.05    90.46    92.07    93.54    96.57
## mu[163]   102.59    0.04 1.64    99.26   101.51   102.59   103.64   105.70
## mu[164]   100.86    0.04 1.50    97.87    99.87   100.85   101.83   103.75
## mu[165]    99.98    0.04 1.43    97.10    99.05    99.98   100.90   102.74
## mu[166]    84.16    0.02 1.10    82.00    83.43    84.14    84.91    86.34
## mu[167]    87.70    0.02 0.96    85.82    87.02    87.69    88.34    89.57
## mu[168]   101.59    0.04 1.56    98.46   100.58   101.59   102.60   104.58
## mu[169]    89.73    0.07 2.22    85.07    88.29    89.79    91.22    94.10
## mu[170]    96.99    0.03 1.21    94.60    96.19    96.99    97.77    99.37
## mu[171]    96.51    0.03 1.19    94.18    95.74    96.53    97.29    98.85
## mu[172]    84.43    0.02 1.08    82.32    83.71    84.42    85.16    86.58
## mu[173]   100.37    0.04 1.46    97.45    99.42   100.38   101.32   103.20
## mu[174]    82.58    0.03 1.19    80.25    81.77    82.56    83.36    85.00
## mu[175]    72.94    0.05 1.90    69.28    71.62    72.92    74.19    76.68
## mu[176]    87.76    0.02 0.96    85.88    87.08    87.75    88.40    89.62
## mu[177]    95.93    0.03 1.15    93.65    95.19    95.95    96.68    98.19
## mu[178]    86.68    0.02 0.99    84.77    86.01    86.68    87.35    88.65
## mu[179]    82.28    0.03 1.21    79.93    81.45    82.26    83.08    84.72
## mu[180]    99.15    0.03 1.37    96.40    98.24    99.14   100.03   101.80
## mu[181]    80.86    0.05 1.87    77.10    79.63    80.91    82.08    84.50
## mu[182]    79.48    0.03 1.41    76.70    78.53    79.47    80.42    82.27
## mu[183]    89.37    0.02 0.95    87.47    88.73    89.35    89.98    91.20
## mu[184]    87.64    0.02 0.97    85.77    86.96    87.63    88.28    89.52
## mu[185]    81.82    0.06 1.89    77.97    80.58    81.87    83.08    85.50
## mu[186]    86.31    0.02 1.00    84.33    85.63    86.30    86.98    88.30
## mu[187]    92.76    0.02 1.00    90.71    92.10    92.77    93.43    94.75
## mu[188]    88.35    0.02 0.95    86.48    87.70    88.33    88.98    90.18
## mu[189]    87.76    0.02 0.96    85.88    87.08    87.75    88.40    89.62
## mu[190]    95.98    0.03 1.15    93.70    95.22    95.99    96.73    98.24
## mu[191]    78.24    0.05 1.84    74.69    77.01    78.27    79.43    81.92
## mu[192]    75.56    0.05 1.85    72.05    74.30    75.57    76.73    79.26
## mu[193]    85.80    0.06 2.02    81.65    84.48    85.85    87.14    89.74
## mu[194]    84.17    0.06 1.96    80.19    82.86    84.22    85.49    87.99
## mu[195]    87.65    0.02 0.97    85.78    86.98    87.64    88.29    89.53
```

```
## mu[196]    86.16    0.06 2.04    81.94    84.83    86.21    87.50    90.13
## mu[197]    87.58    0.02 0.97    85.72    86.91    87.57    88.22    89.46
## mu[198]    89.43    0.02 0.95    87.54    88.80    89.42    90.05    91.26
## mu[199]    84.01    0.02 1.11    81.83    83.27    83.99    84.76    86.22
## mu[200]    97.58    0.03 1.25    95.14    96.77    97.60    98.40   100.04
## mu[201]    79.79    0.03 1.38    77.04    78.85    79.79    80.71    82.53
## mu[202]    91.16    0.07 2.31    86.33    89.68    91.23    92.68    95.67
## mu[203]    95.51    0.03 1.13    93.30    94.78    95.53    96.26    97.72
## mu[204]    91.94    0.02 0.98    89.92    91.30    91.95    92.59    93.87
## mu[205]    85.68    0.02 1.03    83.66    85.00    85.67    86.37    87.75
## mu[206]    69.37    0.05 2.02    65.48    67.96    69.33    70.69    73.44
## mu[207]    81.98    0.03 1.23    79.61    81.15    81.96    82.80    84.45
## mu[208]    95.08    0.03 1.10    92.91    94.36    95.10    95.81    97.25
## mu[209]    94.08    0.02 1.05    91.96    93.39    94.08    94.78    96.17
## mu[210]    93.45    0.02 1.03    91.38    92.77    93.46    94.14    95.49
## mu[211]    86.82    0.06 2.07    82.52    85.47    86.86    88.19    90.83
## mu[212]    90.86    0.02 0.95    88.93    90.23    90.87    91.50    92.70
## mu[213]    86.70    0.06 2.06    82.42    85.35    86.73    88.06    90.72
## mu[214]    93.41    0.02 1.02    91.34    92.73    93.41    94.10    95.44
## mu[215]   107.05    0.05 2.03   103.00   105.69   107.07   108.36   110.94
## mu[216]   104.38    0.05 1.79   100.78   103.18   104.40   105.52   107.78
## mu[217]    88.77    0.02 0.95    86.88    88.12    88.75    89.38    90.59
## mu[218]    80.86    0.03 1.30    78.28    79.98    80.84    81.75    83.45
## mu[219]    98.30    0.03 1.30    95.71    97.44    98.31    99.14   100.86
## mu[220]    99.53    0.03 1.39    96.73    98.62    99.52   100.43   102.23
## mu[221]    82.58    0.03 1.19    80.25    81.77    82.55    83.36    85.00
## mu[222]    76.66    0.04 1.63    73.46    75.56    76.65    77.72    79.87
## mu[223]   108.78    0.06 2.19   104.40   107.34   108.80   110.17   112.95
## mu[224]    89.42    0.02 0.95    87.52    88.79    89.41    90.04    91.25
## mu[225]   106.57    0.05 1.99   102.58   105.25   106.59   107.85   110.37
## mu[226]    85.95    0.02 1.02    83.95    85.27    85.93    86.63    87.98
## mu[227]    98.37    0.03 1.31    95.77    97.50    98.37    99.21   100.93
## mu[228]    98.30    0.03 1.30    95.71    97.44    98.31    99.14   100.86
## mu[229]    88.90    0.02 0.95    87.00    88.27    88.88    89.51    90.73
## mu[230]    79.13    0.05 1.84    75.49    77.89    79.16    80.33    82.77
## mu[231]    79.36    0.03 1.41    76.56    78.41    79.35    80.31    82.18
## mu[232]    80.55    0.03 1.33    77.92    79.66    80.54    81.46    83.19
## mu[233]   104.62    0.05 1.82   100.99   103.42   104.65   105.78   108.08
## mu[234]   103.79    0.04 1.74   100.32   102.63   103.81   104.90   107.12
## mu[235]    91.94    0.02 0.98    89.92    91.30    91.95    92.59    93.87
## mu[236]    91.31    0.02 0.96    89.31    90.67    91.32    91.94    93.18
## mu[237]    97.70    0.03 1.26    95.22    96.88    97.71    98.52   100.18
## mu[238]    99.02    0.03 1.36    96.28    98.12    99.01    99.89   101.64
## mu[239]    89.61    0.02 0.95    87.71    88.97    89.61    90.24    91.44
## mu[240]    86.99    0.06 2.07    82.66    85.63    87.04    88.37    91.03
## mu[241]   103.02    0.04 1.68    99.66   101.92   103.02   104.09   106.24
## mu[242]    89.12    0.02 0.95    87.23    88.49    89.11    89.75    90.94
## mu[243]    81.86    0.06 1.89    78.01    80.62    81.92    83.13    85.55
## mu[244]    83.41    0.03 1.14    81.17    82.64    83.38    84.17    85.72
## mu[245]    79.06    0.03 1.44    76.22    78.09    79.05    80.03    81.92
## mu[246]    79.18    0.03 1.43    76.35    78.22    79.17    80.14    82.02
## mu[247]    76.21    0.04 1.67    72.92    75.09    76.20    77.31    79.50
## mu[248]    83.18    0.03 1.15    80.93    82.40    83.15    83.95    85.54
## mu[249]    82.63    0.03 1.19    80.32    81.83    82.61    83.42    85.05
```

