Australian National University

# Fuzzy Signatures

COMP4660/8420 - Neural Networks, Deep Learning and Bio-inspired Computing

Human Centred Computing

# Overview

- Fuzzy Signatures
  - Example
- Why Fuzzy Signatures?
- Aggregation of Fuzzy Signatures
- Polymorphic Fuzzy Signatures
- Case Studies

# Fuzzy Rule Based Systems

Dense rule based systems suffer from a serious problem called *Rule explosion*:

$$|R| = O\left(T^k\right) \quad (1)$$

3 possible solutions:

- Sparse fuzzy rule based systems
- Hierarchical fuzzy rule based systems
- Sparse hierarchical fuzzy rule based systems
  - Fuzzy signatures

**Definition**

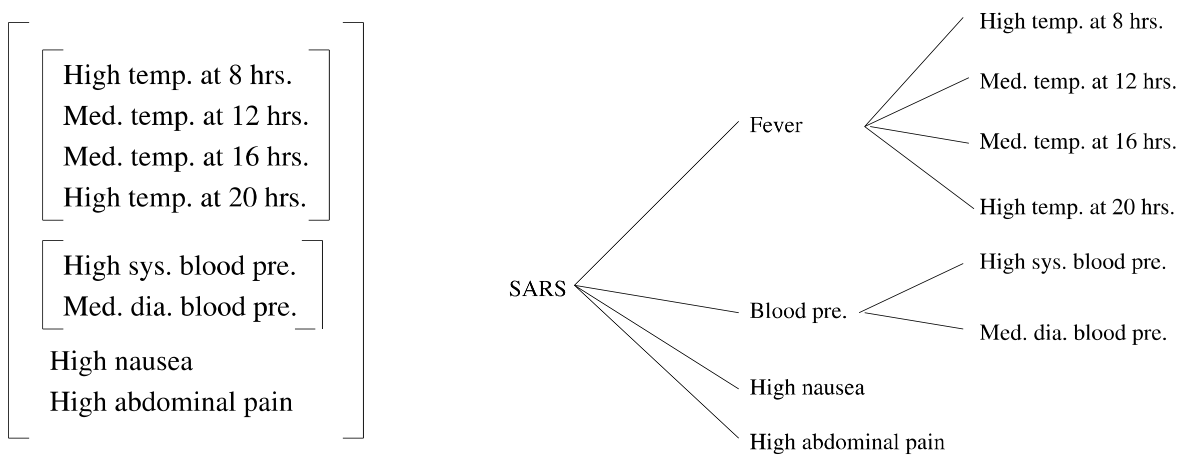
*A Fuzzy Signature is a vector of fuzzy values, where each vector component can be another vector,*

$$\begin{aligned} A \quad &: \quad X \to [a_i]_{i=1}^{k} \\ \textit{where } a_i \quad &= \quad \begin{cases} \left[a_{ij}\right]_{j=1}^{k_i} & ;\textit{if branch} \\ [0,1] & ;\textit{if leaf} \end{cases} \end{aligned}$$

- Aggregate sub-branches to get the final atomic result
- Two fuzzy signatures (even with slightly different structures) can be compared to find the similarity

# An Example Fuzzy Signature

**can be graphically represented as a vector or tree**




(a) As a Vector (b) As a Tree

- Construct an individual fuzzy signature for each data point in hand

# Why Fuzzy Signatures?

**Fuzzy Signatures:**

- Sparse and hierarchical descriptor of an object
- Simplifies the approximation of aggregation for complex structured data
- Very robust and flexible under perturbed input data

# Arbitrary aggregation of fuzzy signatures

Fuzzy signature structure with two arbitrary levels $g$ and $(g+1)$.

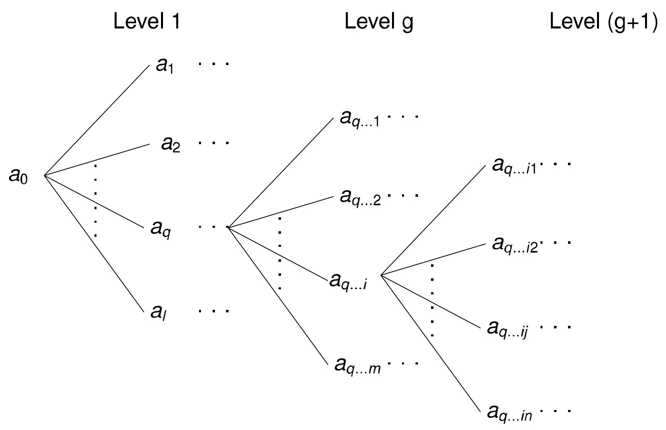



Figure: An Arbitrary Fuzzy Signature

- Aggregation of an arbitrary branch $a_{p\ldots i}$ in level $g$ can be written as:

$$a_{q\ldots i} = @_{q\ldots i}\{a_{q\ldots ij}\}$$

- Where $@_{q\ldots i}$ is an arbitrary aggregation function of the $q\ldots i$th branch, $j = 1, \ldots, n$, and $a_{p\ldots i}, a_{q\ldots ij} \in [0, 1]$.

