Optical Mouse Based Odometry

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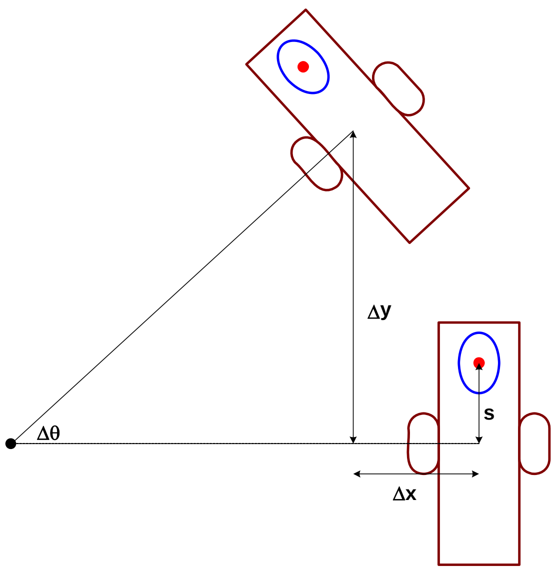
Abstract

In Optical Mouse Based Odometry, one attempts to determine the translation and rotation of a robot chassis using the data from an attached optical mouse. Many have already studied this problem in the context of autonomous mobile robotics and gave a formula for this problem online (for example http://elvis.rowan.edu/~ohara/mps/). In this note we propose another formula that is more precise and also easier to evaluate.

1 Problem

Consider the following configuration of a two-wheeled robot and an optical mouse where the blue ellipse is the optical mouse and the red dot is the camera. The distance between the camera and the line connecting two wheels is s. The picture shows that the robot is rotating around a point.



The problem is as follows. Given the two numbers Δx_m and Δy_m reported by the optical mouse, determine $\Delta\theta$ (the change in the orientation of the robot), and Δx and Δy (the changes in the position of the robot).

2 Solution

Theorem 1 *We have*

$$\Delta\theta = \frac{\Delta x_m}{s}$$

$$\Delta x = \Delta y_m \left(\frac{1-\cos\Delta\theta}{\Delta\theta}\right)$$

$$\Delta y = \Delta y_m \left(\frac{\sin\Delta\theta}{\Delta\theta}\right)$$

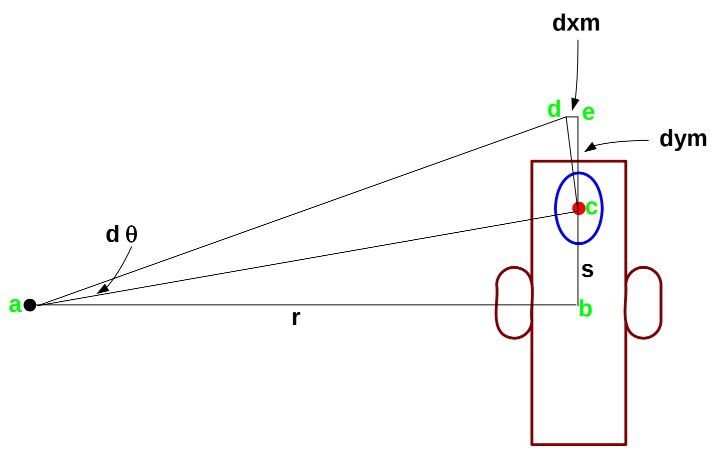
By approximation, we have

$$\Delta\theta = \frac{\Delta x_m}{s}$$
$$\Delta x \approx \Delta y_m \frac{\Delta\theta}{2}$$
$$\Delta y \approx \Delta y_m \left(1 - \frac{\Delta\theta^2}{6}\right)$$

Proof. While the robot is turning by the angle $\Delta\theta$, the optical mouse makes numerous measurements (about 1500 times per second) of x and y displacements. We will idealize it by assuming that it makes infinitely many measurements, thus each time the optical mouse will measure some infinitisimal values dx_m and dy_m after making an infinitisimal rotation of the angle $d\theta$. Then we have

$$\Delta x_m = \int dx_m$$
$$\Delta y_m = \int dx_m$$
$$\Delta\theta = \int d\theta$$

