Discussion 2

visualising data structures and algorithms through animation - VisuAlgo

Agenda

- Asymptotic notation
  - definition
  - Algorithmic Common Runtimes
  - Analyzing Runtime examples
  - a little taste of formal proof
- homework 1
  - data structures to implement
  - the `optional` type
  - Google Test

Asymptotic notation

Definitions

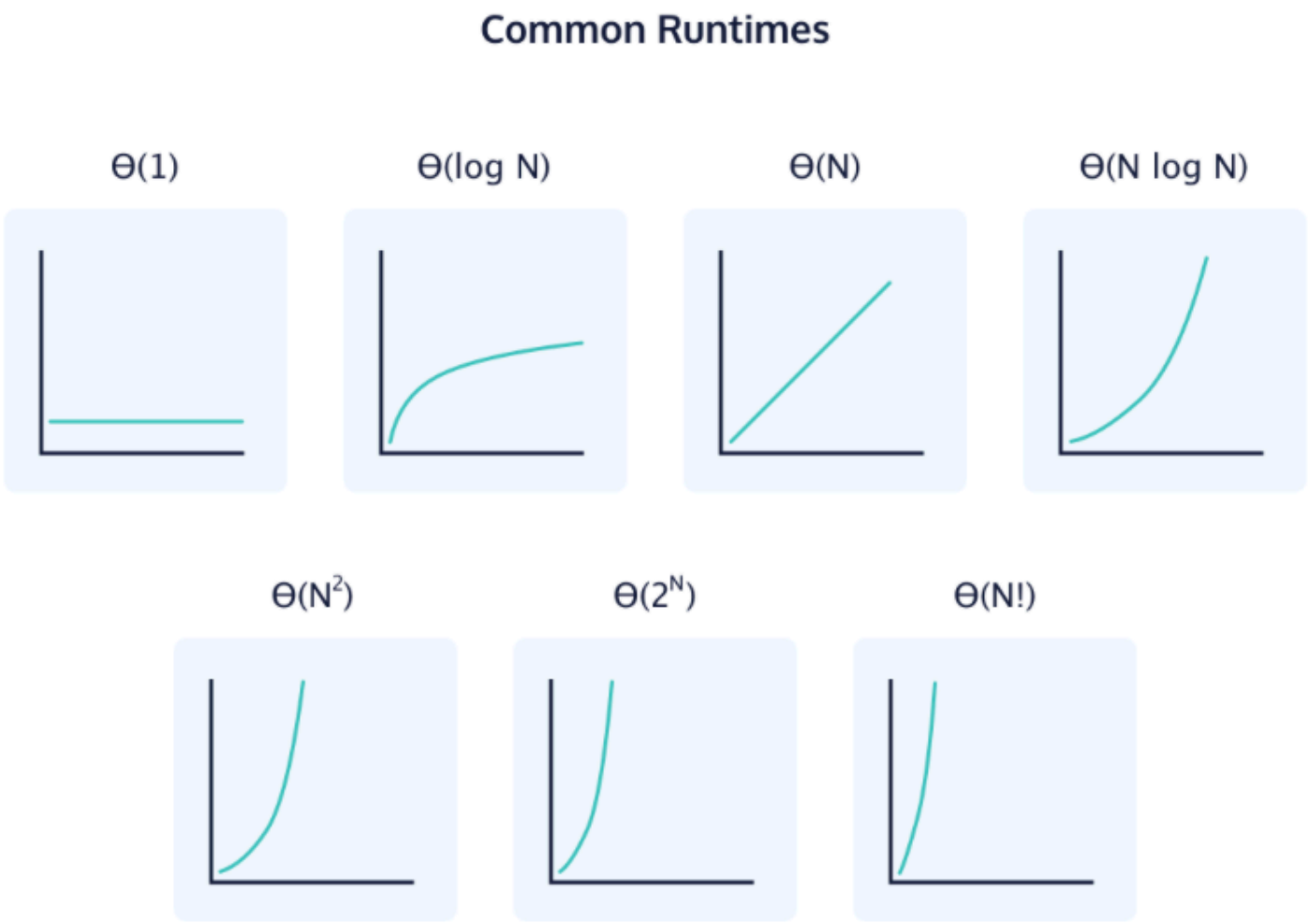
- $T(n)$: run time of the function for $n \in \mathbb{N}$ denote the size of input
- O: asymptotic **upper** bound
- Ω (Omega): asymptotic **lower** bound
- Θ (Theta): asymptotic **tight** bound

Algorithmic Common Runtimes

The common algorithmic runtimes from fastest to slowest:

- constant: Θ(1)
  - Accessing an element of an array by index.
  - Basic arithmetic operations (e.g., addition, subtraction).
  - Checking if a data structure is empty.
- logarithmic: Θ(log N)
  - Binary Search: Finding an element in a sorted array by repeatedly dividing the search interval in half.