# Simple aggregation of fuzzy signatures

- Simple aggregation functions *min* and *max* used
- Each sub-branch can have different aggregation function

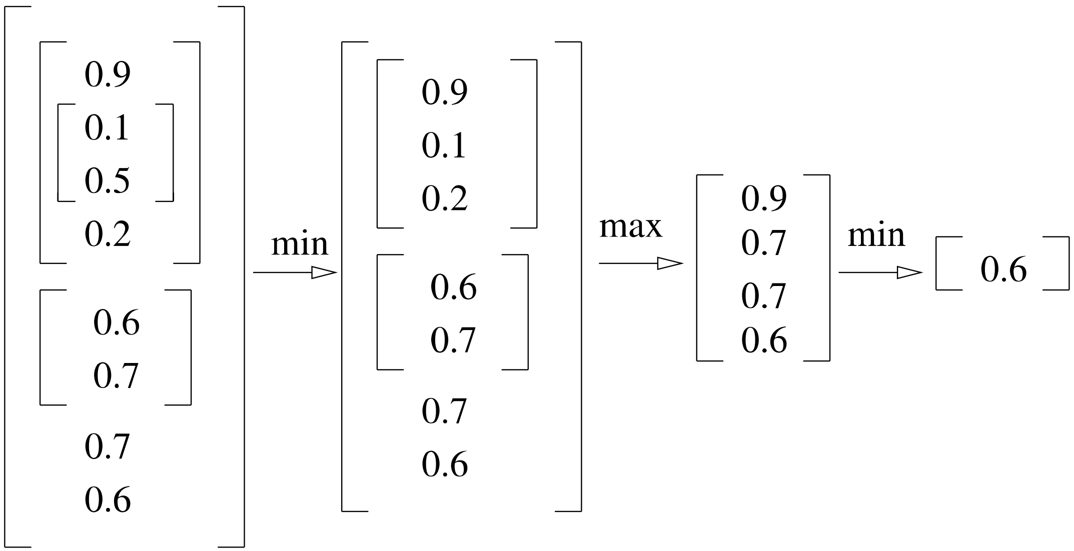



Figure: Aggregation of fuzzy signatures

# Kóczy's Method of Aggregation

$$S = S_1 \wedge S_2$$

$$S = (S_1 \wedge S_2) \vee (\bar{S}_1 \wedge \bar{S}_2)$$

$$S = 1 - |S_1 - S_2|$$

Where $S$, $S_1$, and $S_2$ are Fuzzy Signatures. $\bar{S}_1$ is the complement of the Fuzzy Signature $S_1$

# Example

- Compare Fuzzy signatures with slightly different structures

  Step 1: Truncate to find a common structure
  Step 2: Use operations conjunction or disjunction
  Step 3: Aggregate resulting fuzzy signature

$$S_1 = \begin{bmatrix} \begin{bmatrix} 0.8 \\ 0.6 \\ 0.5 \end{bmatrix} \\ 0.7 \\ \begin{bmatrix} 0.3 \\ 0.4 \\ 0.9 \end{bmatrix} \end{bmatrix} \quad S_2 = \begin{bmatrix} 0.6 \\ 0.2 \\ \begin{bmatrix} 0.8 \\ 0.5 \\ 0.1 \end{bmatrix} \end{bmatrix} \quad S_1' = trunc_{s_2}(s_1) = \begin{bmatrix} 0.8 \\ 0.7 \\ \begin{bmatrix} 0.3 \\ 0.4 \\ 0.9 \end{bmatrix} \end{bmatrix}$$

(a) Truncation

$$\text{(a) } S_1' \wedge S_2 = \begin{bmatrix} 0.6 \\ 0.2 \\ \begin{bmatrix} 0.3 \\ 0.4 \\ 0.1 \end{bmatrix} \end{bmatrix} \quad \text{(b) } S_1' \vee S_2 = \begin{bmatrix} 0.8 \\ 0.7 \\ \begin{bmatrix} 0.8 \\ 0.5 \\ 0.9 \end{bmatrix} \end{bmatrix}$$

