Using Conic Correspondences in Two Images to Estimate the Epipolar Geometry

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Abstract

In this paper it is shown how corresponding conics in two images can be used to estimate the epipolar geometry in terms of the fundamental/essential matrix. The corresponding conics can be images of either planar conics or silhouettes of quadrics. It is shown that one conic correspondence gives two independent constraints on the fundamental matrix and a method to estimate the fundamental matrix from at least four corresponding conics is presented. Furthermore, a new type of fundamental matrix for describing conic correspondences is introduced. Finally, it is shown that the problem of estimating the fundamental matrix from 5 *point correspondences and* 1 *conic correspondence in general has* 10 *different solutions. A method to calculate these solutions is also given together with an experimental validation.*

1 Introduction

During the last years there has been an intense research on multiple view geometry, especially concentrated on using point correspondences to estimate either the epipolar or the trifocal geometry, see [4, 8, 20, 7]. Recently, attention has also turned to the use of other features, such as lines, conics, general curves or even silhouettes of three-dimensional bodies, see [7, 17, 6, 2, 1].

For points the geometric situation is fairly well understood. The uncalibrated case can be found in e.g. [20, 8, 3] and the calibrated case in e.g. [15]. For example, there exist 10 solutions to the problem of estimating the essential matrix from 5 point correspondences in 2 views. These solutions are obtained as the roots of a tenth degree polynomial equation, which in turn is obtained from the so called Kruppa equations, see [5, 12, 15, 9].

The next natural step is to use line correspondences. However, at least three images are needed to obtain constraints on the viewing geometry, see [21, 7]. It is also possible, but much more difficult, to use general space curves to estimate the epipolar geometry, see [16, 2]. The drawback of these methods is that they are computationally expensive and in [16], higher order image derivatives have to be calculated. An even more general situation is to use images of silhouettes of three-dimensional bodies. This approach involves complicated geometrical considerations, see [10, 1, 11].

In [13, 17] conic correspondences were used to reconstruct a conic in space when the epipolar geometry is known. In this case the object in space is assumed to be a degenerate quadric, i.e. a planar conic. When the object in space is a non-degenerate quadric, three images are needed to do reconstruction, see [14]. Observe that none of these methods use conic correspondences to calculate the epipolar geometry. One such attempt was made in [18], but no results are presented.

One motivation for the use of conics is that many man-made objects are built up by conics. It has also been indicated that it is possible to reconstruct general curves by fitting conics to the curves and use conic correspondences to estimate the epipolar geometry, see [18]. Finally, the image of the absolute conic plays a fundamental role in the calibrated case, which also motivates the use of conics.

The purpose of this paper is to use corresponding conics in two images to calculate the epipolar geometry, represented by the fundamental matrix in the uncalibrated case and the essential matrix in the calibrated case. Furthermore, it is not assumed that the object in space is a planar conic, but instead a general quadric. We will show that each conic correspondence gives two independent constraints on the fundamental/essential matrix. Moreover, the minimal case of 5 corresponding points and 1 conic in the uncalibrated

*Supported by the ESPRIT Reactive LTR project 21914, CUMULI

†Supported by the Swedish Research Council for Engineering Sciences (TFR), project 95-64-222

case is shown to have 10 different solutions. A by-product of this result is a new proof of the Kruppa-Demazure theorem, saying that there are 10 different solutions to the essential matrix, given 5 corresponding points in 2 views. We will also show that given 3 or more corresponding conics in the calibrated case and 4 or more conics in the uncalibrated case, it is possible to calculate the epipolar geometry. In fact, there exists a correspondence between n conics in two uncalibrated views and $n-1$ conics in two calibrated views. The results are illustrated by examples.

2 Review of Projective Geometry

Before we start our analysis of conic correspondences, we will recapitulate some of the basic concepts in projective geometry, especially concerning conics and quadrics. For further details, see [19].

