Homework 3

p63 D.1, p64 E.5 and E.6, p65 G.1 and G.2, p75 A.1, A.2, and A.5, p76 B.2

1. **p63 D.1.** Find the composite function, $f \circ g$ and $g \circ f$:

   $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = \sin(x)$
   $g : \mathbb{R} \to \mathbb{R}$ is defined by $g(x) = e^x$

   Solution.

2. **p64 E.5.** f is a bijective function. Describe its inverse.

   $A = \{a, b, c, d\}, B = \{1, 2, 3, 4\}$ and $f : A \to B$ is given by:
   $\begin{pmatrix} a & b & c & d \\ 3 & 1 & 2 & 4 \end{pmatrix}$

   Solution.

3. **p64 E.6.** f is a bijective function. Describe its inverse.

   G is a group, $a \in G$, and $f : G \to G$ is defined by $f(x) = ax$.

   Solution.

4. **p65 G.1.** Let A, B, and C by sets. Prove that if $g \circ f$ is injective, then f is injective.

   Solution.

5. **p65 G.2.** Let A, B, and C by sets. Prove that if $g \circ f$ is surjective, then g is surjective.

   Solution.

6. **p75 A.1.** Consider the following permutations f, g, and h in S_6

   $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 3 & 5 & 4 & 2 \end{pmatrix}$ $g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 1 & 6 & 5 & 4 \end{pmatrix}$

   $h = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 6 & 4 & 5 & 2 \end{pmatrix}$

   Solution. Compute the following:

   $f^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ \_ & \_ & \_ & \_ & \_ & \_ \end{pmatrix}$ $g^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ \_ & \_ & \_ & \_ & \_ & \_ \end{pmatrix}$

   $h^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ \_ & \_ & \_ & \_ & \_ & \_ \end{pmatrix}$

   $f \circ g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ \_ & \_ & \_ & \_ & \_ & \_ \end{pmatrix}$ $g \circ f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ \_ & \_ & \_ & \_ & \_ & \_ \end{pmatrix}$

7. **p75 A.2.** Given p75 A.1, compute the following:

   $f \circ (g \circ h) =$

   Solution.

   $g \circ h = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ \_ & \_ & \_ & \_ & \_ & \_ \end{pmatrix}$

8. **p75 A.5.** Given p75 A.1, compute the following:

   $g \circ g \circ g =$

   Solution.

   $g \circ g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ \_ & \_ & \_ & \_ & \_ & \_ \end{pmatrix}$

9. **p76 B.2.** List the elements of the cyclic subgroup of S_6 generated by:
$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 4 & 1 & 6 & 5 \end{pmatrix}$
Solution.