(b) Comparison

# Example

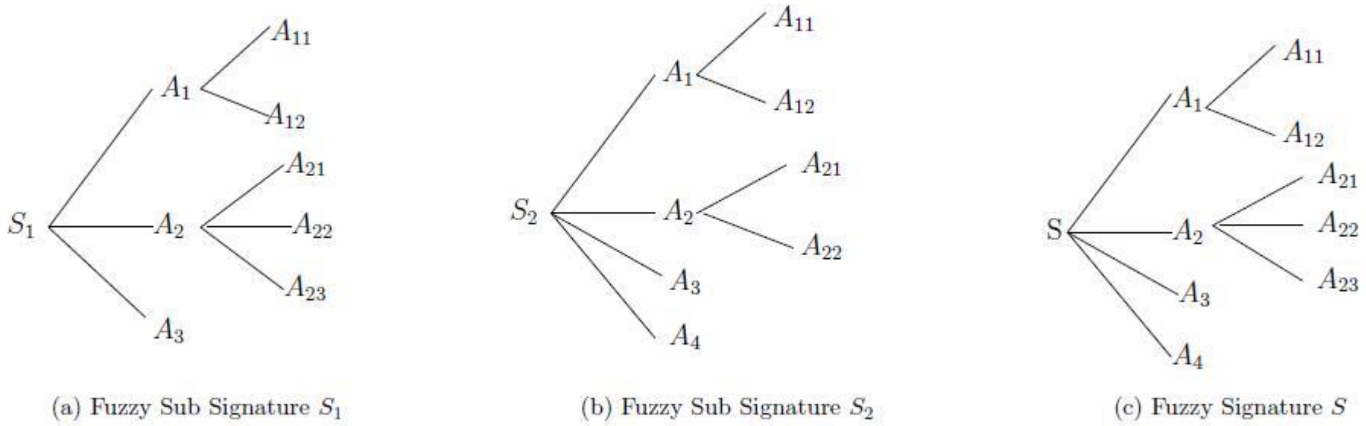



Figure: Fuzzy Sub Signatures

# Polymorphic Fuzzy Signatures

- Find a common signature structure for set of data points
- Use fuzzy constrains/events at leaves
- Use specialised aggregations operators (such as WRAO) to fuse data

