Problem 1: Sec 1.2, #32 (8 points)

(Exercises 28–35 relate to inhabitants of an island on which there are three kinds of people: knights who always tell the truth, knaves who always lie, and spies (called normals by Smullyan [Sm78]) who can either lie or tell the truth. You encounter three people, A, B, and C. You know one of these people is a knight, one is a knave, and one is a spy. Each of the three people knows the type of person each of other two is. For each of these situations, if possible, determine whether there is a unique solution and determine who the knave, knight, and spy are. When there is no unique solution, list all possible solutions or state that there are no solutions.)

A says “I am the knight,” B says “A is not the knave,” and C says “B is not the knave.”

Solution 1: B is the knight, C is the knave, A is the spy
Solution 2: C is the knight, A is the knave, B is the spy

Problem 2: Sec 1.2, #40 (10 points)

Four friends have been identified as suspects for an unauthorized access into a computer system. They have made statements to the investigating authorities. Alice said, “Carlos did it.” John said, “I did not do it.” Carlos said, “Diana did it.” Diana said, “Carlos lied when he said that I did it.”

a) If the authorities also know that exactly one of the four suspects is telling the truth, who did it? Explain your reasoning.

John must have done it because if exactly one person is telling the truth, this person must be Diana as all other options lead to contradictions. If Diana is the only one telling the truth, then Johns statement "I did not do it" must be a lie, so he did do it.

b) If the authorities also know that exactly one is lying, who did it? Explain your reasoning.

In this case, it must be Carlos who is guilty, this is because he must be the one lying as all other options lead to contradictions, if Carlos is lying, Alices statement must be true, so Carlos is guilty.

Problem 3: Sec 1.3, #26 (8 points)

Show that $(p \rightarrow q) \wedge (p \rightarrow r)$ and $p \rightarrow (q \rightarrow r)$ are logically equivalent.

$(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \rightarrow r)$ This is already a logical equivalence involving conditional statements.

Problem 4: Sec 1.3, #30 (10 points)

Show that $\neg p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \vee r)$ are logically equivalent.

$\neg\neg p \vee (q \rightarrow r)$ - Logical equivalences involving conditional statements.
$p \vee (q \rightarrow r)$ - Double negation.
$p \vee (\neg q \vee r)$ - Logical equivalences involving conditional statements.
$\neg q \vee (p \vee r)$ - Associative laws
$q \rightarrow (p \vee v)$ - Logical equivalences involving conditional statements.

Problem$\neg(p \vee q) \wedge \neg(\neg p \vee r) \vee (q \vee r)$ 5: Sec 1.3, #34 (10 points)

Show that $(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$ is a tautology.

$\neg((p \vee q) \wedge (\neg p \vee r)) \vee (q \vee r)$ - Logical equivalences involving conditional statements.
$\neg(p \vee q) \wedge \neg(\neg p \vee r) \vee (q \vee r)$ - DeMorgan's law
$(\neg p \wedge \neg q) \vee (\neg\neg p \wedge \neg r) \vee (q \vee r)$ - DeMorgan's law twice
$(\neg p \wedge \neg q) \vee (p \wedge \neg r) \vee (q \vee r)$ - Double negation
$((\neg p \wedge \neg q) \vee q) \vee ((p \wedge \neg r) \vee r$ - Distributive laws twice
$((\neg p \vee q) \wedge (\neg q \vee q)) \vee ((p \vee r) \wedge (r \vee \neg r))$ - Distributive laws twice
$((\neg p \vee q) \wedge T) \vee ((p \vee r) \wedge T)$ - Negation laws twice
$(\neg p \vee q) \vee (p \vee r)$ - Identity laws twice
$(\neg p \vee p) \vee (q \vee r)$ - Absorption law

$T \lor (q \lor r)$ - Negation law
T - Domination law

Problem 6: Sec 1.4, #8 (12 points)

Translate these statements into English, where $R(x)$ is "x is a rabbit" and $H(x)$ is "x hops" and the domain consists of all animals.

a) $\forall x(R(x) \to H(x))$

For every animal x, if x is a rabbit, then x hops.

b) $\forall x(R(x) \land H(x))$

For every animal x, x is a rabbit and x hops.

c) $\exists x(R(x) \to H(x))$

There exists an animal x, such that if x is a rabbit, then x hops.

d) $\exists x(R(x) \land H(x))$

There exists an animal x, such that x is a rabbit and x hops.

Problem 7: Sec 1.4, #10 (15 points)

Let $C(x)$ be the statement “x has a cat,” let $D(x)$ be the statement “x has a dog,” and let $F(x)$ be the statement “x has a ferret.” Express each of these statements in terms of $C(x)$, $D(x)$, $F(x)$, quantifiers, and logical connectives. Let the domain consist of all students in your class.

a) A student in your class has a cat, a dog, and a ferret.

$\exists x(C(x) \land D(x) \land F(x))$

b) All students in your class have a cat, a dog, or a ferret.

$\forall x(C(x) \vee D(x) \vee F(x))$

c) Some student in your class has a cat and a ferret, but not a dog.

$\exists x(C(x) \wedge F(x) \wedge \neg D(x))$

d) No student in your class has a cat, a dog, and a ferret.

$\neg \exists x(C(x) \wedge D(x) \wedge F(x))$

e) For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has this animal as a pet.

$\exists x \exists y \exists z(C(x) \wedge D(y) \wedge F(z))$

Problem 8: Sec 1.4, #12 (7 points)

Let $Q(x)$ be the statement "$x+1>2x$." If the domain consists of all integers, what are these truth values.

a) $Q(0))$

True

b) $Q(-1)$

True

c) $Q(1)$

False

d) $\exists Q(x)$

True

e) $\forall Q(x)$

False

f) $\exists \neg Q(x)$

True

g) $\forall \neg Q(x)$

False

Problem 9: Sec 1.4, #24 (20 points)

24. Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives. First, let the domain consist of the students in your class and second, let it consist of all people.

A(x) = x has a cellular phone
B(x) = x is in your class
C(x) = x has seen a foreign movie
D(x) = x can swim
E(x) = x can solve quadratic equations
F(x) = x wants to be rich

a) Everyone in your class has a cellular phone.

1. $\forall x(A(x))$
2. $\exists x(B(x) \to A(x))$

b) Somebody in your class has seen a foreign movie.

1. $\exists x(C(x))$
2. $\exists x(B(x) \wedge C(x))$

c) There is a person in your class who cannot swim.

1. $\exists x(\neg D(x))$
2. $\exists x(B(x) \wedge \neg D(x))$

d) All students in your class can solve quadratic equations.

1. $\forall x(E(x))$
2. $\exists x(B(x) \rightarrow E(x))$

e) Some student in your class does not want to be rich.

1. $\exists x(\neg F(x))$
2. $\exists x(B(x) \wedge \neg F(x))$

Challenge 1: Sec 1.3, #50 (2 extra points)

Construct a truth table for the logical operator NAND.

p	q	p nand q
T	T	F
T	F	T
F	T	T
F	F	T

Challenge 2: Sec 1.3, #56 (6 extra points)

Show that {|} is a functionally complete collection of logical operators.

NOT: $\neg A$ equivalent to A|A

AND: $A \land B$ equivalent to $\neg(A|B)$

OR: $A \lor B$ equivalent to $(\neg A|\neg B)$