    - Balanced Binary Search Trees (e.g., AVL trees): Operations like insertion and deletion take O(log n) time because the tree remains balanced.
- linear: Θ(N)
    - Linear Search: Finding an element in an unsorted array by checking each element one by one.
    - Summing the elements of an array.
- polynomial: Θ(N^2)
    - Bubble Sort: Repeatedly steps through the list, compares adjacent elements, and swaps them if they are in the wrong order.
- exponential: Θ(2^N)
- factorial: Θ(N!)



O-notation

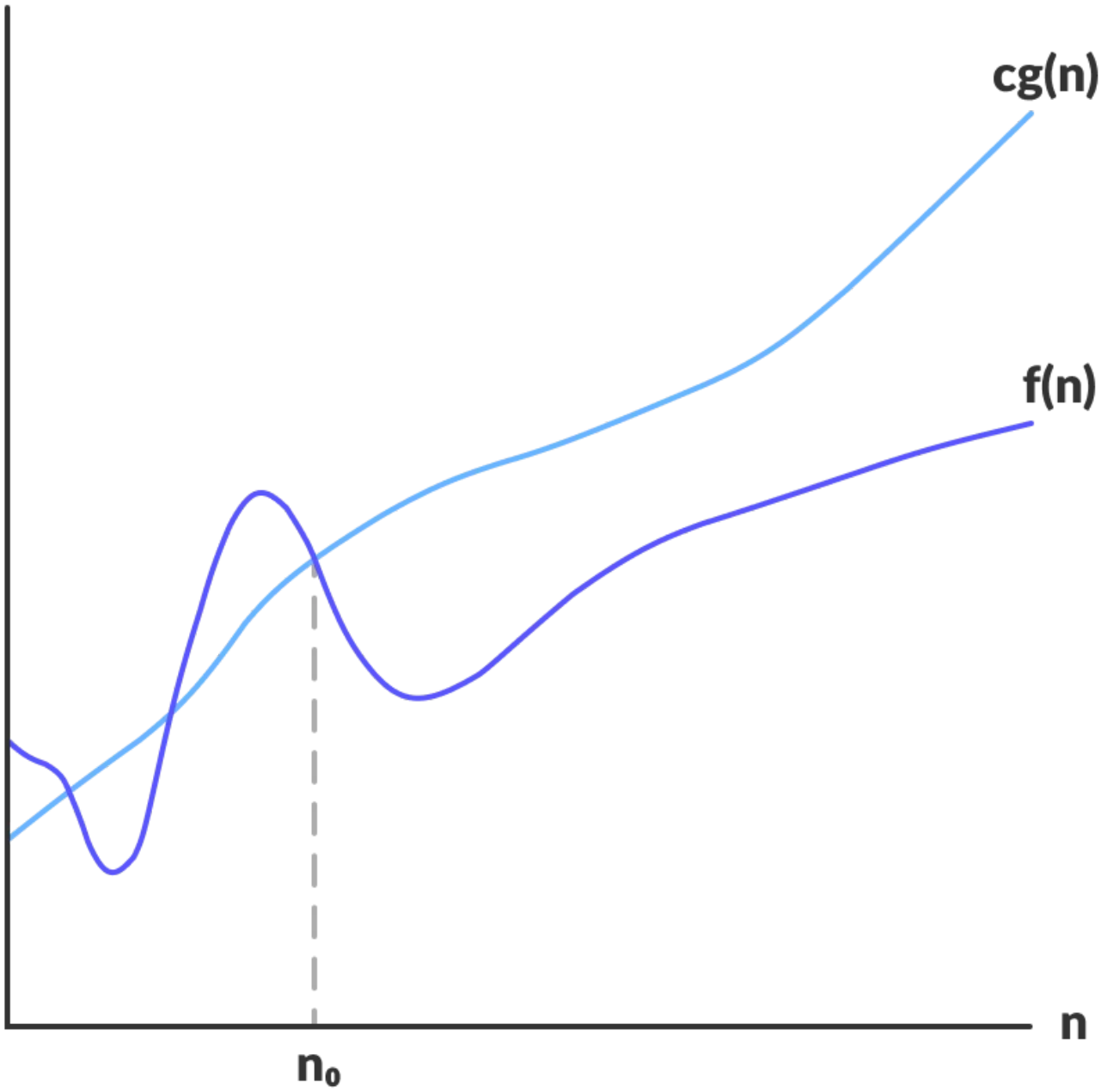
Defn for given function $g(n)$, if there exists constant $c > 0$ and $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$,
$O(g(n)) = f(n) : 0 \leq f(n) \leq cg(n)$.