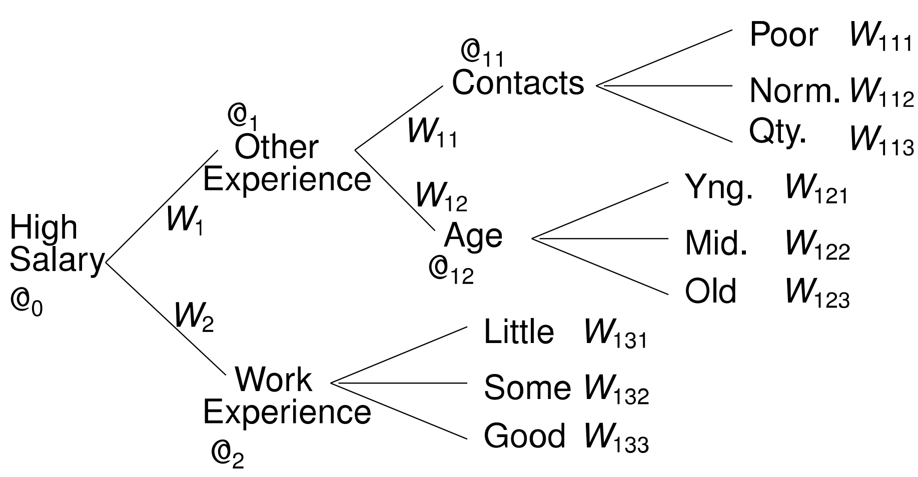



Figure: High Salary Selection PFS

# SARS Polymorphic Fuzzy Signatures

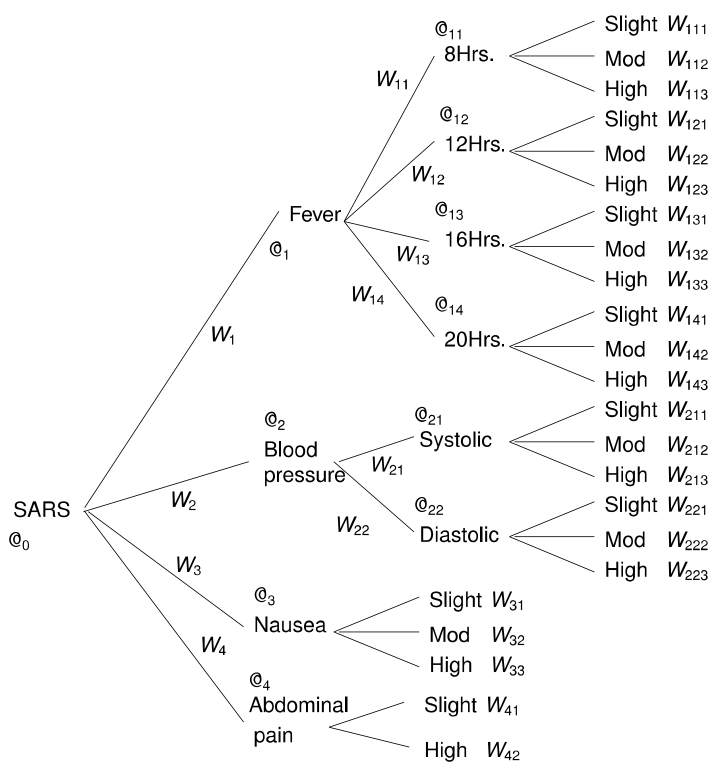
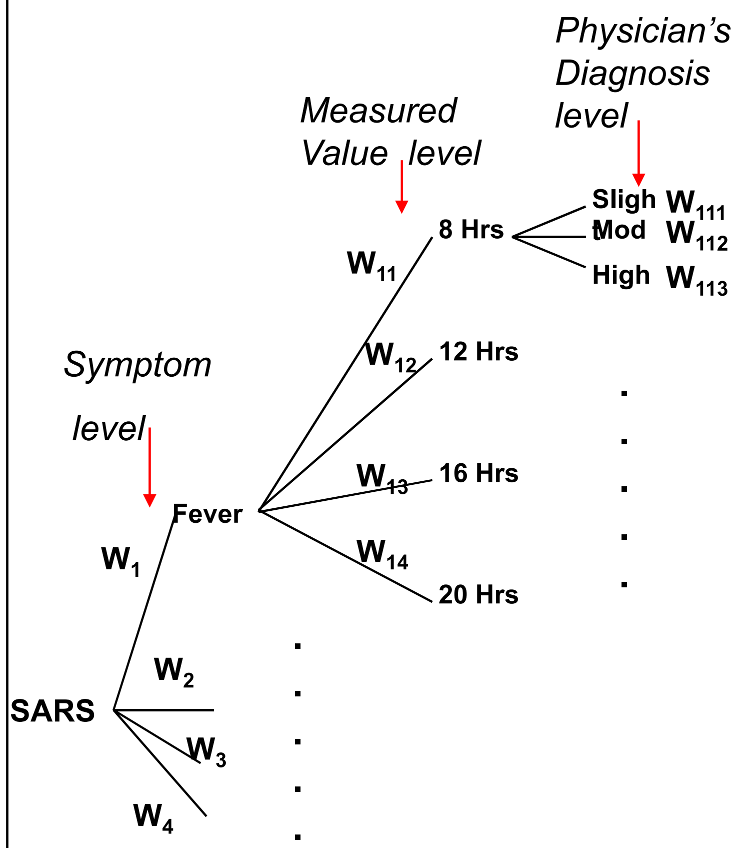



Figure: Weighted SARS Patient Classification PFS

# Weighted Relevance Aggreg. Method

- Weighted Relevance Aggregation method (Mendis *et al. 2005)*
- Some branches contribute more to final result than others at the same level.
- eg. Contribution of ***slight fever***, ***moderate fever***, and ***high fever*** to SARS is linguistically "***less***", "***somewhat***", and "***more***".
- Thus, weights $w_{111}$, $w_{112}$, and $w_{113}$ in Fig.1 were set for these linguistic expressions.




