Problem 1: Sec 1.5, #4

Let P(x, y) be the statement “Student x has taken class y,” where the domain for x consists of all students in your class and for y consists of all computer science courses at your school. Express each of these quantifications in English.

a) $\exists x \exists y P(x, y)$

There exists a student in the class who has taken one of the computer science courses.

b) $\exists x \forall y P(x, y)$

There exists a student in the class who took all the computer science courses.

c) $\forall x \exists y P(x, y)$

Every student in the class has taken a computer science course.

d) $\exists y \forall x P(x, y)$

There exists a computer science class that every student in the class has taken.

e) $\forall y \exists x P(x, y)$

For every computer science course, there exists a student in the class who has taken it.

f) $\forall x \forall y P(x, y)$

Every student in the class has taken all the computer science courses.

Problem 2: Sec 1.5, #8

Let Q(x, y) be the statement “Student x has been a contestant on quiz show y.” Express each of these sentences in terms of Q(x, y), quantifiers, and logical connectives, where the domain for x consists of all students at your school and for y consists of all quiz shows on television.

a) There is a student at your school who has been a contestant on a television quiz show.

$\exists x \exists y Q(x, y)$

b) No student at your school has ever been a contestant on a television quiz show.

$\neg \exists x \forall y Q(x, y)$

c) There is a student at your school who has been a contestant on Jeopardy! and on Wheel of Fortune.

$\exists x(Q(x, \text{Jeopardy!}) \wedge Q(x, \text{Wheel of Fortune}))$

d) Every television quiz show has had a student from your school as a contestant.

$\forall y \exists x Q(x, y)$

e) At least two students from your school have been contestants on Jeopardy!.

$\exists x \exists y \exists z(Q(x, \text{Jeopardy!}) \wedge Q(y, \text{Jeopardy!}) \wedge x \neq y)$

Problem 3: Sec 1.5, #20

Express each of these statements using predicates, quantifiers, logical connectives, and mathematical operators where the domain consists of all integers.

a) The product of two negative integers is positive.

$\forall x \forall y(((x < 0) \wedge (y < 0)) \rightarrow (xy > 0))$

b) The average of two positive integers is positive.

$\forall x \forall y(((x > 0) \wedge (y > 0)) \rightarrow (\frac{x+y}{2} > 0))$

c) The difference of two negative integers is not necessarily negative.

$\exists x \exists y((x < 0) \wedge (y < 0) \wedge ((x - y) > 0))$

d) The absolute value of the sum of two integers does not exceed the sum of the absolute values of these integers.

$\forall x \forall y((|x + y| \leq (|x| + |y|))$

Problem 4: Sec 1.5, #34

Find a common domain for the variables x, y, and z for which the statement $\forall x \forall y((x \neq y) \rightarrow \forall z((z = x) \vee (z = y)))$ is true and another domain for which it is false.

False domain: {z,y,z}
True domain: {x,y}

Problem 5: Sec 1.6, #4

What rule of inference is used in each of these arguments?

a) Kangaroos live in Australia and are marsupials. Therefore, kangaroos are marsupials.

Simplification

b) It is either hotter than 100 degrees today or the pollution is dangerous. It is less than 100 degrees outside today. Therefore, the pollution is dangerous.

Disjunctive syllogism

c) Linda is an excellent swimmer. If Linda is an excellent swimmer, then she can work as a lifeguard. Therefore, Linda can work as a lifeguard.

Modus Ponens

d) Steve will work at a computer company this summer. Therefore, this summer Steve will work at a computer company or he will be a beach bum.

Addition

e) If I work all night on this homework, then I can answer all the exercises. If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, then I will understand the material.

Hypothetical syllogism

Problem 6: Sec 1.6, #14

For each of these arguments, explain which rules of inference are used for each step.

a) “Linda, a student in this class, owns a red convertible. Everyone who owns a red convertible has gotten at least one speeding ticket. Therefore, someone in this class has gotten a speeding ticket.”

Premises:

1. Linda, a student in this class own a red convertible
2. Everyone who owns a red convertible has gotten a speeding ticket
   Conclusion: Someone in this class has gotten a speeding ticket
   Reasoning:
3. Linda owns a red convertible (premise)
4. Everyone who owns a red convertible has gotten at least one speeding ticket (premise)
5. Linda has gotten at least one speeding ticket (Universal Instantiation on premise 2)
6. Therefore, someone in this class has gotten a speeding ticket (Existential generalization)

b) “Each of five roommates, Melissa, Aaron, Ralph, Veneesha, and Keeshawn, has taken a course in discrete mathematics. Every student who has taken a course in discrete mathematics can take a course in algorithms. Therefore, all five roommates can take a course in algorithms next year.”

