Untangling Triangular Meshes in Large Deformations using Edge Flip

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Motivation

Tangled meshes are not suitable for solving PDEs

Existing mesh deformation algorithms often produce inverted elements in large deformations

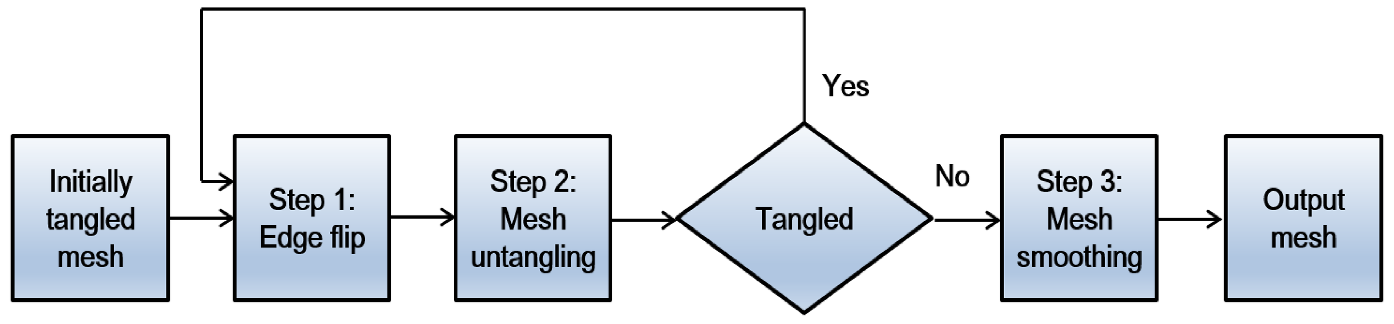
Existing optimization-based mesh untangling algorithms often fail to untangle highly tangled meshes or produce output meshes with poor element qualities

Objectives

Produce meshes with no inverted elements and good element qualities on the deformed domains

Preserve similar element qualities while maintaining the total number of vertices and edges

Proposed Method



Basic idea: Combine topological changes (edge flip) and optimization-based mesh untangling for tangled meshes

Iteratively perform edge flip and optimization-based mesh untangling until all inverted elements are eliminated

Step 1: Perform edge flip [1]: Delaunay flipping criterion is used to perform edge flip on a given edge. A greedy strategy is used to flip edges in decreasing order of opposite angles

Step 2: Perform optimization-based mesh untangling [3], mesh quality metric: $q_i = \frac{1}{2}(|\mathrm{Area} - \beta| - (\mathrm{Area} - \beta))$
(other optimization-based mesh untangling methods can be used)

