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Homework 5

p86 B.1(b), p87 C.1(a,e), p97 A.3, p101 H.5 and I.3

1. **p86 B.1(b)** Compute $\alpha^{-1}, \alpha^2, \alpha^3, \alpha^4, \alpha^5$ where $\alpha = (1234)$

   Solution.

   $\alpha^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix} = (1432) \quad \alpha^2 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix} = (13)(24)$

   $\alpha^3 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix} = (1432) \quad \alpha^4 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} = (1)(2)(3)(4)$

   $\alpha^5 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix} = (1234)$

2. **p87 C.1(a)** Determine if the permutation is even or odd. Justify your answer.

   $\pi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 7 & 4 & 1 & 5 & 6 & 2 & 3 & 8 \end{pmatrix}$

   Solution.

   $\pi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 7 & 4 & 1 & 5 & 6 & 2 & 3 & 8 \end{pmatrix} = (173)(2456) = (17)(37)(24)(45)(56)$

   Since this requires 5 transpositions, the permutation π is odd.

3. **p87 C.1(e)** Determine if the permutation is even or odd. Justify your answer.

   (123)(2345)(1357)

   Solution.

   $(123)(2345)(1357) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 5 & 4 & 3 & 7 & 2 & 6 & 1 \end{pmatrix} = (17)(14)(12)(15)$

   Since this requires 4 transpositions, the permutation (123)(2345)(1357) is even.

4. **p97 A.3** Let G_1, G_2, and G_3 be groups, and let $f : G_1 \to G_2$ and $g : G_2 \to G_3$ be isomorphisms. Prove that $g \circ f : G_1 \to G_3$ is an isomorphism.

   Solution.

5. **p101 H.5** Let c be a fixed element of G. Let H be a group with the same set as G, and with the operation $x * y = xcy$. Prove that the function $f(x) = c^{-1}x$ is an isomorphism from G to H.

   Solution.

6. **p101 I.3** If G is any group, and a is any element of G, prove that $f(x) = axa^{-1}$ is an automorphism of G.

   Solution.