

# State-Space Formulation

- **Intelligent agents: problem solving as search**
- Search consists of
  - state space
  - operators
  - start state
  - goal states
- The search graph
- **A Search Tree is an effective way to represent the search process**
- **There are a variety of search algorithms, including**
  - Depth-First Search
  - Breadth-First Search
  - Others which use heuristic knowledge (in future lectures)

# Uninformed search strategies

## uninformed (or blind) search:

While searching you have no clue whether one non-goal state is better than any other. Your search is blind. You don't know if your current exploration is likely to be fruitful.

## common blind strategies:

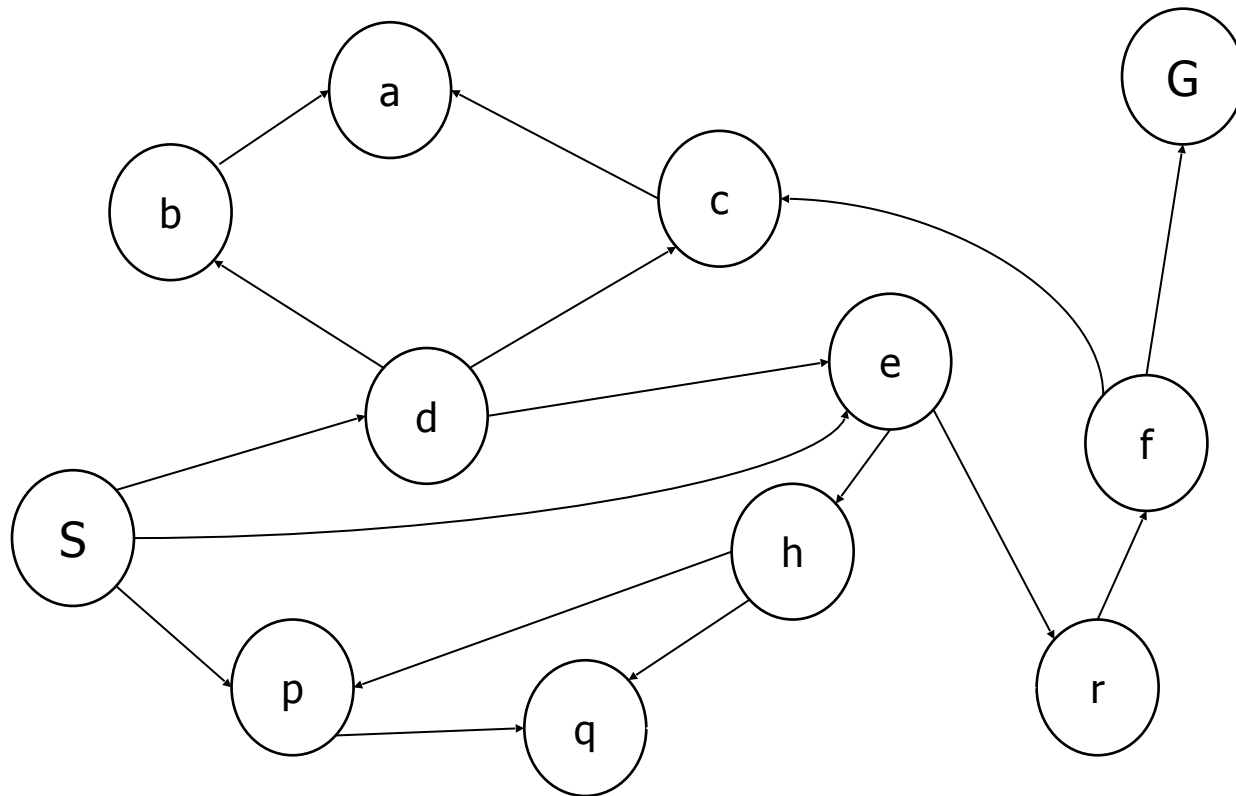
- Breadth-first search
- Uniform-cost search
- Depth-first search
- Iterative deepening search