Premises:

1. Each of the five roommates has taken a course in discrete mathematics.
2. Every student who has taken a course in discrete mathematics can take a course in algorithms.
   Conclusion: All five roommates can take a course in algorithms next year.
   Reasoning:
3. Melissa, Aaron, Ralph, Veneesha, and Keeshawn have all taken a course in discrete mathematics. (Premise 1)
4. Every student who has taken a course in discrete mathematics can take a course in algorithms. (Premise 2)
5. Melissa can take a course in algorithms. (Universal Instantiation on Premise 2 using Melissa)
6. Similarly, Aaron, Ralph, Veneesha, and Keeshawn can all take a course in algorithms. (Universal Instantiation repeatedly)

7. Therefore, all five roommates can take a course in algorithms next year. (Conjunction on steps 3 and 4)

c) “All movies produced by John Sayles are wonderful. John Sayles produced a movie about coal miners. Therefore, there is a wonderful movie about coal miners.”

Premises:

1. All movies produced by John Sayles are wonderful.
2. John Sayles produced a movie about coal miners.
   Conclusion: There is a wonderful movie about coal miners.
   Reasoning:
3. All movies produced by John Sayles are wonderful. (Premise)
4. John Sayles produced a movie about coal miners. (Premise)
5. The movie about coal miners is wonderful. (Universal Instantiation on Premise 1 using the movie about coal miners)
6. Therefore, there is a wonderful movie about coal miners. (Existential Generalization on Step 3)

d) “There is someone in this class who has been to France. Everyone who goes to France visits the Louvre. Therefore, someone in this class has visited the Louvre.”

Premises:

1. There is someone in this class who has been to France.
2. Everyone who goes to France visits the Louvre.
   Conclusion: Someone in this class has visited the Louvre.
   Reasoning:
3. There is someone in this class who has been to France. (Premise)
4. Everyone who goes to France visits the Louvre. (Premise)
5. Let us denote the person from the class as P who has been to France. (Existential Instantiation on Premise 1)
6. P went to the Louvre. (Universal Instantiation on Premise 2 using P)
7. Therefore, someone in this class (P) has visited the Louvre. (Existential Generalization on Step 4)

Problem 7: Sec 1.6, #16

For each of these arguments determine whether the argument is correct or incorrect and explain why.

a) Everyone enrolled in the university has lived in a dormitory. Mia has never lived in a dormitory. Therefore, Mia is not enrolled in the university.

This is correct, modus tollens.

b) A convertible car is fun to drive. Isaac’s car is not a convertible. Therefore, Isaac’s car is not fun to drive.

This is not correct, this only states that convertibles are fun to drive, not that all other cars are not fun to drive.

c) Quincy likes all action movies. Quincy likes the movie Eight Men Out. Therefore, Eight Men Out is an action movie.

This is not correct, Quincy can like other movies other than action.

d) All lobstermen set at least a dozen traps. Hamilton is a lobsterman. Therefore, Hamilton sets at least a dozen traps.

This is correct, modus ponens.

Problem 8: Sec 1.6, #18

What is wrong with this argument? Let $S(x, y)$ be “x is shorter than y.” Given the premise $\exists s S(s, Max)$, it follows that $S(Max, Max)$. Then by existential generalization it follows that $\exists x S(x, x)$, so that someone is shorter than himself.

While $\exists s S(s, Max)$ means there is someone shorter than max, this does not mean that max is shorter than max. The premise does not tell us this, just that someone is shorter than max.

Problem 9: Sec 1.6, #28

Use rules of inference to show that if $\forall x(P(x) \vee Q(x))$ and $\forall x((\neg P(x) \wedge Q(x)) \rightarrow R(x))$ are true, then $\forall x(\neg R(x) \rightarrow P(x))$ is also true, where the domains of all quantifiers are the same.

$\forall x(P(x) \vee Q(x))$
$\forall x((\neg P(x) \wedge Q(x)) \rightarrow R(x)$
$P(c) \vee Q(c)$ - Universal Instantiation
$(\neg P(c) \wedge Q(c)) \rightarrow R(c)$ - Universal instantiation
$\neg(\neg P(c) \wedge Q(c)) \vee R(c)$ - Law of implication
$\neg\neg P(c) \wedge \neg Q(c) \vee R(c)$ - DeMorgan's law
$P(c) \vee \neg Q(c) \vee R(c)$ - Double negation
$P(c) \vee R(c)$ - Resulution
$R(c) \vee P(c)$ - Commutative law
$\neg(\neg R(c) \vee P(c)$ - double negation
$\neg R(c) \rightarrow P(c)$ - Law of implication
$\forall x(\neg R(x) \rightarrow P(x))$ - Universal generalization