# Lab 4 (100 pts.) - Interpretation of Confidence Intervals and Power Analysis for Z tests

# Objectives: A Better Understanding of Confidence Intervals and Power Curves.

**A. (55 points) Interpretation of a Confidence Interval (no data file required).** Use software to generate 40 observations from a normal distribution with $\mu = 10$ and $\sigma = 2$. Repeat this 30 times.

1. (30 points) From each set of observations, compute a 95% confidence interval. No data is required, however, you need to include all 30 confidence intervals.
2. (10 points) Determine how many of these intervals contain the population mean, $\mu = 10$. Please indicate for each confidence interval if it contains the value or not. Is this number that you would expect? Why or why not?
3. (15 points) GROUP PART: This is a group assignment and is due on Blackboard at Midnight on FRIDAY, March 13. Be sure that the names and sections of each person at the top of the page. Combine your data with 3 or 4 other students (in any of my sections) and answer the following questions (no data is required for this part):
    a. Is the number of intervals that contain the mean what you would expect for the combined data?
    b. How are the results from part 2 and part 3 different?

**B. (45 points) Water quality testing (no data file required).** The Deely Laboratory is a drinking-water testing and analysis service. One of the common contaminants it tests for is lead. Lead enters drinking water through corrosion of plumbing materials, such as lead pipes, fixtures, and solder. The service knows that their analysis procedure is unbiased but not perfectly precise, so the laboratory analyzes each water sample three times and reports the mean result. The repeated measurements follow a Normal distribution quite closely. The standard deviation of this distribution is a property of the analytic procedure and is known to be $\sigma = 0.25$ parts per billion (ppb).

The Deely Laboratory has been asked by the university to evaluate a claim that the drinking water in the Student Union has a lead concentration of 6 ppb, well below the Environmental Protection Agency's action level of 15 ppb. Since the true concentration of the sample is the mean $\mu$ of the population of repeated analyses, the hypotheses are

$$H_0: \mu = 6$$
$$H_a: \mu \neq 6$$

The lab chooses the 1% level of significance, $\alpha = 0.01$. They plan to perform three analyses of one specimen (n=3).

1. (30 points, 6 points each part) Using computer software, calculate the following powers:
    a. At the 1% level of significance, what is the power of this test against the specific alternative $\mu = 6.5$?
    b. At the 5% level of significance, what is the power of this test against the specific alternative $\mu = 6.5$?
    c. At the 1% level of significance, what is the power of this test against the specific alternative $\mu = 6.75$?
    d. If the lab performs five analyses of one specimen (n=5), what is the power of this test against the specific alternative $\mu = 6.5$?
    e. Write a short paragraph explaining the consequences of changing the significance level, alternative $\mu$ and sample size on the power.
2. (10 points) Generate a power curve when n =3 at a 1% significance level. Please use an interval length of 3.
3. (5 points) What sample size would be required for the power to be at least 0.90 at the 1% level of significance against the specific alternative $\mu = 6.3$?