The camera is modeled by

$$\lambda \mathbf{x} = P\mathbf{X}, \quad \lambda \neq 0 \ , \tag{1}$$

where P denotes the standard 3×4 camera matrix and λ a scale factor. Here $\mathbf{X} = [X\,Y\,Z\,1]^T$ and $\mathbf{x} = [x\,y\,1]^T$ denote homogeneous point coordinates in 3D space and in the image respectively. Given two cameras, defined by the camera matrices P_1 and P_2, and the images of a point $\mathbf{X}$ in space, i.e. $\lambda_1\mathbf{x_1} = P_1\mathbf{X}$ and $\lambda_2\mathbf{x_2} = P_2\mathbf{X}$, the two corresponding image-points, $\mathbf{x_1}$ and $\mathbf{x_2}$ fulfil the epipolar constraint,

$$\mathbf{x_1}^T F \mathbf{x_2} = 0 \ ,$$

where F denotes the fundamental matrix. The fundamental matrix has seven degrees of freedom, since it is a singular matrix containing 9 homogeneous parameters.

A general *conic curve* or *conic locus* in the plane is defined by the quadratic form

$$\mathbf{x}^T c \mathbf{x} = 0 \ , \tag{2}$$

where c denotes a 3×3 symmetric matrix and $\mathbf{x}$ denotes homogeneous point coordinates. The dual to a conic curve is a *conic envelope*, given by

$$\mathbf{u}^T l \mathbf{u} = 0 \ , \tag{3}$$

where l denotes a 3×3 symmetric matrix and $\mathbf{u} = [u\,v\,1]^T$ denotes homogeneous line coordinates. A conic, (2) or (3), is said to be *proper* if its matrix is nonsingular, otherwise it is said to be *degenerate*. For proper conics, the conic locus and the conic envelope are related by $c = l^{-1}$.

Analogously, a general *quadric surface* or *quadric locus* in space is defined by the quadratic form

$$\mathbf{X}^T C \mathbf{X} = 0 \ ,$$

where C denotes a 4×4 symmetric matrix and $\mathbf{X}$ denotes homogeneous point coordinates. The dual to a quadratic surface is a *quadric envelope*, given by

$$\mathbf{U}^T L \mathbf{U} = 0 \ , \tag{4}$$

where L denotes a 4×4 symmetric matrix and $\mathbf{U} = [U\,V\,W\,1]^T$ denotes homogeneous plane coordinates. For singular matrices the quadric locus degenerates to a cone or a plane pair, while the quadric envelope degenerates to a disk quadric or a point pair. A disk quadric is a conic lying in a plane in space.

The image, under a perspective projection (1), of a quadric is a conic. This relation is conveniently expressed using the envelope forms (3) and (4),

$$\lambda l = PLP^T, \quad \lambda \neq 0 \ ,$$

where λ is a scale factor.

Of special interest in projective geometry is the *absolute conic*. It is defined by the equations $X_1^2 + X_2^2 + X_3^2 = X_4 = 0$. It is a virtual conic, i.e. it contains no real points, positioned in the plane at infinity, $X_4 = 0$. Euclidean transformations in the projective space are exactly those transformations that leave the absolute conic invariant. Knowing the calibration of a camera is equivalent to knowing the image of the absolute conic.

3 Fundamental Matrices for Conics

It is well-known that the epipolar constraint for corresponding points in two images can be expressed by the fundamental matrix. In this section we will derive the constraints imposed by conic correspondences in two images. It turns out that these constraints can be encoded in a similar way as for points. We start by a fundamental lemma.

Lemma 3.1. *The projection of a quadric envelope,*

$$\lambda l = PLP^T$$

can be written

$$\lambda \tilde{l} = \tilde{P}\tilde{L} \ , \tag{5}$$

where $\tilde{l}$ and $\tilde{L}$ denote column vectors of length 6 and 10 obtained from stacking the elements in l and L, respectively, and $\tilde{P}$ is a 6×10 matrix. The entries of $\tilde{P}$ are quadric expressions in the entries of P.