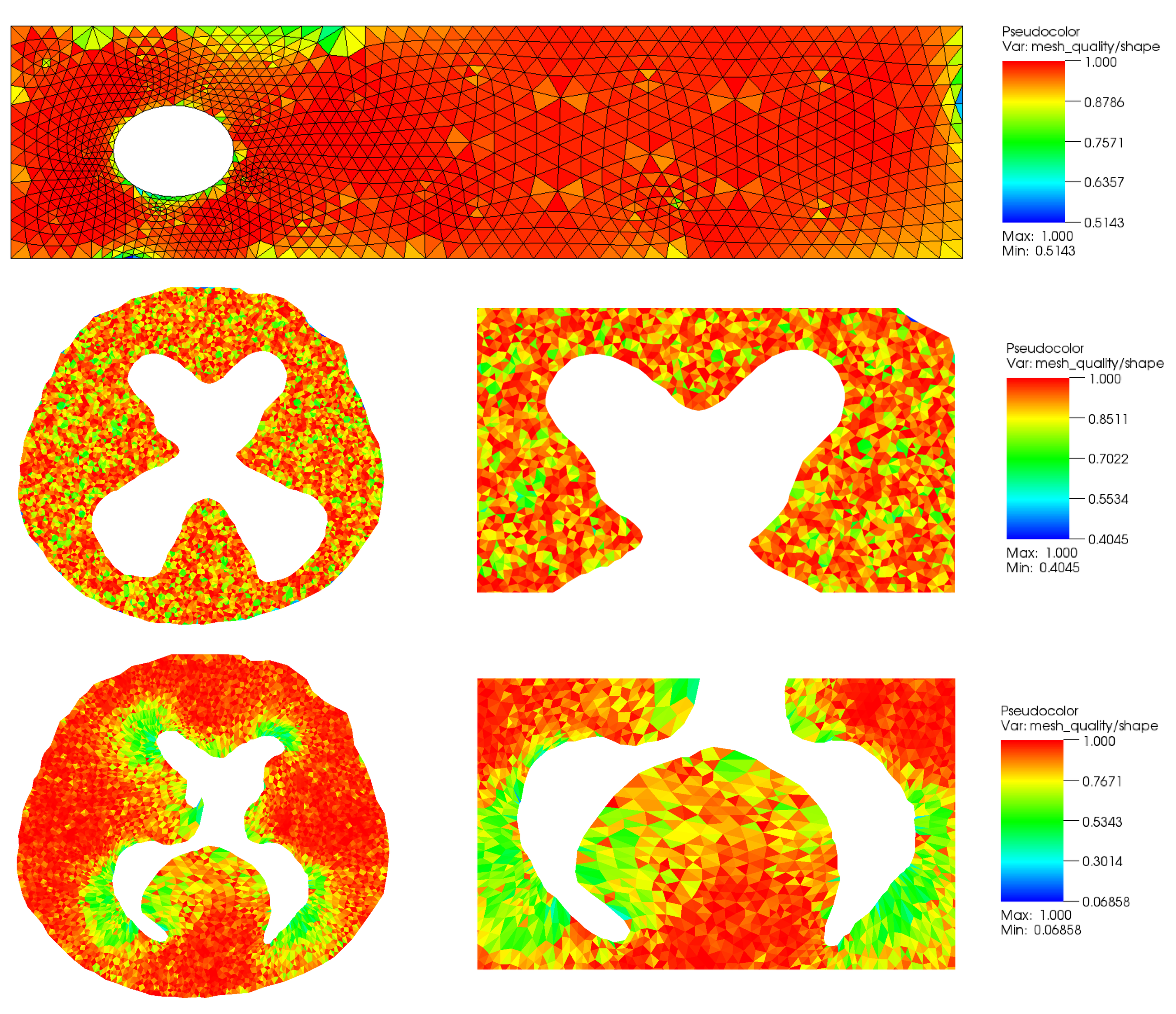
Step 3: Perform optimization-based mesh smoothing, mesh quality metric (inverse mean ratio): $q_i = \frac{\|AW^{-1}\|_F^2}{2|\det(AW^{-1})|}$