The following picture attempts to depict an infinitismal angle rotation:



where $\measuredangle abc = \measuredangle acd = \measuredangle ced = \frac{\pi}{2}$. Note that$\measuredangle cab = \frac{\pi}{2} - \measuredangle acb = \pi - \frac{\pi}{2} - \measuredangle acb = \pi - \measuredangle acd - \measuredangle acb = \measuredangle dce$. Hence we conclude that $\triangle cab$ and $\triangle dce$ are similar. So we have

$$\frac{r}{s} = \frac{dy_m}{dx_m}$$

Since $d\theta$ is inifnitismally small, we have

$$d\theta = \frac{\overline{dc}}{\overline{ac}} = \frac{\sqrt{dy_m^2 + dx_m^2}}{\sqrt{r^2+s^2}} = \frac{\sqrt{\left(\frac{r}{s}\right)^2 dx_m^2 + dx_m^2}}{\sqrt{r^2+s^2}} = dx_m \frac{\sqrt{\left(\frac{r}{s}\right)^2 + 1}}{\sqrt{r^2+s^2}} = \frac{dx_m}{s}\frac{\sqrt{r^2+s^2}}{\sqrt{r^2+s^2}} = \frac{dx_m}{s}$$

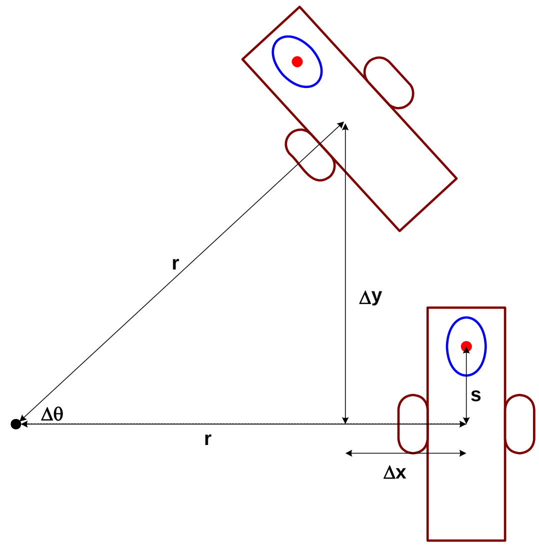
By integrating, we have

$$\frac{\Delta y_m}{\Delta x_m} = \frac{\int dy_m}{\int dx_m} = \frac{\int \frac{r}{s} dx_m}{\int dx_m} = \frac{\frac{r}{s}\int dx_m}{\int dx_m} = \frac{\frac{r}{s}\Delta x_m}{\Delta x_m} = \frac{r}{s}$$
$$\Delta\theta = \int d\theta = \int \frac{dx_m}{s} = \frac{\int dx_m}{s} = \frac{\Delta x_m}{s}$$

Hence we can determine $\Delta\theta$ and r from Δx_m and Δy_m as follows:

$$\Delta\theta = \frac{\Delta x_m}{s}$$
$$r = s\frac{\Delta y_m}{\Delta x_m} = \frac{\Delta y_m}{\Delta\theta}$$

The following picture shows how to determine Δx and Δy from $\Delta\theta$ and r.



We see immediately that

$$\Delta x = r - r\cos\Delta\theta = r\left(1 - \cos\Delta\theta\right)$$
$$\Delta y = r\sin\Delta\theta$$

By rcalling $r = \frac{\Delta y_m}{\Delta\theta}$, we have

$$\Delta x = \frac{\Delta y_m}{\Delta\theta}\left(1 - \cos\Delta\theta\right) = \Delta y_m\left(\frac{1-\cos\Delta\theta}{\Delta\theta}\right)$$
$$\Delta y = \frac{\Delta y_m}{\Delta\theta}\sin\Delta\theta = \Delta y_m\left(\frac{\sin\Delta\theta}{\Delta\theta}\right)$$

Putting everything together, we have

$$\Delta\theta = \frac{\Delta x_m}{s}$$
$$\Delta x = \Delta y_m\left(\frac{1-\cos\Delta\theta}{\Delta\theta}\right)$$
$$\Delta y = \Delta y_m\left(\frac{\sin\Delta\theta}{\Delta\theta}\right)$$