Proof. $\lambda l = PLP^T$ can be written as the tensor product, $\lambda \hat{l} = (P \otimes P)\hat{L}$, where $\hat{l}$ and $\hat{L}$ denote the stacked columns of l and L. Since l and L are symmetric, symmetrisation of the tensor product gives (5). ∎

Now consider two different images of the same quadric, i.e. $\lambda_1 \tilde{l}_1 = \tilde{P}_1 \tilde{L}$ and $\lambda_2 \tilde{l}_2 = \tilde{P}_2 \tilde{L}$, which equivalently can be written

$$Mz = \begin{bmatrix} \tilde{P}_1 & \tilde{l}_1 & 0 \\ \tilde{P}_2 & 0 & \tilde{l}_2 \end{bmatrix} \begin{bmatrix} \tilde{L} \\ -\lambda_1 \\ -\lambda_2 \end{bmatrix} = 0 \ . \tag{6}$$

Definition 3.1. The matrix M defined in (6) will be called the **universal matrix for conics.** ■

Linear algebra tells us that if $\tilde{l}_1$ and $\tilde{l}_2$ are corresponding conics, the nullspace of M is non-empty, i.e. M is singular. This proves the following theorem.

Theorem 3.1. *For two corresponding conics, given by $\tilde{l}_1$ and $\tilde{l}_2$, the universal matrix obeys*

$$\operatorname{rank} M \leq 11 \ .$$

Choose affine coordinate systems in the first image and in space such that $P_1 = [\, I_{3\times 3} \ 0_{3\times 1}\,]$, gives $\tilde{P}_1 = [\, I_{6\times 6} \ 0_{6\times 4}\,]$, where the subscript indicates the size of the matrix. Using these coordinate systems and the notation $\tilde{P}_2 = [\, Q_2 \ R_2\,]$, we obtain

$$M = \begin{bmatrix} I_{6\times 6} & 0_{6\times 4} & \tilde{l}_1 & 0 \\ Q_2 & R_2 & 0 & \tilde{l}_2 \end{bmatrix} \ . \tag{7}$$

According to Theorem 3.1 M has rank less than or equal to 11 if $\tilde{l}_1$ and $\tilde{l}_2$ represent two corresponding conics. However, M has *always* rank 11 or less even if $\tilde{l}_1$ and $\tilde{l}_2$ do not represent corresponding conics. This is due to the fact that R_2 has rank 3, according to the following proposition.

Proposition 3.1. *Transforming a camera matrix P to $\tilde{P} = [\, Q \, R\,]$, defined by (5) in Lemma 3.1, gives*

$$\operatorname{rank} R \leq 3 \ .$$

Proof. Let $\mathbf{e}_i$, $i = 1, ..., 10$ and $\mathbf{n}_j$, $j = 1, ..., 4$ denote the standard bases in $\mathbb{R}^{10}$ and $\mathbb{R}^4$ respectively and let f denote the projection function, cf. (5),

$$f(\tilde{L}) = PLP^T \ .$$

Since the camera matrix $P = [\, \mathbf{p}_1 \, \mathbf{p}_2 \, \mathbf{p}_3 \, \mathbf{p}_4\,]$ is always of rank 3 or less, we can write one column as a linear combination of the others, e.g. $\mathbf{p}_1 = \alpha \mathbf{p}_2 + \beta \mathbf{p}_3 + \gamma \mathbf{p}_4$. Then,

$$\begin{aligned} f(\mathbf{e}_7) &= P(\mathbf{n}_4 \mathbf{n}_1^T + \mathbf{n}_1 \mathbf{n}_4^T) P^T = \mathbf{p}_4 \mathbf{p}_1^T + \mathbf{p}_1 \mathbf{p}_4^T \\ &= \alpha(\mathbf{p}_4 \mathbf{p}_2^T + \mathbf{p}_2 \mathbf{p}_4^T) + \beta(\mathbf{p}_4 \mathbf{p}_3^T + \mathbf{p}_3 \mathbf{p}_4^T) + 2\gamma \mathbf{p}_4 \mathbf{p}_4^T \\ &= \alpha f(\mathbf{e}_8) + \beta f(\mathbf{e}_9) + 2\gamma f(\mathbf{e}_{10}) \ . \end{aligned}$$

Thus, the images of $\mathbf{e}_7$, $\mathbf{e}_8$, $\mathbf{e}_9$ and $\mathbf{e}_{10}$ are linearly dependent and this asserts that $\operatorname{rank} R \leq 3$. ■

The proposition implies that the nullspace of M, that always exists, has the form $[\, 0\,0\,0\,0\,0\,0 \times \times \times \times 0\,0\,]^T$. This is an uninteresting solution to (6) since $\lambda_1 = \lambda_2 = 0$. Thus, we have the following sharpening of Theorem 3.1, following from the discussion above.

