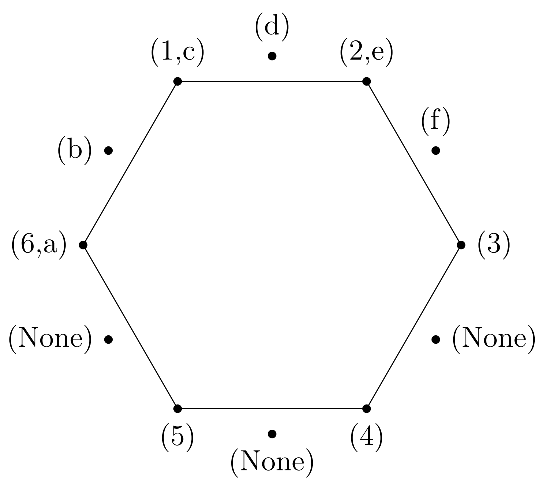
Homework 4

p77 F.1, p78 H.1, p86 A.1 (a,b,c), A.2 (b,c,d), and A.4 (d)

1. **p77 F.1.** Let G be the group of symmetries of the regular hexagon. List the elements of G (there are 12 of them).



(No need to write the group table of G.)

Solution.

$$R_0 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 3 & 4 & 5 & 6 \end{pmatrix} \; R_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 4 & 5 & 6 & 1 \end{pmatrix} \; R_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 1 & 2 \end{pmatrix}$$

$$R_3 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 1 & 2 & 3 \end{pmatrix} \; R_4 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 6 & 1 & 2 & 3 & 4 \end{pmatrix} \; R_5 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 2 & 3 & 4 & 5 \end{pmatrix}$$

$$R_6 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 4 & 3 & 2 & 1 \end{pmatrix} \; R_7 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 4 & 3 & 2 & 1 & 6 \end{pmatrix} \; R_8 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 3 & 2 & 1 & 6 & 5 \end{pmatrix}$$

$$R_9 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 2 & 1 & 6 & 5 & 4 \end{pmatrix} \; R_{10} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 6 & 5 & 4 & 3 \end{pmatrix} \; R_{11} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 6 & 5 & 4 & 3 & 2 \end{pmatrix}$$

2. **p78 H.1.** Let A be a set of $a \in A$. Let G be the subset of S_A consisting of all the permutation F of A such that $F(a) = a$. Prove that G is a subgroup of S_A.

   Solution.

   Proof. To prove G is a subgroup of S_A, we must show:

   - if $f \in S$ then $f^{-1} \in S$
     Because $F(a)$ is the identity permutation, it follows that $F^{-1}F(a) = F(a) = a$. Thus, it is closed under inverses.
   - if $f, g \in S$ then $fg \in S$
     Because $F(a) = a \forall a \in S_A$, then $\forall f, g \in G \rightarrow fg(a) = f(g(a)) = f(a) = a$. Thus, it is closed under multiplication.

   Thus, G is a subgroup of S_A.

   □

3. **p86 A.1 (a,b,c).** Compute each of the following products in S_9. (Write your answer as a singular permutation.)

   Solution.

   (a) $(145)(37)(682) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 4 & 6 & 7 & 5 & 1 & 8 & 3 & 2 & 9 \end{pmatrix}$

   (b) $(17)(628)(9354) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 8 & 5 & 9 & 4 & 2 & 1 & 6 & 3 \end{pmatrix}$

   (c) $(71825)(36)(49) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 8 & 5 & 6 & 9 & 7 & 3 & 1 & 2 & 4 \end{pmatrix}$

4. **p86 A.2 (b,c,d).** Write each of the following permutations in S_9 as a product of disjoint cycles:

   Solution.

   (a) Not Assigned

   (b) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 4 & 9 & 2 & 3 & 8 & 1 & 6 & 5 \end{pmatrix} = (17)(24)(395)(68)$

   (c) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 9 & 5 & 3 & 1 & 2 & 4 & 8 & 6 \end{pmatrix} = (17435)(296)$

   (d) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 9 & 8 & 7 & 4 & 3 & 6 & 5 & 1 & 2 \end{pmatrix} = (1928)(375)$

5. **p86 A.4 (d).** If $\alpha = (3714), \beta = (123), and \gamma = (24135)$ in S_7, express each of the following as a product of disjoint cycles:

   (d) $\beta^2\alpha\gamma$

   Solution.

$$
\begin{aligned}
\beta^2\alpha\gamma &= (123)(123)(3714)(24135) \\
&= (132)(3714)(24135) \\
&= (17)(243)(24135) \\
&= (173452)
\end{aligned}
$$