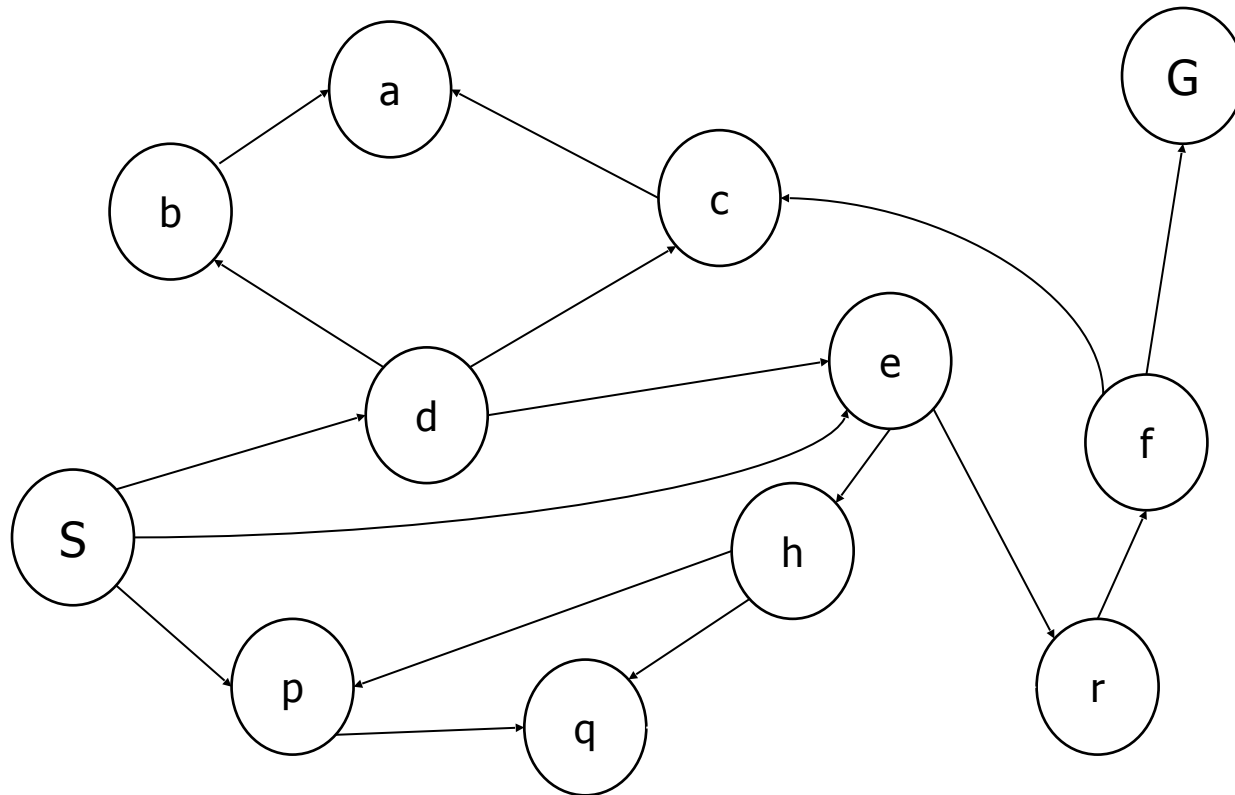
# General Tree Search

```
function TREE-SEARCH(problem, strategy) returns a solution, or failure
  initialize the search tree using the initial state of problem
  loop do
    if there are no candidates for expansion then return failure
    choose a leaf node for expansion according to strategy
    if the node contains a goal state then return the corresponding solution
    else expand the node and add the resulting nodes to the search tree
  end
```

- Important details:
  - Fringe (frontier)
  - Expansion
  - Exploration strategy
- Main question: which fringe nodes to explore?