Theorem 3.2. *If $\tilde{l}_1$ and $\tilde{l}_2$ represent images of the same quadric, then the nullspace of the universal matrix for conics, M, must be of dimension at least 2, i.e. $\operatorname{rank} M \leq 10$.*

After operations on the rows of M we obtain the 6×6 matrix

$$\begin{bmatrix} R_2 & -Q_2 \tilde{l}_1 & \tilde{l}_2 \end{bmatrix} \ , \tag{8}$$

which according to the previous theorem has rank less than or equal to 4 since it has a nontrivial nullspace. Since $\operatorname{rank} R_2 \leq 3$ and in general no three of the four columns has rank 2 or less, we can eliminate one of the columns in R_2, say the first one.

Definition 3.2. Denote the four columns in R_2 by c_1, c_2, c_3 and c_4. The matrix

$$M_r = \begin{bmatrix} c_2 & c_3 & c_4 & -Q_2 \tilde{l}_1 & \tilde{l}_2 \end{bmatrix} \tag{9}$$

will be called the **reduced universal matrix for conics.** ■

Corollary 3.1. *If $\tilde{l}_1$ and $\tilde{l}_2$ represent images of the same quadric*

$$\operatorname{rank} M_r \leq 4 \ .$$

The inequality means that all 5×5 subdeterminants of M_r vanish. This leads to the following theorem.

Theorem 3.3. *Let $\tilde{l}_1$ and $\tilde{l}_2$ denote corresponding conics and $\tilde{P}_1 = [\, I \, 0\,]$ and $\tilde{P}_2 = [\, Q_2 \, R_2\,]$ the matrices obtained from Lemma 3.1 by choosing coordinate systems such that $P_1 = [\, I \, 0\,]$. Then the constraints imposed by the correspondence can be written*

$$\tilde{l}_1^T F_i \tilde{l}_2 = 0, \quad i = 1, 2, ..., 6 \ . \tag{10}$$

Proof. There are a total of six 5×5 subdeterminants of M_r that equal zero. Since these determinants are bilinear in $\tilde{l}_1$ and $\tilde{l}_2$, the equations in (10) follow. ■

This formulation is very similar to the point case. Therefore we introduce:

Definition 3.3. The matrices F_i in (10) will be called **the fundamental matrices for conics.** ■

The ordinary fundamental matrix has a one-dimensional nullspace. The dimension of the null-spaces of the fundamental matrices for conics is even higher. In fact, we have the following somewhat surprising result.

Theorem 3.4. *All six fundamental matrices for conics, F_i, have rank ≤ 2. There are three such linearly independent matrices (in the entries in F_i). However, there are only two algebraic independent constraints, in the sense that the algebraic manifold defined by the 6 equations in* (10) *has dimension* 4.

Proof. Use Maple or some other computer algebra system to find the rank of the fundamental matrices for conics and the number of linearly independent constraints. The last statement, that there are only two algebraic independent is shown by calculating resultants of the polynomial equations in (10). ∎

The fact that each conic correspondence yields two constraints on the epipolar geometry has important implications. First of all, we can conclude that we need at least 4 conic correspondences in order to calculate the epipolar geometry. Then, some minimal cases arise that are of theoretical interest, e.g. one conic and five points. This case is treated in the next section.

4 5 Points and 1 Conic

We consider the minimal case of 5 corresponding points and 1 corresponding conic in two images. In detail, we want to find the number of possible reconstructions given the image coordinates of the corresponding points and the conic. The derivation has been made using Maple.

In the generic case, we can always choose an affine base in space such that the first 4 points have homogeneous coordinates $[1\,0\,0\,0]^T$, $[0\,1\,0\,0]^T$, $[0\,0\,1\,0]^T$ and $[0\,0\,0\,1]^T$ and a projective base in each image such that these points are projected to $[1\,0\,0]^T$, $[0\,1\,0]^T$, $[0\,0\,1]^T$ and $[1\,1\,1]^T$. Furthermore the plane at infinity in space can be chosen such that all scale factors λ in the first image are equal to 1. This projective reduction gives the following camera matrices, cf. [3],

$$P_1 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \qquad P_2 = \begin{bmatrix} p_1 & 0 & 0 & p_4 \\ 0 & p_2 & 0 & p_4 \\ 0 & 0 & p_3 & p_4 \end{bmatrix}$$