```
## mu[250]    76.15    0.04 1.68    72.84    75.02    76.13    77.25    79.44
## mu[251]    93.30    0.02 1.02    91.23    92.63    93.30    93.98    95.32
## mu[252]    74.87    0.04 1.79    71.30    73.67    74.86    76.05    78.36
## mu[253]    86.68    0.02 0.99    84.77    86.01    86.68    87.35    88.65
## mu[254]   104.38    0.05 1.79   100.78   103.18   104.40   105.52   107.78
## mu[255]    90.08    0.07 2.24    85.36    88.63    90.13    91.57    94.46
## mu[256]    90.39    0.07 2.26    85.65    88.93    90.46    91.90    94.80
## mu[257]    95.14    0.03 1.11    92.96    94.41    95.16    95.87    97.31
## mu[258]    68.36    0.05 2.07    64.46    66.92    68.31    69.70    72.51
## mu[259]    71.87    0.05 1.93    68.19    70.50    71.84    73.14    75.66
## mu[260]    89.54    0.02 0.95    87.64    88.90    89.54    90.17    91.37
## mu[261]    88.72    0.02 0.95    86.83    88.08    88.70    89.33    90.54
## mu[262]    94.97    0.03 1.10    92.80    94.26    94.99    95.70    97.13
## mu[263]    95.61    0.08 2.61    90.19    93.92    95.68    97.25   100.68
## mu[264]    80.43    0.03 1.33    77.79    79.54    80.43    81.34    83.09
## mu[265]    88.11    0.02 0.96    86.22    87.45    88.10    88.75    89.95
## mu[266]    89.37    0.02 0.95    87.47    88.73    89.35    89.98    91.20
## mu[267]    81.22    0.03 1.28    78.72    80.36    81.20    82.09    83.76
## mu[268]    71.15    0.05 1.95    67.42    69.76    71.12    72.44    75.01
## mu[269]    71.15    0.05 1.95    67.42    69.76    71.12    72.44    75.01
## mu[270]    82.99    0.03 1.16    80.72    82.20    82.96    83.76    85.37
## mu[271]    79.87    0.03 1.38    77.13    78.95    79.87    80.80    82.60
## mu[272]    95.51    0.03 1.13    93.29    94.77    95.52    96.26    97.72
## mu[273]    94.88    0.03 1.09    92.71    94.16    94.89    95.60    97.04
## mu[274]    88.77    0.02 0.95    86.88    88.12    88.75    89.38    90.59
## mu[275]    72.04    0.05 1.92    68.36    70.68    72.01    73.31    75.85
## mu[276]    71.63    0.05 1.94    67.95    70.25    71.59    72.90    75.43
## mu[277]    77.47    0.04 1.56    74.42    76.42    77.45    78.50    80.57
## mu[278]    99.73    0.04 1.41    96.88    98.82    99.73   100.64   102.45
## mu[279]    89.43    0.02 0.95    87.54    88.80    89.42    90.05    91.26
## mu[280]    82.93    0.03 1.17    80.66    82.15    82.91    83.71    85.32
## mu[281]    90.15    0.02 0.95    88.24    89.51    90.14    90.79    91.98
## mu[282]    89.37    0.02 0.95    87.47    88.73    89.35    89.98    91.20
## mu[283]   100.44    0.04 1.47    97.51    99.49   100.44   101.38   103.28
## mu[284]    75.91    0.05 1.84    72.42    74.68    75.94    77.09    79.63
## mu[285]    76.69    0.05 1.84    73.20    75.47    76.72    77.88    80.40
## mu[286]    73.01    0.05 1.90    69.36    71.69    72.99    74.25    76.74
## mu[287]    83.47    0.03 1.14    81.24    82.70    83.44    84.23    85.77
## mu[288]    80.79    0.03 1.31    78.20    79.91    80.78    81.69    83.39
## mu[289]    71.27    0.05 1.95    67.53    69.88    71.24    72.55    75.12
## mu[290]    73.45    0.05 1.88    69.86    72.14    73.44    74.69    77.15
## mu[291]    69.72    0.05 2.01    65.84    68.32    69.69    71.05    73.76
## mu[292]    72.29    0.05 1.92    68.61    70.94    72.26    73.55    76.08
## mu[293]    75.23    0.04 1.75    71.72    74.05    75.22    76.38    78.66
## mu[294]    82.22    0.03 1.21    79.87    81.39    82.20    83.03    84.67
## mu[295]    70.95    0.05 1.96    67.20    69.55    70.92    72.25    74.83
## mu[296]    77.93    0.04 1.53    74.96    76.91    77.91    78.95    80.97
## mu[297]    71.15    0.05 1.95    67.43    69.76    71.13    72.44    75.01
## mu[298]    69.52    0.05 2.02    65.64    68.12    69.49    70.86    73.58
## mu[299]    72.58    0.05 1.91    68.93    71.25    72.57    73.85    76.34
## mu[300]    81.93    0.03 1.23    79.56    81.10    81.92    82.76    84.41
## mu[301]    81.86    0.03 1.23    79.49    81.03    81.85    82.70    84.35
## mu[302]    76.10    0.05 1.84    72.62    74.87    76.13    77.28    79.81
## mu[303]    98.77    0.03 1.34    96.08    97.88    98.78    99.64   101.37
```