# Graph representation of the landscape explored

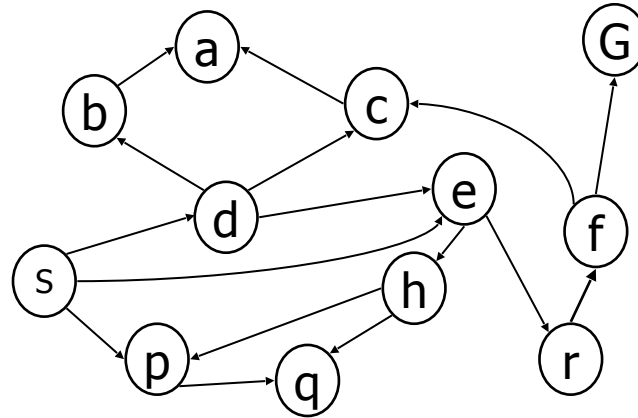




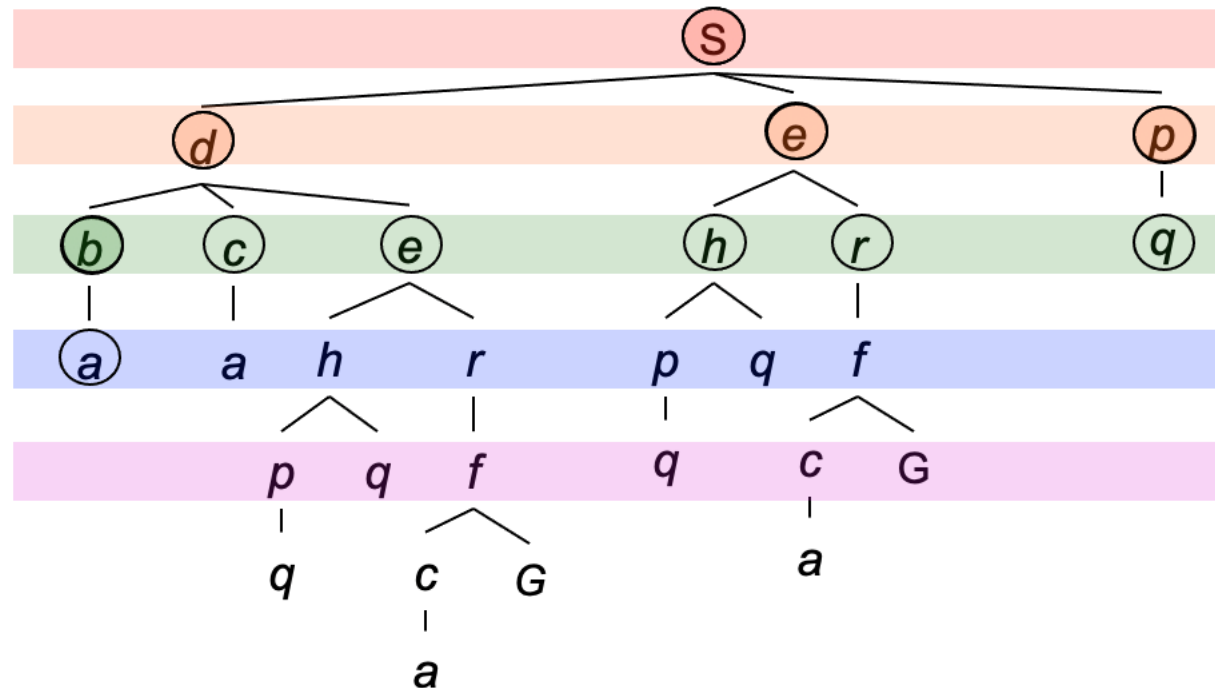
# Breadth-First Search

*Strategy: expand  
a shallowest  
node first*

*Implementation:  
Fringe is a FIFO  
queue*



Search  
Tiers



```

function BREADTH-FIRST-SEARCH(problem) returns a solution, or failure
    node  $\leftarrow$  a node with STATE = problem.INITIAL-STATE, PATH-COST = 0
    if problem.GOAL-TEST(node.STATE) then return SOLUTION(node)
    frontier  $\leftarrow$  a FIFO queue with node as the only element
    explored  $\leftarrow$  an empty set
    loop do
        if EMPTY?(frontier) then return failure
        node  $\leftarrow$  POP(frontier) /* chooses the shallowest node in frontier */
        add node.STATE to explored
        for each action in problem.ACTIONS(node.STATE) do
            child  $\leftarrow$  CHILD-NODE(problem, node, action)
            if child.STATE is not in explored or frontier then
                if problem.GOAL-TEST(child.STATE) then return SOLUTION(child)
                frontier  $\leftarrow$  INSERT(child, frontier)

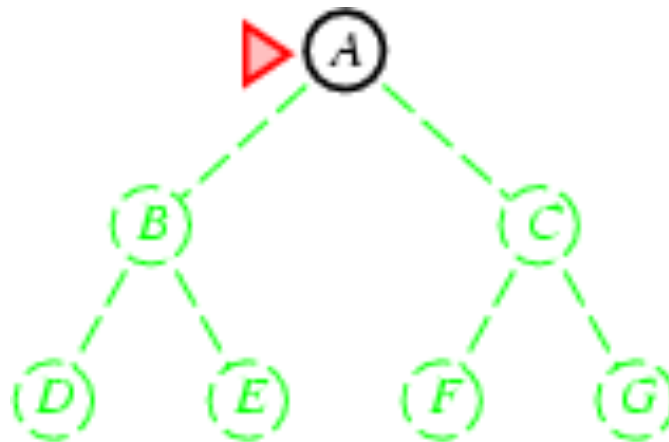
```

**Figure 3.11** Breadth-first search on a graph.

# Breadth-first search

- Expand shallowest unexpanded node
- Fringe: nodes waiting in a queue to be explored, also called **OPEN**
- **Implementation:**
  - *fringe* is a first-in-first-out (FIFO) queue, i.e., new successors go at end of the queue.