If $\Delta\theta$ is small, we can approximate Δx and Δy using the Taylor expansion as

$$\Delta x = \Delta y_m\left(\frac{1-\left(1-\frac{\Delta\theta^2}{2!}+\cdots\right)}{\Delta\theta}\right) = \Delta y_m\left(\frac{\frac{\Delta\theta^2}{2!}+\cdots}{\Delta\theta}\right) = \Delta y_m\frac{\Delta\theta}{2}+\cdots$$
$$\Delta y = \Delta y_m\left(\frac{\Delta\theta-\frac{\Delta\theta^3}{3!}+\cdots}{\Delta\theta}\right) = \Delta y_m\left(1-\frac{\Delta\theta^2}{6}\right)+\cdots$$

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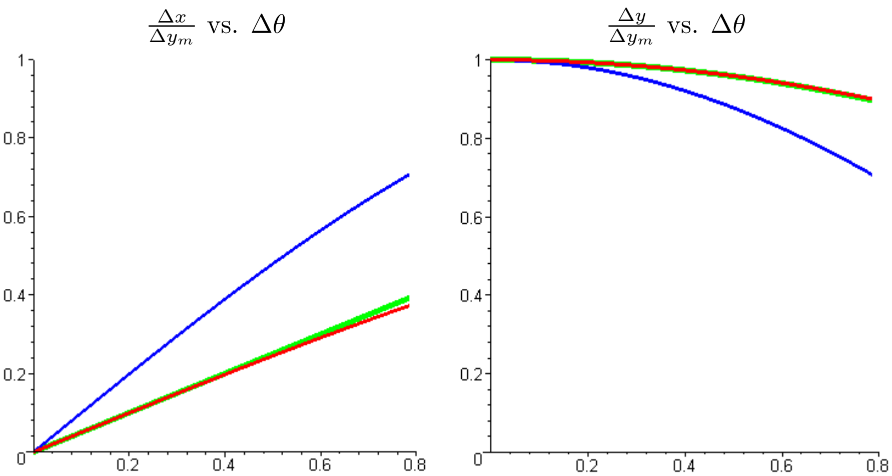
3 Comparison with Previous Solutions

In this section, we compare the approximation suggested in this note and the previously suggested approximation, against the exact solution found above. From left to right the formulas are: the exact solution, our approximation, the previous approximation.

Exact	Our Approximation	Previous Approximation
$\Delta\theta = \frac{\Delta x_m}{s}$	$\Delta\theta = \frac{\Delta x_m}{s}$	$\Delta\theta = \frac{\Delta x_m}{s}$
$\Delta x = \Delta y_m \left(\frac{1-\cos\Delta\theta}{\Delta\theta}\right)$	$\Delta x \approx \Delta y_m \frac{\Delta\theta}{2}$	$\Delta x \approx \Delta y_m \sin\Delta\theta$
$\Delta y = \Delta y_m \left(\frac{\sin\Delta\theta}{\Delta\theta}\right)$	$\Delta y \approx \Delta y_m \left(1 - \frac{\Delta\theta^2}{6}\right)$	$\Delta y \approx \Delta y_m \cos\Delta\theta$

First note that our approximation is easier to compute than the previous approximation, since it does not involve transcendental functions. Next, we compare the accuracy. For convenience we compare $\frac{\Delta x}{\Delta y_m}$ and $\frac{\Delta y}{\Delta y_m}$. The left graphs compares $\frac{\Delta x}{\Delta y_m}$. The right graph compares $\frac{\Delta y}{\Delta y_m}$. The red curves are the precise solution, the green curve are our approximation and the blue curve is the previous approximation.



It seems that our approximation of $\frac{\Delta x}{\Delta y_m}$ is very accurate up to $\Delta\theta = \frac{\pi}{8} \approx 0.4$. Our approximation of $\frac{\Delta y}{\Delta y_m}$ is accurate up to $\Delta\theta = \frac{\pi}{4} \approx 0.8$. In actual robots, we expect that $\Delta\theta$ will be much smaller than $\frac{\pi}{8}$. Hence our approximations are very accurate and save precious cpu time.