```
## mu[304]    78.89    0.03 1.45    76.03    77.92    78.88    79.87    81.78
## mu[305]    79.00    0.03 1.44    76.16    78.03    78.99    79.98    81.87
## mu[306]    90.91    0.02 0.96    88.98    90.28    90.91    91.54    92.75
## mu[307]    92.58    0.02 0.99    90.56    91.92    92.59    93.24    94.56
## mu[308]    86.56    0.02 0.99    84.63    85.89    86.56    87.23    88.53
## mu[309]    90.20    0.02 0.95    88.29    89.56    90.19    90.83    92.02
## mu[310]    80.98    0.05 1.87    77.22    79.75    81.03    82.21    84.62
## mu[311]    81.62    0.03 1.25    79.20    80.78    81.61    82.47    84.13
## mu[312]    92.15    0.02 0.98    90.13    91.51    92.16    92.81    94.10
## mu[313]    82.29    0.03 1.21    79.94    81.47    82.27    83.09    84.74
## mu[314]    93.10    0.02 1.01    91.04    92.44    93.10    93.77    95.12
## mu[315]    88.41    0.02 0.95    86.53    87.77    88.39    89.04    90.23
## mu[316]    98.73    0.03 1.33    96.05    97.84    98.73    99.59   101.32
## mu[317]    85.50    0.02 1.03    83.47    84.81    85.48    86.19    87.57
## mu[318]    79.42    0.03 1.41    76.62    78.47    79.41    80.37    82.22
## mu[319]    88.47    0.02 0.95    86.59    87.83    88.45    89.10    90.28
## mu[320]    80.68    0.03 1.32    78.07    79.80    80.67    81.59    83.30
## mu[321]    69.79    0.05 2.00    65.92    68.39    69.75    71.12    73.82
## mu[322]    79.72    0.03 1.39    76.96    78.78    79.72    80.64    82.46
## mu[323]    83.95    0.02 1.11    81.76    83.21    83.93    84.71    86.18
## mu[324]    73.80    0.05 1.88    70.00    72.55    73.80    75.04    77.48
## mu[325]    96.69    0.03 1.20    94.35    95.91    96.70    97.46    99.05
## mu[326]    92.23    0.02 0.98    90.20    91.58    92.23    92.88    94.18
## mu[327]    89.30    0.02 0.95    87.40    88.66    89.28    89.91    91.13
## mu[328]    86.86    0.02 0.99    84.95    86.20    86.87    87.52    88.81
## mu[329]    86.27    0.02 1.00    84.29    85.59    86.25    86.94    88.26
## mu[330]    94.55    0.02 1.08    92.41    93.83    94.55    95.25    96.66
## mu[331]    93.41    0.02 1.02    91.34    92.73    93.41    94.10    95.44
## mu[332]    77.99    0.04 1.52    75.02    76.97    77.97    79.00    81.01
## mu[333]    83.95    0.02 1.11    81.76    83.21    83.93    84.71    86.18
## mu[334]    96.87    0.03 1.21    94.50    96.08    96.88    97.66    99.25
## mu[335]    73.30    0.05 1.89    69.70    71.98    73.29    74.53    77.01
## mu[336]    97.23    0.03 1.23    94.81    96.43    97.24    98.03    99.64
## mu[337]    87.65    0.02 0.97    85.78    86.98    87.64    88.29    89.53
## mu[338]    80.79    0.03 1.31    78.20    79.91    80.78    81.69    83.39
## mu[339]    74.85    0.04 1.79    71.27    73.65    74.83    76.03    78.34
## mu[340]    77.11    0.04 1.59    73.99    76.04    77.09    78.15    80.26
## mu[341]    72.40    0.05 1.91    68.73    71.06    72.38    73.67    76.17
## mu[342]    79.65    0.03 1.39    76.89    78.70    79.65    80.58    82.40
## mu[343]    84.12    0.02 1.10    81.96    83.39    84.10    84.87    86.30
## mu[344]    81.57    0.03 1.25    79.14    80.74    81.56    82.42    84.09
## mu[345]    89.78    0.02 0.95    87.87    89.14    89.78    90.41    91.60
## mu[346]    74.90    0.05 1.86    71.38    73.61    74.91    76.09    78.60
## mu[347]    81.16    0.06 1.87    77.35    79.92    81.21    82.40    84.81
## mu[348]    99.91    0.04 1.42    97.04    98.98    99.90   100.83   102.66
## mu[349]    83.36    0.03 1.14    81.12    82.59    83.33    84.12    85.68
## mu[350]    80.97    0.03 1.30    78.42    80.10    80.95    81.85    83.55
## mu[351]    82.63    0.03 1.19    80.32    81.83    82.61    83.42    85.05
## mu[352]    79.07    0.03 1.44    76.23    78.11    79.06    80.04    81.93
## mu[353]    86.15    0.02 1.01    84.16    85.48    86.14    86.83    88.16
## mu[354]    85.61    0.02 1.03    83.59    84.93    85.59    86.30    87.69
## mu[355]    81.68    0.03 1.25    79.28    80.85    81.67    82.53    84.19
## mu[356]    77.89    0.05 1.84    74.37    76.68    77.92    79.09    81.56
## mu[357]    92.58    0.02 0.99    90.56    91.92    92.59    93.25    94.57
```

