Alpha Beta Procedure

- Idea:
  - Do Depth first search to generate partial game tree.
  - Give static evaluation function to leaves.
  - compute bound on internal nodes.
- Alpha, Beta bounds:
  - Alpha value for Max node means that Max real value is at least alpha.
  - Beta for Min node means that Min can guarantee a value below Beta.
- Computation:
  - Alpha of a Max node is the maximum value of its seen children.
  - Beta of a Min node is the lowest value seen of its child node .

When to Prune

- Pruning
  - Below a Min node whose beta value is lower than or equal to the alpha value of its ancestors.
  - Below a Max node having an alpha value greater than or equal to the beta value of any of its Min nodes ancestors.

The Alpha-Beta Procedure

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- Use two variables alpha (associated with MAX nodes) and beta (associated with MIN nodes).
- These variables contain the best (highest or lowest, resp.) e(p) value at a node p that has been found so far.
- Notice that alpha can never decrease, and beta can never increase.

The Alpha-Beta Procedure

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Alpha-Beta procedure

```
def value(state, α, β):
  if (state is a max node):
    max-value((state, α, β)
  else:
    min-value((state, α, β)
```

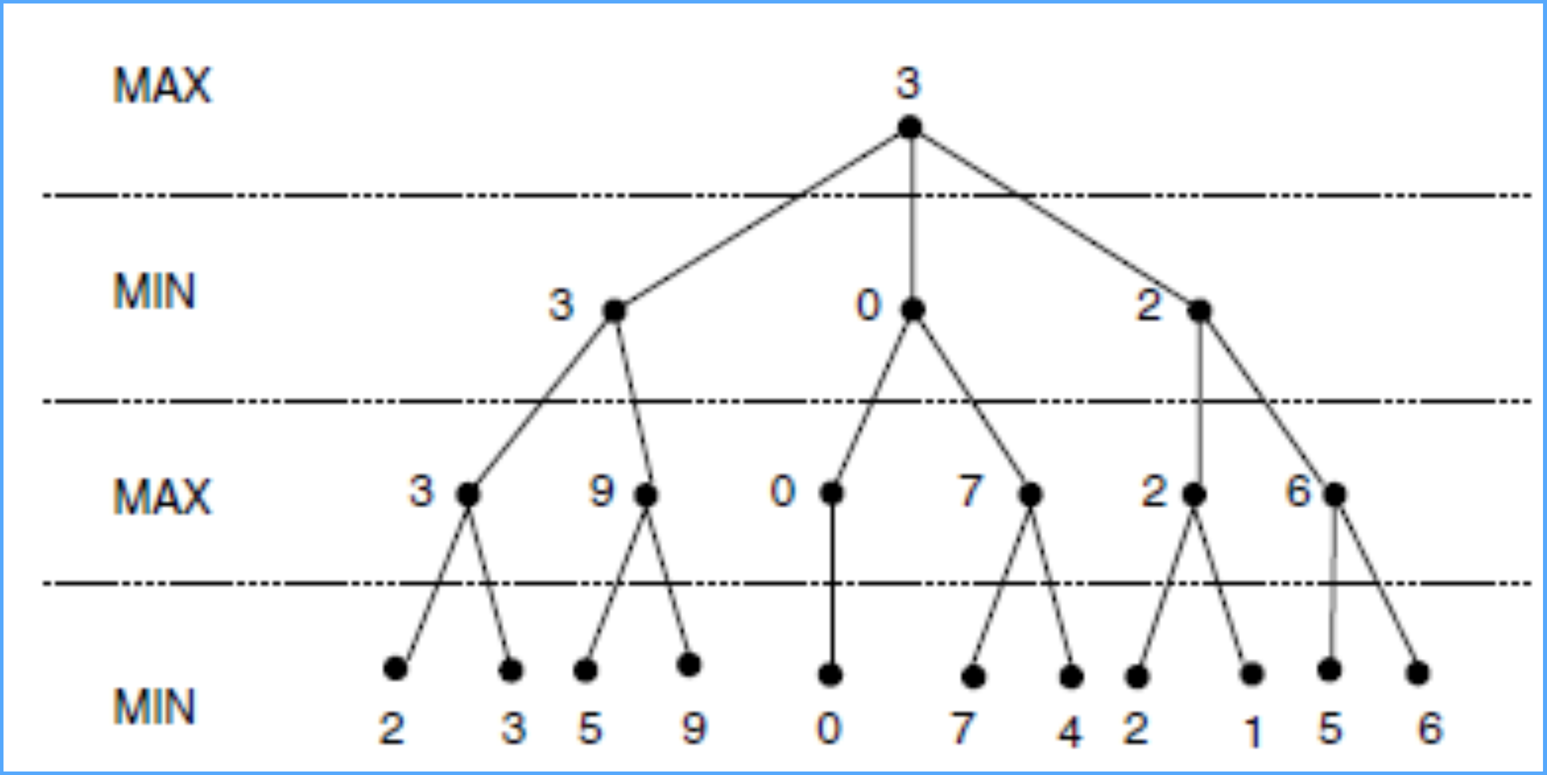
α: MAX's best option on path to root

β: MIN's best option on path to root

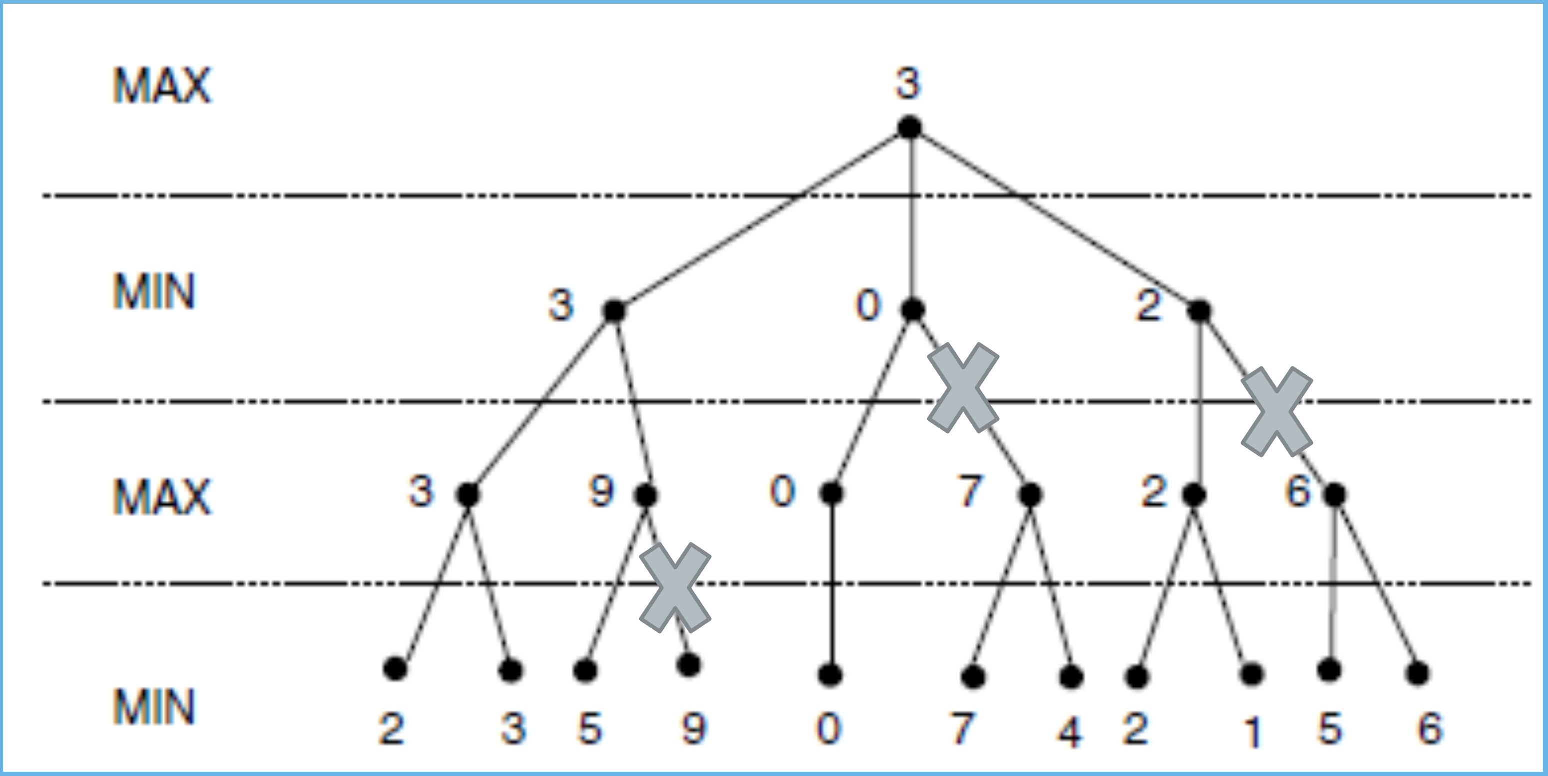
```
def max-value(state, α, β):
    initialize v = -∞
    if leaf(state):
        return value(state)
    for each successor of state:
        v = max(v, value(successor, α, β))
        if v ≥ β return v
        α = max(α, v)
    return v
```

```
def min-value(state , α, β):
    initialize v = +∞
    if leaf(state):
        return value(state)
    for each successor of state:
        v = min(v, value(successor, α, β))
        if v ≤ α return v
        β = min(β, v)
    return v
```