Is A a goal state?





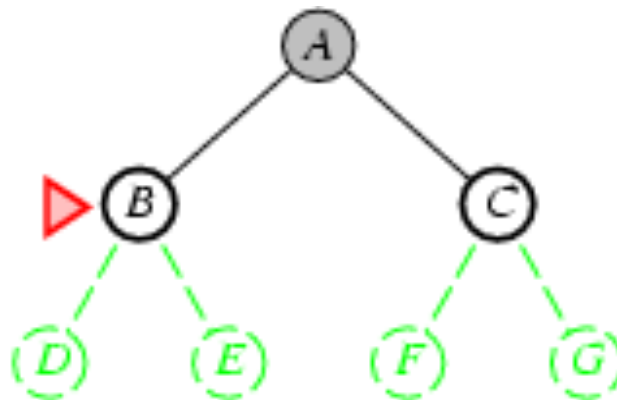
# Breadth-first search

- Expand shallowest unexpanded node
- Implementation:
  - *fringe* is a FIFO queue, i.e., new successors go at end

Expand:

fringe = [B,C]

Is B a goal state?

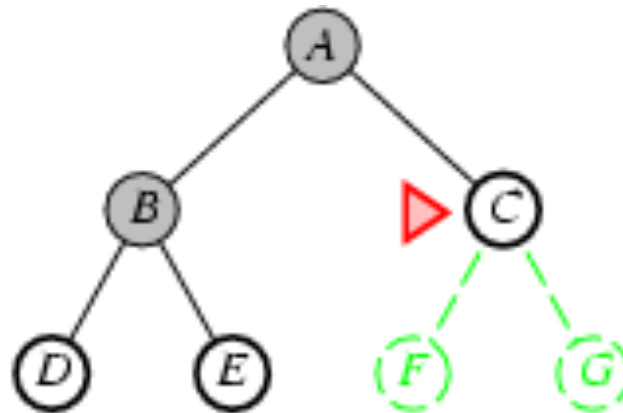


# Breadth-first search

- Expand shallowest unexpanded node
- Implementation:
  - *fringe* is a FIFO queue, i.e., new successors go at end

Expand:  
fringe=[C,D,E]

Is C a goal state?

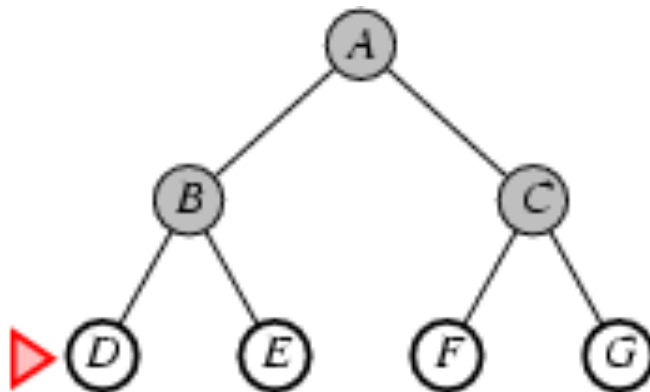


# Breadth-first search

- Expand shallowest unexpanded node
- Implementation:
  - *fringe* is a FIFO queue, i.e., new successors go at end