```
## mu[358]    76.93    0.04 1.61    73.78    75.85    76.92    77.98    80.11
## mu[359]    82.23    0.06 1.90    78.35    80.97    82.28    83.49    85.94
## mu[360]    82.40    0.03 1.20    80.06    81.58    82.38    83.19    84.83
## mu[361]    84.61    0.02 1.07    82.53    83.89    84.60    85.35    86.75
## mu[362]    90.15    0.02 0.95    88.24    89.51    90.14    90.79    91.98
## mu[363]    75.07    0.04 1.77    71.53    73.89    75.06    76.24    78.53
## mu[364]    98.00    0.03 1.28    95.46    97.15    98.00    98.83   100.51
## mu[365]    76.62    0.04 1.64    73.41    75.52    76.61    77.68    79.84
## mu[366]    78.76    0.04 1.46    75.88    77.77    78.75    79.74    81.67
## mu[367]    81.66    0.03 1.25    79.25    80.83    81.65    82.51    84.17
## mu[368]    78.29    0.04 1.50    75.36    77.29    78.27    79.30    81.27
## mu[369]    80.61    0.03 1.32    77.98    79.72    80.59    81.52    83.24
## mu[370]    81.66    0.03 1.25    79.25    80.83    81.65    82.51    84.17
## mu[371]    83.65    0.02 1.13    81.44    82.89    83.63    84.41    85.93
## mu[372]    82.69    0.03 1.18    80.39    81.89    82.67    83.48    85.11
## mu[373]    79.06    0.03 1.44    76.22    78.09    79.05    80.03    81.92
## mu[374]    84.12    0.02 1.10    81.96    83.39    84.10    84.87    86.30
## mu[375]    74.31    0.05 1.87    70.78    73.01    74.32    75.52    78.02
## mu[376]    87.40    0.02 0.97    85.55    86.73    87.39    88.04    89.30
## mu[377]    85.50    0.02 1.03    83.47    84.81    85.48    86.19    87.57
## mu[378]    88.65    0.02 0.95    86.76    88.01    88.63    89.27    90.47
## mu[379]    80.07    0.03 1.36    77.37    79.16    80.07    81.00    82.78
## mu[380]    78.29    0.04 1.50    75.36    77.29    78.27    79.30    81.27
## mu[381]    87.29    0.02 0.97    85.43    86.62    87.29    87.94    89.20
## mu[382]    72.22    0.05 1.92    68.53    70.86    72.19    73.49    76.01
## mu[383]    74.19    0.05 1.87    70.66    72.89    74.19    75.40    77.90
## mu[384]    70.69    0.05 1.97    66.91    69.29    70.65    71.99    74.61
## mu[385]    68.36    0.05 2.07    64.46    66.92    68.31    69.70    72.51
## mu[386]    77.22    0.04 1.59    74.12    76.16    77.20    78.26    80.36
## mu[387]    96.99    0.03 1.21    94.60    96.19    96.99    97.77    99.37
## mu[388]    86.44    0.02 1.00    84.48    85.76    86.42    87.11    88.42
## mu[389]    71.51    0.05 1.94    67.83    70.13    71.48    72.79    75.33
## mu[390]    76.43    0.04 1.65    73.18    75.32    76.42    77.52    79.68
## mu[391]    86.92    0.02 0.98    85.02    86.26    86.93    87.58    88.86
## mu[392]    93.07    0.02 1.01    91.02    92.42    93.07    93.75    95.09
## mu[393]    71.40    0.05 1.94    67.70    70.02    71.37    72.69    75.24
## mu[394]    88.47    0.02 0.95    86.59    87.83    88.45    89.10    90.28
## mu[395]    83.00    0.03 1.16    80.73    82.22    82.98    83.78    85.39
## mu[396]    78.40    0.04 1.49    75.48    77.40    78.39    79.40    81.36
## mu[397]    82.22    0.03 1.21    79.87    81.39    82.20    83.03    84.67
## mu[398]   105.62    0.05 1.90   101.81   104.36   105.64   106.84   109.23
## mu[399]   101.94    0.04 1.59    98.75   100.91   101.94   102.96   104.99
## mu[400]    83.29    0.03 1.15    81.05    82.52    83.26    84.06    85.63
## mu[401]    78.37    0.05 1.84    74.80    77.13    78.39    79.56    82.04
## mu[402]    77.74    0.05 1.84    74.23    76.54    77.77    78.94    81.40
## mu[403]    75.93    0.05 1.84    72.45    74.70    75.96    77.11    79.65
## mu[404]   102.84    0.04 1.66    99.49   101.75   102.84   103.90   106.01
## mu[405]    73.83    0.05 1.88    70.28    72.52    73.83    75.06    77.53
## mu[406]    72.70    0.05 1.90    69.05    71.37    72.68    73.96    76.46
## mu[407]    89.06    0.02 0.95    87.17    88.43    89.04    89.68    90.88
## mu[408]    88.19    0.02 0.96    86.30    87.52    88.17    88.82    90.03
## mu[409]    93.72    0.02 1.04    91.62    93.04    93.72    94.41    95.79
## mu[410]    75.87    0.05 1.84    72.37    74.63    75.89    77.04    79.58
## mu[411]    91.87    0.02 0.97    89.85    91.23    91.89    92.52    93.80
```