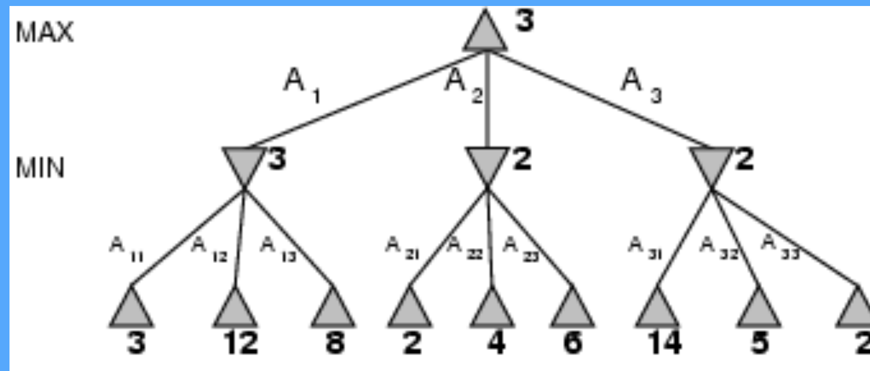
Pruning the tree Example 1

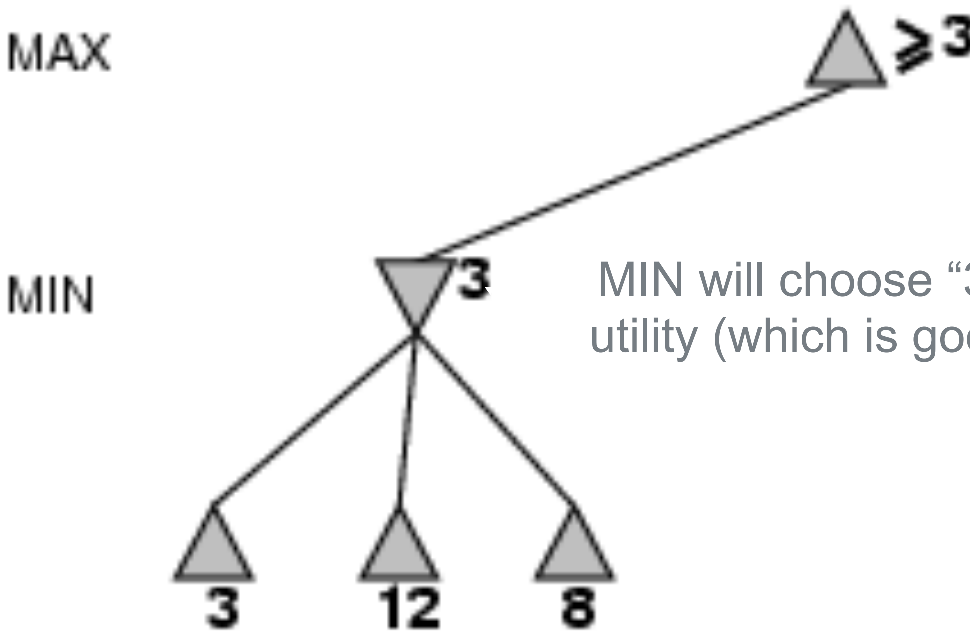


Pruning the tree Example



α-β pruning example 2

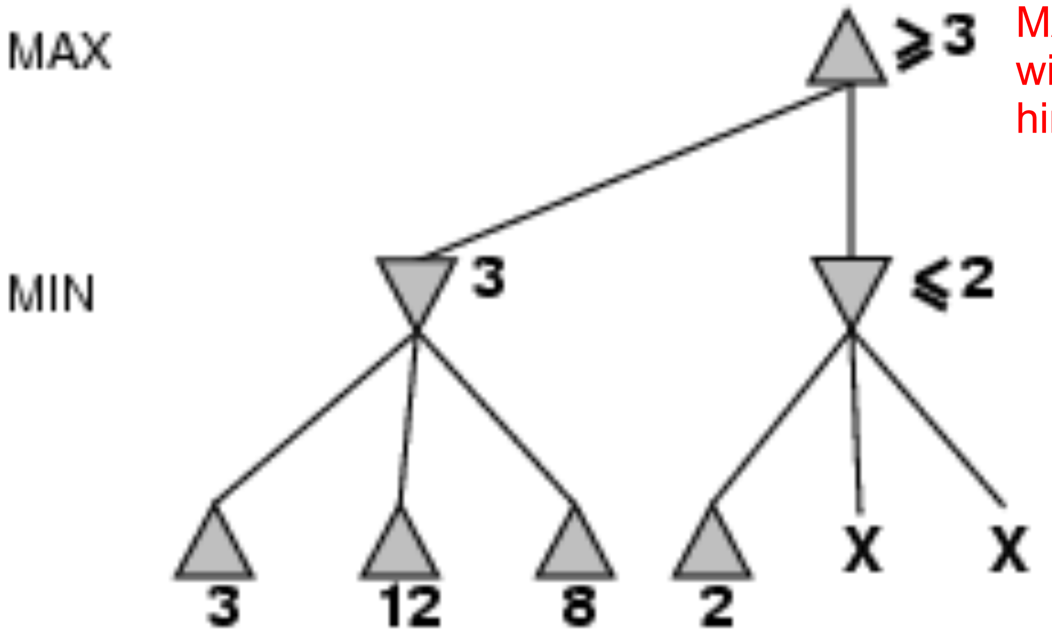




MAX knows that it can at least get "3" by playing this branch

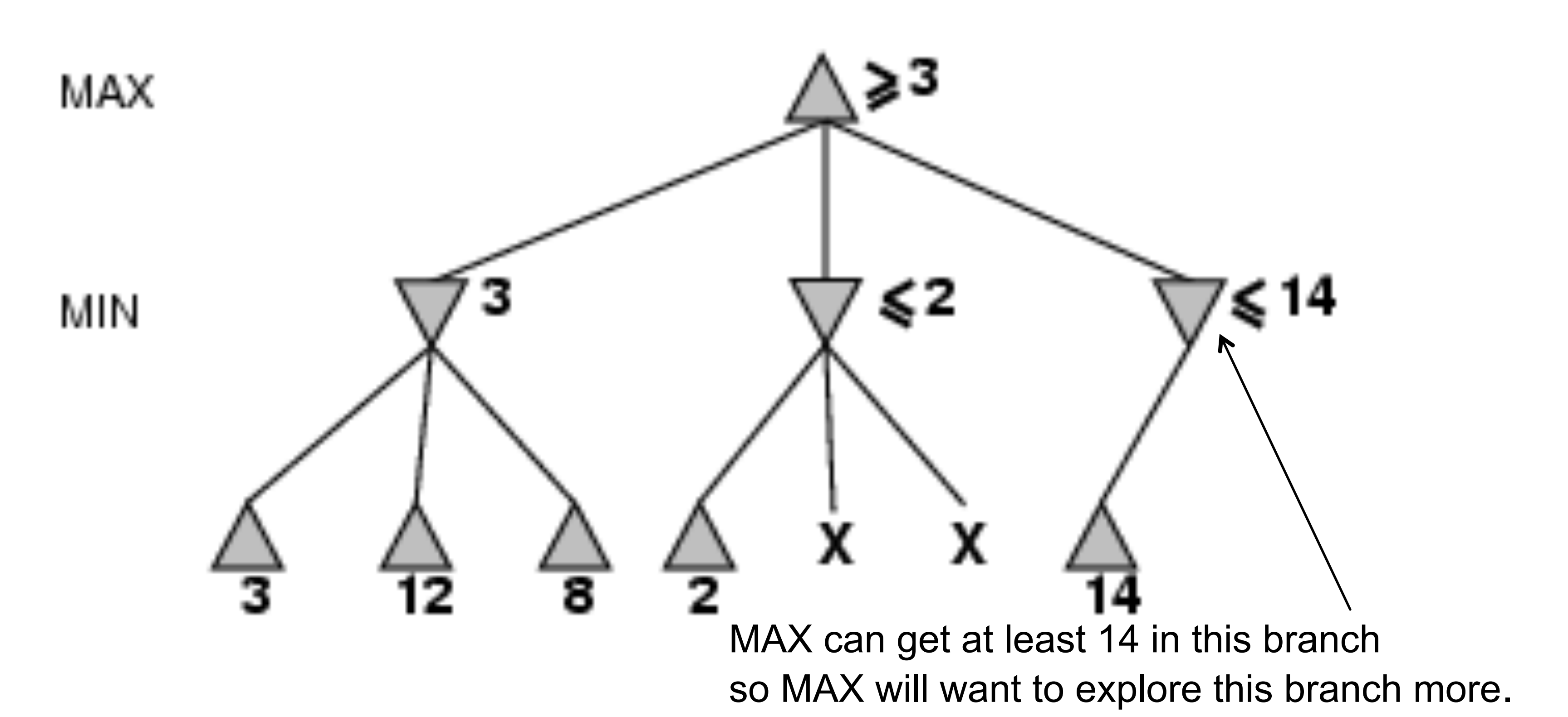
MIN will choose "3", because it minimizes the utility (which is good for MIN)

α-β pruning example 2

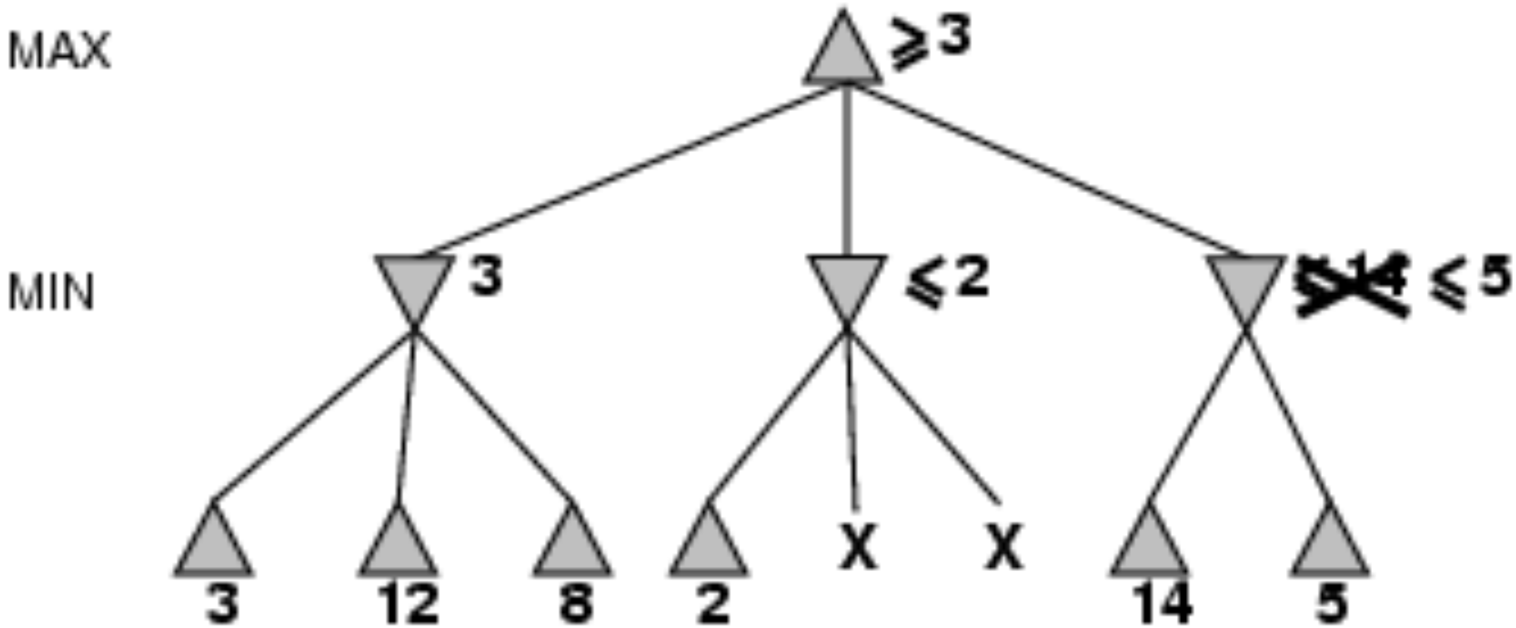


MAX knows that the new branch will never be better than 2 for him. It can *ignore* it.