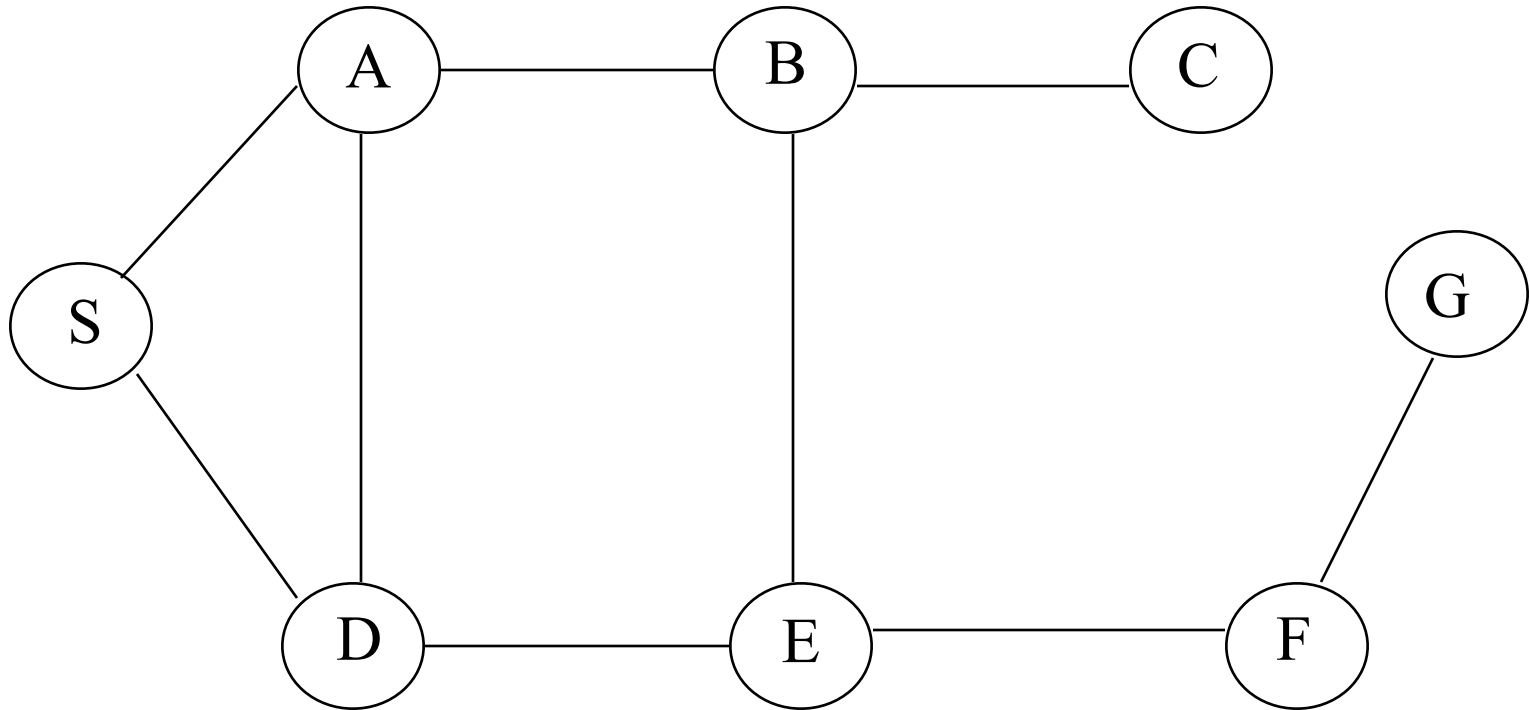
Expand:  
fringe=[D,E,F,G]

Is D a goal state?