In other words, $f(n)$ is $O(g(n))$ if $f(n) \leq cg(n)$ for some c and sufficiently large n.

Another way of explanation:

- We want to use a function $g(n)$ to emulate/approach the algorithm function $f(n)$.
- c, n are positive constant

```
O(g(n)) = { f(n): there exist positive constants c and n0
            such that 0 ≤ f(n) ≤ cg(n) for all n ≥ n0 }
```



Analyzing Runtime

factorial

```
def factorial(x: Int) -> Int:
  if x == 1
    return 1
  return x * factorial(x - 1)
```

Calculating time bound for recursive algorithms

$$
\begin{aligned}
T(n) &= T(n-1) + c \\
&= T(n-2) + c + c \\
&\cdots \\
&= T(1) + (n-1)c \in O(n)
\end{aligned} \tag{1}
$$

Proving upper bound

Recall the definition,

> there exists constant $c > 0$ and $n_0 \in \mathbb{N}$ such that ...

Find these constants! Find the c, n_0. *Proof* (factorial is $O(n)$).

For $T(n) = T(1) + (n-1)c_1$,

Let $c_2 = c_1 + 1$ and $n_0 = 1$, then
for any $n \geq n_0$, we have

$$
T(n) = (n-1)c_1 \leq nc_2 \tag{2}
$$

thus $T(n) \in O(n)$.

---

selection sort

```
def sort(xs: [a]) -> [a]:
  for i in [1..n]
    key = xs[i]
    for j in [i..n]
      swap(xs, i, j)
  return xs
```

Calculating time bound directly

$$T(n) = n \times n + n = n^2 + n \in O(n^2). \tag{3}$$

linked list search (hw1)

```
def search(head: LinkedListNode<a>, v: a) -> bool:
  ptr = head
  while (ptr is not null)
    if ptr.value == v:
      return true
    ptr = ptr.next
  return false
```

Optional Quick Sort

```
def qsort(xs: [a]) -> [a]:
  pivot = some element in xs
  left = [x | x in xs, x < pivot]
  right = [x | x in xs, x > pivot]
  return qsort(left) ++ [x] ++ qsort(right)
```

For best case partitioning,

$$\begin{aligned} T(n) &= 2T(n/2) + n + c \\ &= 2(2T(n/4) + n/2) + \cdots \\ &\cdots \\ &= 2^k T(n/(2^k)) + kn + c \end{aligned} \implies \frac{n}{2^k} = 1 \implies k = \log_2 n \tag{4}$$

$$\implies T(n) = 2^{\log_2 n} T(1) + \log_2 n \cdot n = n + n \log n \in O(n \log n) \tag{5}$$

Proving not an upper bound

Proof (quick sort is not $O(n)$).

Basic idea: Select a contradiction.

$T(n) = n + n \log n$.
Suppose $T(n) \in O(n)$, then there exists some $c, n_0 > 0$
such that for all $n \geq n_0$,

$$n + n \log n \leq cn$$

$$
\begin{aligned}
& n + n \log n \geq cn \\
\implies & n \log n \leq (c-1)n \\
\implies & \log n \leq c - 1
\end{aligned} \tag{6}
$$

Choose $n \geq 2^c$, then

$$
\log_2 2^c = c \leq \log_2 n \leq c - 1 \tag{7}
$$

which is a contradiction.
Thus the time complexity of quick sort cannot be $O(n)$.

