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Homework 7

p131 D.1 and D.2, p143 C.1, C.3, C.4, and C.8

1. **p131 D.1** Let G be a finite group, and let H and K be subgroups of G. Prove the following:

   Suppose $H \subseteq K$ (therefore H is a subgroup of K). Then $(G : H) = (G : K)(K : H)$.

   Solution.

2. **p131 D.2** Let G be a finite group, and let H and K be subgroups of G. Prove the following:

   The order of $H \cap K$ is a common divisor of the order of H and the order of K.

   Solution.

3. **p143 C.1** Let G, H, and K be groups. Prove the following:

   If $f : G \to H$ and $g : H \to K$ are homomorphisms, then their composite $g \circ f : G \to K$ is a homomorphism.

   Solution.

4. **p143 C.3** Let G, H, and K be groups. Prove the following:

   If $f : G \to H$ is a homomorphism and K is any subgroup of G, then $f(K) = \{f(x) : x \in K\}$ is a subgroup of H.

   Solution.

5. **p143 C.4** Let G, H, and K be groups. Prove the following:

   If $f : G \to H$ is a homomorphism and J is any subgroup of H, then

   $$f^{-1}(J) = \{x \in G : f(x) \in J\}$$

   is a subgroup of G. Furthermore, ker $f \subseteq f^{-1}(J)$.

   Solution.

6. **p143 C.8** Let G, H, and K be groups. Prove the following:

   The function $f : G \to G$ defined by $f(x) = x^2$ is a homomorphism iff G is abelian.

   Solution.