```
## mu[412]    85.38    0.02 1.04    83.35    84.69    85.36    86.07    87.45
## mu[413]    92.76    0.02 1.00    90.71    92.10    92.77    93.43    94.75
## mu[414]    93.90    0.02 1.05    91.78    93.21    93.90    94.60    95.98
## mu[415]    97.59    0.03 1.25    95.14    96.77    97.60    98.40   100.04
## mu[416]    74.79    0.05 1.86    71.27    73.51    74.80    75.98    78.49
## mu[417]    86.27    0.02 1.00    84.29    85.59    86.25    86.94    88.26
## mu[418]    76.81    0.05 1.84    73.33    75.60    76.85    78.01    80.50
## mu[419]    81.98    0.03 1.23    79.61    81.15    81.96    82.80    84.45
## mu[420]    67.74    0.05 2.10    63.83    66.28    67.70    69.10    71.95
## mu[421]    77.74    0.05 1.84    74.23    76.54    77.77    78.94    81.40
## mu[422]    80.33    0.05 1.86    76.61    79.11    80.38    81.54    83.96
## mu[423]    80.87    0.05 1.87    77.11    79.64    80.92    82.09    84.51
## mu[424]   105.92    0.05 1.93   102.04   104.63   105.94   107.16   109.59
## mu[425]    75.72    0.04 1.71    72.33    74.57    75.70    76.84    79.07
## mu[426]   103.73    0.04 1.74   100.27   102.58   103.75   104.84   107.05
## mu[427]   101.94    0.04 1.59    98.75   100.91   101.94   102.96   104.99
## mu[428]    71.27    0.05 1.95    67.53    69.88    71.24    72.55    75.12
## mu[429]    68.09    0.05 2.08    64.21    66.65    68.04    69.44    72.26
## mu[430]    73.77    0.05 1.88    70.22    72.46    73.76    75.00    77.47
## mu[431]    84.07    0.02 1.10    81.91    83.34    84.05    84.83    86.27
## mu[432]    79.43    0.05 1.84    75.78    78.19    79.45    80.64    83.04
## mu[433]    86.27    0.02 1.00    84.29    85.59    86.25    86.94    88.26
## mu[434]    83.09    0.03 1.16    80.83    82.31    83.06    83.86    85.45
## lp__    -1474.01    0.05 1.38 -1477.41 -1474.74 -1473.69 -1472.95 -1472.29
##         n_eff Rhat
## alpha    1129 1.00
## beta[1]  1114 1.00
## beta[2]  1367 1.00
## sigma    1549 1.00
## mu[1]    1595 1.00
## mu[2]    1898 1.00
## mu[3]    1746 1.00
## mu[4]    2384 1.00
## mu[5]    2057 1.00
## mu[6]    1073 1.00
## mu[7]    1435 1.00
## mu[8]    1532 1.00
## mu[9]    1669 1.00
## mu[10]   2182 1.00
## mu[11]   1867 1.00
## mu[12]   2171 1.00
## mu[13]   1882 1.00
## mu[14]   1796 1.00
## mu[15]   1124 1.00
## mu[16]   1609 1.00
## mu[17]   1784 1.00
## mu[18]   1910 1.00
## mu[19]   1458 1.00
## mu[20]   1161 1.00
## mu[21]   1989 1.00
## mu[22]   1513 1.00
## mu[23]   2331 1.00
## mu[24]   2390 1.00
## mu[25]   1157 1.00
```

```
## mu[26]   1583 1.00
## mu[27]   2368 1.00
## mu[28]   2332 1.00
## mu[29]   1645 1.00
## mu[30]   1789 1.00
## mu[31]   2043 1.00
## mu[32]   2384 1.00
## mu[33]   1668 1.00
## mu[34]   1070 1.00
## mu[35]   2262 1.00
## mu[36]   2063 1.00
## mu[37]   2228 1.00
## mu[38]   2167 1.00
## mu[39]   1817 1.00
## mu[40]   1898 1.00
## mu[41]   2369 1.00
## mu[42]   1482 1.00
## mu[43]   1709 1.00
## mu[44]   1524 1.00
## mu[45]   1360 1.00
## mu[46]   1515 1.00
## mu[47]   1625 1.00
## mu[48]   1836 1.00
## mu[49]   1977 1.00
## mu[50]   2384 1.00
## mu[51]   2372 1.00
## mu[52]   1535 1.00
## mu[53]   2171 1.00
## mu[54]   2037 1.00
## mu[55]   2384 1.00
## mu[56]   1479 1.00
## mu[57]   2246 1.00
## mu[58]   1430 1.00
## mu[59]   1539 1.00
## mu[60]   1994 1.00
## mu[61]   1910 1.00
## mu[62]   1815 1.00
## mu[63]   1118 1.00
## mu[64]   2390 1.00
## mu[65]   2385 1.00
## mu[66]   2371 1.00
## mu[67]   2096 1.00
## mu[68]   1821 1.00
## mu[69]   1821 1.00
## mu[70]   1785 1.00
## mu[71]   1918 1.00
## mu[72]   1057 1.00
## mu[73]   1065 1.00
## mu[74]   1995 1.00
## mu[75]   1945 1.00
## mu[76]   1880 1.00
## mu[77]   1842 1.00
## mu[78]   1513 1.00
## mu[79]   2200 1.00
```