Results

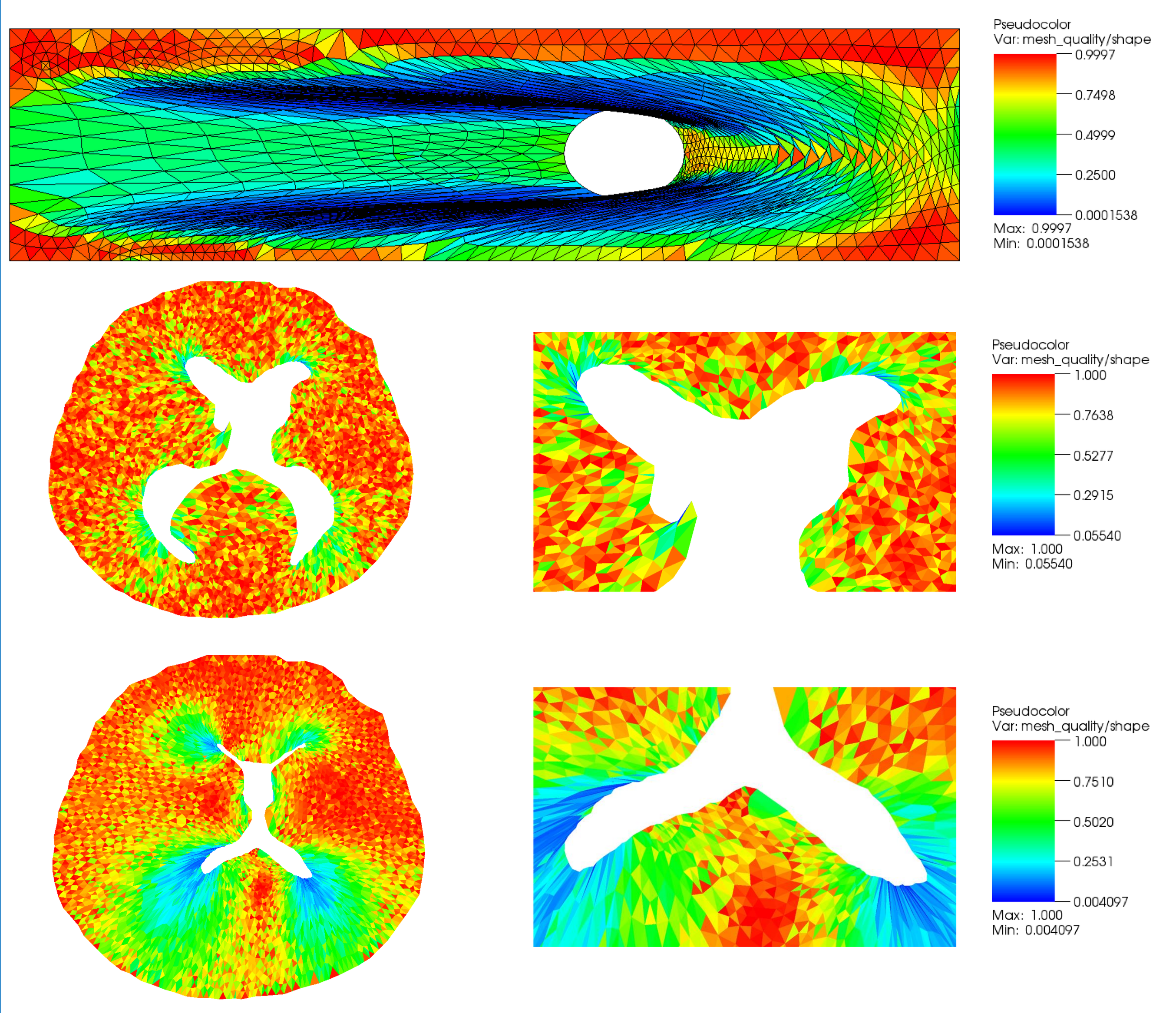
When large deformations occur, deformed meshes using FEMWARP mesh deformation algorithm [2] has many inverted elements with skinny triangles

Proposed method is able to simultaneously untangle inverted elements and improve element qualities in large deformations. Edge flip yields large improvement in the element quality and accelerates the mesh untangling