α-β pruning example 2

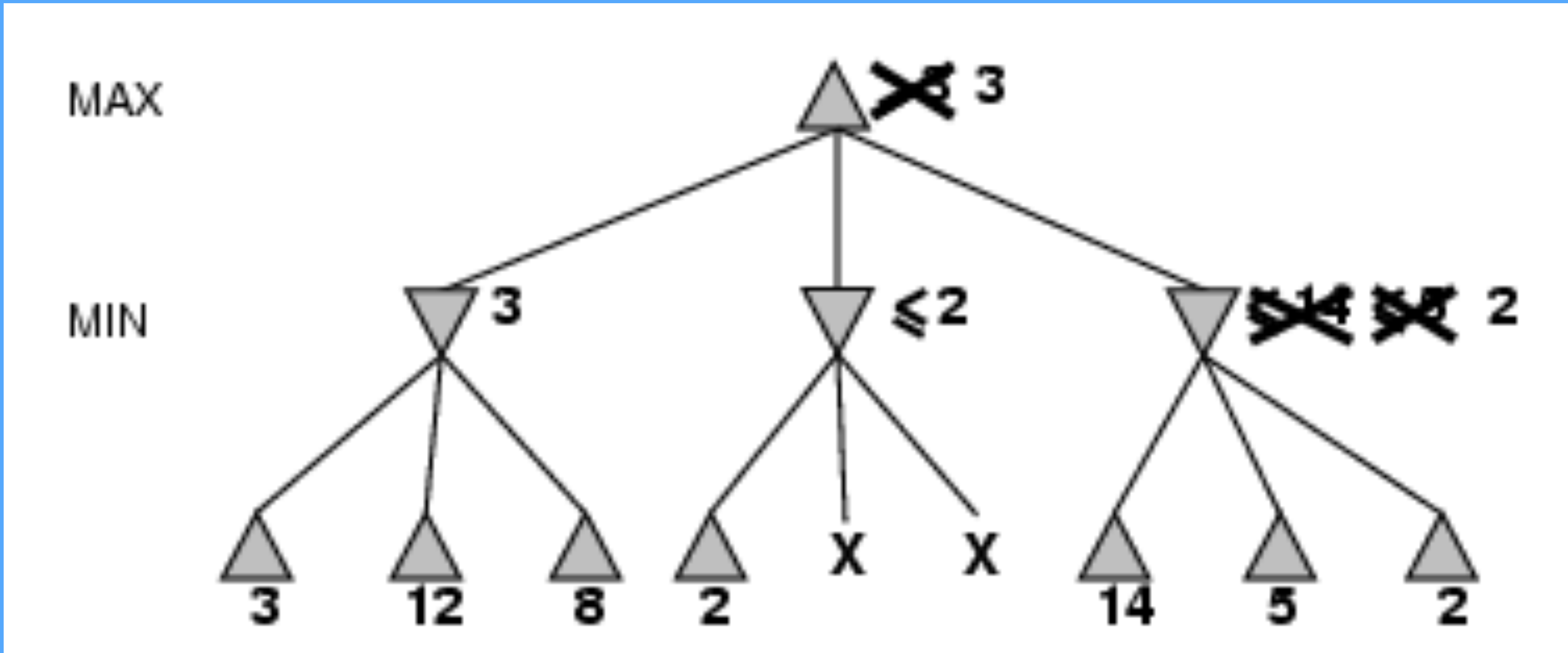


α-β pruning example 2



Should continue to explore

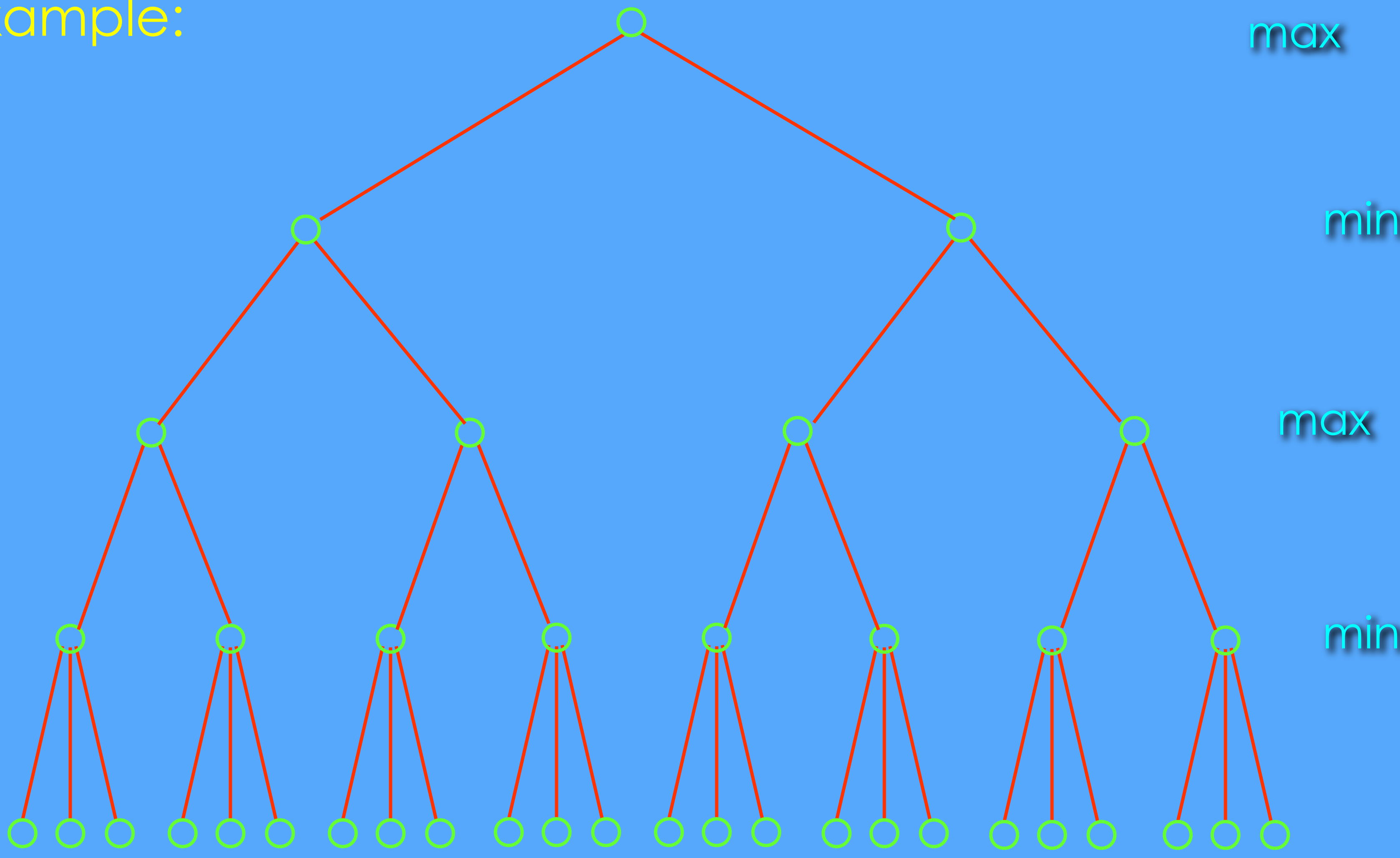
α-β pruning example 2



Search completed without exploring two branches

The Alpha-Beta Procedure Example 3

Example:



The Alpha-Beta Procedure

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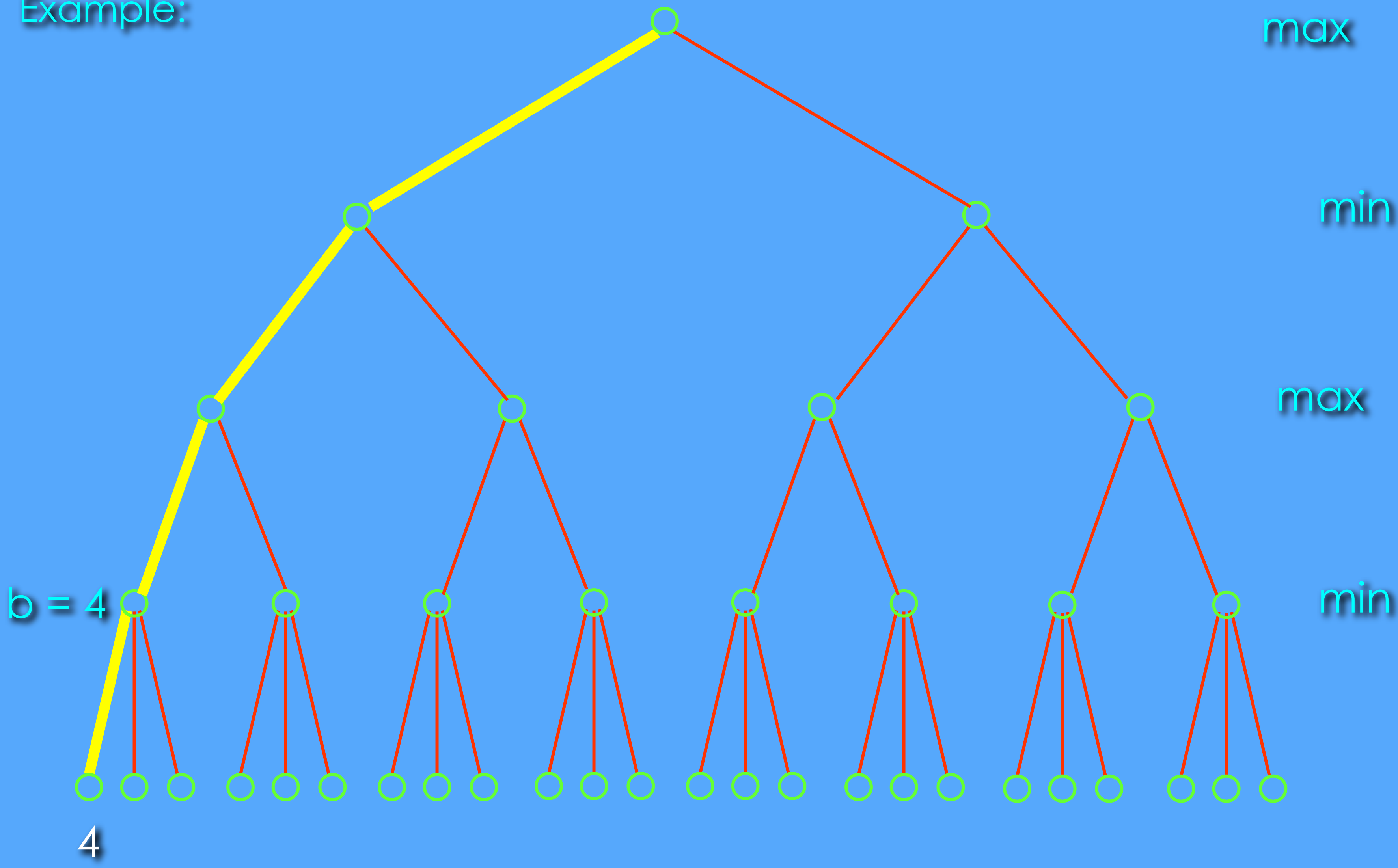

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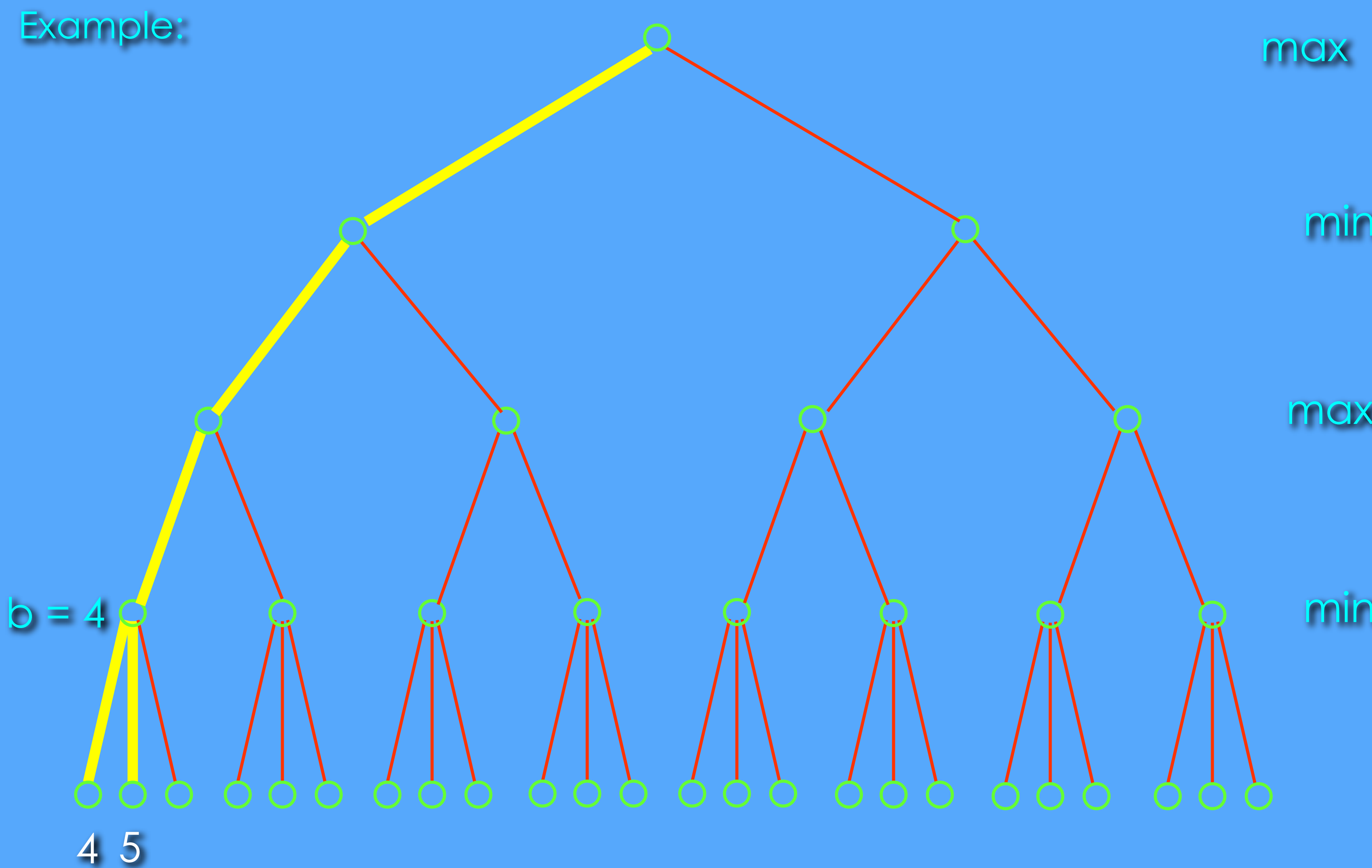

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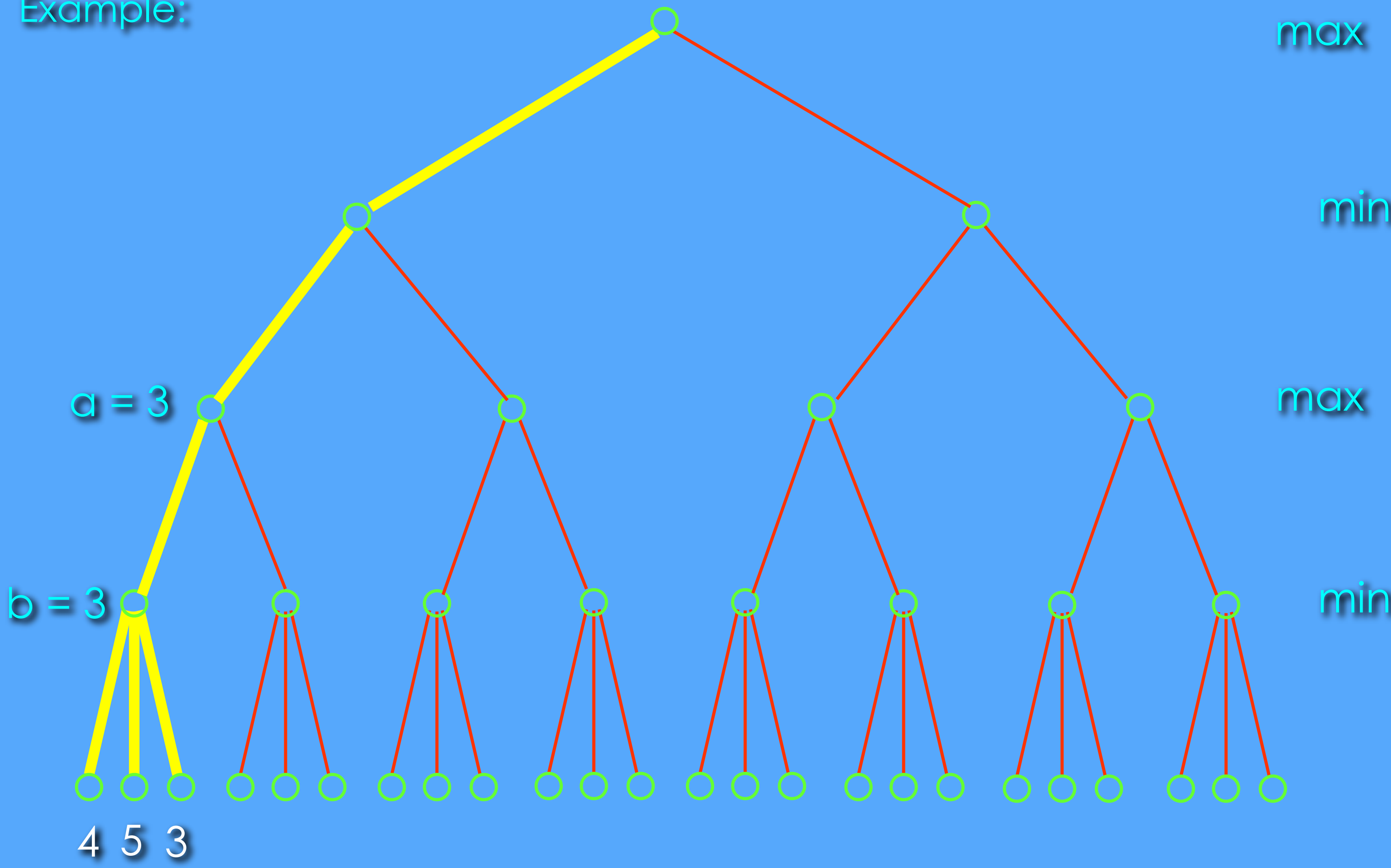