**Part of SARS Fuzzy Signature Structure**

# Weight. Relev. Aggreg. Operator (WRAO)

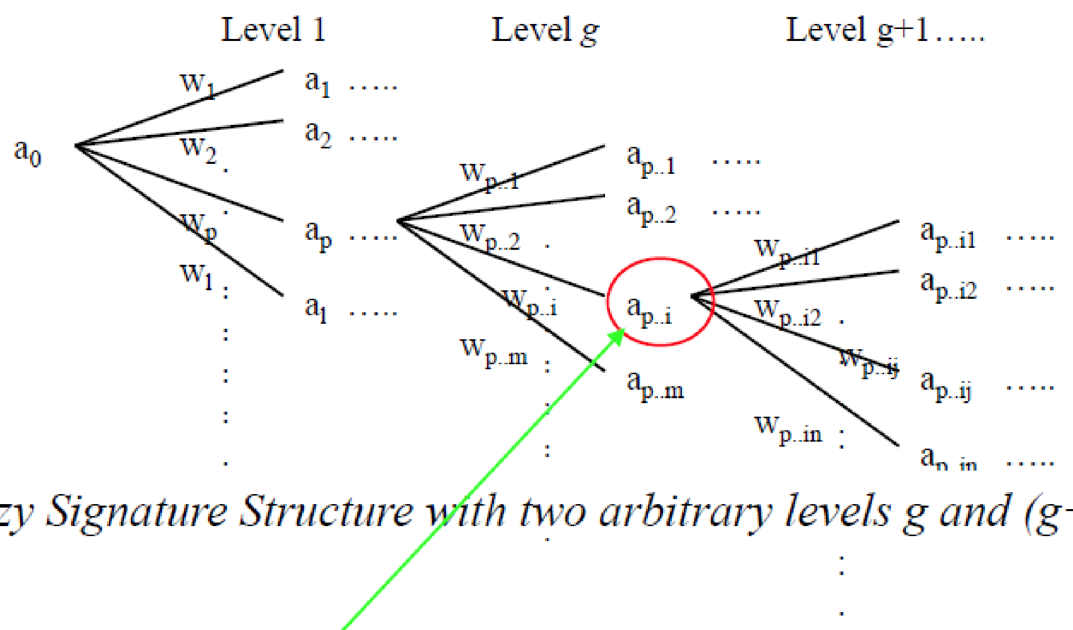
**Definition**: The generalised WRAO of *n* branches $s_1, s_2, \ldots, s_n \in [0,1]$ with *n* weighted relevancies $w_1, w_2, \ldots, w_n \in [0,1]$, in a fuzzy signature, is a function $g:[0,1]^{2n} \rightarrow [0,1]$ such that,

$$g(s_1, s_2, \ldots, s_n; w_1, w_2, \ldots, w_n) = \left( \frac{1}{n} \sum_{i=1}^{n} (w_i s_i)^p \right)^{\frac{1}{p}}$$

where $p \in \Re$, $p \neq 0$, $i \in [1, n]$ and $\sum_{i=1}^{n} w_i$ is not necessarily equal to 1.

- Further, we replaced $w_i$ by the following sigmoid function, $w_i = \dfrac{1}{1 + e^{-\lambda_i}}$
- Now for any $\lambda_i$ the weighted relevance, $w_i \in [0,1]$
- WRAO can be rewritten as follows, $a_{q\ldots i} = \left[ \dfrac{1}{n} \sum_{j=1}^{n} \left( \left\{ \dfrac{1}{1 + e^{-\lambda_{q\ldots ij}}} \right\} a_{q\ldots ij} \right)^{p_{q\ldots i}} \right]^{\frac{1}{p_{q\ldots i}}}$

We called *p* is the aggregation factor and λ is the weighted relevance factor of the WRAO.




*Fuzzy Signature Structure with two arbitrary levels g and (g+1)*

$$a_{q\ldots i} = \left[ \frac{1}{n} \sum_{j=1}^{n} \left( \left\{ \frac{1}{1+e^{-\lambda_{q\ldots ij}}} \right\} a_{q\ldots ij} \right)^{p_{q\ldots i}} \right]^{\frac{1}{p_{q\ldots i}}}$$

where $\dfrac{1}{1+e^{-\lambda_{q\ldots ij}}} = w_{q\ldots ij}$

# Levenberg-Marquardt Learning of WRAO

- Why Levenberg-Marquardt Learning?
  - WRAO learning is a local search within the scope of the fuzzy signature structure
- Minimize the Sum of Squared Errors (SSE)

$$f(x) = \frac{1}{2}\sum_{i=1}^{m} r_i(x)^2 = \frac{1}{2} \parallel r(x) \parallel_2^2 = r^T r$$

