

Homework 1

p24 B.5 and B.6, p29 A.1 and A.2, p39 A.1 and A.2, p40 C.3 and C.5

1. **p24 B.5.** $x * y = xy + 1$

- **Commutative** Yes
- **Associative** Yes
- **Identity** Yes
- **Inverses** No

2. **p24 B.6.** $x * y = \max\{x, y\}$ = the larger of the two numbers x and y

- **Commutative** Yes
- **Associative** Yes
- **Identity** Yes
- **Inverses** No

3. **p29 A.1.** Prove the following set is an abelian group:

$x * y = x + y + k$ (k is a fixed constant), on the set \mathbb{R} of the real numbers

Solution.

Let $x, y \in \mathbb{R}$ be real numbers. Forms in $*$ are **commutative** iff $x * y = y * x$.

$$x * y = x + y + k$$

$$y * x = y + x + k$$

$$x + y + k = y + x + k$$

Based upon the commutative nature of addition in the real numbers the above equations are equal \square

Let $x, y, z \in \mathbb{R}$ be real numbers. Forms in $*$ are **associative** iff $(x * y) * z = y * (x * z)$.

$$(x * y) * z = (x + y + k) + z + k$$

$$x * (y * z) = x + (y + z + k) + k$$

$$x + y + k = y + x + k$$

Based upon the commutative nature of addition in the real numbers the above equations are equal \square

Let $x, e \in \mathbb{R}$ be real numbers. Forms in $*$ have an **identify** iff $x * e = x$.
 $x * e = x + e + k = x$ is true for all x with $e = -k$ \square

Let $x, x^{-1} \in \mathbb{R}$ be real numbers. Forms in $*$ have an **inverse** iff $x * x^{-1} = 1$.
 $x * x^{-1} = x + x^{-1} + k = 1$ is true for all x with $x^{-1} = -x - k + 1$ \square

Thus, because the group is true for commutative, associative, identify, and inverse, it is an abelian group. \square

4. **p29 A.2.** Prove the following set is an abelian group:

$x * y = \frac{xy}{2}$ on the set $\{x \in \mathbb{R} : x \neq 0\}$

Solution.

Let $x, y \in \{x \in \mathbb{R} : x \neq 0\}$ be real numbers. Forms in $*$ are **commutative** iff $x * y = y * x$.

$$x * y = \frac{xy}{2}$$

$$y * x = \frac{yx}{2}$$

$$\frac{xy}{2} = \frac{yx}{2}$$

Based upon the commutative nature of multiplication in the real numbers the above equations are equal \square

Let $x, y, z \in \{x \in \mathbb{R} : x \neq 0\}$ be real numbers. Forms in $*$ are **associative** iff $(x * y) * z = y * (x * z)$.

$$(x * y) * z = \frac{(\frac{xy}{2})z}{2}$$

$$x * (y * z) = \frac{x(\frac{yz}{2})}{2}$$

$$\frac{(\frac{xy}{2})z}{2} = \frac{x(\frac{yz}{2})}{2}$$

Based upon the commutative nature of multiplication in the real numbers the above equations are equal \square

Let $x, e \in \{x \in \mathbb{R} : x \neq 0\}$ be real numbers. Forms in $*$ have an **identify** iff $x * e = x$.
 $x * e = \frac{xe}{2} = x$ is true for all x with $e = 2$

Let $x, x^{-1} \in \{x \in \mathbb{R} : x \neq 0\}$ be real numbers. Forms in $*$ have an **inverse** iff $x * x^{-1} = 1$.

$$x * x^{-1} = \frac{xx^{-1}}{2} = 1 \text{ is true for all } x \text{ with } x^{-1} = 2/x \quad \square$$

Thus, because the group is true for commutative, associative, identify, and inverse, it is an abelian group. \square

5. **p39 A.1.** Solve in terms of a , b , and c :

$$axb = c$$

Solution.

$$a = c/bx$$

$$b = c/ax$$

$$c = abx$$

6. **p39 A.2.** Solve in terms of a , b , and c :

$$x^2b = xa^{-1}c$$

Solution.

$$a = \frac{xc}{x^2b}$$

$$b = \frac{xa^{-1}c}{x^2}$$

$$c = \frac{x^2b}{xa^{-1}}$$

7. **p40 C.3.** Assuming that a and b commute, prove the following:

a commutes with ab

Solution. Answer

8. **p40 C.5.** Assuming that a and b commute, prove the following:

axa^{-1} commutes with xbx^{-1} , for any $x \in G$

Solution. Answer