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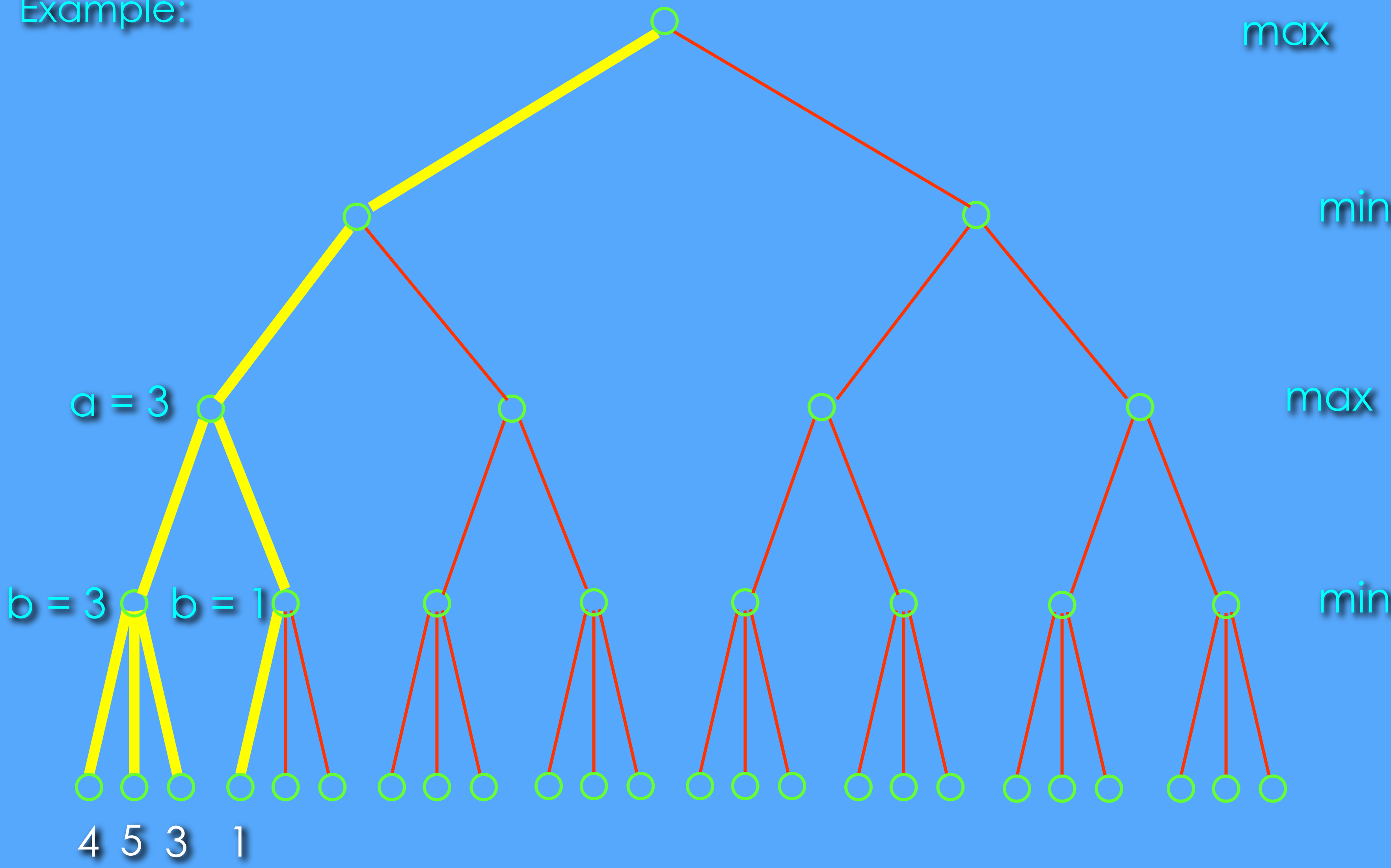

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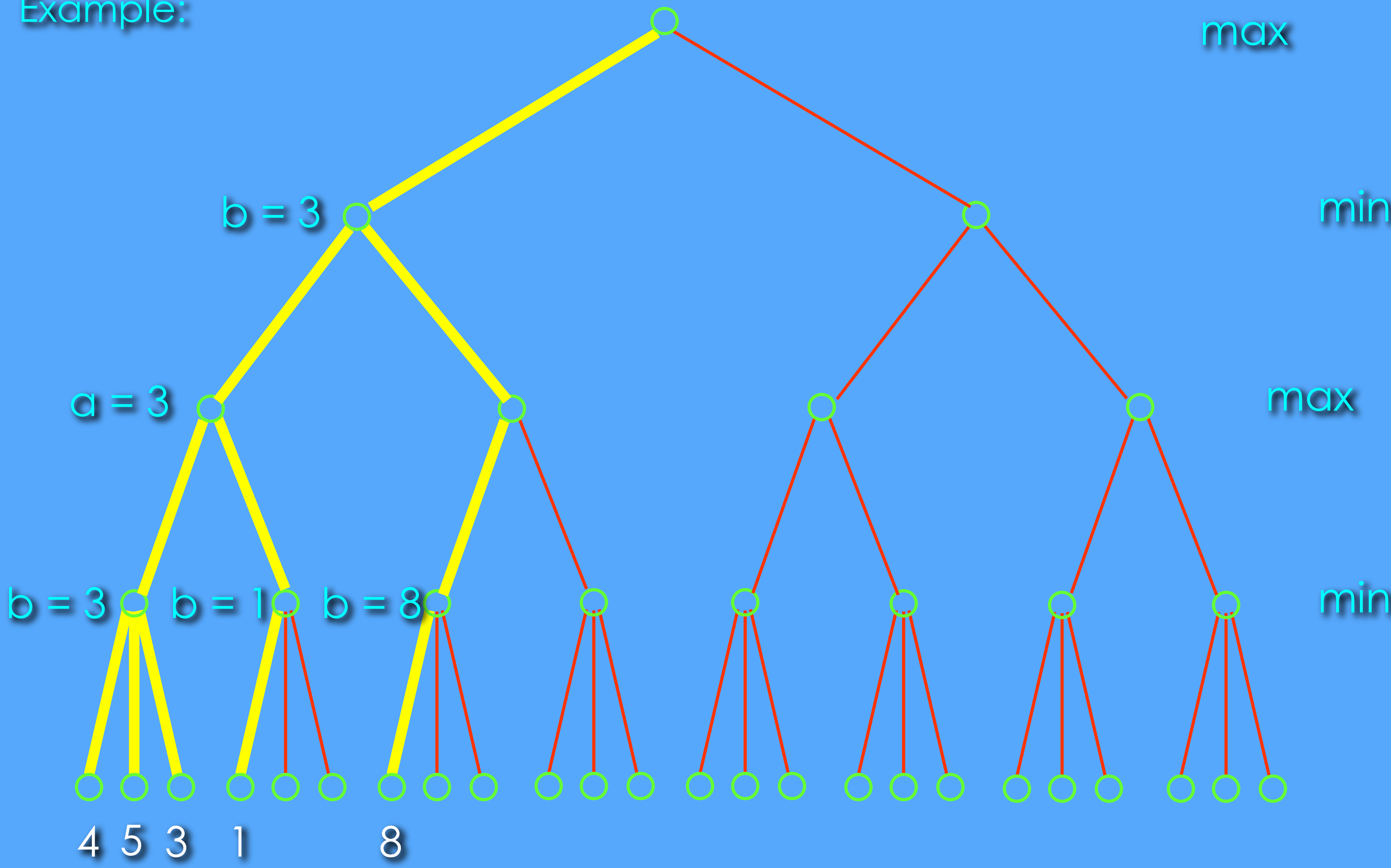

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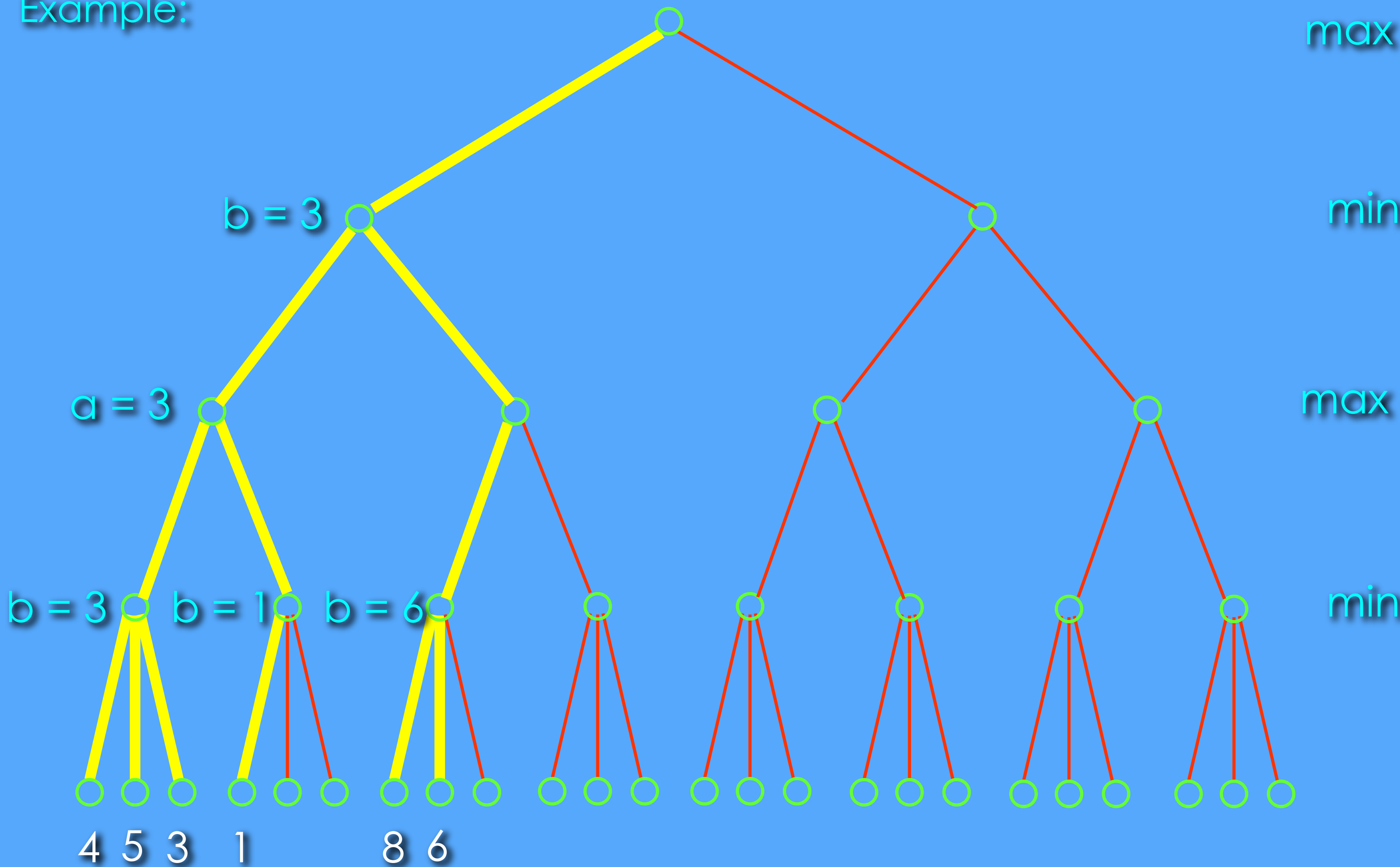