Let $[\,a_1\,a_2\,a_3]^T$ and $[\,b_1\,b_2\,b_3]^T$ denote the image coordinates of the fifth point in image 1 and image 2, respectively, and let $[\,X\,Y\,Z\,W\,]^T$ and λ denote the unknown coordinates of the last point and its scale factor in image 2, respectively. Finally, let $\tilde{l}_1$ and $\tilde{l}_2$ denote the image coordinates of the conic envelope in image 1 and image 2 respectively. The objective is to determine the unknown variables p_1, p_2, p_3, p_4, X, Y, Z, W and λ. If we can do that, we have a reconstruction. Using (1) for the last point in image 1

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = P_1 \begin{bmatrix} X \\ Y \\ Z \\ W \end{bmatrix} \Leftrightarrow \begin{cases} X = a_1 - W \\ Y = a_2 - W \\ Z = a_3 - W \end{cases} \tag{11}$$

Repeating the same procedure for image 2 gives

$$\lambda \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = P_2 \begin{bmatrix} X \\ Y \\ Z \\ W \end{bmatrix} \Leftrightarrow \begin{cases} p_1 = \frac{\lambda b_1 - p_4 W}{a_1 - W} \\ p_2 = \frac{\lambda b_2 - p_4 W}{a_2 - W} \\ p_3 = \frac{\lambda b_3 - p_4 W}{a_3 - W} \end{cases} \tag{12}$$

Here we have assumed that $W \neq a_1$, $W \neq a_2$ and $W \neq a_3$, which is in general true. Let us now turn to the six epipolar constraints for conics (10). Insert X, Y and Z from (11) and p_1, p_2 and p_3 from (12). This leaves us with the unknown variables p_4, W and λ. The elimination of p_1, p_2 and p_3 introduced fractions, so we start by extending the equations in order to obtain polynomials. This results in 6 different homogeneous polynomial equations of degree 5 in p_4 and λ, with coefficients that are polynomial expressions in W of degree 4. Dividing these equations by λ^5, we can substitute $\alpha = p_4/\lambda$ and 2 variables remain, W and α. According to Theorem 3.4, the 6 equations give two independent constraints, and thus it is possible to determine the two unknown variables. Taking the resultant of two of the polynomial equations in W and α with respect to α, we get a new equation in the remaining unknown W. Repeating the same procedure for every pair out of the six equations we get in total 15 polynomial equations in W. The common solutions to these equations are the only solutions to the six epipolar constraints for conics in (10). Factorising the polynomial equations, we see that the common solutions are obtained from the following polynomial equation,

$$q(W)(W - a_1)(W - a_2)(W - a_3) = 0 \ , \tag{13}$$

where $q(W)$ denotes a 10:th degree polynomial expression in W. The last three factors are spurious solutions, due to the substitutions in (12). However, the tenth degree factor, $q(W)$, corresponds to true solutions. Some of the roots may be complex and some may be discarded if, for example, a point is behind the camera. A numerical example is presented in Section 5.2.

Theorem 4.1. *The problem of estimating the epipolar geometry in the uncalibrated case between two views, given 5 point correspondences and 1 conic correspondence, in general has 10 different solutions, obtained as the roots of a tenth degree polynomial equation.*

If we choose the corresponding conics in the above theorem as the images of the absolute conic, which is equivalent to calibrated cameras, we get the following well-known result, cf. [5].

Corollary 4.1. *The problem of estimating the epipolar geometry with calibrated cameras between two views, given 5 point correspondences, in general has 10 different solutions, obtained as the roots of a tenth degree polynomial equation.*

The problem of estimating the epipolar geometry in the calibrated case from n conic correspondences is similar to the problem of estimating the epipolar geometry in the uncalibrated case using $n+1$ conic correspondences. This can again be seen by choosing one conic as the image of the absolute conic.

5 Experimental Validations

In this section, we present some experimental results on the theory of conic correspondences. The experiments have been performed on synthetic data.