```
## mu[80]   1404 1.00
## mu[81]   1351 1.00
## mu[82]   2200 1.00
## mu[83]   1928 1.00
## mu[84]   1510 1.00
## mu[85]   1923 1.00
## mu[86]   2382 1.00
## mu[87]   1660 1.00
## mu[88]   1966 1.00
## mu[89]   1445 1.00
## mu[90]   1549 1.00
## mu[91]   1454 1.00
## mu[92]   1975 1.00
## mu[93]   2348 1.00
## mu[94]   1518 1.00
## mu[95]   1505 1.00
## mu[96]   1233 1.00
## mu[97]   1744 1.00
## mu[98]   1124 1.00
## mu[99]   1072 1.00
## mu[100]  1499 1.00
## mu[101]  1595 1.00
## mu[102]  2355 1.00
## mu[103]  2393 1.00
## mu[104]  1866 1.00
## mu[105]  1758 1.00
## mu[106]  1630 1.00
## mu[107]  1950 1.00
## mu[108]  2251 1.00
## mu[109]  1504 1.00
## mu[110]  1621 1.00
## mu[111]  1060 1.00
## mu[112]  2045 1.00
## mu[113]  2235 1.00
## mu[114]  1990 1.00
## mu[115]  2354 1.00
## mu[116]  2392 1.00
## mu[117]  1917 1.00
## mu[118]  1760 1.00
## mu[119]  1169 1.00
## mu[120]  2247 1.00
## mu[121]  2322 1.00
## mu[122]  1470 1.00
## mu[123]  2384 1.00
## mu[124]  2096 1.00
## mu[125]  2320 1.00
## mu[126]  2332 1.00
## mu[127]  1075 1.00
## mu[128]  2046 1.00
## mu[129]  2148 1.00
## mu[130]  2077 1.00
## mu[131]  1709 1.00
## mu[132]  1529 1.00
## mu[133]  1504 1.00
```

```
## mu[134]  1738 1.00
## mu[135]  1259 1.00
## mu[136]  1463 1.00
## mu[137]  1542 1.00
## mu[138]  2393 1.00
## mu[139]  1464 1.00
## mu[140]  2360 1.00
## mu[141]  1738 1.00
## mu[142]  1693 1.00
## mu[143]  1445 1.00
## mu[144]  2194 1.00
## mu[145]  1456 1.00
## mu[146]  2292 1.00
## mu[147]  2383 1.00
## mu[148]  2393 1.00
## mu[149]  1905 1.00
## mu[150]  1599 1.00
## mu[151]  1837 1.00
## mu[152]  1517 1.00
## mu[153]  1566 1.00
## mu[154]  1748 1.00
## mu[155]  1999 1.00
## mu[156]  1870 1.00
## mu[157]  2158 1.00
## mu[158]  2278 1.00
## mu[159]  2326 1.00
## mu[160]  1989 1.00
## mu[161]  1307 1.00
## mu[162]  1055 1.00
## mu[163]  1525 1.00
## mu[164]  1569 1.00
## mu[165]  1596 1.00
## mu[166]  2077 1.00
## mu[167]  2383 1.00
## mu[168]  1549 1.00
## mu[169]  1058 1.00
## mu[170]  1735 1.00
## mu[171]  1764 1.00
## mu[172]  2103 1.00
## mu[173]  1583 1.00
## mu[174]  1941 1.00
## mu[175]  1408 1.00
## mu[176]  2385 1.00
## mu[177]  1805 1.00
## mu[178]  2316 1.00
## mu[179]  1917 1.00
## mu[180]  1627 1.00
## mu[181]  1160 1.00
## mu[182]  1746 1.00
## mu[183]  2374 1.00
## mu[184]  2382 1.00
## mu[185]  1139 1.00
## mu[186]  2283 1.00
## mu[187]  2091 1.00
```

```
## mu[188]  2393 1.00
## mu[189]  2385 1.00
## mu[190]  1801 1.00
## mu[191]  1228 1.00
## mu[192]  1315 1.00
## mu[193]  1082 1.00
## mu[194]  1100 1.00
## mu[195]  2382 1.00
## mu[196]  1078 1.00
## mu[197]  2380 1.00
## mu[198]  2371 1.00
## mu[199]  2063 1.00
## mu[200]  1701 1.00
## mu[201]  1762 1.00
## mu[202]  1055 1.00
## mu[203]  1836 1.00
## mu[204]  2176 1.00
## mu[205]  2224 1.00
## mu[206]  1529 1.00
## mu[207]  1896 1.00
## mu[208]  1871 1.00
## mu[209]  1960 1.00
## mu[210]  2021 1.00
## mu[211]  1073 1.00
## mu[212]  2278 1.00
## mu[213]  1074 1.00
## mu[214]  2025 1.00
## mu[215]  1459 1.00
## mu[216]  1493 1.00
## mu[217]  2390 1.00
## mu[218]  1821 1.00
## mu[219]  1664 1.00
## mu[220]  1613 1.00
## mu[221]  1941 1.00
## mu[222]  1637 1.00
## mu[223]  1444 1.00
## mu[224]  2372 1.00
## mu[225]  1464 1.00
## mu[226]  2250 1.00
## mu[227]  1661 1.00
## mu[228]  1664 1.00
## mu[229]  2388 1.00
## mu[230]  1203 1.00
## mu[231]  1741 1.00
## mu[232]  1803 1.00
## mu[233]  1489 1.00
## mu[234]  1502 1.00
## mu[235]  2176 1.00
## mu[236]  2237 1.00
## mu[237]  1694 1.00
## mu[238]  1633 1.00
## mu[239]  2363 1.00
## mu[240]  1072 1.00
## mu[241]  1517 1.00
```