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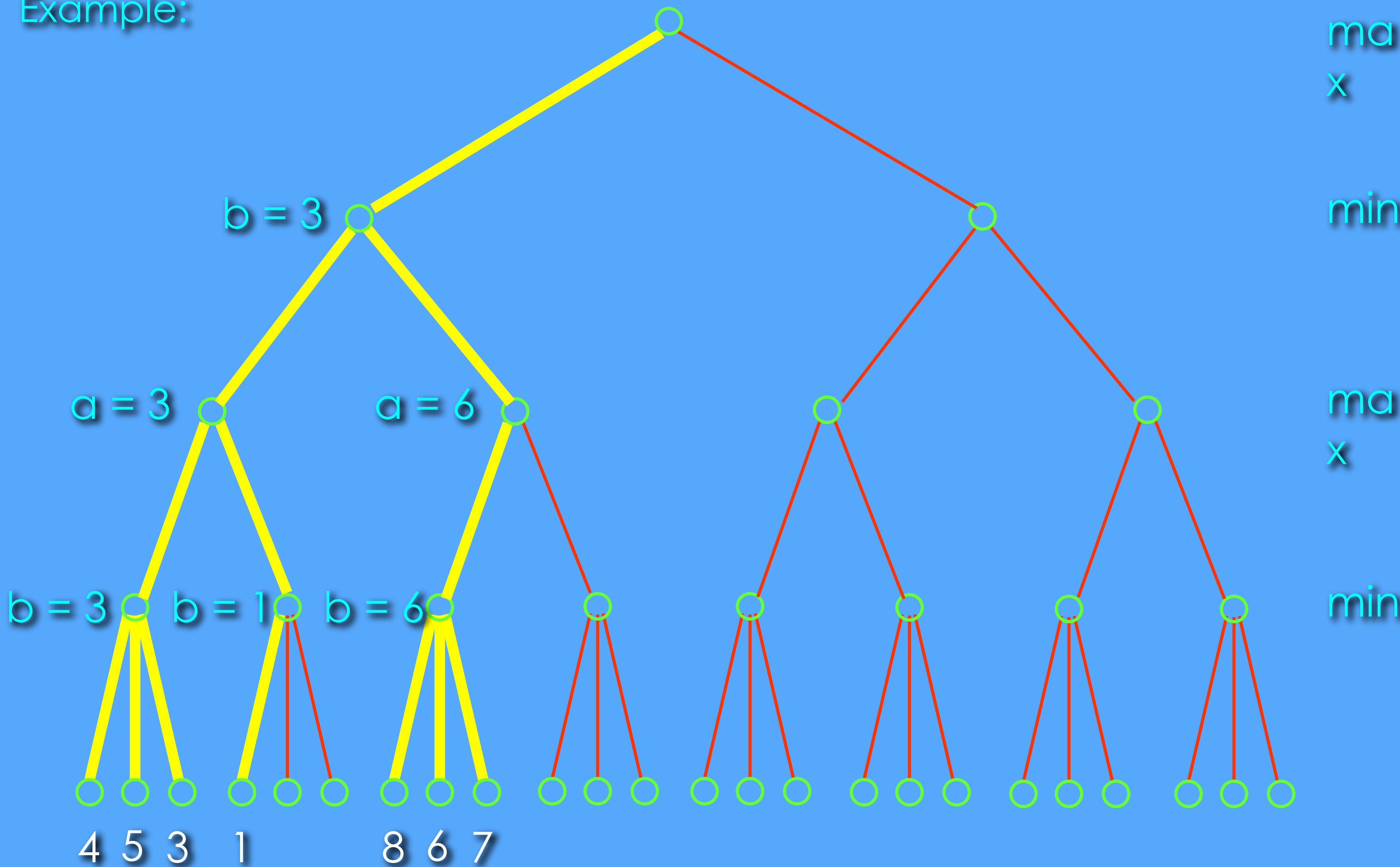

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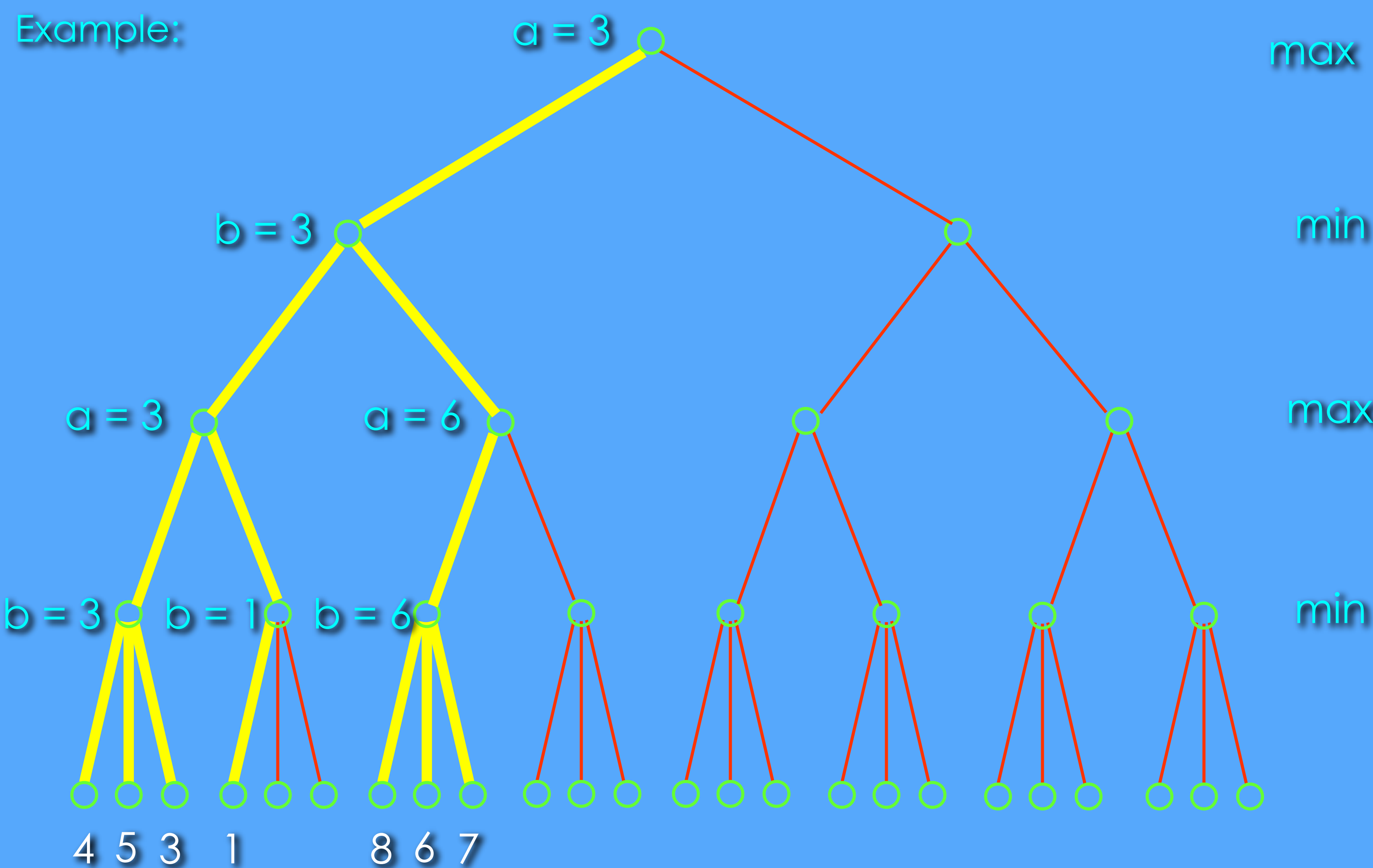

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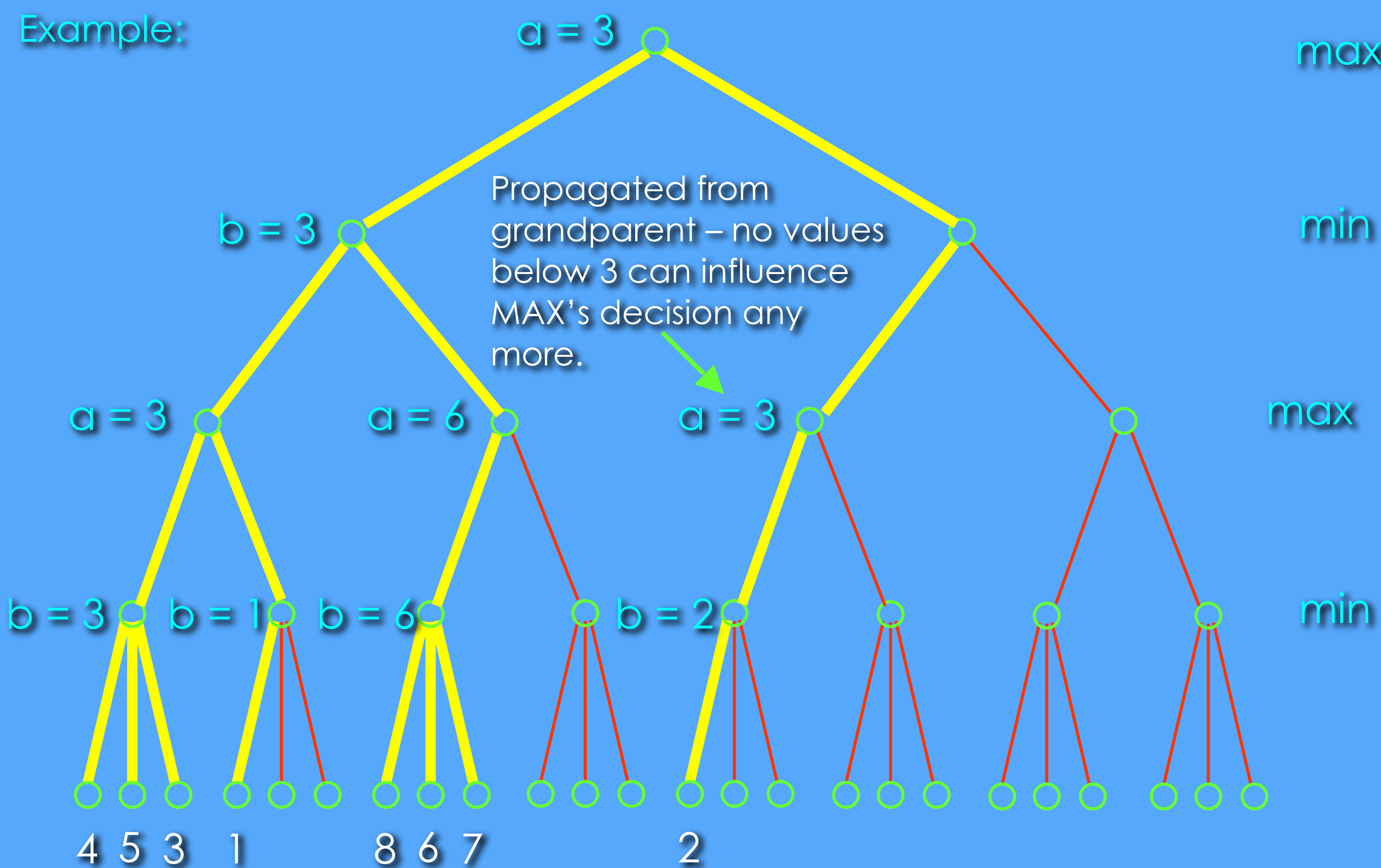

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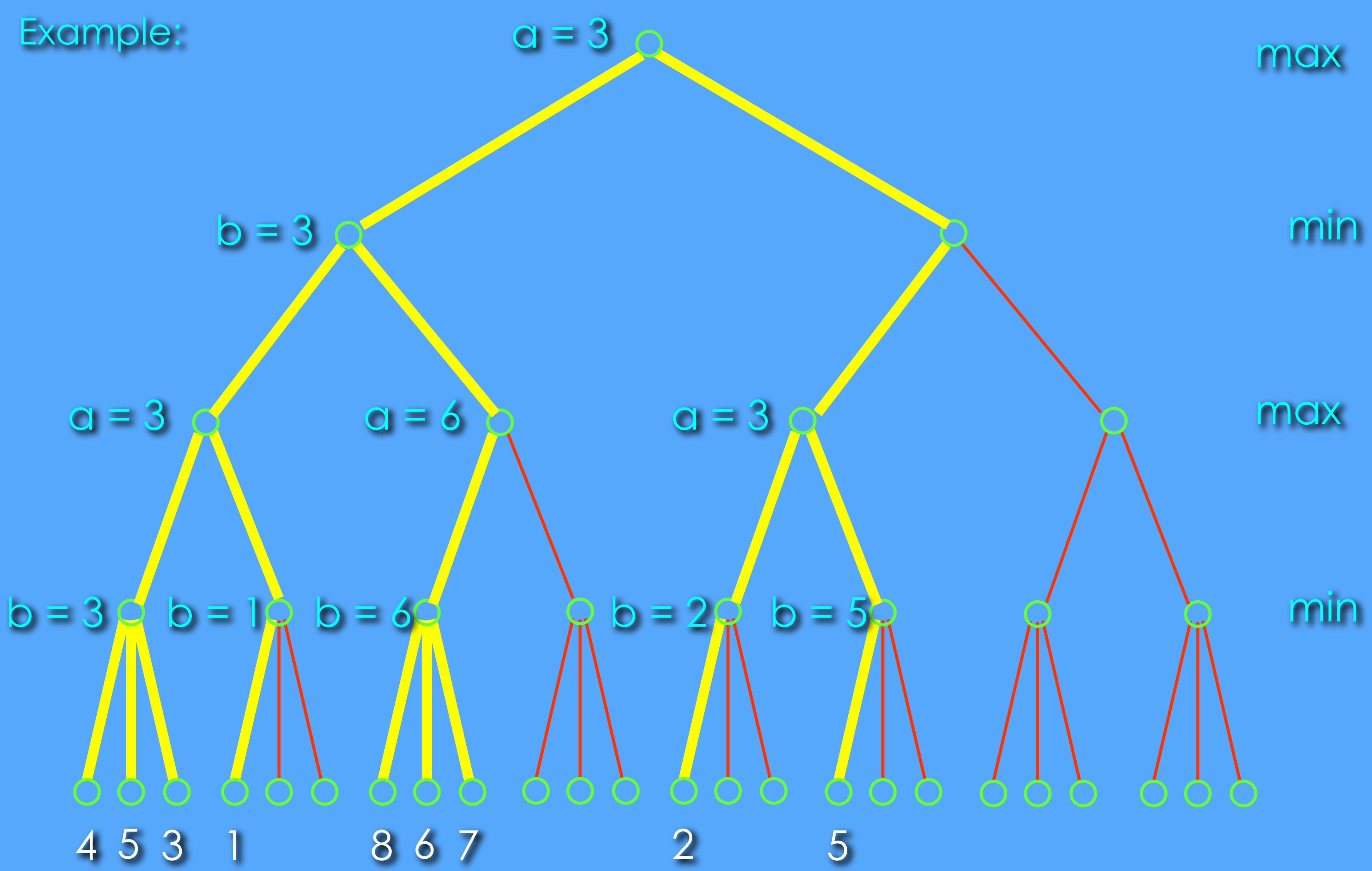

