Problem 1: Sec 1.7, #4

Show that the additive inverse, or negative, of an even number is an even number using a direct proof.

1. Let y be the additive inverse of an even number x
2. $x = 2k$ where k is an integer
3. Since y is the additive inverse of x, $y + 2k = 0$
4. Simplifying, $y = 2 \cdot (-k)$
5. Let $z = -k$
6. $y = 2z$
7. If z is an integer, y must be an even number by definition

Problem 2: Sec 1.7, #8

Prove that if n is a perfect square, then $n + 2$ is not a perfect square.

1. Assuming the contradiction that n is a perfect square and $n + 2$ is a perfect square
2. Let $n = a^2$ for some integer a$
3. Then $n + 2 = a^2 + 2$
4. Assuming $n + 2$ is a perfect square, $n + 2 = k^2$ for some integer k
5. Considering $a^2 - k^2, k^2 - a^2 = (k + m)(k - m) = (n + 2) - n = 2$
6. So, $(k + m)(k - m) = 2$
7. Since k and m are integers, $(k + m) \text{ and } (k - m)$ must also be integers
8. So the only possible factors are $1 \cdot 2$ and $-1 \cdot -2$
9. Neither of these factors work since if $k + m = 1$ then $k - m$ cannot be 2, and if $k + m = 2$, then $k - m$ cannot equal 1, and the same for the negatives
10. This shows a contradiction, if n is a perfect square, then $n + 2$ is not a perfect square

Problem 3: Sec 1.7, #16

Prove that if x, y, and z are integers and $x + y + z$ is odd, then at least one of x, y, and z is odd.

1. To proof this by contrapositive, assume that if none of x,y, and z are odd, then $x+y+z$ is even
2. Let $x=2a, y=2b, z=2c$ where a, b and c are integers
3. $x+y+z=2a+2b+2c=2(a+b+c)$
4. Since $a+b+c$ must be an integer, by definition $x+y+z$ is even because it is a multiple of 2
5. Therefore, if none of x,y, and z are odd, then $x+y+z$ is even
6. by contrapositive, if x,y, and z are integers and $x+y+z$ is odd, then at least one of x,y, and z is odd

Problem 4: Sec 1.7, #30

Prove that $m^2=n^2$ if and only if $m=n$ or $m=-n$.

If $m=n$ or $m=-n$, then $m^2=n^2$

1. Squaring both sides of $m=n$, you get $m^2=n^2$
2. Squaring both sides of $m=-n$, you get $m^2=-n^2=m^2=n^2$
3. Therefore, if $m=n$ or $m=-n$, then $m^2=n^2$
   If $m^2=n^2$, then $m=n$ or $m=-n$
4. If $m^2=n^2$, $m^2-n^2=0=(m+n)(m-n)=0$
5. $m+n=0$ or $m-n=0$
6. If $m+n=0, m=-n$
7. If $m-n=0, m=n$
8. Therefore, if $m^=n^2$, then $m=n$ or $m=-n$
   Since both direction have been proven, $m^2=n^2$ if and only if $m=n$ or $m=-n$

Problem 5: Sec 1.8, #14

Show that the product of two of the numbers $65^{1000}-8^{2001}+3^{177}, 79^{1212}-9^{2399}+2^{2001}$, and $24^{4493}-5^{8192}+7^{1777}$ is non negative. Is your proof constructive or nonconstructive? [Hint: Do not try to evaluate these numbers!]

Consider two cases:

1. Assume at least one of the numbers is 0, so multiplying 0 by any other number results in 0, a nonnegative product
2. None of the numbers are 0 and all numbers are iether postive or negative, If there are two positive numbers, then those numbers will prodice a nonnegative product together, if there are two negative numbers, then those numbers will produce a nonnegative product. If one number is positive, then one other number must be positive, or both are negative. Therefore, there must be a product between two positives or two negatives, resulting in a nonnegative product. if one number is negative, then the other number must be negative, or two must be positive, both resulting in a nonnegative product. This is a non-constructive proof.

Problem 6: Sec 1.8, #20

Show that if r is an irrational number, there is a unique integer n such that the distance between r and n is less than $\frac{1}{2}$.

1. For any irration number r, there are two unique integers a and b such that $a < b, |r-a| < 1, |r-b| < 1$. This is because between any two consecutive integers, there lies an irration number.
2. The point directly between a and b can be show as $\frac{a+b}{2}$/ Which is also an integer.
3. Since any integer plus $\frac{1}{2}$ is rational, the irrational number r must be between $a,a+\frac{1}{2},\text{ and } b(cannot include these number as they are rational).
4. All of these spaces are less than $\frac{1}{2}$ away from a or b$
5. Therefore, if r is an irration number, there is a unique integer n such that the distance between r and n is less than $\frac{1}{2}$.

Problem 7: Sec 1.8, #26

The quadratic mean of two real numbers x and y equals $\sqrt{\frac{x^2+y^2}{2}}$. By computing the arithmetic and quadratic means of different pairs of positive real numbers, formulate a conjecture about their relative sizes and prove your conjecture.

x	1	2	1	3
y	2	3	1	4
Quadratic mean	$\sqrt{\frac{1^2+2^2}{2}} = 1.581$	$\sqrt{\frac{2^2+3^2}{2}} = 2.549$	$\sqrt{\frac{1^2+1^2}{2}} = 1$	$\sqrt{\frac{3^2+4^2}{2}} = 3.535$
Arithmetic mean	$\frac{1+2}{2} = 1.5$	$\frac{2+3}{2} = 2.5$	$\frac{1+1}{2} = 1$	$\frac{3+4}{2} = 3.5$

Problem 8: Sec 1.8, #32

Prove that there are no solutions in integers x and y to the equation $2x^2 + 5y^2 = 14$.

Proof by contradiction:

1. Assume you found a pair (x,y) for the equation
2. 14 is even
3. $2x^2$ is even by definition
4. Since 14 and $2x^2$ are even, $5y^2$ must be even
5. Since $5y^2$ is even, y must be even.
6. Therefore, $y = 2z$ for some integer z
7. $2x^2 + 20z^2 = 14$ simplifies to $x^2 + 10z^2 = 7$
8. $10z^2 > \$if\$z \neq 0$
9. Therefore, z must be 0, showing that $x^2 = 7$
10. This is a contradiction that there are integers x and y to solve the equation
11. Therefore, there are no solutions in integers x and y to the equation $2x^2 + 5y^2 = 14$

Challenge 1: Sec 1.8, #36

Prove that $\sqrt[3]{2}$ is irrational

1. Assume $\sqrt[3]{2}$ is rational
2. So it can be express as a ratio of two integers, $\frac{a}{b}$. which are rational numbers
3. Then, $\sqrt[3]{2} = \frac{p}{q}$
4. So, $2 = \frac{p^3}{q^3}$
5. And, $p^3 = 2q^3$
6. $2q^3$ is even by definition
7. So, p^3 must be even

8. And p must be even
9. Thus, $p = 2n$ for some integer n
10. Therefore, $(2n)^3 = 2q^3$
11. $8n^3 = 2q^3$
12. $q^3 = 4n^3$
13. $4n^3$ is even by definition
14. So, q^3 must be even
15. So q must be even
16. Since q and p are both even, they are not coprime, creating a contradiction
17. So, $\sqrt[3]{2}$ must be irrational.