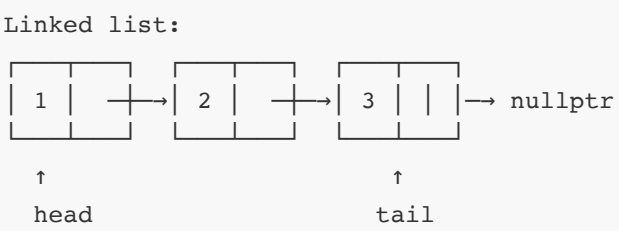
Homework 1

Linked List Node

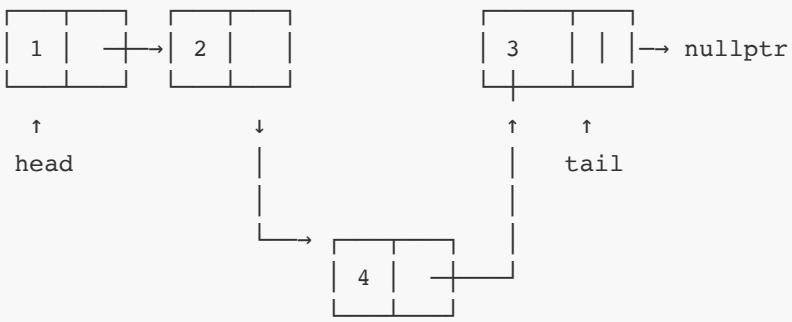
```cpp
template <typename T>
class LinkedListNode
{
private:
  // make constructors and `_next` field only available to `LinkedList` class
  // to avoid instantiating node and mutating `_next` outside `LinkedList` class.

  template <typename U>
  friend class LinkedList;

  LinkedListNode<T> *_next;

  explicit LinkedListNode(T value) : value(value), _next(nullptr) {}
  LinkedListNode(T value, LinkedListNode<T> *next) : value(value), _next(next) {}

public:
  T value;
  LinkedListNode<T> *next() { return _next; }
};
```

this definition is provided in lib/LinkedListNode.hpp.

Linked List Insert

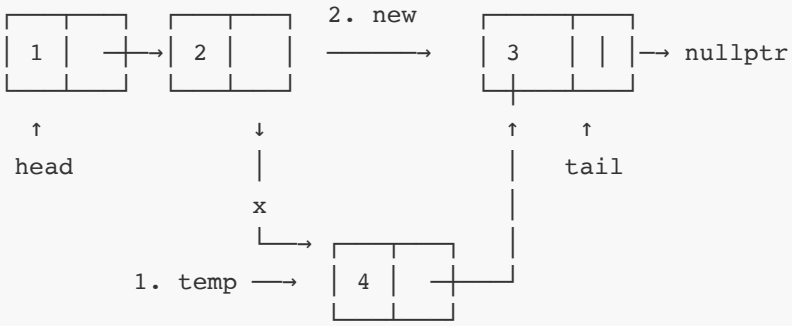


add a node 4 after node 2



Linked List Remove

deleting node



what to do next?

Remember to free your heap memory!

```
delete temp;
```

`std::optional` manages an *optional* contained value

```
/// @brief remove the first element from the list
/// @return the value of head it just removed
T removeHead();
```

what's wrong? The current list **maybe** empty, then what should it return?

```
data Maybe a = Just a | Nothing
```

If a operation may fail, the return type of the function should be its desired type **or** nothing.

In out case, `removeHead()` should return Just a value of type `T` or nothing,
so

```
/// @brief remove the first element from the list
/// @return the removed element if there was at least one element in the list;
///         std::nullopt otherwise
std::optional<T> removeHead();
```

Sum/union Type

In general, viewing types as sets,
we can say a **union type** is the union of these types,
so a instance of the union type can be of any type in that union.

```
def square(number: int | float) -> int | float:
    return number ** 2
```

GoogleTest

A unit testing framework for C++ (and other languages).

running tests for `hw1`

```
> mkdir build
> cd build
> cmake ..
> make
> ./run_tests
Running main() from /ECS36C/hw1/build/_deps/googletest-src/googletest/src/gtest_main.cc
[----------] 1 test from QueueTest (0 ms total)

[----------] Global test environment tear-down
[----------] 24 tests from 3 test suites ran. (0 ms total)
```

```
[----------] 24 tests from 3 test suites ran. (0 ms total)
[  PASSED  ] 22 tests.
[  FAILED  ] 2 tests, listed below:
[  FAILED  ] StackTest.BadTest
[  FAILED  ] QueueTest.BadTest

 2 FAILED TESTS
```

Adding your new tests

suppose adding a new test for linked list

```
TEST(LinkedListTest, DoesSomething) {
  // init the linked list
  LinkedList<char> ll;
  ...
  ASSERT_EQ(do_something, its_rnt);
  ASSERT_NE(do_something, what_it_should_not_be)
  ...
}
```

to test other things,

- change name of test suite (`LinkedListTest`)
- init your object
- assert its behaviors