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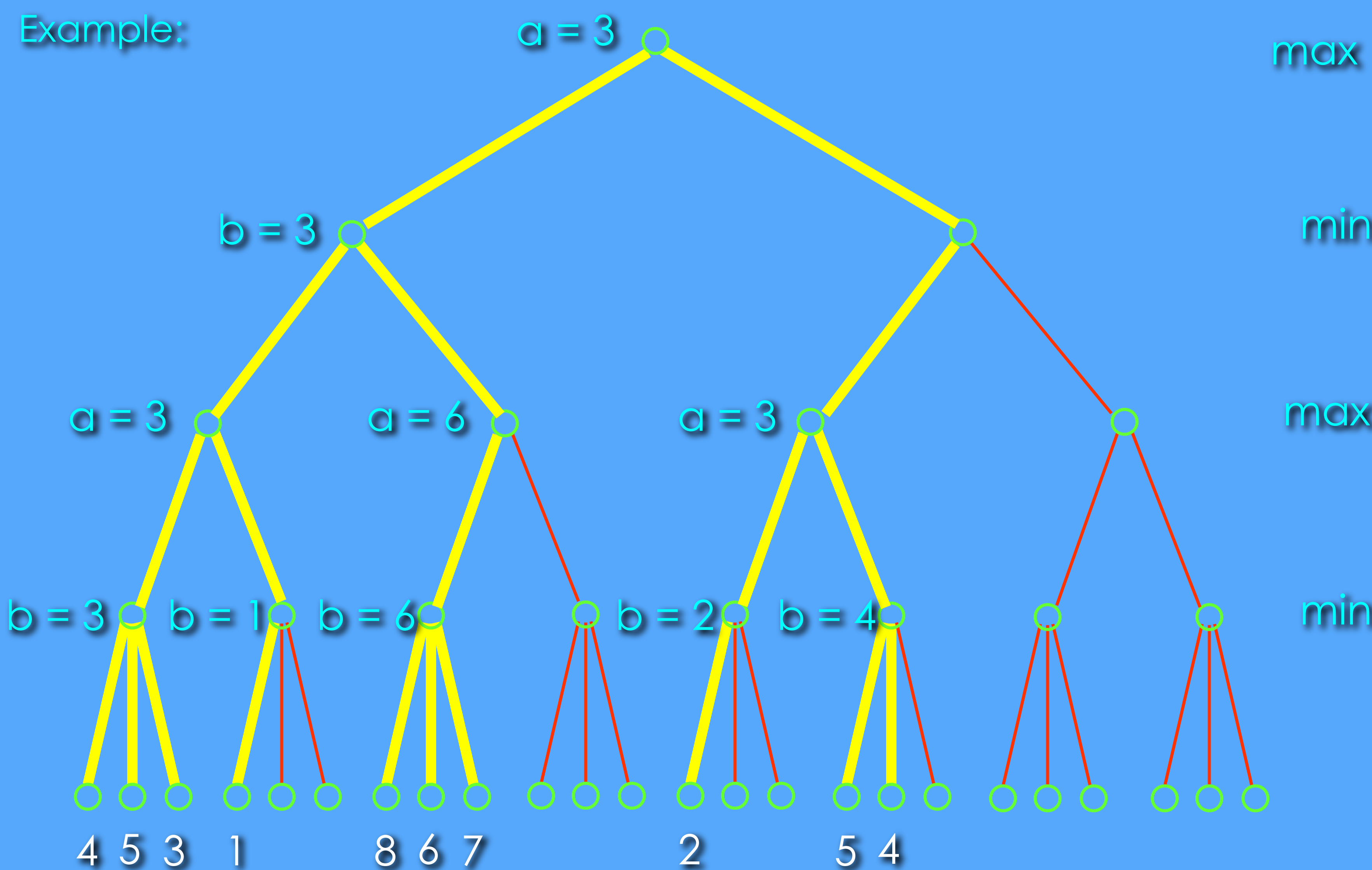

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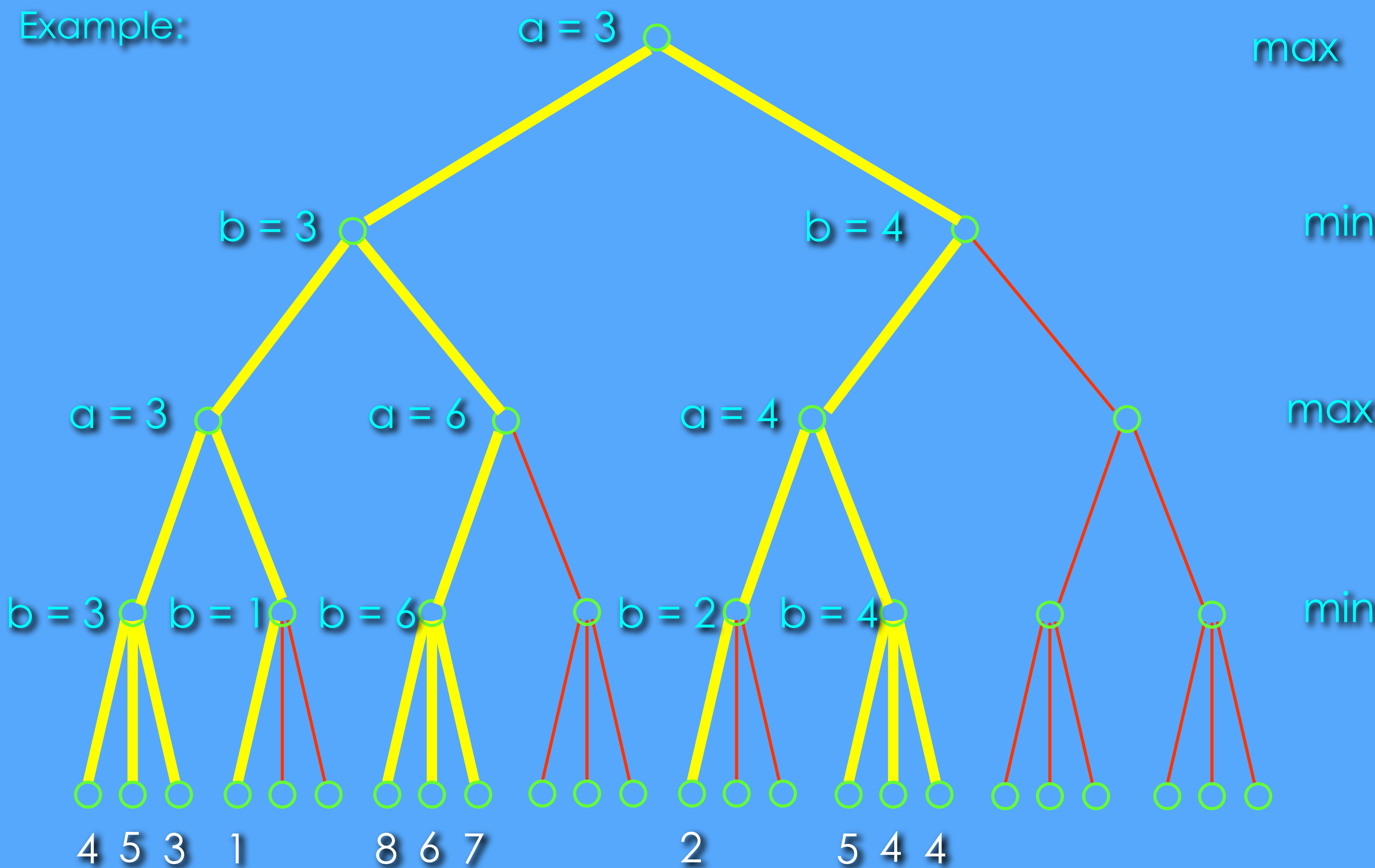

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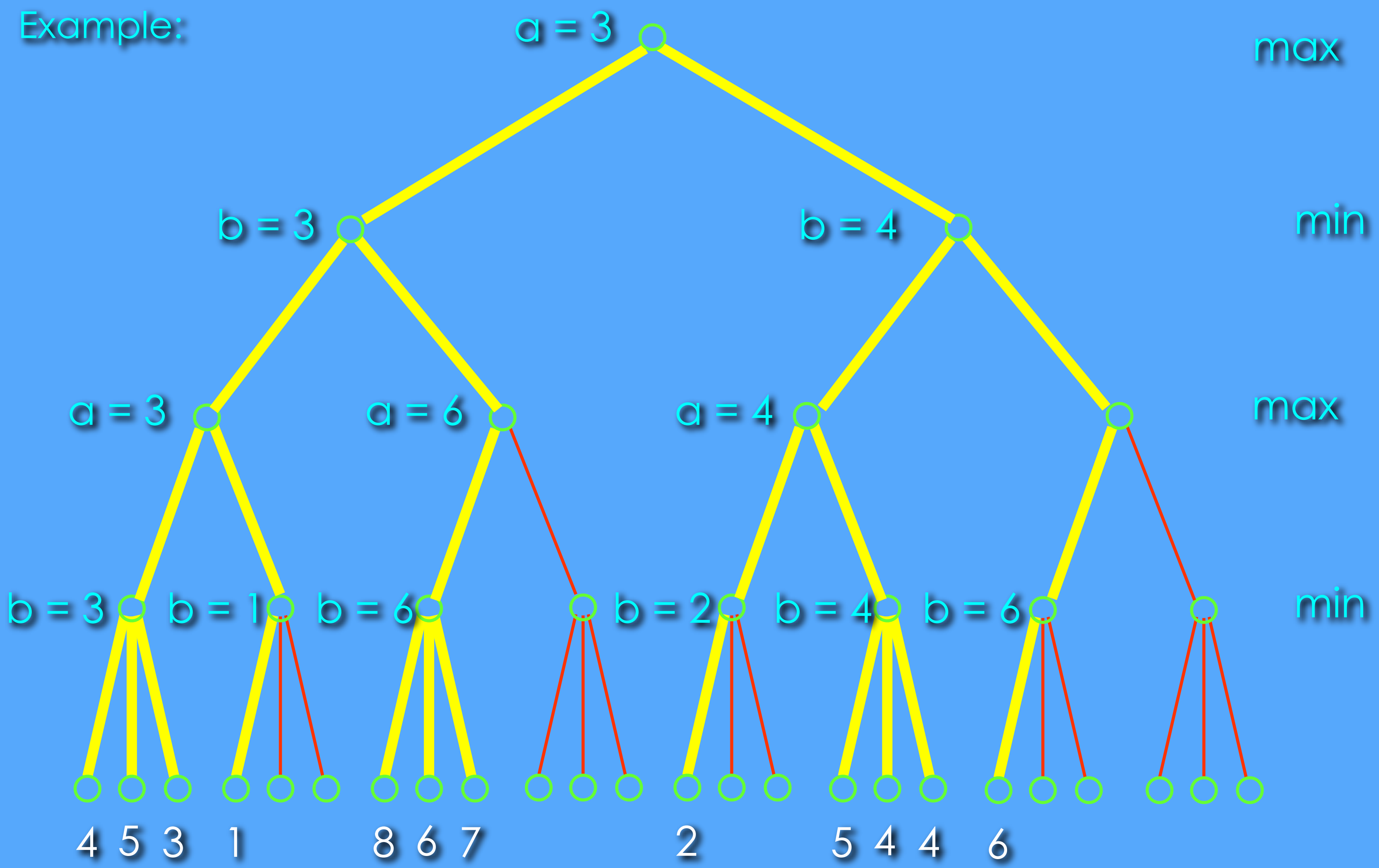