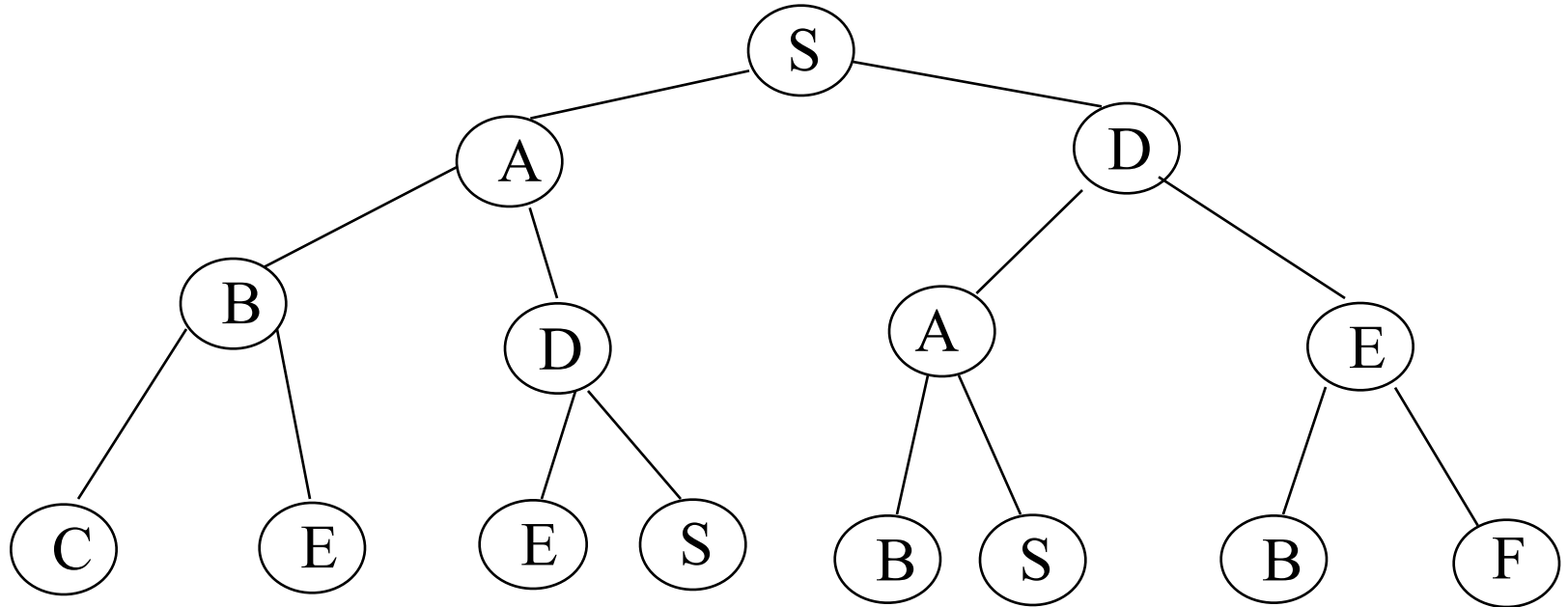
# BFS for 8 puzzle

# Example: Map Navigation



S = start, G = goal, other nodes = intermediate states, links = legal transitions

# Breadth First Search Tree (BFS)



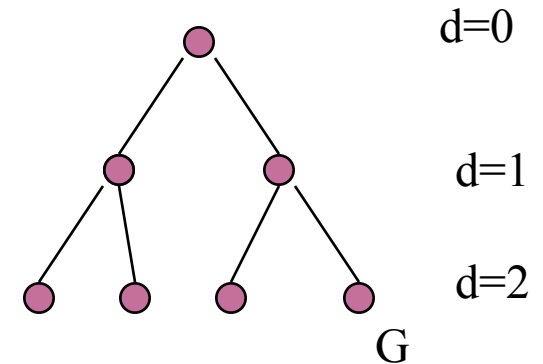
Here BFS is implemented as a tree search with only parent node not added as a child node.

# What is the Complexity of Breadth-First Search?

- Time Complexity

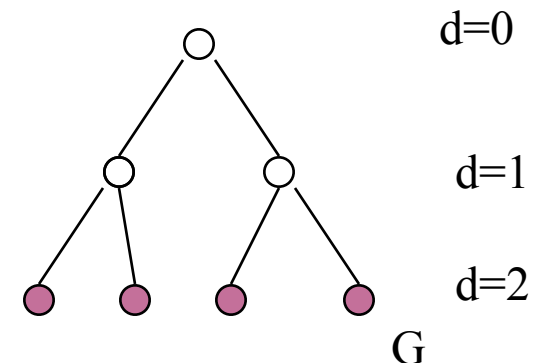
- assume (worst case) that there is 1 goal leaf at the RHS
- so BFS will **expand** all nodes

$$\begin{aligned} &= 1 + b + b^2 + \dots + b^d \\ &= \mathbf{O(b^{d+1})} \end{aligned}$$



- Space Complexity

- how many nodes can be in the queue (worst-case)?
- at depth  $d$  there are  $b^d$  **unexpanded** nodes in the Q =  $\mathbf{O(b^d)}$



- Time and space of number of generated nodes is  $\mathbf{O(b^{d+1})}$

Examples of Time and Memory Requirements for *tree search version* of Breadth-First Search

Depth of Solution	Nodes Expanded	Time	Memory
0	1	1 millisecond	100 bytes
2	111	0.1 seconds	11 kbytes
4	11,111	11 seconds	1 megabyte
8	$10^8$	31 hours	11 giabytes
12	$10^{12}$	35 years	111 terabytes

Assuming  $b=10$ , 1000 nodes/sec, 100 bytes/node



# Breadth-First Search (BFS) Properties

- **Complete** (will find a solution in a finite number of steps if one exists)
- Solution Length: **optimal** (assuming unit cost per move)
- (Can) expand each node once (if checks for duplicates)
- Search Time:  $O(b^d)$  which is the size of the state space
- Memory Required:  $O(b^d)$
- Drawback: requires space proportional to the state-space (Search time is unavoidable)