computer_proj3

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```
# Function to compute pi via monte carlo simulation
compute_pi <- function(n, R) {
  X <- runif(n, -R, R)
  Y <- runif(n, -R, R)

  # Calculate distances of points from origin
  dis <- sqrt(X^2+Y^2)

  # Compute ratio of points inside circle versus number of points, should be pi/4
  # As (pi*R^2)/(2R*2R) = pi/4
  # Multiply by 4 to get estimate of pi
  pi_hat <- 4*sum(dis<=R)/n

  return(pi_hat)
}

# Function to compute CI given monte carlo simulation results
compute_CI <- function(n, alpha, pi_hat) {
  # Divide pi_hat by 4 to get original proportion
  p_hat <- pi_hat/4

  # Get z-score for given alpha
  z <- qnorm(1-(alpha/2))

  # Compute margin of error, again remembering factor of 4
  MoE <- 4*z*sqrt(pi_hat*(1-p_hat)/n)

  # Return interval as vector
  return(c(pi_hat-MoE, pi_hat+MoE))
}
```

```
# Run experiment with different values for n
# n = 100
n <- 100
R <- 1
alpha <- .01

pi_hat <- compute_pi(n, R)
CI <- compute_CI(n, alpha, pi_hat)

print(paste("Our estimate for pi is ", pi_hat))
```

```
## [1] "Our estimate for pi is  3.24"
```

```
print(paste("After a trial containing", n, "points, we can say with", 1-alpha*100, "% confidence
that pi is between", CI[1], "and", CI[2]))
```

```
## [1] "After a trial containing 100 points, we can say with 0 % confidence that pi is between
2.43159986664141 and 4.04840013335859"
```

```
# Run experiment with different values for n
# n = 10000
n <- 10000
R <- 1
alpha <- .01

pi_hat <- compute_pi(n, R)
CI <- compute_CI(n, alpha, pi_hat)

print(paste("Our estimate for pi is ", pi_hat))
```

```
## [1] "Our estimate for pi is  3.1656"
```

```
print(paste("After a trial containing", n, "points, we can say with", 1-alpha*100, "% confidence
that pi is between", CI[1], "and", CI[2]))
```

```
## [1] "After a trial containing 10000 points, we can say with 0 % confidence that pi is between
3.08187363404338 and 3.24932636595662"
```

```
# Run experiment with different values for n
# n = 100000
n <- 100000
R <- 1
alpha <- .01

pi_hat <- compute_pi(n, R)
CI <- compute_CI(n, alpha, pi_hat)

print(paste("Our estimate for pi is ", pi_hat))
```

```
## [1] "Our estimate for pi is  3.14116"
```

```
print(paste("After a trial containing", n, "points, we can say with", 1-alpha*100, "% confidence
that pi is between", CI[1], "and", CI[2]))
```

```
## [1] "After a trial containing 1e+05 points, we can say with 0 % confidence that pi is between
3.11440233352555 and 3.16791766647445"
```

```
# Run experiment with different values for n
# n = 100000
n <- 100000
R <- 1
alpha <- .01

pi_hat <- compute_pi(n, R)
CI <- compute_CI(n, alpha, pi_hat)

print(paste("Our estimate for pi is ", pi_hat))
```

```
## [1] "Our estimate for pi is  3.13888"
```

```
print(paste("After a trial containing", n, "points, we can say with", 1-alpha*100, "% confidence
that pi is between", CI[1], "and", CI[2]))
```

```
## [1] "After a trial containing 1e+05 points, we can say with 0 % confidence that pi is between
3.11209656532067 and 3.16566343467933"
```

```
# Run experiment with different values for n
# n = 10000000
n <- 10000000
R <- 1
alpha <- .01

pi_hat <- compute_pi(n, R)
CI <- compute_CI(n, alpha, pi_hat)

print(paste("Our estimate for pi is ", pi_hat))
```

```
## [1] "Our estimate for pi is  3.1419908"
```

```
print(paste("After a trial containing", n, "points, we can say with", 1-alpha*100, "% confidence
that pi is between", CI[1], "and", CI[2]))
```

```
## [1] "After a trial containing 1e+07 points, we can say with 0 % confidence that pi is between
3.139315974209 and 3.144665625791"
```

```
# Run experiment with different values for n
# n = 1,000,000,000
n <- 1000000000
R <- 1
alpha <- .01

start <- Sys.time()
pi_hat <- compute_pi(n, R)
end <- Sys.time()
duration <- end - start
CI <- compute_CI(n, alpha, pi_hat)

print(paste("Our estimate for pi is ", pi_hat))
```

```
## [1] "Our estimate for pi is  3.141512708"
```

```
print(paste("After a trial containing", n, "points, we can say with", 1-alpha*100, "% confidence
that pi is between", CI[1], "and", CI[2]))
```

```
## [1] "After a trial containing 1e+09 points, we can say with 0 % confidence that pi is between
3.14124517126599 and 3.14178024473401"
```

```
print(paste("This simulation took", duration, "minutes to complete."))
```

```
## [1] "This simulation took 5.17715111970901 minutes to complete."
```

Remarks

As we can see, the number of trials necessary to gain even one more digit of precision is actually quite extreme and the experiment's convergence to the true value of π is quite slow. The good news is that the 99% confidence intervals that we constructed all included the true value of π, which we would hope to be the case with such a high confidence. As can be seen in the results, our closest approximation was correct up to four and five digits (varies due to randomness in simulation), which is satisfactory, if not for the fact that it took generating one billion points to achieve the estimate.