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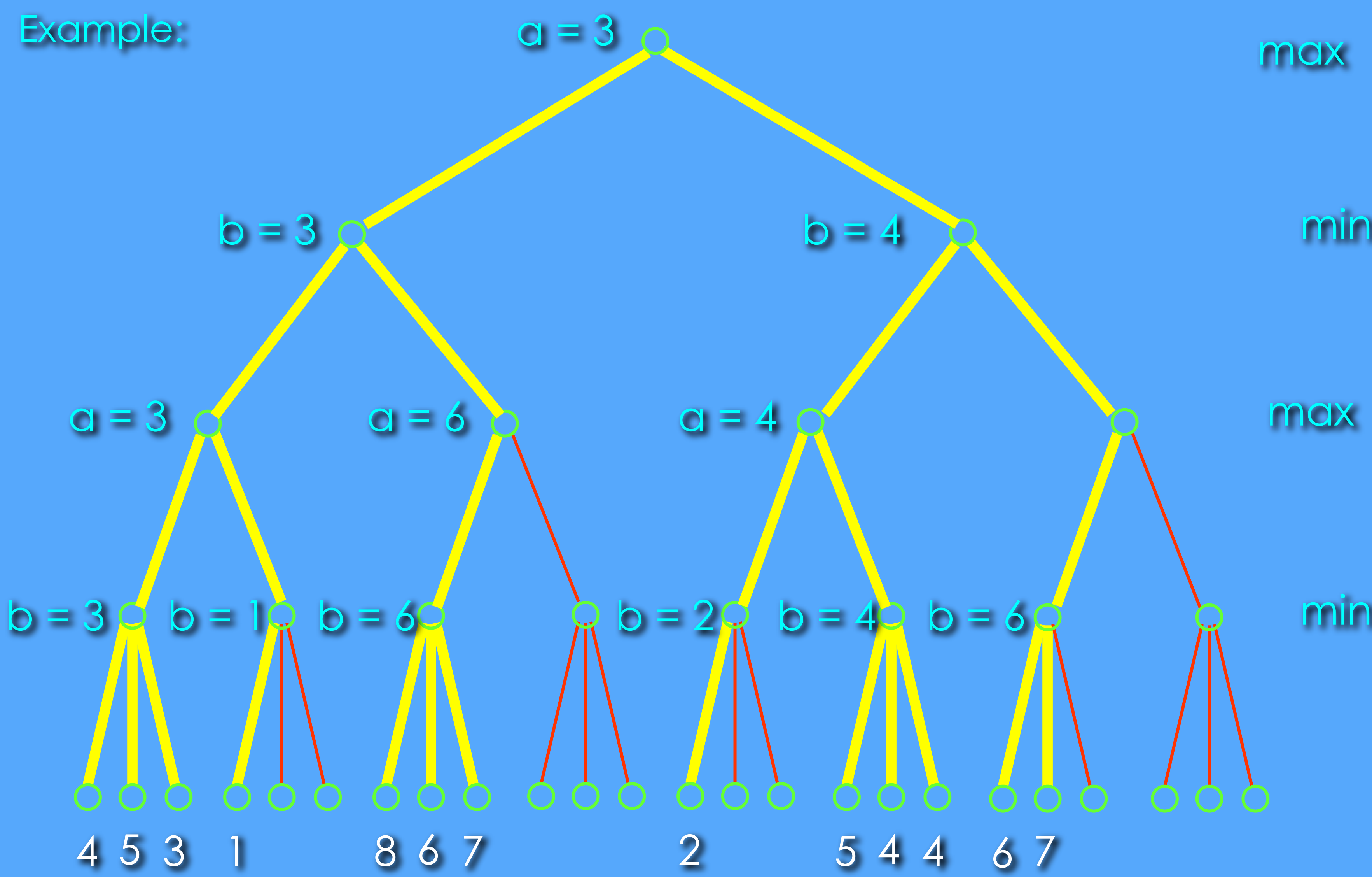

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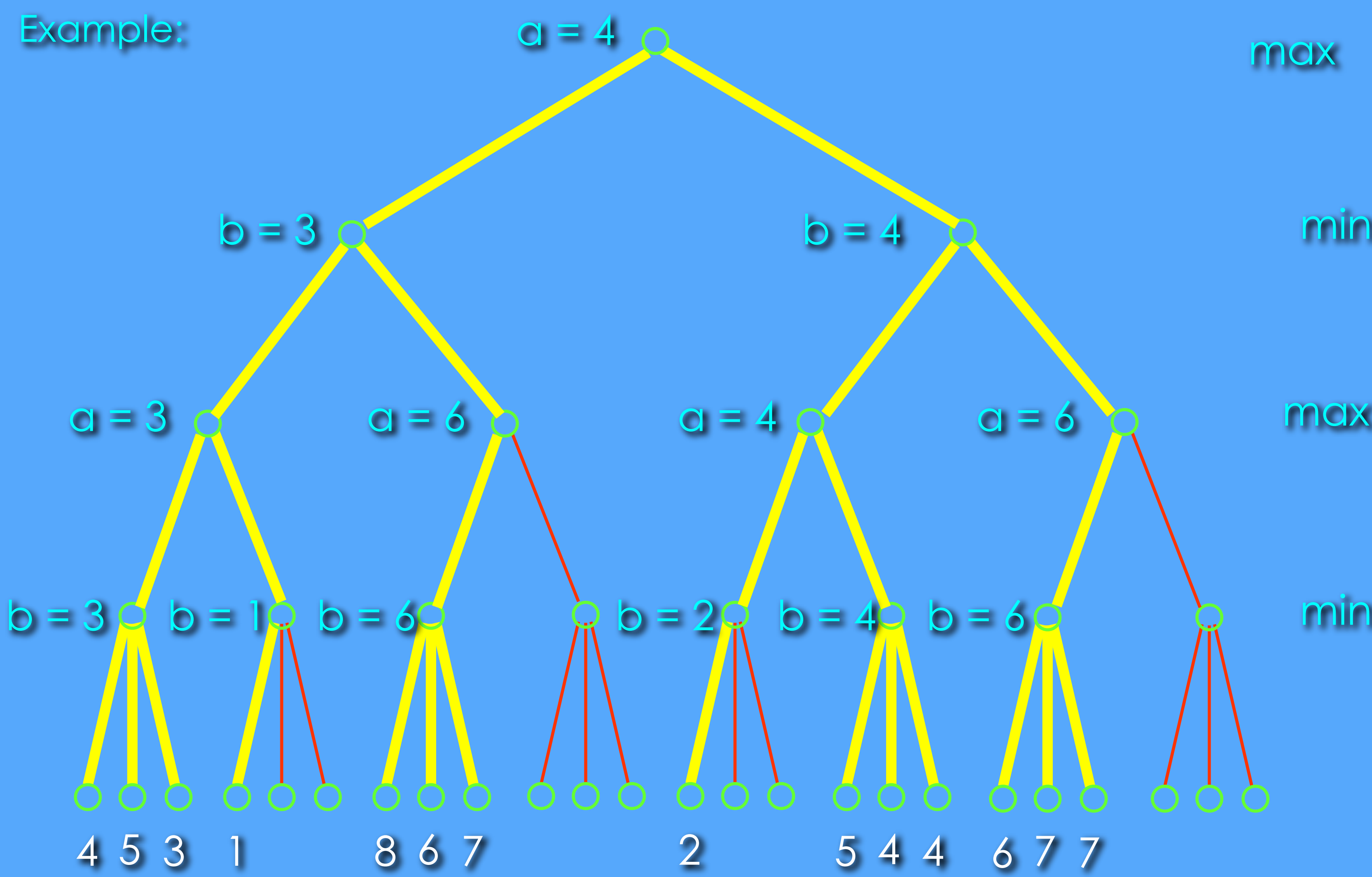

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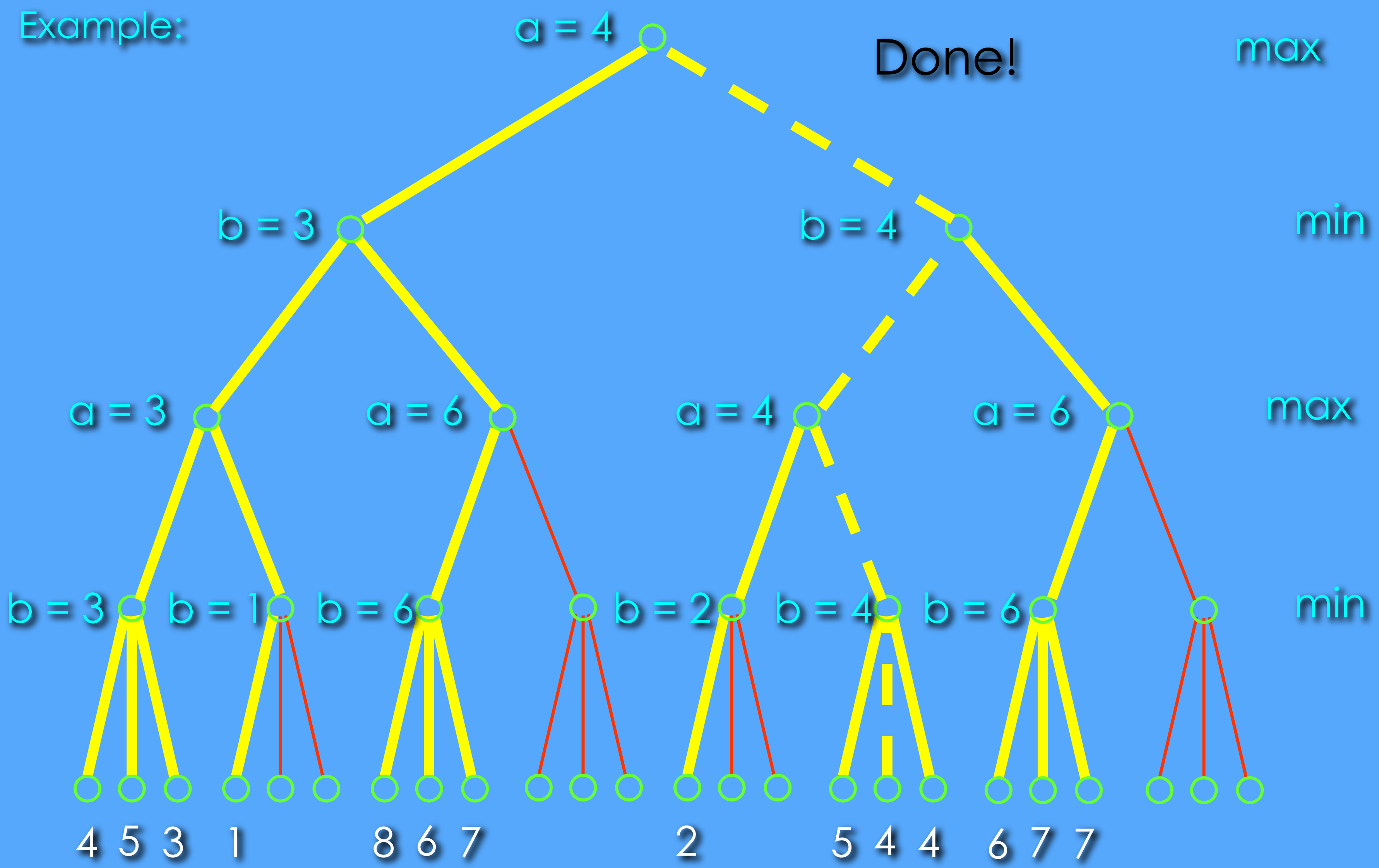
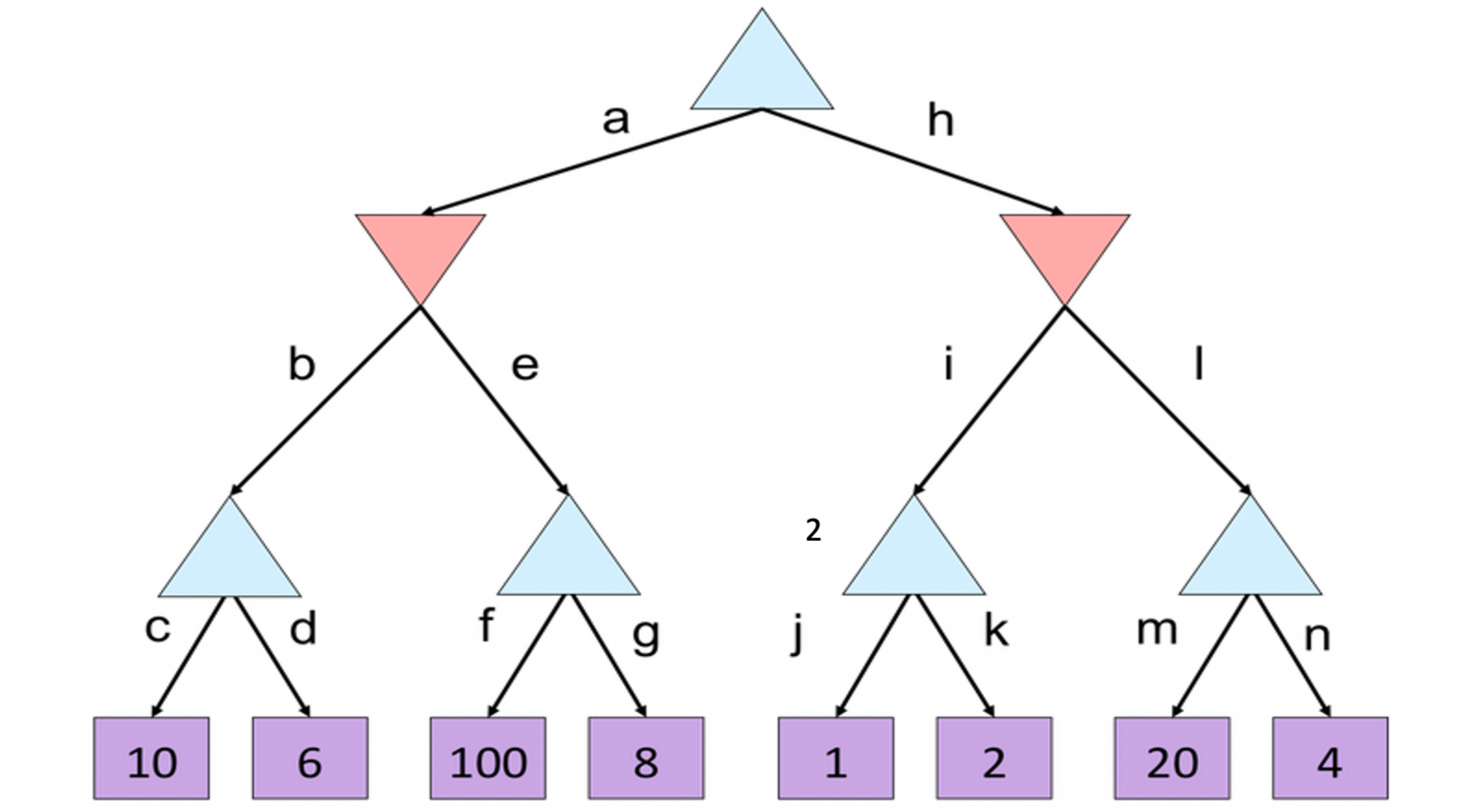


α-β pruning quiz



Properties of α-β pruning

- Pruning does not affect final result (it is exact).
- Good move ordering improves effectiveness of pruning (see last branch in example).
- The values at intermediate nodes may not be the same as the values computed by the minmax algorithm.