```
## mu[242]  2382 1.00
## mu[243]  1139 1.00
## mu[244]  2009 1.00
## mu[245]  1727 1.00
## mu[246]  1732 1.00
## mu[247]  1624 1.00
## mu[248]  1990 1.00
## mu[249]  1945 1.00
## mu[250]  1622 1.00
## mu[251]  2036 1.00
## mu[252]  1589 1.00
## mu[253]  2316 1.00
## mu[254]  1493 1.00
## mu[255]  1057 1.00
## mu[256]  1056 1.00
## mu[257]  1866 1.00
## mu[258]  1559 1.00
## mu[259]  1447 1.00
## mu[260]  2367 1.00
## mu[261]  2391 1.00
## mu[262]  1880 1.00
## mu[263]  1059 1.00
## mu[264]  1796 1.00
## mu[265]  2391 1.00
## mu[266]  2374 1.00
## mu[267]  1844 1.00
## mu[268]  1471 1.00
## mu[269]  1471 1.00
## mu[270]  1974 1.00
## mu[271]  1766 1.00
## mu[272]  1836 1.00
## mu[273]  1887 1.00
## mu[274]  2390 1.00
## mu[275]  1440 1.00
## mu[276]  1455 1.00
## mu[277]  1663 1.00
## mu[278]  1605 1.00
## mu[279]  2371 1.00
## mu[280]  1969 1.00
## mu[281]  2332 1.00
## mu[282]  2374 1.00
## mu[283]  1581 1.00
## mu[284]  1303 1.00
## mu[285]  1277 1.00
## mu[286]  1406 1.00
## mu[287]  2015 1.00
## mu[288]  1817 1.00
## mu[289]  1467 1.00
## mu[290]  1390 1.00
## mu[291]  1518 1.00
## mu[292]  1432 1.00
## mu[293]  1598 1.00
## mu[294]  1913 1.00
## mu[295]  1478 1.00
```

```
## mu[296]  1680 1.00
## mu[297]  1471 1.00
## mu[298]  1525 1.00
## mu[299]  1421 1.00
## mu[300]  1892 1.00
## mu[301]  1887 1.00
## mu[302]  1297 1.00
## mu[303]  1643 1.00
## mu[304]  1719 1.00
## mu[305]  1724 1.00
## mu[306]  2273 1.00
## mu[307]  2110 1.00
## mu[308]  2306 1.00
## mu[309]  2329 1.00
## mu[310]  1157 1.00
## mu[311]  1870 1.00
## mu[312]  2154 1.00
## mu[313]  1918 1.00
## mu[314]  2057 1.00
## mu[315]  2393 1.00
## mu[316]  1645 1.00
## mu[317]  2207 1.00
## mu[318]  1743 1.00
## mu[319]  2393 1.00
## mu[320]  1811 1.00
## mu[321]  1516 1.00
## mu[322]  1758 1.00
## mu[323]  2058 1.00
## mu[324]  1566 1.00
## mu[325]  1753 1.00
## mu[326]  2146 1.00
## mu[327]  2377 1.00
## mu[328]  2331 1.00
## mu[329]  2279 1.00
## mu[330]  1917 1.00
## mu[331]  2025 1.00
## mu[332]  1682 1.00
## mu[333]  2058 1.00
## mu[334]  1742 1.00
## mu[335]  1396 1.00
## mu[336]  1720 1.00
## mu[337]  2382 1.00
## mu[338]  1817 1.00
## mu[339]  1589 1.00
## mu[340]  1651 1.00
## mu[341]  1428 1.00
## mu[342]  1755 1.00
## mu[343]  2074 1.00
## mu[344]  1867 1.00
## mu[345]  2355 1.00
## mu[346]  1338 1.00
## mu[347]  1153 1.00
## mu[348]  1599 1.00
## mu[349]  2005 1.00
```

```
## mu[350]  1828 1.00
## mu[351]  1945 1.00
## mu[352]  1727 1.00
## mu[353]  2269 1.00
## mu[354]  2217 1.00
## mu[355]  1875 1.00
## mu[356]  1239 1.00
## mu[357]  2109 1.00
## mu[358]  1646 1.00
## mu[359]  1132 1.00
## mu[360]  1927 1.00
## mu[361]  2120 1.00
## mu[362]  2332 1.00
## mu[363]  1594 1.00
## mu[364]  1679 1.00
## mu[365]  1636 1.00
## mu[366]  1714 1.00
## mu[367]  1873 1.00
## mu[368]  1694 1.00
## mu[369]  1807 1.00
## mu[370]  1873 1.00
## mu[371]  2031 1.00
## mu[372]  1950 1.00
## mu[373]  1727 1.00
## mu[374]  2074 1.00
## mu[375]  1359 1.00
## mu[376]  2372 1.00
## mu[377]  2207 1.00
## mu[378]  2392 1.00
## mu[379]  1777 1.00
## mu[380]  1694 1.00
## mu[381]  2365 1.00
## mu[382]  1434 1.00
## mu[383]  1364 1.00
## mu[384]  1487 1.00
## mu[385]  1559 1.00
## mu[386]  1655 1.00
## mu[387]  1735 1.00
## mu[388]  2295 1.00
## mu[389]  1459 1.00
## mu[390]  1630 1.00
## mu[391]  2336 1.00
## mu[392]  2059 1.00
## mu[393]  1463 1.00
## mu[394]  2393 1.00
## mu[395]  1975 1.00
## mu[396]  1699 1.00
## mu[397]  1913 1.00
## mu[398]  1475 1.00
## mu[399]  1540 1.00
## mu[400]  1999 1.00
## mu[401]  1224 1.00
## mu[402]  1243 1.00
## mu[403]  1302 1.00
```