- The update, $u_k$, for the $k^{th}$ iteration can be calculated as

$$u_k = (J^T(x_k)J(x_k) + \alpha_k I)^{-1}(-J^T(x_k)r(x_k))$$

- Unconstrained WRAO:

$$a_{q\ldots i} = \left[\frac{1}{n}\sum_{j=1}^{n}\left(a_{q\ldots ij} \bullet \left[\frac{1}{1+e^{-\lambda_{q\ldots ij}}}\right]\right)^{p_{q\ldots i}}\right]^{\frac{1}{p_{q\ldots i}}}$$

# Generalized OWA (GOWA)

## Definition

$$M\left(a_1, a_2, \ldots, a_n\right) = \left(\sum_{j=1}^{n} w_j b_j^p\right)^{\frac{1}{p}}$$

- $p \in [-\infty, \infty]$
- $b_j$ *is the* $j$*th largest input*
- $w_j$ *are collection of weights satisfying*
  - (i) $w_j \in [0, 1]$
  - (ii) $\sum_{j=1}^{n} w_j = 1$

# Generalized OWA (GOWA)

- Used Levenberg-Marquardt Learning for both GOWA and WRAO
- Both methods use fuzzy signature structure
- Unconstrained WRAO:

$$a_{q\ldots i} = \left[ \frac{1}{n} \sum_{j=1}^{n} \left( a_{q\ldots ij} \bullet \left[ \frac{1}{1 + e^{-\lambda_{q\ldots ij}}} \right] \right)^{p_{q\ldots i}} \right]^{\frac{1}{p_{q\ldots i}}}$$

- Unconstrained GOWA:

$$a_{q\ldots i} = \left( \sum_{j=1}^{n} \left[ \frac{e^{\lambda_{q\ldots ij}}}{\sum_{k=1}^{n} e^{\lambda_{q\ldots ik}}} \right] b_{q\ldots ij}^{p_{q\ldots i}} \right)^{\frac{1}{p_{q\ldots i}}}$$

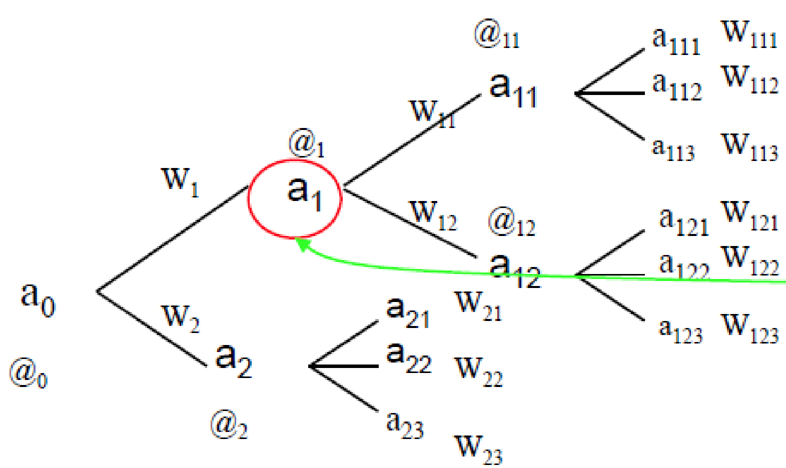
# Expt 1: High Salary Selection Fuzzy Sig.

$$High\_Salary\begin{bmatrix}\begin{bmatrix}Other\_Exp & \begin{bmatrix}Contacts & \begin{bmatrix}[Poor]\\ [Norm]\\ [Qty]\end{bmatrix}\\ Age & \begin{bmatrix}[Yng]\\ [Mid]\\ [Old]\end{bmatrix}\end{bmatrix}\end{bmatrix}\\ \begin{bmatrix}Work\_Exp & \begin{bmatrix}[Little]\\ [Some]\\ [Good]\end{bmatrix}\end{bmatrix}\end{bmatrix}$$