5.1 More than 3 conics

For convenience, we have parametrised the epipolar geometry using the fundamental matrix as follows,

$$F = \begin{bmatrix} e_3d_1 & e_3d_2 & e_3d_3 \\ e_3d_4 & e_3d_5 & e_3d_6 \\ -e_1d_1-e_2d_4 & -e_1d_2-e_2d_5 & -e_1d_3-e_2d_6 \end{bmatrix} .$$

This parametrisation has only 9 variables and F is automatically rank 2 with the left epipole $\mathbf{e} = [\,e_1\, e_2\, e_3]^T$. In fact the variables are grouped into two different sets, each homogeneous, which makes the number of variables equal to 7, when the homogeneity is taken into account. One possible choice of camera matrices is then $P_1 = [\,S\,\mathbf{e}\,]$ and $P_2 = [\,I\,0\,]$, where

$$S = -\frac{1}{||\mathbf{e}||^2}\begin{bmatrix} 0 & -e_3 & e_2 \\ e_3 & 0 & -e_1 \\ -e_2 & e_1 & 0 \end{bmatrix} F \ .$$

The epipolar geometry was estimated using the epipolar constraints for conics (10) as *residuals*. To avoid the trivial solution, the constraints $||\mathbf{e}|| = 1$ and $||F|| = 1$ were added. A Gauss-Newton technique was used to minimise the residuals. The simulated data consisted of randomly distributed quadrics in space, and randomly chosen views. All data was rescaled to lie in the range $[-1, 1]$ to improve numerical conditioning.

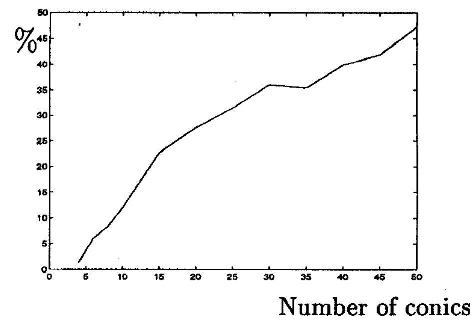

Figure 1: Convergence rate of epipolar recovery vs. number of conics.

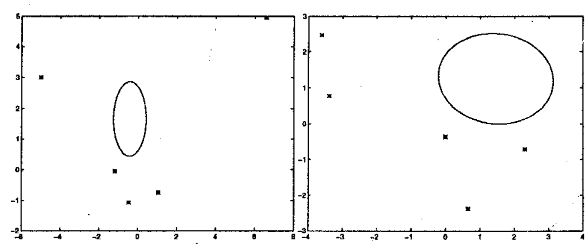

Figure 2: Two images of 5 points and 1 quadric.

We performed some experiments with different numbers of conics, starting with 4 conics, since 3 is not sufficient. The initial estimate needed in order to start the minimisation was randomly chosen. In Figure 1 it is shown how many percent of the experiments converged to the correct minimum. The experiments show that a good initial estimate is absolutely necessary. The situation will, of course, be even worse when the data is perturbated with noise.

5.2 5 points and 1 conic

The experiment was carried out in the following way. 5 points and 1 quadric were randomly distributed in space. These primitives were then projected onto two images with randomly chosen camera matrices. The resulting images are shown in Figure 2.

The polynomial $q(W)$ in (13) was solved numerically with Maple, yielding 4 real and 6 complex solutions. The other variables are obtained by back-substitution. The camera matrices for one of the real solution correspond indeed to the original camera matrices.

6 Conclusions

In this paper, we have shown that conic correspondences in two views can be used to calculate the epipolar geometry. In the calibrated case, the essential matrix can be calculated from at least three conic correspondences and in the uncalibrated case the fundamental matrix can be calculated from at least four conic correspondences. It has furthermore been shown that each conic correspondence gives two independent conditions on the essential/fundamental matrix. These conditions can conveniently be expressed by the introduced new *fundamental matrices for conics*. These six fundamental matrices for conics express the epipolar constraint in the case of corresponding conics in the same way as the ordinary fundamental matrix expresses the epipolar constraint for point correspondences.

The minimal case of two uncalibrated views, with 5 point correspondences and 1 conic correspondence has been solved using the previously described methods. It turns out that there exist 10 different solutions to this problem, given as the solutions to a 10:th degree polynomial equation. However, some may be complex and some not physically realisable.

Acknowledgements

The authors thank Kalle Åström and Sven Spanne for interesting discussions and valuable comments.

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