```
## mu[404]  1520 1.00
## mu[405]  1376 1.00
## mu[406]  1417 1.00
## mu[407]  2384 1.00
## mu[408]  2392 1.00
## mu[409]  1994 1.00
## mu[410]  1304 1.00
## mu[411]  2183 1.00
## mu[412]  2195 1.00
## mu[413]  2091 1.00
## mu[414]  1977 1.00
## mu[415]  1700 1.00
## mu[416]  1342 1.00
## mu[417]  2279 1.00
## mu[418]  1273 1.00
## mu[419]  1896 1.00
## mu[420]  1576 1.00
## mu[421]  1243 1.00
## mu[422]  1172 1.00
## mu[423]  1159 1.00
## mu[424]  1472 1.00
## mu[425]  1610 1.00
## mu[426]  1503 1.00
## mu[427]  1540 1.00
## mu[428]  1467 1.00
## mu[429]  1566 1.00
## mu[430]  1379 1.00
## mu[431]  2069 1.00
## mu[432]  1195 1.00
## mu[433]  2279 1.00
## mu[434]  1982 1.00
## lp__      763 1.01
##
## Samples were drawn using NUTS(diag_e) at Fri Feb 12 09:35:31 2021.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).
```

4.4 Question 5

Confirm the results from Stan agree with `lm()`

```
fitlm2<-lm(kid_score~mom_hs+mom_iqc,data=kidiq)
summary(fitlm2)
```

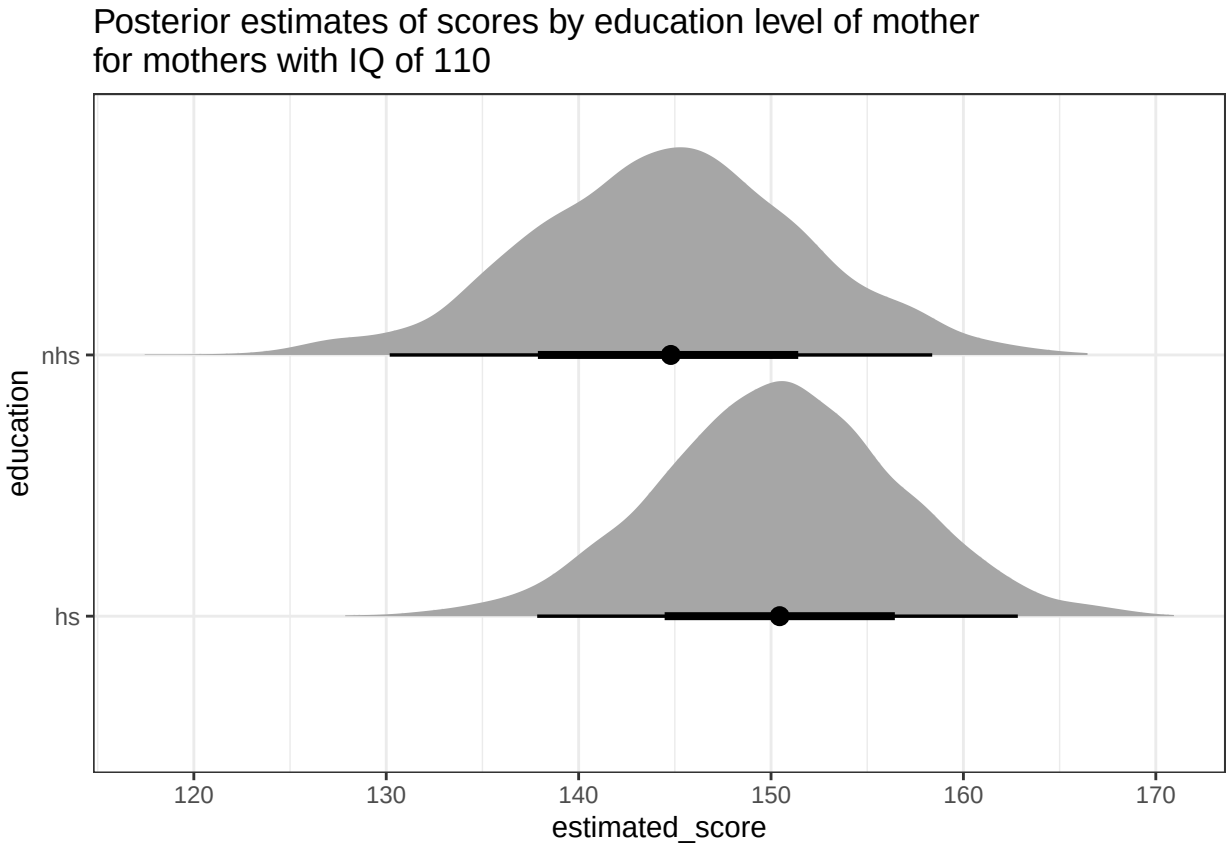
```
##
## Call:
## lm(formula = kid_score ~ mom_hs + mom_iqc, data = kidiq)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -52.873 -12.663   2.404  11.356  49.545
##
```

```
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 82.12214    1.94370  42.250  < 2e-16 ***
## mom_hs       5.95012    2.21181   2.690  0.00742 **
## mom_iqc      0.56391    0.06057   9.309  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 18.14 on 431 degrees of freedom
## Multiple R-squared:  0.2141, Adjusted R-squared:  0.2105
## F-statistic: 58.72 on 2 and 431 DF,  p-value: < 2.2e-16
```

4.5 Question 6

Plot the posterior estimates of scores by education of mother for mothers who have an IQ of 110.

```
fit3 %>%
  spread_draws(alpha, beta[k], sigma) %>%
  pivot_wider(names_from=k,values_from=beta,names_prefix="beta")%>%
     mutate(nhs = alpha + beta2*110, # no high school
          hs = alpha + beta1 + beta2*110) %>%
  pivot_longer(nhs:hs, names_to = "education", values_to = "estimated_score") %>%
  ggplot(aes(y = education, x = estimated_score)) +
  stat_halfeye() +
  theme_bw() +
  ggtitle("Posterior estimates of scores by education level of mother \nfor mothers with IQ of 110")
```


Posterior estimates of scores by education level of mother
for mothers with IQ of 110
education
nhs
hs
120
130
140
150
160
170
estimated_score