High Salary Selection Fuzzy Signature Structure

High Salary Selection Fuzzy Signature Structure with all notations of

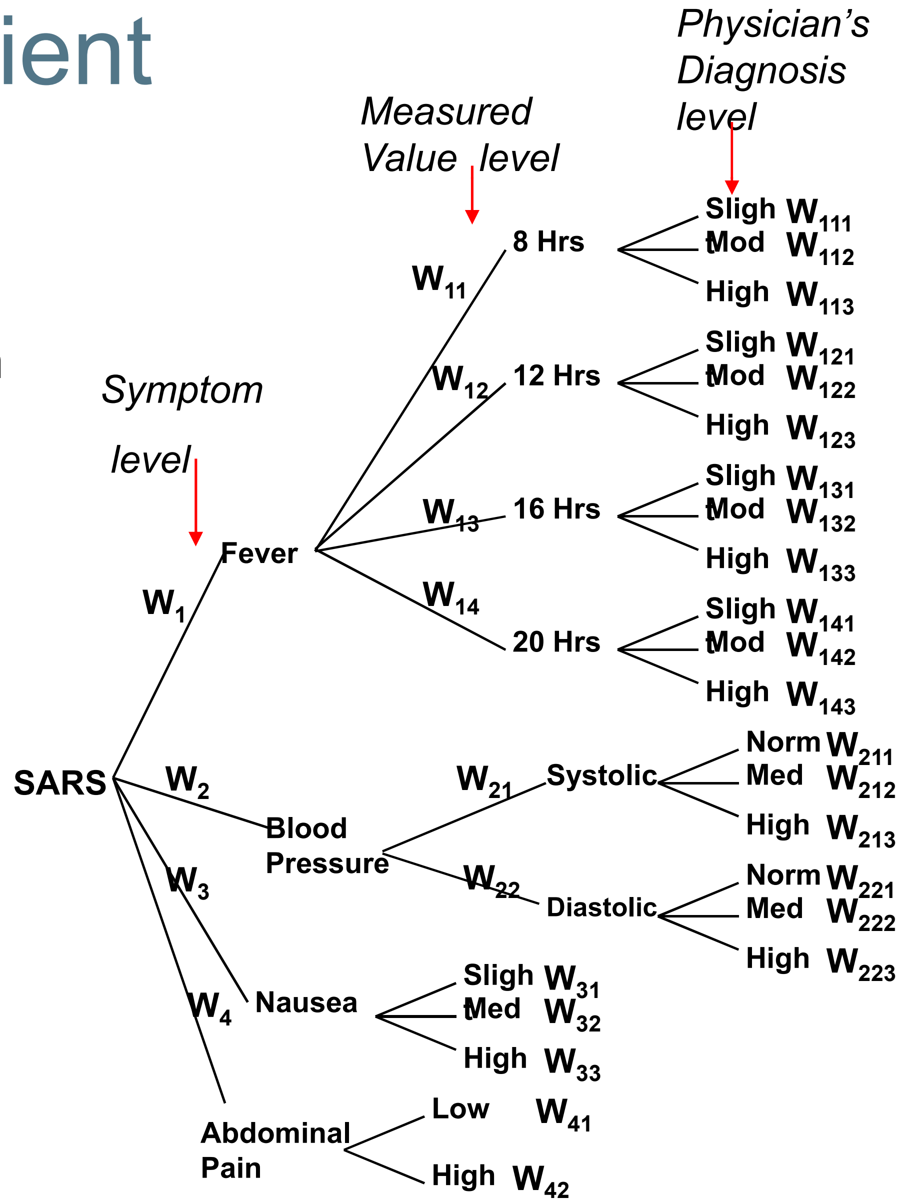
- input values
- weights
- aggregation functions




$$a_1 = \left[\frac{1}{2}\sum_{j=1}^{2}\left(\left\{\frac{1}{1+e^{-\lambda_{1j}}}\right\}a_{1j}\right)^{p_1}\right]^{\frac{1}{p_1}}$$

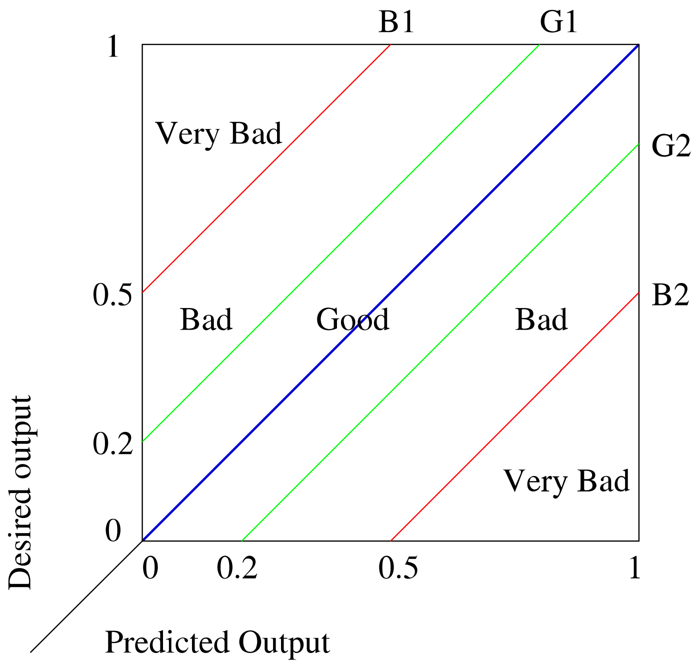
# Expt 2: SARS Patient Classification

- 4 Symptoms in input test/train Data
  1. SARS
  2. Pneumonia
  3. Hypertension
  4. Normal