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- Then our search tree has **b^d leaves**.

- If we do not use alpha-beta pruning, we would have to apply the static evaluation function $N_d = b^d$ times.

The Alpha-Beta Procedure

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- if we assume that new children of a node are explored in the "most beneficial" order - those nodes p are explored first that will yield maximum values e(p) at depth d for MAX and minimum values for MIN - the number of nodes to be evaluated is:

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- This is ***continued*** until the time runs out. Then the currently remembered best move is executed.

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- Then e(p) is simply computed as the **scalar product** of f(p) and w(p).
- Both the identification of the most relevant features and the correct estimation of their relative importance are **crucial** for the strength of a game-playing program.

Evaluation function for chess

- For games like chess, typically linear weighted sum of features
- $Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$

e.g. $w_1 = 9$ with

- $f_1(s)$ = (number of white queens) – (number of black queens), etc.

- Weights 9 for queen, 5 for rook, 3 for bishop and knight and 1 for pawn – suggested by Shannon is still widely used.

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Heuristics and Game Tree Search

- How to determine the depth of evaluation? (clearly the time or other resource is one guide.)

- The Horizon Effect
  - sometimes there’s a major “effect” (such as a piece being captured) which is just “below” the depth to which the tree has been expanded
  - the computer cannot see that this major event could happen
  - it has a “limited horizon”
  - there are heuristics to try to follow certain branches more deeply to detect to such important events.
  - this helps to avoid catastrophic losses due to “short-sightedness”

Heuristics and Game Tree Search

Heuristics for Tree Exploration

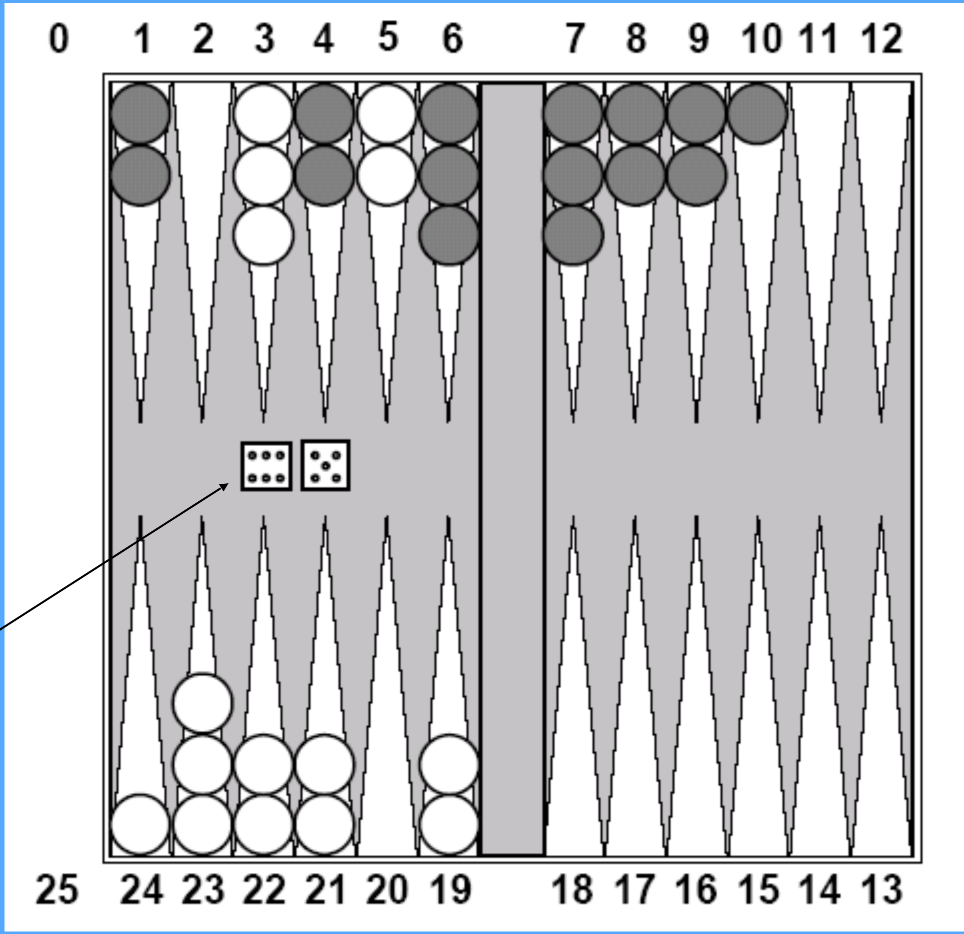
- it may be better to explore some branches more deeply in the allotted time (forward pruning)
- various heuristics exist to identify "promising" branches
  - expand to some depth k, and rank the nodes by the static evaluation function and choose the best m nodes to expand deeper.
  - for each node, determine the uncertainty of the evaluation function and go deeper on the most uncertain ones.
  - determine if a position is "active" or "passive". For active nodes, explore deeper.

Forward Pruning & Lookup tables

- Humans don’t consider all possible moves.
- Can we prune certain branches immediately?
- Use estimates (from past experience) of the uncertainty in the estimate of the node’s value and uses that to decide if a node can be pruned.
- Instead of search one can also store game states.
- Openings in chess are played from a library
- Endgames have often been solved and stored as well.

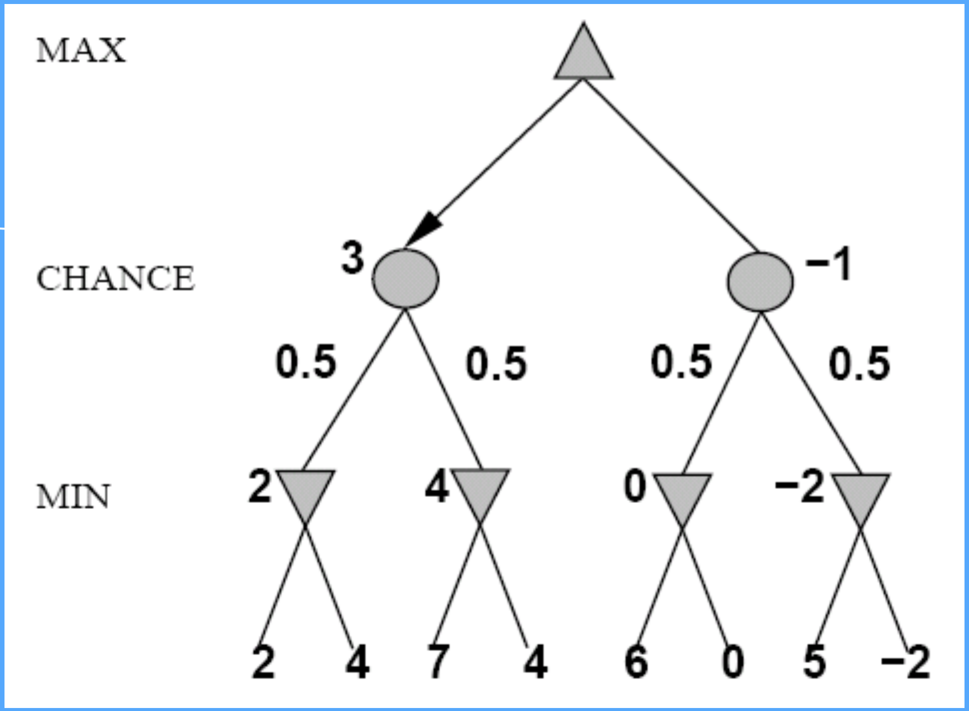
Chance Games

Backgammon



element of chance

Expected Minimax (Expectimax)



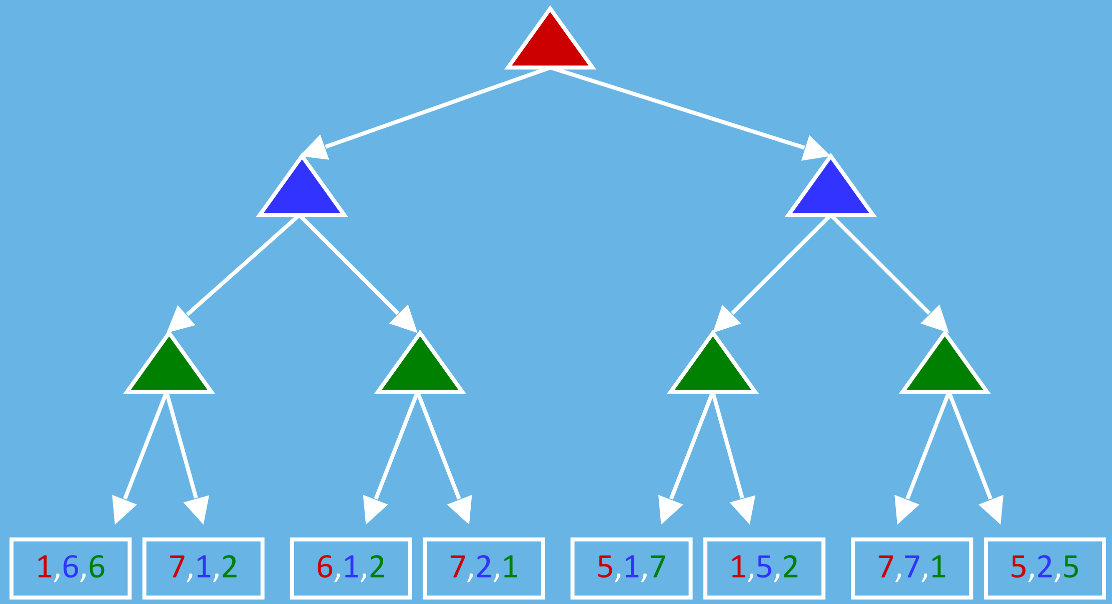
Again, the tree is constructed bottom-up.

Now we have even more nodes to search!

Multi-Agent Utilities

- What if the game is not zero-sum, or has multiple players?

- Generalization of minimax:
  - Terminals have utility tuples
  - Node values are also utility tuples
  - Each player maximizes its own component
  - Can give rise to cooperation and
  - competition dynamically…



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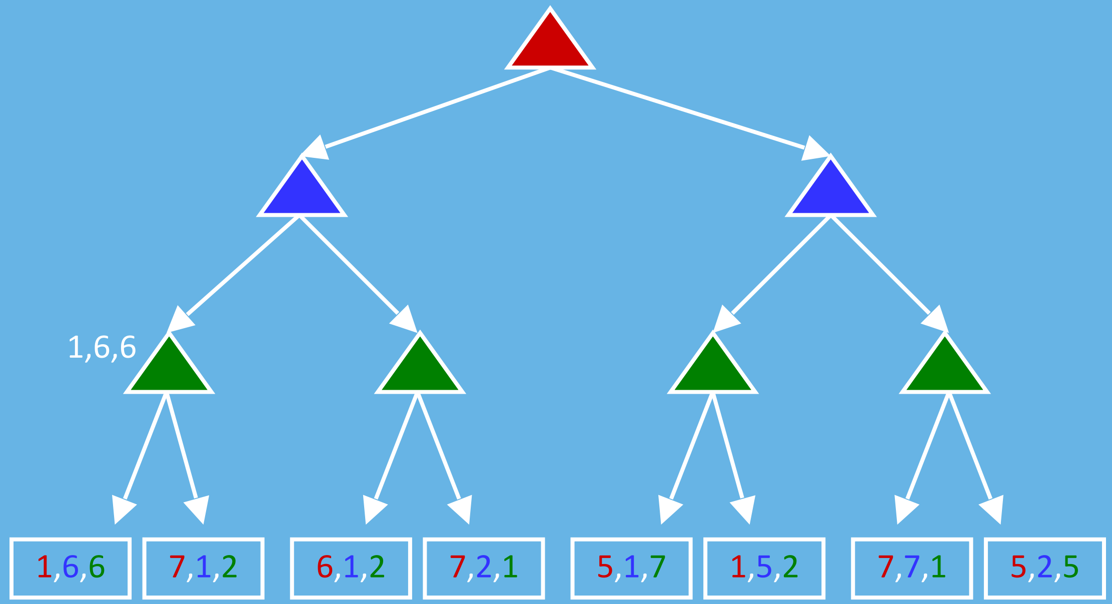

Summary

- Game playing is best modeled as a search problem
- Game trees represent alternate computer/opponent moves
- Evaluation functions estimate the quality of a given board configuration for the Max player.
- Minmax is a procedure which chooses moves by assuming that the opponent will always choose the move which is best for them
- Alpha-Beta is a procedure which can prune large parts of the search tree and allow search to go deeper
- For many well-known games, computer algorithms based on heuristic search match or out-perform human world experts.