Homework 6

p108: B1, B2, B3, B4, B6

1. **p108. B1** What is the order of 10 in $\mathbb{Z}_{25}$?

   Solution. The order of 10 in $\mathbb{Z}_{25}$ is 5.

2. **p108. B2** What is the order of 6 in $\mathbb{Z}_{16}$?

   Solution. The order of 6 in $\mathbb{Z}_{16}$ is 4.

3. **p108. B3** What is the order of the following in S_6?

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 3 & 2 & 5 & 4 \end{pmatrix}$$

   Solution. The order in S_6 is 3.

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 3 & 2 & 5 & 4 \\ 4 & 6 & 3 & 1 & 5 & 2 \\ 1 & 2 & 3 & 4 & 5 & 6 \end{pmatrix}$$

4. **p108. B4** What is the order of 1 in $\mathbb{R}^*$? What is the order of 1 in $\mathbb{R}$?

   Solution. The order of 1 (the identity) in $\mathbb{R}^*$ is 1. In $\mathbb{R}$, 1 has no order as the group is not closed under multiplication.

5. **p108. B6** Can an element of an *infinite* group have *finite* order? Explain.

   Solution. Yes, there exists many infinite groups which contain an element or elements of finite order. For example, in the infinite group $\mathbb{R}^*$, the element 1 has finite order 1.

   There also exists infinite groups where all elements have finite order.