# Fuzzy classification error

- Mean Squared Error
- Sum of Fuzzy Classification Error (SYCLE)



# High Salary Selection Polymorphic Fuzzy Signature

Table: High Salary Selection Experiment: WRAO and GOWA

| | MSE Train | SYCLE Train | MSE Test | SYCLE Test |
|---|---|---|---|---|
| GOWA | 0.1639 | 104.5 | 0.2003 | 81.0 |
| WRAO | 0.0130 | 8.5 | 0.0130 | 6 |

# SARS Polymorphic Fuzzy Signature

Table: Test Results of SARS Data Classification: WRAO and GOWA

| | Train | | Test | |
|---|---|---|---|---|
| | MSE | SYCLE | MSE | SYCLE |
| GOWA | 0.2185 | 2500 | 0.2192 | 2500 |
| WRAO | 0.0020 | 33.5 | 0.0018 | 25.5 |

# Eye Gaze path Analysis

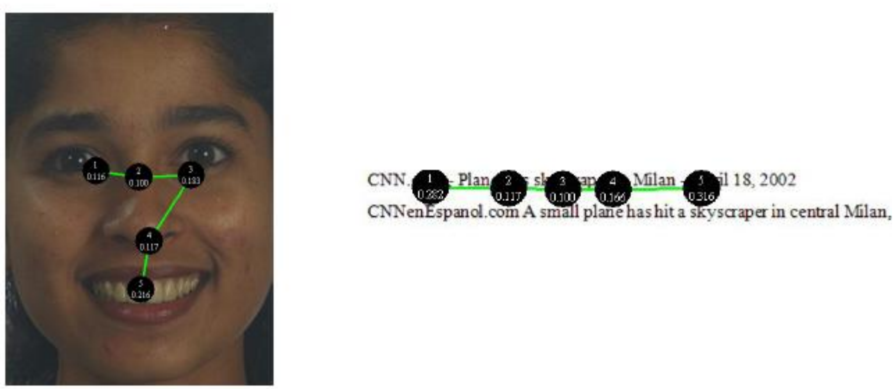



Figure: Eye Gaze Experiment

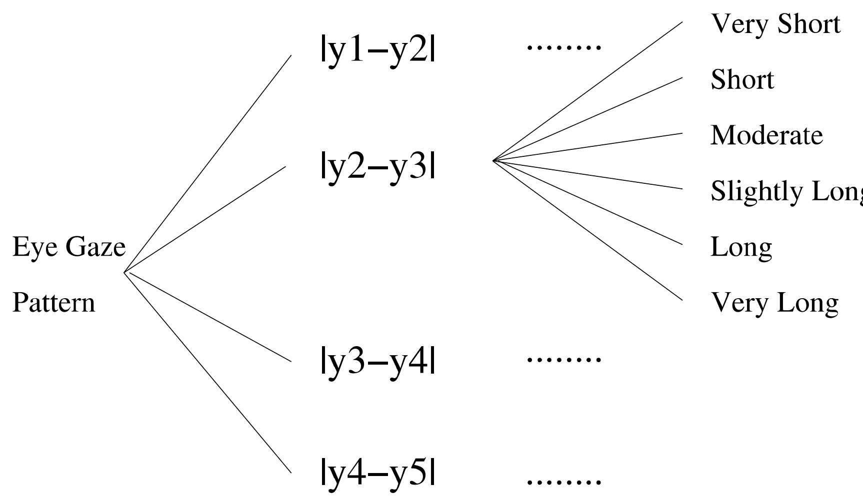



Figure: Eye Gaze PFS

# PFS Vs SVM for Eye Gaze Path Analysis

| Experiment | FS (3 classes) | FS (2 classes) | SVM (2 classes) |
|---|---|---|---|
| Training | 19.77% | 3.04% | 1.18% |
| Test | 20.00% | 5.00% | 4.55% |

- Fuzzy C-Means clustering to identify better subspaces for PFS
- Fuzzy Signatures for single document analysis