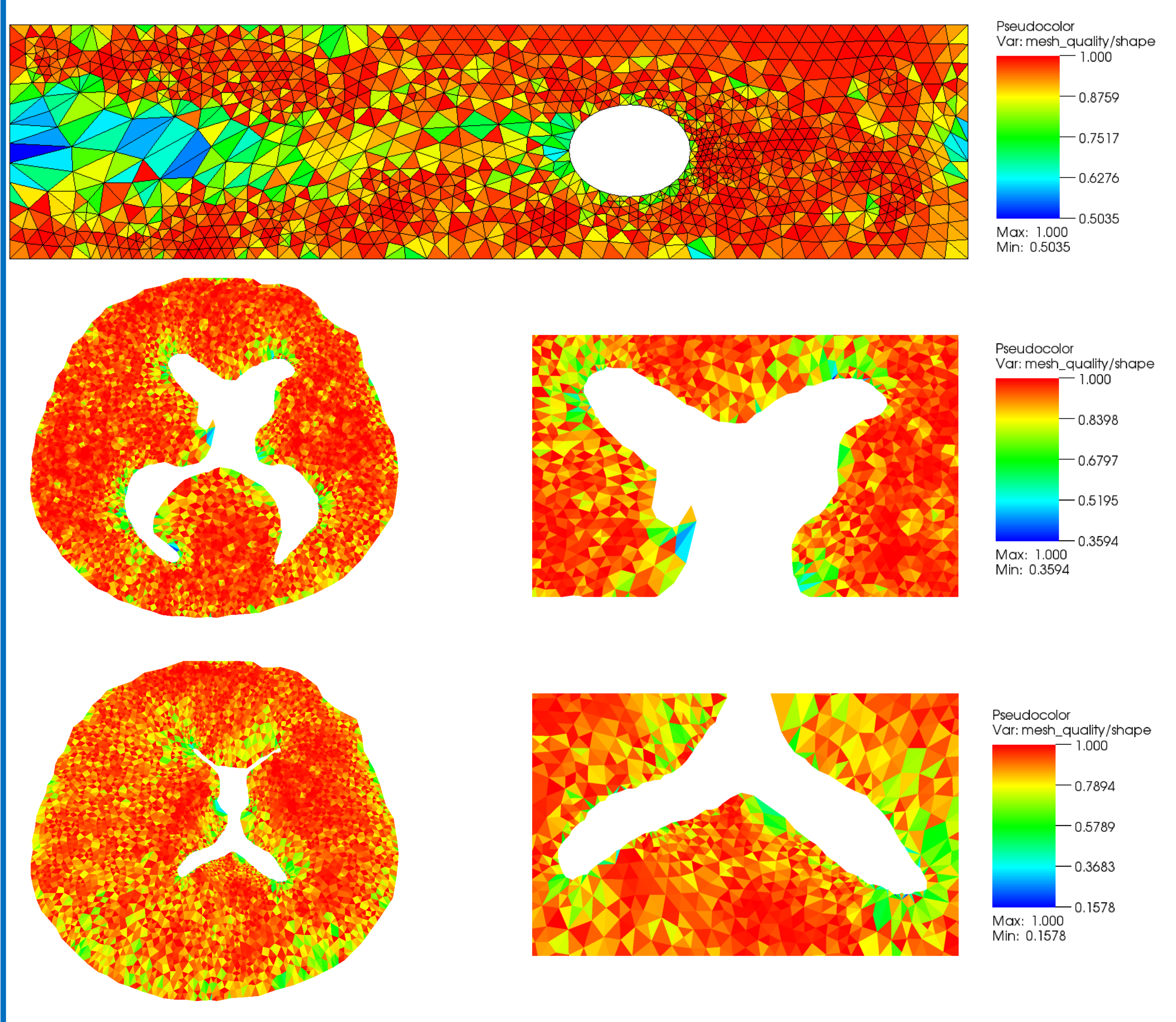
Initial meshes (undeformed)



Deformed meshes



Final output meshes



Mesh quality statistics

Moving cylinder example	# of inverted elements	Mesh quality (worst)
Initial mesh	0	0.51
Deformed mesh	53	0.00015
Final output mesh	**0**	**0.50**
Knupp [3]	67	0.000036

Hydrocephalus example 1	# of inverted elements	Mesh quality (worst)
Initial mesh	0	0.40
Deformed mesh	4	0.055
Final output mesh	**0**	**0.36**
Knupp [3]	3	0.0025

Hydrocephalus example 2	# of inverted elements	Mesh quality (worst)
Initial mesh	0	0.068
Deformed mesh	8	0.0041
Final output mesh	**0**	**0.15**
Knupp [3]	6	0.00024

Future Work

Apply other optimization-based mesh untangling methods in step 2

Add other topological changes for extremely large boundary deformations

References

[1] J. Kim, D. McLaurin, and S. M. Shontz. A 2D topology adaptive mesh deformation framework for mesh warping. Proceedings of the 4th TetrahedronWorkshop on GridGeneration for Numerical Computation, May 2014.

[2] S.M. Shontz and S.A. Vavasis. Analysis of and workarounds for element reversal for a finite element-based algorithm for warping triangular and tetrahedral meshes. BIT, Numerical Mathematics, 50: 863-884, 2010.

[3] P.M. Knupp. Hexahedral and tetrahedral mesh untangling. Engineering with Computers, vol 17, no. 3, pp. 261-268, 2001.